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The Implied Longevity Yield: A Note on Developing an Index for Life Annuities

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Abstract

I develop an index for tracking the dynamic behavior of life (pension) annuity payouts over time, based on the concept of self-annuitization. Our *implied longevity yield* (ILY) value is defined equal to the internal rate of return (IRR) over a fixed deferral period that an individual would have to earn on their investable wealth if they decided to self-annuitize using a systematic withdrawal plan. A larger ILY number indicates a greater relative benefit from immediate annuitization. I use age 65—with a 10-year period certain—compared against the same annuity at age 75 as the standard benchmark for the index, and calibrate to a comprehensive time series of weekly (Canadian) life annuity quotes from 2000 through 2004. I find that during this period the ILY varied from 5.45 percent to 6.90 percent for males and from 5.00 percent to 6.42 percent for females and was highly correlated with a duration-weighted average yield of 10-year and long-term Government of Canada bonds. I believe our ILY metric can help promote and explain the benefits of acquiring lifetime payout annuities by translating the abstract-sounding longevity insurance into more concrete and measurable financial rates of return.

Introduction

In this article, I develop a financial metric and index for tracking the time series behavior of life annuity payouts. Indeed, as North American baby boomers approach age 65 and their so-called retirement years there is a growing interest in pension and annuity issues, especially given the apparent liability crises in defined benefit (DB) pension plans. Most retirees lack the actuarial intuition needed to understand the longevity insurance benefits of annuitization compared with traditional alternatives in the market. It is also difficult to position the rate of return from life annuities within a

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portfolio's risk and return context. I therefore believe that a properly designed annuity payout annuity index might contribute to a greater appreciation and intuition for these products.

Against this demographic backdrop, a number of recent articles in the pensions, insurance, and actuarial literature[1] have explored the properties of *self-annuitization*. This retirement strategy is a consumption and investment plan that attempts to closely mimic the payout from a generic life annuity while allocating investable assets to minimize or limit the probability of lifetime ruin. This plan is not necessarily optimal within a classical life-cycle model with no bequest motives—in which continuously renegotiated tontine annuities are available—as originally demonstrated by Yaari (1965) and recently extended by Davidoff, Brown, and Diamond (2003). However, as pointed out by Yagi and Nishigaki (1993) and others, incomplete annuity markets is just one of the many theoretical justifications for consumers who shun annuitization. In practice, the popularity and interest in "drawdown" and "annuity alternative" continues to grow amongst practitioners.

Our proposed index goes beyond a (trivial) cross-sectional average of life annuity payouts offered by different insurance companies. Rather, our contemporaneous index value is defined equal to the internal rate of return (IRR) that an individual would have to earn on their financial portfolio during a deferral period, if they choose to self-annuitize, instead of purchasing a life annuity. I define this IRR—which is based on the current term structure of annuity payouts—as the *implied longevity yield* (ILY) at a given age and for a given deferral period. Later, I discuss the relationship between ILY values and the traditional actuarial concept of mortality credits.

The (unique) ILY value solves a nonlinear equation that is at the core of the article. I also present an approximation that provides a relatively simple and intuitive expression for the ILY that is the root of a quadratic equation.

From a practical perspective I suggest using age 65 against age 75, as the standard benchmark for the ongoing index, since this age range appears to be common, at which annuitization decisions are made. As an illustration of the concept, I calibrate the index and ILY to a comprehensive time series of weekly (Canadian) life annuity quotes from 2000 through 2004. During this period, the ILY value varied from a low of 5.45 percent to a high of 6.90 percent for males, and from 5.00 percent to 6.42 percent for females.

The remainder of the article is organized as follows. The next section provides a basic numerical example to explain the mechanics of the index. "Annuity Index Analytics" follows up with the analytic representation. "Data Calibration" provides an examination of the historical behavior of the ILY index. "Additional Issues" provides some additional insights and derives an easy to use approximation, and the last section concludes the article.

[1] See Khorasanee (1996), Milevsky (1998), Kapur and Orszag (1999), Milevsky and Robinson (2000), Albrecht and Maurer (2002), Blake, Cairns, and Dowd (2003), Gerrard, Haberman, and Vigna (2003) as well as recent work by Huang, Milevsky, and Wang (2004), Dushi and Webb (2003), Young (2004), and Reichenstein (2003).

UNDERSTANDING THE ANNUITY INDEX: EXAMPLE

On November 26, 2003, a 65-year-old Canadian male would have been able to convert a $100,000 lump-sum (tax sheltered) premium into a life annuity by going to any one of ten or so insurance companies that offer competitive quotes. According to data compiled by CANNEX Financial Exchanges and The IFID Centre, these companies would have quoted him a payout ranging from a high of $690 per month (Empire Life) to a low of $633 per month (Great West Life). These numbers assumed he was interested in acquiring 10 years of guaranteed payments, and that the remaining payments would continue as long as he lived. If he wanted a longer guarantee period, or perhaps payments to go to a spouse in the event of his death, the monthly payout would be lower. In contrast, if he was willing to settle for a lower guarantee period, he would receive more income per month.

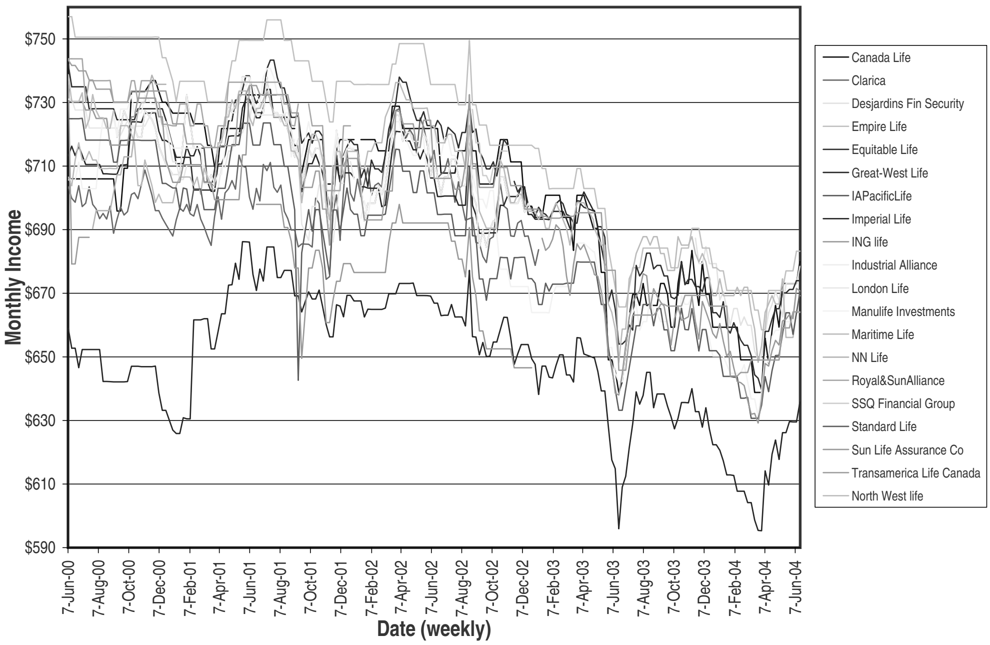
Recall that an annuity with a 10-year (payment certain) guarantee can be broken into two components. The guaranteed portion is similar to a portfolio of zero-coupon bonds. The other portion continues to make payments to the annuitant after the end of the payment certain period, only if the annuitant survives the guaranteed period. Insurance companies pool risk and—as a direct result of the possibility, the individual will not receive a full return of their original payment—the actual payments will be higher to annuitants that survive the guarantee period. The ILY index I am proposing, measures how much higher those payments would be relative to a product in which this risk pooling is not available.

Figure 1 displays the payouts (per month) for all insurance companies quoting such annuities on a weekly basis during the last 4 years. Notice the wide dispersion of up to 15 percent between the highest and lowest companies. Part of this can be attributed to the credit rating of the company (higher-rated companies pay less) and part can be attributed to the general appetite of the company for taking on more annuity business. For example, Empire Life tends to show up at the top of most income comparisons, but the company is ranked a solitary "A" by the rating agency *A.M. Best*. In contrast, Great West Life appears on the bottom, but has a coveted "A+" credit rating. Nevertheless, if I (arbitrarily) take the average of five highest annuity payouts quoted to a 65-year-old male with a $100,000 premium, I get $678.22 per month. This consisted of Empire Life ($690), Maritime Life ($679), Desjardins ($679), Equitable Life ($672), and Transamerica Life ($669). The $678.22 number will form the basis of our index on November 26, 2003.

On the same date, a 75-year-old male would have been able to convert a $100,000 premium into a much higher monthly payment ranging from $1,002 per month (Empire Life) to $948 per month (Sun Life). In this case, the average of five best quotes was $975.90 per month. Stated differently—and this is the key to the ILY index—if a 75-year-old male wanted to purchase a life annuity with a zero-year guarantee paying the original $678.22 per month, he would only have to pay ($678/$976) $\times$ $100,000 = $69,396 or roughly 70 percent of the original cost. The same annuity would be cheaper if purchased later. A 65 year old requires a $100,000 premium to generate $678 for life (with 10 years of certain payments), while a 75 year old requires only $69,396.

What would happen if the 65-year-old male decided to forgo the purchase of a life annuity and instead invested the $100,000 and withdrew the same exact $678.22 per

FIGURE 1
Life Annuity Payouts per $100,000 Premium Male 65 with 10-yr Payment Certain



month for the next 10 years? This strategy is called self-annuitization. What would be the required portfolio investment return needed to successfully withdraw $678.216 per month and still have $69,396 at the end of 10 years to purchase an identical annuity?

This number is precisely the ILY at age 65. In the above example the number works out to 5.90 percent. I will demonstrate how to compute this number in the subsequent section. But, if the 65 year old can earn an annual return of 5.90 percent, he will be able to purchase the exact same life annuity at age 75 as he could have at age 65. If I go through the same exact calculation for a female, the ILY would be 5.46 percent. As a means of comparison, the ILY values can be compared with the 10-year Government of Canada Bond yield, which on November 26 was 4.73 percent (stated as a continuously compounded rate). The ILY value for males (females) was approximately 117 (73) basis points above the bond yield.[2] On the same date the average yield on a long-term high-quality corporate bond (proxied by the Scotia Capital AA bond index) was 6.27 percent. Stated differently, if the retiree could lock-in the 6.27 percent return on investment for the next 10 years and at the same time lock-in a forward price for the

[2] Please note that the bond yield was not actually computed from market prices. Rather, the yield was taken from the Bank of Canada Website that reports these and other official statistics on an ongoing basis. Quite likely they make a number of approximations with regard to the precise maturity, treatment-accrued semi-annual coupons as well as the bid-ask spread. Therefore, the 4.73 percent—which is not necessarily the focus of our article—should be taken as a rough approximation to the true yield in a strict mathematical sense.

life annuity at age 75, they could (stochastically) dominate the payoff from purchasing the annuity at age 65.

How can this number be used? There are several important uses for such a metric, and therefore, good reasons for it to be computed and reported on an ongoing basis. The ILY should help consumers understand (and decompose) exactly what they are getting when they purchase a life annuity. In fact, one can obtain ILY values using the same algorithm to compare any two ages. One might compute the ILY for someone aged 70 or 75 who is contemplating purchasing a life annuity versus waiting to age 80 or 85. In the same manner, consumers can compute the ILY from taking a defined-benefit pension at any age.

As a general fact the older the age group the higher the ILY. It will be very hard to beat the returns from a life annuity using any other financial instrument. Section 5.1 discusses this point further. The next section will derive the analytics.

ANNUITY INDEX ANALYTICS

With some painful abuse of actuarial notation I let the symbol $a^{\tau}_{(x,y)}$ denote the price of a life annuity—sold to an individual aged x who is born in year y—that pays \$1 per annum for life (in continuous time) starting at time τ. If the annuitant does not survive to age $(x+\tau)$ the estate or beneficiaries receive nothing. I will suppress the superscript when $\tau = 0$ and the life annuity commences payment immediately. Along the same lines, I let c^{τ}_{t} denote the price of a term-certain (no mortality component) annuity at time t, which pays \$1 per annum (in continuous time) for τ years. Using our notation, a life annuity that pays \$5,000 per annum but with 10-year payment certain, purchased by a 65 year old in January 2005 would be denoted by: $5,000(c^{10}_{0} + a^{10}_{(65,1940)})$, where time $t = 0$ corresponds with January 2005. In January 2006, the identical life annuity stream with 9-year payment certain would be $5,000(c^{9}_{1} + a^{9}_{(66,1940)})$.

Although I do not use or require any specific pricing model in our analysis, one can think of the annuity as satisfying the following valuation equation:

$$\begin{aligned} c^{\tau}_{0} &= E^{Q}\left[\int_{0}^{\tau} \exp\left\{-\int_{0}^{t} (\mathbf{r}_s)\,ds\right\} dt \,\middle|\, \mathcal{F}_0\right], \\ a^{\tau}_{(x,y)} &= E^{Q}\left[\int_{\tau}^{\infty} \exp\left\{-\int_{0}^{t} (\mathbf{r}_s + \mathbf{h}_{(x+s,y)})\,ds\right\} dt \,\middle|\, \mathcal{F}_0\right], \end{aligned} \tag{1}$$

where $E^{Q}[.]$ denotes a mathematical expectation with respect to pricing (as opposed to biometrical) Q-measure given the information set $\mathcal{F}_0$ available at pricing time zero, while $\mathbf{r}_s$ denotes the (stochastic) instantaneous short-term interest rate, and $\mathbf{h}_{(x+s,y)}$ denotes the (stochastic) instantaneous hazard rate at age $(x+s)$ for a cohort born in year y. For comparison purposes, recall that under a traditional actuarial (deterministic) approach to insurance and annuity pricing, $\mathbf{h}_{(x,y)}$ is the continuous force of mortality at age x, which is sometimes denoted by the symbol μ_x.

Equation (1) might initially seem like "actuarial over kill" compared to traditional annuity pricing formulae; however, our approach is consistent with *financial economic* pricing of life annuities by allowing for both stochastic mortality and interest. A more complete analysis of a no arbitrage pricing relationship is alluded to in Carriere (1999) and is discussed more fully in Dahl (2003), Biffis and Millossovich (2004), as well as Milevsky, Promislow, and Young (2005). Our main point in presenting Equation (1) is to acknowledge up front that the future price of the life annuity—after the deferral period is over—is unknown due to uncertainty in both future interest rates and mortality pricing assumptions. And, while this article does not require an actual parameterization of the continuous law of mortality across different ages and cohorts, a typical example would be a process whose expectation $E[\mathbf{h}_{(x+t,y)}]$ obeys the Gompertz–Makeham law. See Carriere (1994), Frees, Carriere, and Valdez (1996), and Forfar, McCutcheon, and Wilkie (1988) for discussions regarding calibration and estimation of this process.

The theoretical basis of our ILY index is as follows. I compute the *internal rate of return* that an x year old (born in year y) would have to earn on the nonannuitized portfolio over the next τ years in order to replicate the income payout from the annuity and still be able to acquire the same income pattern at age $x+\tau$, assuming current pricing remains unchanged.

Self-annuitization that is the basis of our index, was proposed as a normative alternative to annuitization by Khorasanee (1996) and Milevsky (1998) and subsequently investigated by Kapur and Orszag (1999), Milevsky and Robinson (2000), Albrecht and Maurer (2002), Blake, Cairns, and Dowd (2003), Gerrard, Haberman, and Vigna (2003), as well as recent work by Huang, Milevsky, and Wang (2004), Dushi and Webb (2003), Young (2004), and a practitioner-oriented article by Reichenstein (2003).

To understand the analytic dynamics of self-annuitization, I begin with a hypothetical retiree who has $W_0 = w$ dollars in marketable wealth. If this individual were to annuitize—that is, to convert a stock of wealth w into a lifetime flow—he or she would be entitled to w/a_1 per annum for life, where a_1 abbreviates the annuity factor. If, in contrast, the retiree decided to forgo the purchase of the life annuity and instead self-annuitized—by investing the funds at a force of interest denoted by δ and consuming at the annuity rate w/a_1—the wealth dynamics would satisfy the Ordinary Differential Equation (ODE):

$$dW_t = \left(\delta W_t - \frac{w}{a_1}\right) dt, \quad W_t \geq 0 \tag{2}$$

In other words, the instantaneous change in the value of the portfolio would be the sum of the interest gain (δW_t) minus the withdrawal for consumption purposes (w/a_1). The δ is assumed constant (nonstochastic) over time.

The solution to the Ordinary Differential Equation (ODE) in Equation (2) is

$$W_t = \left(w - \frac{w}{\delta a_1}\right) e^{\delta t} + \frac{w}{\delta a_1}, \quad W_t \geq 0, \tag{3}$$

where δ can always be selected so that $W_t > 0$ for all values of t. But, if this investment portfolio is to contain enough funds to purchase the *same exact annuity flow* at age $x+\tau$, the following relationship must hold:

$$\frac{w}{a_1}a_2 = \left(w - \frac{w}{\delta a_1}\right)e^{\delta\tau} + \frac{w}{\delta a_1}, \tag{4}$$

where a_2 is short-hand notation for the relevant annuity factor at age $(x + \tau)$. The intuition behind Equation (4) is as follows. The right-hand side describes the evolution of wealth under a consumption rate of (w/a_1) and an interest force δ. The annuity factor a_2 represents the cost of acquiring a dollar-for-life at some future age $x + \tau$. The cost of acquiring the original life annuity flow (w/a_1) at age $x + \tau$, is exactly the left-hand side value of $(w/a_1)a_2$. I am then searching for a value of δ that equates both sides. If δ is "too small" then the left-hand side will be "too expensive." In contrast, if δ is "too large" then the individual can afford a better annuity. Finally, dividing by w and multiplying by a_1, I am left with

$$a_2 - \left(a_1 - \frac{1}{\delta}\right)e^{\delta\tau} - \frac{1}{\delta} = 0. \tag{5}$$

The value of δ^* that solves the above equation will be the ILY. It is the rate that must be earned on nonannuitized wealth to be as well-off after τ years, *assuming a_2 is known with certainty.* Just to make sure this point is clear, using the notation I introduced earlier around Equation (1), I am implicitly assuming that the current life annuity factor $a_{(x+\tau,\,y-\tau)}$ can be used as a proxy for the (random) future annuity factor $a_{(x+\tau,\,y)}$.

We demonstrate Equation (5) with the help of the numerical example we presented in the previous section. On November 26, 2003, a 65-year-old male quoted an average monthly payout of $678.22 per initial premium of $100,000 with a 10-year payment certain period. The continuous-time annuity factor is approximated as $100,000/(12 \times 678.216) = 12.2871$, which is $a_1 = 12.2871$ per $1-for-life using our notation. On the same date, a 75 year old quoted an average monthly payout of $977 per premium of $100,000 with a zero-year payment certain period. The annuity factor is $100,000/(12 \times 975.904) = 8.5391$, which is $a_2 = 8.5391$ per $1-for-life.[3]

We are searching for the δ that the 65 year old would have to earn on their discretionary investment portfolio to beat the return from the annuity, but still consume the exact same income on an ongoing basis. The situation I am faced with is Equation (5) with $\tau = 10$ years, $x = 65$, and δ being the unknown return variable.

$$8.5391 - \left(12.2871 - \frac{1}{\delta}\right)e^{10\delta} - \frac{1}{\delta} = 0. \tag{6}$$

The solution must be computed numerically due to the nonlinearity of the equation, and is $\delta^* = 0.0590$ that is an ILY value of 5.90 percent. As stated earlier, the 65-year-old

[3] I am fully cognizant that our formula requires a continuously paying annuity and the data involve monthly pay annuities. For the sake of transparency, we decided to forgo some of the actuarial approximations used to convert monthly into continuous annuity factors since it makes a minimal difference to the final delta values. For example, in the above case adding an additional 1/24 units to both annuity factors—which is the approximation needed to convert into a continuous factor—would decrease δ by less than 3 basis points.

male would have to earn 5.90 percent per annum each year for the next 10 years to beat the return from the annuity. Ergo, the value of the ILY index on November 26, 2003 is 5.90 percent for males. The same calculation can be done for females using the average payouts listed earlier. In this case, $a_1 = 13.3706$ and $a_2 = 9.7875$ for a value of $\delta^* = 5.465$ percent. Naturally, the δ^* value is lower since hazard rates are lower, and the (expected) horizon over which the payments are being returned is longer.

Relation to Actuarial Mortality Credits

There is a close relationship between our ILY values and the concept often referred to as *actuarial mortality credits*. See Bowers et al. (1984) for the actuarial background. The "mortality credit" is defined as the additional incremental return the annuitant receives above and beyond the pricing rate due to risk pooling. To see this connection explicitly, I analyze the simplest possible case of Equation (1), namely when $\mathbf{r}_s = r$, mortality is exponential with a constant hazard rate $\mathbf{h}_{(x,y)} = \lambda$ at all ages, and all annuities are life-only with no guarantee period. In this case annuity pricing Equation (1) collapses to

$$a_{(x,y)} = \int_0^\infty e^{-(r+\lambda)s}\, ds = \frac{1}{\lambda + r}, \tag{7}$$

regardless of the age x, or the cohort birth year y, of the annuitant. Using our short-hand notation, both a_1 and a_2 are therefore equal to $(r + \lambda)^{-1}$ since exponential mortality (and a constant hazard rate) is synonymous with no aging. The fundamental equation for the ILY is then

$$\frac{1}{\lambda + r} - \left(\frac{1}{\lambda + r} - \frac{1}{\delta}\right) e^{\delta\tau} - \frac{1}{\delta} = 0, \tag{8}$$

whose solution is precisely $\delta = r + \lambda$ regardless of the value of τ. In other words, the self-annuitization strategy must earn (and the ILY value must be) at least λ above the pricing rate r in order to purchase the same annuity income flow in the future.

In sum, under the special exponential mortality case, the ILY "spread" above the pricing rate $(\delta - r)$ is exactly the instantaneous hazard rate λ. Under a more general law of mortality the relationship would not be as direct and would obviously depend on the deferral period τ, which is why I consider the ILY an extension of the traditional concept of mortality credits.

Numerical Solution of ILY Equation

I can solve for the unknown δ value using numerical techniques by taking the left-hand side of Equation (5) and treating it as a function $f(\delta)$ and then searching for the root of $f(\delta) = 0$. I use a Newton–Raphson (NeRa) algorithm to locate the δ. The NeRa algorithm is based on Taylor's expansion of the function $f(x)$ in the neighborhood of a point x

$$f(x + \varepsilon) \approx f(x) + f'(x)\varepsilon + \frac{f''}{2}\varepsilon^2 + \cdots \tag{9}$$

For small enough values of ε, the terms beyond $f'(x)\varepsilon$ are of second-order importance, hence $f(x+\varepsilon)=0$ implies

$$\varepsilon = \frac{-f(x)}{f'(x)}. \tag{10}$$

Thus, when I am trying to locate a value of δ such that $f(\delta)=0$, I start with an initial $\delta=\delta_0$, and then by the NeRa algorithm I pick the next value of δ so that

$$\delta_{i+1} = \delta_i - \frac{f(\delta_i)}{f'(\delta_i)}. \tag{11}$$

I follow this process until $|\delta_{i+1}-\delta_i|<\varepsilon$, where ε is a small enough value, which in our case is three significant digits after the decimal point. In a later section, I will provide a quadratic approximation for δ^* that yields some additional insight into the structure of the ILY.

DATA CALIBRATION

This section uses data provided by CANNEX Financial Exchanges and The IFID Centre—based in Toronto, Canada—to calibrate the index I described in the previous section. CANNEX compiles ongoing quotes from, what I believe to be, *most* insurance companies in Canada that market and sell life annuities. On any given day, a (subscribed) user can log into CANNEX's secure website and query their system for a list of available quotes. Their system displays all companies that are offering to sell that particular product type (i.e., age, gender, joint life, payment certain, premium size, etc.). The user can then perform their own comparison of these quotes, and contact the "best" company directly to actually purchase the annuity contract. To get a sense of their influence on the market, in the year 2002 alone, subscribed[4] users requested over 46,000 paid "queries."

The IFID Centre's annuity database captures annuity quotes for ages 55, 60, 65, 70, 75, and 80 for single males, females, and a variety of joint-life scenarios and guarantee periods. This database has been operational for the last 4 years, and I have used these numbers as the primary source for this article. Once again, Figure 1 displays a sample of these quotes for all companies. Table 1 provides some summary statistics for yields on 10-year Government of Canada bonds as well as long-term Government of Canada bonds over the same time period.

The first step in creating the ILY index values is to average the five best annuity quotes for males and females at age 65. As mentioned earlier, the highest quotes tend to be from insurance companies with relatively lower credit ratings. Of the companies that were consistently in the top five—namely Empire Life, Equitable Life, Maritime

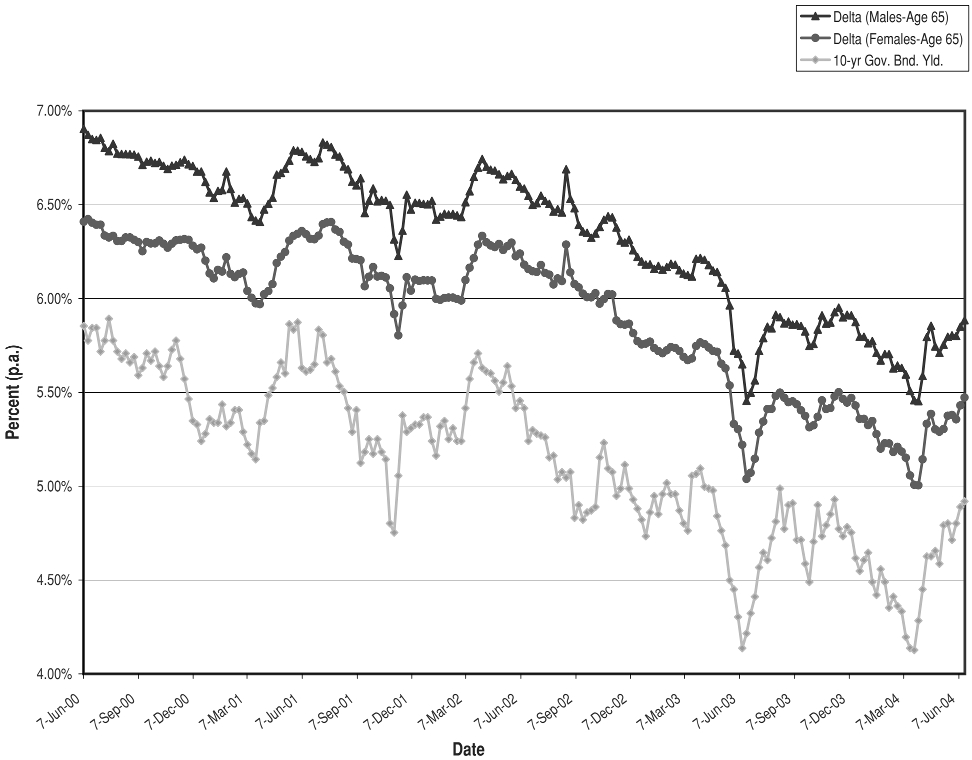
[4] To provide a further sense of the magnitude of the (payout) life annuity market in Canada, according to the industry publication *The Insurance Journal*, in the first quarter of 2003 over \$542.5 million (\$1 CAD = \$0.77 USD) of single premium immediate annuities were sold in Canada. This represented a growth rate of 68 percent over the sales during the first quarter of 2002.

TABLE 1
Summary Statistics for ILY Values

	ILY Value: Male	ILY Value: Female	Canada 10-Year Bond Yield	Spread Off 10-Year Bond: Male	Spread Off 10-Year Bond: Female	Canada Long-Term Bond Yield	Spread Off Long-Term Bond: Male	Spread Off Long-Term Bond: Female	Gender Gap: Male-Female ILY
Average	6.33%	5.90%	5.13%	1.19%	0.77%	5.47%	0.85%	0.43%	0.43%
Min	5.45%	5.00%	4.13%	0.87%	0.43%	4.81%	0.41%	−0.04%	0.32%
10%	5.75%	5.31%	4.56%	1.00%	0.56%	5.13%	0.56%	0.12%	0.39%
25%	5.91%	5.48%	4.80%	1.09%	0.66%	5.32%	0.69%	0.24%	0.41%
50%	6.45%	6.03%	5.16%	1.17%	0.75%	5.48%	0.84%	0.43%	0.43%
75%	6.67%	6.26%	5.50%	1.28%	0.87%	5.63%	0.96%	0.57%	0.45%
90%	6.76%	6.32%	5.68%	1.38%	0.96%	5.80%	1.22%	0.81%	0.46%
Max	6.90%	6.42%	5.89%	1.70%	1.25%	6.00%	1.40%	0.94%	0.50%

This table displays the summary statistics for the implied longevity yield (ILY) value over the period from June 2000 through June 2004 for both males and females at age 65 (10 yr.g.) against age 75. I compare this number against the (c.c.) yield on a 10-year Government of Canada bond and a long-term Government of Canada bond.

FIGURE 2
Implied Longevity Yield (Delta) Value Versus Yield on Benchmark 10-Year Government Bond



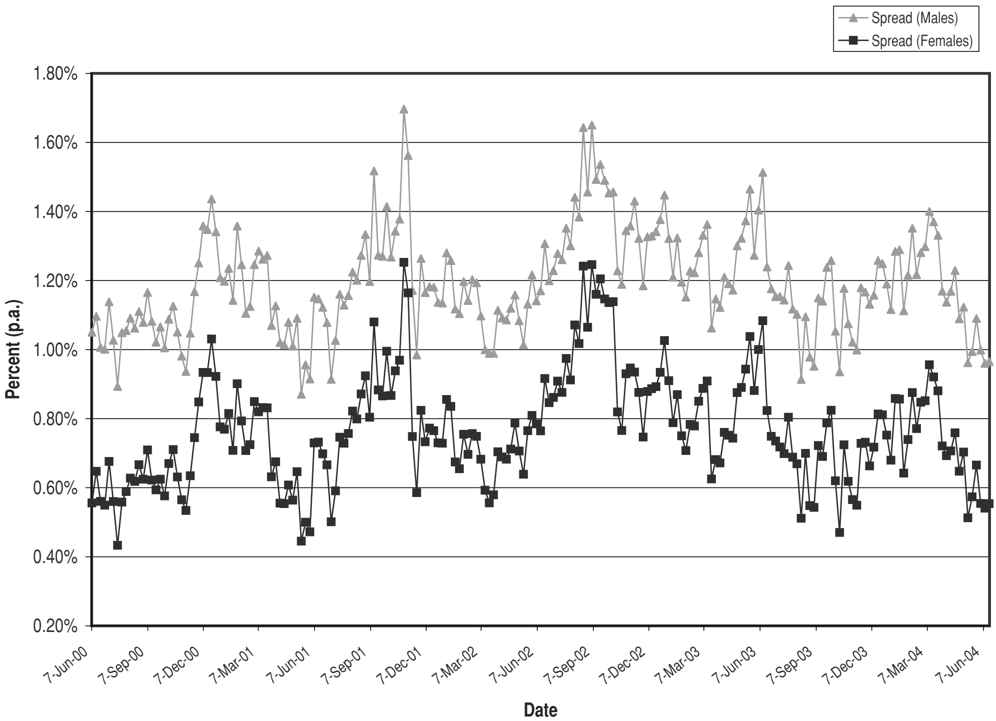
Life, Transamerica Life, and Canada Life—three are rated single A or lower by the credit-rating agency A.M. Best.

For each and every Wednesday, from June 2000 through June 2004, I computed a_1 and a_2 annuity factors for both males and females by dividing the \$100,000 into twelve times the monthly payout. I thus generated a series of 208 male and female pairs $\{a_{65}^{10}(i), a_{75}(i); i = 1.208\}$, which were then inserted into Equation (5) to solve for the relevant δ^* on that date. According to Table 1, the average ILY value from June 2000 through June 2004 period was 6.33 percent for males and 5.90 percent for females. The gap between males and females of approximately 43 basis points is relatively consistent during the entire period. Males must earn a higher benchmark return to successfully "beat" the return from a life annuity.

Figure 2 illustrates the evolution of the ILY values, compared to the yield on a 10-year Government of Canada Bond. I have chosen the yield on this particular fixed-income instrument[5] given its centrality in many of the insurance companies'

[5] An interesting point of note is the sharp decline and then increase in the bond yield around September 11, 2001. Annuity quotes (per \$100,000) declined as well—as evidenced by Figure 1 —and the ILY declined, but quickly recovered within 2 or 3 weeks of the terrorist attacks.

FIGURE 3
Implied Longevity Yield (Delta) Value Minus Yield on Benchmark 10-Year Government Bond



pricing algorithms, as well as the fact that it is a reasonable investment alternative to purchasing a life annuity at age 65. To be more precise, I also compared cash flow durations. For example, on June 23, 2004, the duration of the life annuity quote (with 10-year payment certain) at age 65 was approximately 9.3 years for males and 10.1 years for females. On the same date, the duration of the benchmark 10-year bond was 7.25 years and the duration of the benchmark long-term bond was 13.7 years.[6] Thus, to match durations, the comparable bond yield would consist of 68.25 percent 10-year and 31.75 percent 30-year for the male annuitant and 56.35 percent 10-year and 43.65 percent 30-year for the female annuitant. I will return to this duration-weighted yield later in the analysis.

Figure 3 subtracts the ILY values from the yield on a 10-year Government of Canada Bond. Note the mean reverting nature of these numbers around the 114 basis points for males and 72 basis point for females. In sum, the ILY index is relatively stable over

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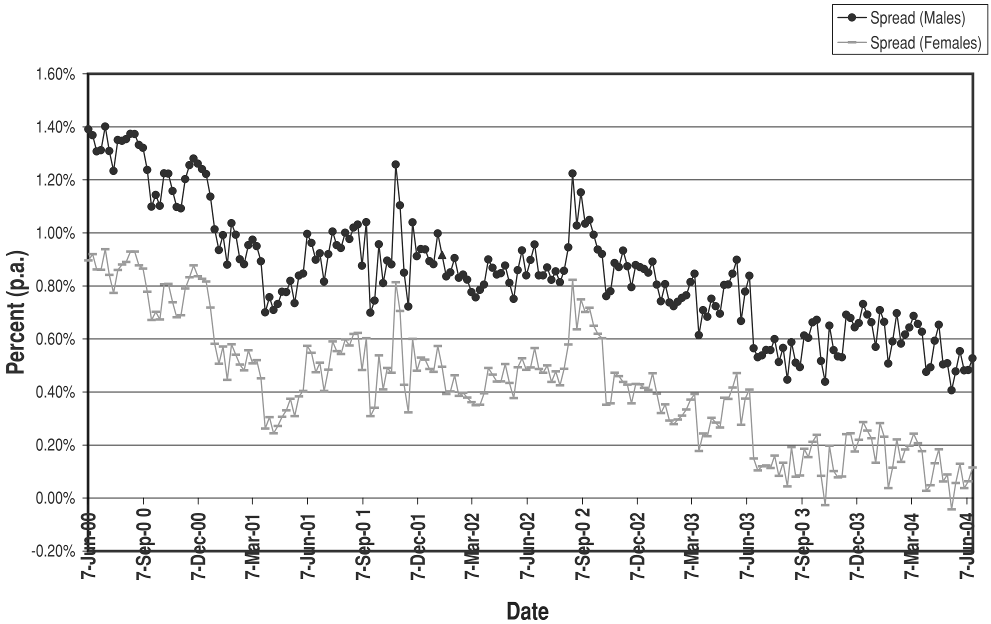
6 Actually, the maturity date of the so-called 10-year bond is June 2013 and the maturity date of the so-called long-term bond is June 2029. The 9 and 25 years to maturity—on bonds that are labeled 10- and 30-year, respectively—might help explain the lower than expected duration numbers of 7.25 and 13.7, respectively.

FIGURE 4
Implied Longevity Yield (Delta) Value Minus Yield on Benchmark Long-Term Government Bond



short periods of time, highly correlated with prevailing interest rates, and relatively easy to derive and explain to the nonspecialist.

Figure 4 compares the ILY values to the long-term (often called 30-year) bond yield. The main insight from this picture is the slowly declining spread between yields on long-term government bonds and the ILY values. In mid 2000, the spread for male (females) was 1.40 percent (0.90 percent) and this declined to 0.50 percent (0.00 percent) by mid 2004. This declining trend was not detected when ILY values were compared to the 10-year bond yield, and I suspect this is because the duration on the 10-year bond (e.g., 7.25 years in late 2004) is closer to the duration of the life annuity I am tracking (9.3 years for males and 10.09 for females) compared to the duration on the long-term bond (13.8 in late 2004).

ADDITIONAL ISSUES

There are a number of additional corollary issues that are raised by the above methodology as well as some *ad hoc* assumptions I have made along the way. I now address each one separately.

Other Age Points

There are a large number of possible ILY values that can be computed. All that is needed are two distinct annuity age/quotes and Equation (5) provides a delta value. In fact, the ILYs should be properly indexed by both age as well as the implicit delay period. There is nothing special about δ_{65}^{10}, other than that it represents one of the most popular life annuities purchased (and quoted) in practice. For example, on the same November 26 date, the payout per \$100,000 premium for a joint-and-last-survivor annuity with a 10-year payment certain would be \$571 per month, if purchased from Maritime Life (which was the best company for that particular quote). At age 75, a couple could obtain \$727 per month for life on a joint-and-last-survivor, but with zero payment certain. Putting these numbers through Equation (5) leads to a $\delta^* =$ 5.21 percent, which is a mere 38 basis points above the yield on the risk-free 10-year Government of Canada bond and much lower than the 6.27 percent yield on long-term corporate bonds. Once again, this illustrates the versatility of the ILY concept in explaining the relatively "low" longevity insurance value embedded within a life annuity when full guarantees and survivor periods are imposed. Along the same lines—but perhaps in the other direction—a 75-year-old male with \$100,000 would have secured \$958.50 per month (best quote from Empire Life) with a 5-year payment certain. An 80-year-old male would have obtained \$1,1234.29 per month (best quote from Maritime Life) assuming a zero payment certain. This works out to $a_1 = 8.6941$ (annuity factor at age 75) and $a_2 = 6.7515$ (annuity factor at age 80), for an ILY value of $\delta^* = 10.281$ percent. The 75-year-old would have to earn a 10.281 percent return over the next 5 years to "beat" the life annuity. This far exceeds the 5.90 percent value listed above (at age 65) and appropriately illustrates the power of longevity insurance. I believe that these types of illustrations—comparing the ILY values of age x against age y – are one of the main benefits of creating and maintaining such an index.

Along these lines, Table 2 displays ILY values across different ages for both males and females in 5-year increments. As one would expect intuitively, higher ages produce higher ILY values. For example, over the 4-year period I studied, a 75-year-old male would have to earn 7.14 percent per year for the next 5-years in order to be able to afford the same annuity income flow—assuming current prices continue—in 5-years. Over the 5-year period, a 75-year-old male would have to earn 8.29 percent. This, once again, can be contrasted with bond yields (or even historical returns on mutual funds) to illustrate the value of annuitization.

How Do Interest Rates Impact ILY Values?

Clearly, a large portion of the ILY value is related to the "pricing" interest rate embedded within the life annuity quote. And, if the typical (or average) insurance company prices life annuities using a static mortality table and based solely on the yield of a bond with a fixed duration, then deviations in δ over relatively short periods of time would be fully explained by changes in that particular bond yield, perhaps plus a random error term. To test whether this is the case, I regressed changes in the ILY values $\delta(i)$, on changes in the (continuously compounded) bond yields $y(i)$, where the i denotes the time/date variable on which the yield was measured. I also included terms for additional time lags that are meant to capture the possibility that annuity factors are not adjusting instantaneously (i.e., on the same week) to changes in interest rates. In general, I ran a number of specifications similar to

$$\ln\left[\frac{\delta(i)}{\delta(i-1)}\right] = b_0 + b_1 \ln\left[\frac{y(i)}{y(i-1)}\right] + b_2 \ln\left[\frac{y(i-1)}{y(i-2)}\right] + b_3 \ln\left[\frac{y(i-2)}{y(i-3)}\right] + \varepsilon_i, \quad (12)$$

TABLE 2
ILY Values at Different Ages: 5-Year Increments

	55(5)–60	60(5)–65	65(5)–70	70(5)–75	75(5)–80
			Males		
Average	5.77%	5.95%	6.43%	7.14%	8.29%
Min	4.85%	5.01%	5.51%	6.21%	7.50%
10%	5.17%	5.33%	5.83%	6.52%	7.72%
25%	5.38%	5.55%	6.05%	6.72%	7.95%
50%	5.91%	6.08%	6.58%	7.27%	8.38%
75%	6.10%	6.29%	6.77%	7.51%	8.57%
90%	6.19%	6.38%	6.88%	7.60%	8.71%
Max	6.31%	6.55%	7.02%	7.77%	8.90%
			Females		
Average	5.57%	5.68%	5.96%	6.35%	7.30%
Min	4.67%	4.80%	5.01%	5.40%	6.49%
10%	4.98%	5.09%	5.34%	5.75%	6.78%
25%	5.17%	5.30%	5.53%	5.98%	7.02%
50%	5.71%	5.81%	6.11%	6.47%	7.34%
75%	5.91%	6.05%	6.32%	6.71%	7.59%
90%	5.98%	6.09%	6.39%	6.80%	7.73%
Max	6.06%	6.22%	6.47%	6.93%	8.00%

Table displays summary statistics for ILY index values from June, 2000 through June, 2004 for both males and females at various ages (x) against age ($x + 5$).

with a variety of risk-free Government of Canada bond (5, 10, and 30 year) maturities for y, in addition to duration-weighted yields. The first thing I noticed is that b_1 rarely came out significant in our regressions, and the coefficient with the greatest explanatory power (by far) was b_2. To understand why the 1-week lag provided the greatest impact, I remind the reader that our data are collected on Wednesday afternoon while the bond yields are closing values at day end. It is quite conceivable that insurance companies are using "stale" yield curves, even by a few days, to price these annuities. Once a week or two has gone by, they have all updated their prices to reflect changes in the curve. I do find systematic deviations in the practices of various insurance companies, with some adjusting quickly (within a day) to changes in interest rates, while others lag by a few days, or keep the same price for even longer periods of time. Averaging the best five quotes between "slowly" and "rapidly" adjusting companies might impact our results on what drives δ.

In the end, I settled on the following regression equation, whose results are displayed in Table 3:

$$\ln\left[\frac{\delta(i)}{\delta(i-1)}\right] = b_0 + b_1 \ln\left[\frac{y_a(i-1)}{y_a(i-2)}\right] + b_2 \ln\left[\frac{y_b(i-1)}{y_b(i-2)}\right] + \varepsilon_i, \tag{13}$$

where y_a and y_b denote the (continuously compounded) yields on different bond maturities and durations. High-level results are as follows: The regression specification that regresses ILY changes on changes in a yield with a duration matching the annuity to the

TABLE 3
Time Series Regression Results for ILY Values versus Canadian Bond Yields

Independent Variable	R^2	Adjusted R^2	Standard Error: S_e	Parameter Estimate: b_o	b_o: p Values	Parameter Estimate: b_1	b_1: p Values	Parameter Estimate: b_2	b_2: p Values
Males—Age 65, 10-year guarantee period									
10-year bond	0.46908	0.46652	4.2930×10^{-4}	−0.00003	0.31547	0.37594	0.00000		
Long-term (30-yr) bond	0.41506	0.41223	4.5061×10^{-4}	−0.00004	0.15925	0.45958	0.00000		
68% 10 year + 32% LT	0.46914	0.46657	4.2928×10^{-4}	−0.00003	0.26283	0.41212	0.00000		
Multiple: 10 year and LT	0.47042	0.46527	4.2980×10^{-4}	−0.00003	0.29090	0.32902	0.00001	0.06629	0.47270
Females—Age 65, 10-year guarantee period									
10-year bond	0.41552	0.41269	4.2533×10^{-4}	−0.00003	0.30850	0.33410	0.00000		
Long-term (30 yr) bond	0.34638	0.34322	4.4978×10^{-4}	−0.00004	0.17142	0.39643	0.00000		
56% 10-year + 44% LT	0.40414	0.40126	4.2945×10^{-4}	−0.00003	0.24531	0.37344	0.00000		
Multiple: 10-year and LT	0.41564	0.40997	4.2631×10^{-4}	−0.00003	0.31902	0.34752	0.00000	−0.01897	0.83582

The above table displays the result from regressing changes in the ILY values against (1) changes in the 10-year bond yield, (2) changes in the long-term bond yield, (3) changes in the duration-weighted bond yield, and (4) changes in both bond yields.
LT = long term.

relevant portfolio of long-term and 10-year bonds, resulted in the highest-adjusted R^2 values, in the case of males. For females, the best fit came from changes in the 10-year bond, although the multiple regression produced similarly high R^2 values. The standard error statistic was also the lowest for both of these models. Conversely, the regression results suggest that although a correlation exists, the weakest fit is with the long-term bond, for both males and females. Regarding the intercept or b_0 parameter, I cannot reject the null hypothesis that its value is equal to zero. The potential for this parameter to have a negative value implies a possible gradual reduction in ILY "spread" values over time. Figure 5 provides a graphical illustration of the "goodness of fit" that graphically illustrate the (relatively) low R^2 values I obtained.

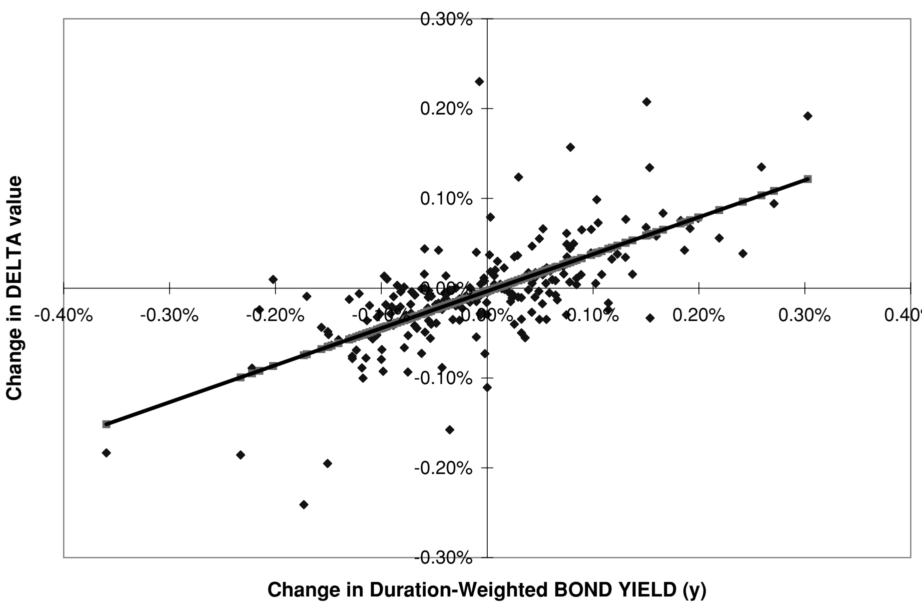
It seems that changes in relative values of δ are clustered around zero, more than one would expect from the normal error term in Equation (13). I believe this is consistent with a slight lag in adjusting to changes in interest rates, as expressed by some of the pricing actuaries in private conversations. In other words, even though interest rates went up (or down) in a given week, the insurance company does not rush to increase (or decrease) annuity payouts until this trend is continued for 1 or 2 more weeks.

The Risk in Self-Annuitizing

It is important to stress once again that there is no guarantee that if the individual decided to self-annuitize at age $x = 65$ and actually earn an internal rate of return greater than our ILY value, they will be able to purchase the same exact life annuity

FIGURE 5

Plot of Regression Results (Male): How Do Interest Rate Movements Impact Changes in the ILY?



stream at age $x = 75$. Indeed, interest rates might decline and/or the company might decide to change (worsen) their annuity pricing basis within the 10-year "waiting" period. It is not even clear if δ_{65}^{10} is an unbiased estimate (expectation) of the random variable on an "expected return" basis. A full investigation of random portfolio returns in life insurance portfolios—such as originally suggested by Boyle (1976)—is beyond the scope of this article. In brief, the stochastic analogue of the differential Equation (2) would be

$$d\tilde{W}_t = \left(\delta\tilde{W}_t - \frac{w}{a_1}\right) dt + \sigma\tilde{W}_t dB_t, \tag{14}$$

and the retiree would face the random prospect—using our full-fledged notation—that in $\tau = 10$ years

$$\Pr\left[\tilde{W}_{10} < w\frac{a_{(75,y)}}{c_0^{10} + a_{(65,y)}^{10}}\right], \tag{15}$$

which represents the case in which the investment portfolio does not contain sufficient funds to purchase the exact same life annuity income flow. I refer the interested reader to the recent articles by Albrecht and Maurer (2002), Young (2003), or Gerrard, Haberman, and Vigna (2003) for a detailed exploration of these and related issues. Our objective is to develop a metric for comparing the relative value from annuitization across ages and over time as opposed to providing a guarantee that earning the ILY will assure the ability to acquire the same exact annuity income flow.

Quadratic Approximation to Delta

Looking back at Equation (5), I can approximate the exponential term $e^{\delta\tau}$ over small values of δ with the quadratic form $(1 + \delta t + 0.5(\delta t)^2)$. Using this approximation and then collecting terms, the ILY is the value of δ that solves

$$-\left(\frac{1}{2}a_1\tau^2\right)\delta^2 + \left(\frac{1}{2}\tau^2 - a_1\tau\right)\delta + (a_2 + \tau - a_1) \approx 0. \tag{16}$$

The solution to this quadratic equation in δ is

$$\delta^* \approx \frac{(\tau - 2a_1) + \sqrt{\tau^2 + 4a_1(\tau + 2a_2 - a_1)}}{2\tau a_1}. \tag{17}$$

In our earlier case (male 65) for which $a_1 = 12.2871$ and $a_2 = 8.5391$, the exact value of the ILY is $\delta^* = 5.900$ percent using the NeRa method. According to the approximation in Equation (17) I obtain

$$\delta^* \approx \frac{(10 - 24.5742) + \sqrt{100 + 49.1484(10 + 17.0782 - 12.2871)}}{2(10)(12.2871)} = 0.05771,$$

which is an ILY value of 5.771 percent, a mere 12 basis points lower than the true value. Or, in the joint-life case mentioned above, the precise value of the ILY was

$\delta^* = 5.21$ percent while the approximate value was $\delta^* = 5.14$ percent. Our quadratic approximation consistently underestimates the true value of δ^* by between 10 and 20 basis points.

CONCLUSION

A number of authors have recently examined the properties of a *consume-term and invest-the-difference* strategy that self-annuitizes instead of purchasing an irreversible life annuity at retirement. In this brief article, I use that idea as the basis for constructing a life annuity index to track and explain the benefits of life annuity payouts over time. And, while the concept underlying the rate of return from a life annuity has been investigated in a number of actuarial and insurance papers such as Broverman (1986), our index does not require any knowledge of the company's mortality tables, rates, loads, or pricing basis.

In fact, it might actually represent a more robust method of calibrating and monitoring the change in money's worth of a generic life annuity, in contrast to the highly-cited work by Mitchell et al. (1999). Namely, by tracking the ILY values over time and across countries, one can obtain a better measure of the investment return from annuities without having to make distributional assumptions regarding mortality, or relying on the accuracy of a given day's yield curve.

I calibrated our index to a comprehensive database containing the last 4 years of annuity quotes from insurance companies in Canada, and confirmed the accuracy of an easy-to-use approximation to the ILY, which is presented in Equation (17). Aside from describing the mechanics of such an index, our main actionable conclusion is that a 65-year-old retiree would have had to earn at least 75–125 basis points over the yield on a risk-free 10-year government bond to "beat" the rate of return from a life annuity during the period from 2000 through 2004. For a 75-year-old retiree who decided to forego annuitization, this number would be closer to 150–200 basis points, depending on gender. Ongoing research will monitor the ILY values over longer periods of time, and in different countries, to measure relative changes in the ILY index.

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