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The Effects of Uncertainty on the Demand for Health Insurance

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Abstract

This article analyzes the effects of uncertainty and increases in risk aversion on the demand for health insurance using a theoretical model that highlights the interdependence between insurance and health care demand decisions. Two types of uncertainty faced by the individuals are examined. The first one is the uncertainty in the consumer's pretreatment health and the second is the uncertainty surrounding the productivity of health care. Comparative statics results are reported indicating the impact on the demand for insurance of shifts in the distributions of pretreatment health and productivity of health care in the form of first-order stochastic dominance, Rothschild–Stiglitz mean-preserving spreads, and second-order stochastic dominance. The demand for insurance increases in response to a Rothschild–Stiglitz increase in risk in the distribution of the pretreatment health provided that the health production function is in a special class and the price elasticity of health care is nondecreasing in the pretreatment health. Provided also that the demand for health care is own-price inelastic, the same conclusion is obtained when the uncertainty is about the productivity of health care.

Introduction

The presence of uncertainty in the health care industry is well established. Arrow (1963) points it out in the incidence of disease and in the efficacy of treatment. Consequently, much of an individual's demand for health care is not steady, but irregular and unpredictable. This implies that the costs of health care act as a random deduction from an individual's income. Therefore, under uncertainty, risk-averse individuals demand risk-bearing goods, such as health insurance, to safeguard their income against possible shocks.

A health insurance policy is a contract between an insurer and an individual. Within the duration of this contract, the individual agrees to exchange income in the form of

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a premium for lower prices for certain health care services in the event of poor health. Individuals seeking to enter into such a contract are not selected at random. Individual characteristics, such as health status, may influence the decision to enter into a contract and thus create a self-selection bias. In other words, the individual may have information about the probability of poor health, which the insurer cannot observe. Thus, individuals with low expectations about their future health status may have an incentive to select more comprehensive insurance coverage. In this article, we analyze how changes in risk surrounding future health status[1] affect the demand for insurance. One focus of our inquiry is into the conditions under which the identification of low- (high-) risk consumers with smaller (greater) demand for health insurance is plausible.

In an attempt to analyze the effects of uncertainty surrounding the incidence of disease and the efficacy of treatment on the demand for health insurance, we consider two shifts in the distributions of these variables. The first is a *mean-preserving spread* in which the individual keeps his/her original estimate of the related variable but becomes less confident about it. The second is a *second-order stochastic dominant shift* in which both the uncertainty surrounding the related variable and its expected value change simultaneously. Moreover, the utility function is parameterized by an ordinal risk aversion parameter to analyze the effect of increases in risk aversion on the demand for insurance.

It is well known that health care demand is influenced by health insurance coverage; the resulting moral hazard and attempts to deal with it have been a central focus of many studies of the demand for health care. Arrow (1963) was the first to identify the moral hazard phenomenon in the health care industry. Pauly (1968) shows that the effect of moral hazard can cause insurance among some types of uncertain events to be nonoptimal even if all individuals are risk averters. As Pauly (1968) mentions, insurance writers have tended to look upon the moral hazard effect as a moral or ethical problem. This effect, however, is "a result not of moral perfidy, but of rational economic behavior" (Pauly, 1968). Since an insurance policy decreases the individual's effective price of health care, the individual has the incentive to increase the quantity of health care demanded at all health states (Zweifel and Manning, 2000).

In health care, moral hazard combines with the interdependence between utilization and the insurance policy decisions.[2] On the one hand, demand for health care is affected by the insurance policy of the consumer. On the other hand, the choice of health insurance plan could be affected by demand considerations, in particular by planned health expenditure and expectations about health care utilization. Obvious examples are anticipation of demand for obstetrics and psychotherapy services, which may lead to selection of an insurance policy having relatively broad and generous coverage. Our modeling approach highlights this interdependence between health insurance and health care demand decisions and emphasizes the role of expectations formulations about future health status.

Numerous studies have analyzed the decisions about health insurance and health care demand. Cameron et al. (1988) model the interaction between health insurance and

[1] The term health status refers to the incidence of disease and the efficacy of treatment.

[2] See, for example, Arrow (1963), Cameron et al. (1988), Nyman (1999a, 1999b), Cutler and Zeckhauser (2000), Zweifel and Manning (2000), and Nyman and Maude-Griffin (2001).

health care demand decisions. In their model, the individual may choose only between a discrete number of mutually exclusive health insurance policies.[3] Blomqvist (1997) analyzes the properties of optimal nonlinear health insurance schedules. Liljas (1998) introduces uncertainty (in the incidence of illness) and insurance in the context of a multi-period mixed investment-consumption model of Grossman (1972). He assumes, however, that no moral hazard exists. Cutler and Zeckhauser (2000) study the demand for health care under uncertainty given an insurance policy. They also analyze the properties of optimal nonlinear health insurance policies with moral hazard using an interdependent model of insurance and health care demand.

Our modeling approach extends the modeling efforts of Cameron et al. (1988) and Cutler and Zeckhauser (2000) by highlighting the distinction between current and future health status; and by formally modeling the formulations of expectations about health status under uncertainty. In our model, we characterize insurance policies by the co-insurance rate or the fraction of costs paid out-of-pocket. Using this model, we then analyze the effects of uncertainty in the incidence of illness and the efficacy of treatment on the demand for health insurance.

In our model, the health care demand decision problem, given the choice of insurance policy and the individual's health state, uses Wagstaff's (1986) simple one-period model of the demand for health, which offers many of the insights afforded by Grossman's (1972) more complex pure consumption model. In this model, health is a choice variable because it is a source of utility. Individuals value health, with health care just a means to producing health. This simplified version of Grossman's (1972) pure consumption model of the demand for health has been extensively used in the health economics literature.[4]

The modeling of health insurance in this article follows closely the modeling efforts of Cameron et al. (1988) and Cutler and Zeckhauser (2000). In these articles, health insurance is differentiated from other types of insurance by incorporating one aspect of moral hazard, which as Pauly (2000) mentions, by an order of magnitude, is virtually unique to health insurance. Pauly (2000) points out that because of the difficulty of measuring or specifying whether a loss has occurred, the size of the loss is affected by health insurance as well. For almost all other insurances, however, it is relatively easy to set up procedures to assess the size of the loss.[5] In the above-mentioned articles, and in this article, the incorporation of this aspect of moral hazard is achieved by the form of the insurance benefits; a subsidy for the purchase of health care, not a simple contingent claim.

Although one of the main features that distinguishes the demand for health care from the demand for other goods and services is the uncertainty in the incidence of illness

[3] This is a special feature of their model which is intended to reflect actual choices confronting Australians in the period covered by their data.

[4] Dardanoni and Wagstaff (1990) and Picone et al. (1998) employ this model to analyze the effects of uncertainty on the demand for health care. Using the setup in the pure consumption model, Cameron et al. (1988) and Cutler and Zeckhauser (2000) analyze the interdependent decisions of health insurance and health care; and Blomqvist (1997) analyzes the properties of optimal health insurance policies.

[5] As an example, Pauly (2000) argues that the amount or type of reimbursement for tornado insurance would not affect the number of tornados or the damage they do.

and the efficacy of treatment, there is little theoretical literature analyzing the role of uncertainty in the demand for health care and insurance. Dardanoni and Wagstaff (1990) analyze the effects of uncertainty (in the incidence of illness and the efficacy of treatment) on the demand for health care using a static version of Grossman's (1972) pure consumption model of the demand for health and find that, under plausible assumptions, greater uncertainty results in an increase in the demand for health care. Picone et al. (1998) examine the effects of uncertainty (in the incidence of illness) on the demand for health care using a stochastic dynamic model based on Grossman's (1972) pure consumption model.[6] Their simulation results suggest, *inter alia*, that a first-order stochastic dominant shift of uncertainty causes the individual to purchase health care and build health capital. More often than not, however, individuals first choose their insurance and then frequently choose their health care utilization after they discover their state of health. The related uncertainty under this scenario is with respect to future health status at the time the insurance policy is chosen. Since expectations about this variable affect the demand for insurance, an analysis of the effects of uncertainty surrounding future health status on the demand for insurance would be useful.

We proceed as follows: The next section outlines a model of individual behavior relevant to interdependent decisions about health care and insurance. This is followed by a description of the modeling of health status and an examination of its effect on the demand for health care. A further section reports comparative statics results indicating the impact on the demand for insurance of shifts in the distributions of pretreatment health and productivity of health care in the form of first-order stochastic dominance, mean-preserving spreads, and second-order stochastic dominance. This is followed by a narration of the changes in the demand for health insurance to increased risk aversion.The final section concludes the discussion.

THE MODEL

We specify a model of individual behavior relevant to decisions about health insurance and health care. We assume that the individual faces a one-period, two-date planning horizon. At time zero, the individual has exogenous income Y. He/She knows his/her current health state, $s_0 \in S_0$, and chooses his/her health insurance, $\sigma \in [0, 1]$. σ can be interpreted as the co-insurance rate, or the fraction of health care costs paid out-of-pocket given an insurance policy. At the time the insurance policy is chosen, time one (i.e., future) health state $s \in S$ is unknown. It is assumed that an increase in s represents a higher level of health status. The individual's time zero-budget constraint is

$$Y = Y' + R(\sigma),$$

where $R(\sigma) : [0, 1] \to \Re_+$ is the insurance policy premium,[7] which, in general, is a function of σ, with $R(0) = K$, $0 < K < \infty$, $R(1) = 0$ and $R'(\sigma) < 0$. Y' is the remaining income.

[6] This article emphasizes the behavior of individuals in their retirement years.

[7] The results in this article do not depend on exactly how the insurance premium is calculated. However, it can be determined, for example, by a constraint, which requires that premiums must cover expected costs.

At the beginning of time one, the individual learns his/her new health state, s, and applies Y' toward the purchase of the consumption good and health care. The individual's time one budget constraint is

$$Y' = pc(s) + \sigma p_m m(s),$$

where c is the consumption good[8] whose price is denoted by $p \in \Re_{++}$, and $p_m \in \Re_{++}$ is the price of health care, m.[9]

The individual's utility function, which is assumed to be continuous, bounded,[10] and concave, is represented by[11]

$$U = U(c, H(m, s), \rho),$$

with U_1, $U_2 > 0$ and U_{11}, $U_{22} < 0$. $H : \Re^2_+ \to \Re_+$ is the health production function with $\frac{\partial H}{\partial m} > 0$, $\frac{\partial^2 H}{\partial m^2} \leq 0$, and $\frac{\partial H}{\partial s} > 0$, as in Dardanoni and Wagstaff (1990). This function determines the amount of health produced with input m in state s. ρ is an ordinal risk aversion parameter, and it is assumed that increases in ρ represent increases in risk aversion as in Diamond and Stiglitz (1974).

The individual may have information about the probability of poor health, which the insurer cannot observe. Thus, the choice of health insurance may be affected by expectations about health state and health care utilization. For example, individuals with low expectations about their future health status may have an incentive to select more comprehensive insurance policy than those with higher expectations. Formally, since the consequences of the insurance choice σ depend crucially on s, the individual forecasts his/her health care utilization according to his/her expectation about the time one health state. We assume that expectations about future health states, s, will be based on current period values, s_0. This expectation is depicted by the conditional subjective probability distribution $F(s \mid s_0)$.[12]

8 Since the individual's expenditures on c is what is left over after paying health care expenditures, there is uncertainty in the consumption good due to random health events.

9 Quantity of health care, m, refers to health care services such as the number of specialist visits, or the length of stay in a hospital.

10 If the individual's utility function is continuous and bounded, the expected utility of insurance is well defined.

11 Assuming that health care spending restores the individual to perfect health, the state of health being in the utility function (via the health production function) suggests that the motivation for the purchase of insurance is not only the preservation of income but also the preservation of utility from the commodity health, since, as discussed below, the individual with health insurance who becomes ill not only utilizes health care services that would not have been purchased without the moral hazard effect, but also may utilize services that would not otherwise be affordable.

12 For each $s_0 \in S_0$, the individual's forecast takes the form of a probability measure defined on S. We assume that both S_0 and S are compact sets. Let $\boldsymbol{B}(S)$ be the Borel algebra of S. Let $\boldsymbol{M}(S)$ be the set of probability measures defined on the measurable space $(S, \boldsymbol{B}(S))$. We can then depict the individual's expectation by a mapping F which takes S_0 into $\boldsymbol{M}(S)$. Then, for every $B \in \boldsymbol{B}$, $F(B \mid s_0)$ is the conditional subjective probability that the individual attaches to the Borel

The individual's allocation problem is to choose his/her insurance policy, health care utilization, and the quantity of consumption good so as to maximize his/her expected utility subject to time zero and time one budget constraints. This is solved by backward induction. First, the individual solves for the optimizing values of m and c conditional upon the choice of insurance policy σ, and each possible realization of the unknown health state s. Next, substituting these optimums in the utility function and integrating this function over s gives the indirect conditional expected utility associated with insurance policy σ. This indirect conditional expected utility function forms the basis of the insurance policy choice at time zero. Given a premium schedule of various insurance policies, the individual maximizes this function yielding the optimal insurance policy.

The Health Care Decision

Given the choice of insurance policy, and any $s \in S$, the individual's problem at time one is

$$\max_{m} U\left(\frac{Y'}{p} - \frac{\sigma p_m}{p} m, H(m, s), \rho\right), \tag{1}$$

where we have used the time one budget constraint to rewrite the first argument. The first-order condition for this problem is[13]

$$\Gamma = -U_1 \frac{\sigma p_m}{p} + U_2 H_1 = 0. \tag{2}$$

Rearranging the first-order condition gives the optimality condition for this problem:

$$\frac{U_2}{U_1} = \frac{\frac{\sigma p_m}{H_1}}{p} = \frac{\pi}{p}, \tag{3}$$

where $\pi = \frac{\sigma p_m}{H_1}$, the shadow price of health, is the insurance-adjusted (net) cost to the consumer of producing an extra unit of health. Not surprisingly, this condition requires that at the optimum the consumer's marginal rate of substitution between health and the consumption good must be equal to their price ratio.

Given any σ and $s \in S$, we define the set of feasible consumption streams as $A(\sigma, s) = \{\varphi = (c, m) \in \Re^2_+$: the consumer has taken the feasible action σ at time zero and the time

set B when he/she observes s_0 at time zero. To introduce the continuity of the consumer's expectation mapping, and to ensure the continuity of the expected utility function, we assume that the set $\boldsymbol{M}(S)$ is endowed with the topology of weak convergence and that F is weakly continuous on S_0.

[13] We assume interior solutions for all endogenous variables throughout the model. This assumption may potentially be restrictive. As Santos Silva and Windmeijer (2001) point out, there are cases where it is natural to assume that there is *at most* one spell of illness, i.e., there is at most one set of health care services received continuously by an individual in response to a particular request. They argue that an example would be the analysis of the demand for health care generated by self-immunizing diseases like varicella.

one budget constraint is satisfied}. The solution to the health care decision problem gives a set $\phi(\sigma, s) \subseteq A(\sigma, s)$, namely the set of conditional consumption programs associated with σ and s. We now define the individual's preferences among actions at the beginning of time zero.

Definition 1: *For all* $(\sigma, s) \in [0, 1] \times S$,

$$U(\phi(\sigma, s)) = U(c, H(m, s), \rho) \quad \textit{for some feasible bundle } \varphi \in A.$$

Definition 2: *The expected utility function that is defined on* $[0, 1] \times S_0$ *is*

$$V(\sigma, s_0) = \int_S U(\phi(\sigma, s)) dF(\cdot \mid s_0).$$

Because $U(\phi(\cdot, \cdot))$ is a well-defined, continuous, bounded utility function, the von Neumann–Morgenstern utility function $V(\sigma, s_0)$ is also well defined.[14]

The Health Insurance Decision

Given the insurance premium function and the utility-maximizing health care schedule, the individual's optimization problem can now be represented as

$$\max_{\sigma} \int_S U\left(\frac{Y'}{p} - \frac{\sigma p_m}{p} m(\sigma, s, Y'), H(m(\sigma, s, Y'), s), \rho\right) dF(\cdot \mid s_0). \tag{4}$$

The first-order condition is given by

$$\int_S \left\{ U_1 \left[-\frac{R'(\sigma)}{p} - \frac{m p_m}{p} \right] + \left[-U_1 \frac{\sigma p_m}{p} + U_2 H_1 \right] \left[\frac{\partial m}{\partial \sigma} - R'(\sigma) \frac{\partial m}{\partial Y'} \right] \right\} dF(\cdot \mid s_0) = 0, \tag{5}$$

which, after using Equation (2), yields the optimality condition for the health insurance decision problem:

$$\int_S \frac{m p_m}{p} U_1 \, dF(\cdot \mid s_0) = \int_S -\frac{R'(\sigma)}{p} U_1 \, dF(\cdot \mid s_0).$$

The right-hand side represents the cost of additional insurance in terms of expected marginal utility. This cost appears in the form of foregone consumption when the individual uses some of his/her income to purchase more insurance. The left-hand side represents the benefit of additional insurance in terms of expected marginal utility. The value of the additional insurance appears as the benefit of extra health care service. The consumer with health insurance who becomes ill utilizes more health care services because of three effects: First, additional insurance reduces the cost of health care services. Second, the individual receives an income transfer from those who remain

[14] By its construction, it inherits the usual regularity properties (i.e., continuity, concavity, "more is better," etc.) and satisfies the axioms of expected utility.

healthy (Nyman and Maude-Griffin, 2001). Consequently, this income transfer effect results in additional utilization of health care services. Third, additional benefits are derived from insurance's ability to make available health care services that would not otherwise be affordable; i.e., access value of insurance (Nyman, 1999a).[15]

HEALTH STATUS AND ITS IMPACT ON THE DEMAND FOR HEALTH CARE

The random effect s reflects the two types of uncertainty identified by Arrow (1963). We first interpret s as the consumer's pretreatment or *ex ante* level of health. Uncertainty arises because the consumer is uncertain as to what his/her level of health will turn out to be due to randomness in the exogenous determinants of health. It is assumed that $\frac{\partial H}{\partial s} = H_2 > 0$ and that $\frac{\partial^2 H}{\partial m \partial s} = H_{12} = 0$. These assumptions state that though the consumer's ex post level of health (H) is an increasing function of his/her *ex ante* level of health, the marginal product of health care does not change with changes in the consumer's *ex ante* level of health.

It is important to understand the effect of the *ex ante* level of health on the health production for what follows. As the *ex ante* level of health improves, at any level of health care more health is produced. This is represented by an upward shift in the health production function. The marginal product of health care, however, remains the same. Consequently, the health production function shifts upward in a parallel way changing the individual's ex post level of health but leaving the marginal product of health care unchanged (see Figure 1).

We next interpret s as an index of health care productivity. Uncertainty arises because the consumer is uncertain as to what his/her level of health will be due to the unpredictability of the treatment outcomes. It is assumed that $H_2 > 0$ and $H_{12} > 0$. Thus, as the index of health care productivity increases, it produces two effects on the amount of health produced. First, at any level of health care more health is produced. This is represented by an upward shift in the health production function. Second, the marginal product of health care increases compared to the pre-shift case. Consequently, the health production function does not only shift upward, but at any level of health care the slope of the function increases as well (see Figure 1).

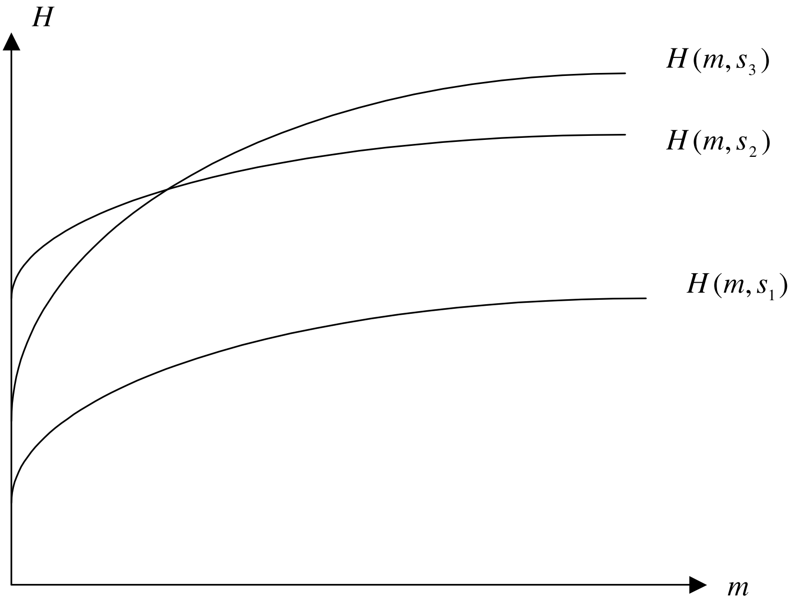
It is crucial to know the relationship between health care demand (m) and health status (s) to understand the effects of changes in risk surrounding the incidence of disease and the efficacy of treatment on the demand for health insurance. Comparative statics analysis of the optimality of the health care decision problem yields

$$\frac{\partial m}{\partial s} = \frac{\left[-\frac{\sigma p_m}{p} U_{12} + H_1 U_{22}\right] H_2 + H_{12} U_2}{-\frac{\partial \Gamma}{\partial m}}, \tag{7}$$

where $\frac{\partial \Gamma}{\partial m}$ is the second-order condition with respect to the health care decision problem. Assuming that this second-order condition is satisfied, the sign of $\frac{\partial m}{\partial s}$ agrees

[15] As Nyman (1999a) argues, this value of insurance is different from the risk-avoidance value since, by definition, there is no financial risk for unaffordable health care purchases.

FIGURE 1
The Impact of Health Status on the Amount of Health Produced



Notes: $s_2 > s_1$ represents an increase in the *ex ante* level of health. $s_3 > s_1$ represents an increase in the index of health care productivity. $H(m, s_1)$ is the pre-shift health production function.

with the sign of $(-\frac{\sigma p_m}{p} U_{12} H_2 + H_1 H_2 U_{22} + H_{12} U_2)$. Thus, there is no unambiguous relationship between health care demand and health status. It is shown in the following proposition that, under a certain restriction on the health production function, the effect of a change in health status on health care demand depends on whether the price elasticity of the demand for health care is greater or less than unity.

Proposition 1: *If*

$$\frac{mH_{11}}{H_1} = \frac{mH_{12}}{H_2} - 1, \tag{8}$$

then the relation between the marginal effect of health status on health care demand and the price elasticity of the demand for health care is given by

$$\left|\varepsilon_{p_m m}\right| - 1 = \frac{\partial m}{\partial s} \frac{H_1}{H_2}. \tag{9}$$

Proof: The price elasticity of health care is $\varepsilon_{p_m m} = -\frac{\frac{\partial \Gamma}{\partial p_m}}{\frac{\partial \Gamma}{\partial m}} \frac{p_m}{m}$, where we have used $\frac{\partial m}{\partial p_m} = -\frac{\frac{\partial \Gamma}{\partial p_m}}{\frac{\partial \Gamma}{\partial m}}$, which is true by the implicit function theorem. Plugging in $\frac{\partial \Gamma}{\partial p_m}$ and $\frac{\partial \Gamma}{\partial m}$ into the numerator of $|\varepsilon_{p_m m}| - 1$ and using the first-order condition (2) of the health care decision problem yields

$$|\varepsilon_{p_m m}| - 1 = \frac{\left(\dfrac{\sigma p_m}{p} U_{12} - H_1 U_{22}\right) H_1}{\dfrac{\partial \Gamma}{\partial m}} - \frac{U_2(H_1 + H_{11} m)}{\dfrac{\partial \Gamma}{\partial m} m}. \tag{10}$$

After multiplying and dividing through by H_1, the impact of changes in health status on the demand for health care given by Equation (7) becomes

$$\frac{\partial m}{\partial s} = \frac{\dfrac{H_2}{H_1}\left(\dfrac{\sigma p_m}{p} U_{12} - H_1 U_{22}\right) H_1}{\dfrac{\partial \Gamma}{\partial m}} - \left[\frac{H_2}{H_1}\right] \frac{H_{12} U_2 H_1}{\dfrac{\partial \Gamma}{\partial m} H_2}. \tag{11}$$

Comparing Equations (10) and (11), it can be easily shown that $|\varepsilon_{p_m m}| - 1 = \frac{\partial m}{\partial s}\frac{H_1}{H_2}$, if $\frac{mH_{11}}{H_1} = \frac{mH_{12}}{H_2} - 1$. Q.E.D.

The assumption in the previous proposition is a restriction on the health production function. The next proposition, for which the proof is given in Appendix A, demonstrates that this assumption characterizes a special class of health production functions.

Proposition 2: *Let the health production function $H(m, s) : \Re^2_+ \to \Re_+$ be twice continuously differentiable with $H_1 > 0$, $H_2 > 0$, and $H_{11} \leq 0$.*

1. *Suppose that $H_{12} > 0$. Then*

$$\frac{mH_{11}}{H_1} = \frac{mH_{12}}{H_2} - 1,$$

if and only if

$$H(m, s) = m^{-C} \exp\left(\int \frac{-C}{\exp[\Theta(s)]} ds\right) C_3, \tag{12}$$

where $-1 \leq C < 0$ and $C_3 > 0$ are constants of integration, and $\Theta(s)$ is an arbitrary function of health status.

2. *Suppose that $H_{12} = 0$. Then $\frac{mH_{11}}{H_1} = -1$ if and only if*

$$H(m, s) = \exp(-C_4) \ln(m) + \xi(s) + C_5, \tag{13}$$

where C_4 and C_5 are constants of integration and $\xi(s)$ is an arbitrary function of health status with $\xi'(s) > 0$.[16]

[16] To illustrate the result of the first part of the proposition, suppose that $C_3 = 1, C = -\alpha$ with $0 < \alpha < 1$, and $\Theta(s) = \ln(\frac{s\alpha}{1-\alpha})$. Then, one gets $H(m, s) = m^{\alpha} s^{1-\alpha}$, which is the general Cobb–Douglas class. To illustrate the result of the second part of the proposition, let $C_4 = C_5 = 0$ and $\xi(s) = \ln(s)$. Then, one gets $H(m, s) = \ln(m) + \ln(s)$, which is the health production function in which inputs are expressed in logarithms in an additive fashion.

These two propositions establish that under a certain restriction on the health production function, if the demand for health care is own-price inelastic, then health care utilization is decreasing in health status. The empirical literature in health economics suggests that the demand for health care is inelastic. Newhouse (1981) provides a review of this literature encompassing the 1970s and reports that the price elasticity of health care is almost certainly smaller than one;[17] and the path-breaking work undertaken by RAND (Manning et al., 1987) estimates the price elasticity of health care to be in the range from −0.10 to −0.20 in a randomized social experiment.[18]

The effects of uncertainty surrounding future health status on the demand for health insurance depends on whether the demand for health care increases at an increasing rate as health status gets worse, i.e., $\frac{\partial^2 m}{\partial s^2} > 0$. The following lemma, using the relation between the marginal effect of health status on health care demand and the price elasticity of the demand for health care given by Equation (9), provides an empirically supported assumption for $\frac{\partial^2 m}{\partial s^2} > 0$.

Lemma 1: *Suppose that the marginal effect of health status on the demand for health care is negative, i.e.,* $\frac{\partial m}{\partial s} < 0$*. Suppose also that the ex post level of health is increasing in health status at a nonincreasing rate,* $H_{22} \leq 0$.[19] *If the price elasticity of health care is nondecreasing in health status, then the demand for health care increases at an increasing rate as health status gets worse,*

$$\frac{\partial |\varepsilon_{p_m m}|}{\partial s} \geq 0 \Rightarrow \frac{\partial^2 m}{\partial s^2} > 0. \tag{14}$$

Proof: Differentiating $|\varepsilon_{p_m m}| - 1 = \frac{\partial m}{\partial s}\frac{H_1}{H_2}$ with respect to health status yields

$$\frac{\partial |\varepsilon_{p_m m}|}{\partial s} = \frac{H_2\left[\frac{\partial^2 m}{\partial s^2}H_1 + \left(\frac{\partial m}{\partial s}\right)^2 H_{11} + \frac{\partial m}{\partial s}H_{12}\right] - \left(\frac{\partial m}{\partial s}\right)^2 H_1 H_{12} - \frac{\partial m}{\partial s}H_1 H_{22}}{[H_2]^2}. \tag{15}$$

If $\frac{\partial m}{\partial s} < 0$ and $H_{22} \leq 0$, then $\frac{\partial |\varepsilon_{p_m m}|}{\partial s} \geq 0$ implies that $\frac{\partial^2 m}{\partial s^2} > 0$. Q.E.D.

There is some evidence in the empirical health economics literature suggesting that the price elasticity of health care is nondecreasing in health status. Wedig (1988) finds that the price response for individuals in fair or poor health is almost half that of individuals with better health in terms of whether they visit a physician at all. He finds no statistically significant differences in how much these groups visit a physician,

[17] Cutler and Zeckhauser (2000) detail estimates of the elasticity of the demand for health care documented by the substantial literature in the 1970s.

[18] A thorough overview of the estimates of the elasticity of the demand for health care can be found in Zweifel and Manning (2000).

[19] If $H_{12} = 0$, a necessary and sufficient condition for $H_{22} \leq 0$ is $\xi''(s) \leq 0$. If $H_{12} > 0$, a necessary condition for $H_{22} \leq 0$ is $\Theta'(s) > 0$.

given any visit. Thus, these results suggest that the price sensitivity of less healthy individuals is smaller over the first few units of health care utilization. The RAND study reaches the same conclusion (Manning et al., 1987).

Throughout the rest of this article, we assume that individual preferences over consumption and health are represented by a quasilinear utility function, i.e.,[20]

$$U(c, H) = c + U^1(H(m, s), \rho).$$

We make the following additional assumptions.

Assumption 1: The health production function is given by Equation (12) when health status is interpreted as the health care productivity and by Equation (13) when health status is interpreted as the pretreatment health.

As is obvious now, one needs this assumption to reach the conclusions of Proposition 1 and Lemma 1.

Assumption 2: The demand for health care is own-price inelastic.

Proposition 1 yields that under these two assumptions the demand for health care is decreasing in health status.

Assumption 3: The price elasticity of health care is nondecreasing in health status.

Using the quasilinear utility function, comparative statics analysis of the optimality of the health care decision problem yields that the sign of $\frac{\partial m}{\partial s}$ agrees with the sign of $(H_1 H_2 U_{22} + H_{12} U_2)$. When health status is interpreted as the ex ante level of health, $H_{12} = 0$ by the derivative assumptions on the health production function. Thus, an increase in the ex ante level of health decreases the demand for health care. Consequently, Assumption 2 is redundant for the analysis of the changes in risk surrounding the pretreatment health.[21]

CHANGES IN RISK SURROUNDING FUTURE HEALTH STATUS

The Effects of Improvements in the Pretreatment Health

The effects of improvements in the pretreatment health on the demand for health insurance can be analyzed by considering a first-order stochastic dominant shift in the distribution of this variable. The following definition provides a condition for the statement that the distribution $F(\cdot)$ yields unambiguously higher returns than the distribution $G(\cdot)$.

Definition 3: *The distribution $F(\cdot)$ is said to first-order stochastically dominate the distribution $G(\cdot)$ if $F(x) \le G(x)\, \forall\, x$.*

[20] This assumption, which is also employed by Jack (1998), is made for tractability and simplicity; however, the results in the next section also hold for utility functions which are additively separable.

[21] The income effect for the consumption good, $\frac{\partial c}{\partial Y'}$, agrees in sign with $[U_{12}\frac{\sigma p_m}{p} - U_{22} H_1]$. If c is a normal good, then $\frac{\partial m}{\partial s} < 0$ if $H_{12} = 0$, independent of the form of the utility function.

A calculus characterization of first-order stochastic dominance would be useful for the purposes of this article. Let $F(x, \tau)$ be a distribution function with compact support, where x is the random variable and τ is an exogenous parameter. For convenience assume that $F(0, \tau) = 0$ and $F(1, \tau) = 1$ for all τ.[22]

Definition 4 (Machina, 1983): *Increases in τ induce first-order stochastically dominating shifts in $F(.,\tau)$ if and only if* $\frac{\partial F(x,\tau)}{\partial \tau} \leq 0$.

Thus, a first-order stochastically dominating shift in $F(s, \tau)$ means that there is an increase in the expected pretreatment level of health.

The first-order condition for the health insurance decision problem is given by

$$\Psi = \int_S \left[-\frac{R'(\sigma)}{p} - \frac{mp_m}{p} \right] F_s(s, \tau)\, ds.$$

Assuming that the second-order condition is satisfied, the effect on the demand for health insurance of changes in the distribution of s is found by the implicit function theorem. Thus, $\frac{\partial \sigma}{\partial \tau} = -\frac{\frac{\partial \Psi}{\partial \tau}}{\frac{\partial \Psi}{\partial \sigma}}$, and the sign of $(\frac{\partial \sigma}{\partial \tau})$ agrees with the sign of $(\frac{\partial \Psi}{\partial \tau})$. Differentiating Ψ with respect to τ, we get

$$\frac{\partial \Psi}{\partial \tau} = \int_S \left[-\frac{R'(\sigma)}{p} - \frac{mp_m}{p} \right] F_{s\tau}(s, \tau)\, ds. \tag{16}$$

Integrating Equation (16) by parts once yields

$$\frac{\partial \Psi}{\partial \tau} = -\frac{R'(\sigma)}{p} F_\tau(s, \tau) \mathrm{I}_0^1 - \frac{mp_m}{p} F_\tau(s, \tau) \mathrm{I}_0^1 - \frac{p_m}{p} \int_S -F_\tau(s, \tau) \frac{\partial m}{\partial s}\, ds, \tag{17}$$

which, after using the facts that $F(0, \tau) = 0$, $F(1, \tau) = 1$ for all τ, becomes

$$\frac{\partial \Psi}{\partial \tau} = -\frac{p_m}{p} \int_S -F_\tau(s, \tau) \frac{\partial m}{\partial s}\, ds. \tag{18}$$

With the quasilinear utility function, if $H_{12} = 0$, we know that comparative statics analysis of the optimality of the health care decision problem yields that $\frac{\partial m}{\partial s} < 0$. Since $F_\tau(s, \tau) \leq 0$ by the definition of first-order stochastic dominance, we get $\frac{\partial \Psi}{\partial \tau} \geq 0$, which means that a first-order stochastically dominating shift in $F(s, \tau)$ induces a decrease in the demand for insurance, since increases in the co-insurance rate represent decreases in demand.[23]

[22] Without loss of generality, we assume that the support of the distribution function is the unit interval.

[23] We assume that the cost of insurance coverage remains constant by assuming that the individual's risk distribution changes but that the insured population's risk distribution does not. The cost of insurance coverage, however, may change if the insured population's risk distribution changes, since the insurance premium should be the expected loss based on the risk distribution of the insured population under the assumption of rational expectations. We thank an anonymous referee for emphasizing this point.

This prediction accords with intuition. When the pretreatment health improves, the consumer can reach a given level of health by utilizing less health care. This decreases the expenditure risk against which the individual insures and thus decreases his/her demand for insurance. In other words, an improvement in the *ex ante* level of health is simply equivalent to a decrease in the expenditure risk. This result may have some appeal to the literature on asymmetric information and signaling where it is argued that low-risk consumers are more likely to signal their characteristics by demanding less insurance. As is demonstrated, first-order stochastically dominating shifts in the distribution of the pretreatment health induces a decrease in the demand for insurance. Thus, the above result supports the identification of low- (high-) risk consumers with smaller (greater) demand for health insurance.[24]

The Effects of Increased Uncertainty Surrounding the Pretreatment Health

A first-order stochastic dominant shift in the distribution of a random variable does not reveal anything about the effects of uncertainty, i.e., it is not a comparison of risks. A mean-preserving spread, on the other hand, permits an analysis of the effect of increased uncertainty. It examines the consequences of increases in risk in terms of a change in the distribution of a random variable that keeps its mean constant but moves the probability density from the center to the tails of the distribution. Following Dardanoni and Wagstaff (1990), we interpret a mean-preserving spread in the distribution of the pretreatment health as the consumer keeping his/her original estimate of his/her pretreatment health but becoming less confident about it. The following definition provides the integral conditions that are necessary and sufficient for an increase in τ to induce a mean-preserving spread in the distribution of the pretreatment health.

Definition 5 (Rothschild and Stiglitz, 1970): *Comparing members of the family of distributions which are close to each other, an increase in τ is said to represent a mean-preserving increase in risk if*

(1) $\int_0^1 F_\tau(s,\tau)\,ds = 0$, *and*

(2) $T(y,\tau) = \int_0^y F_\tau(s,\tau)\,ds \geq 0,\ \forall\, 0 \leq y \leq 1.$

It would be useful to tie these integral conditions to the notion of risk aversion. If two distributions $F(\cdot)$ and $G(\cdot)$ have the same mean, then a condition that is equivalent to these integral conditions is

$$\int u(x)\,dG(x) \leq \int u(x)\,dF(x) \tag{19}$$

for all concave utility functions $u(\cdot)$. This condition states that $G(\cdot)$ should be considered as riskier than $F(\cdot)$, if and only if $F(\cdot)$ is preferred to $G(\cdot)$ by all individuals with risk-averse utility functions.

[24] This result also holds for utility functions which are additively separable, if the net benefit of additional insurance exceeds some negative threshold value for all health states.

The effect of a mean-preserving spread in $F(s, \tau)$ on the optimal value of health insurance is found by integrating by parts Equation (16) twice. This yields

$$\frac{\partial \Psi}{\partial \tau} = \frac{p_m}{p}\left[\frac{\partial m}{\partial s}\int_0^1 F_\tau(s,\tau)\,ds - \int_S \frac{\partial^2 m}{\partial s^2}\left[\int_0^s F_\tau(y,\tau)\,dy\right]ds\right]. \tag{20}$$

Using the first part of Definition 5 yields

$$\frac{\partial \Psi}{\partial \tau} = -\frac{p_m}{p}\int_S \frac{\partial^2 m}{\partial s^2}\left[\int_0^s F_\tau(y,\tau)\,dy\right]ds. \tag{21}$$

Using the second part of Definition 5 and assuming that $\frac{\partial|\varepsilon_{p_m m}|}{\partial s} \geq 0$, we get $\frac{\partial \Psi}{\partial \tau} \leq 0$, which means that the demand for health insurance is increasing since decreases in the co-insurance rate represent increases in demand.[25]

In order to understand the intuition behind this result, the following interpretation of the Rothschild–Stiglitz increase in risk would be useful. As the uncertainty about the pretreatment health increases, its expected value does not change. Equation (19) shows, however, that the consumer's expected utility decreases. This means that the certainty equivalent value of the pretreatment health reduces. Consequently, the consumer views this increase in uncertainty as equivalent to a reduction in the mean of the pretreatment health. In other words, in the case of a mean-preserving spread, though the expected value of the pretreatment health does not change, the consumer perceives that it decreases. This result could be relevant to a generalization of the Rothschild and Stiglitz (1976) screening model in which high-risk consumers would have a riskier loss distribution than low-risk consumers in the sense of a mean-preserving spread.[26] As in the first-order stochastic dominance case, this result also supports the identification of low- (high-) risk consumers with smaller (greater) demand for health insurance.

The Effects of a Mean-Increasing Reduction in Spread in the Distribution of the Pretreatment Health

In most instances where the degree of uncertainty surrounding the pretreatment health decreases, there is also an increase in the consumer's expected pretreatment health. An actual clinical example might be that after a good preventative check-up, the individual is reassured to be in good health and more confident of his/her *ex ante* level of health.[27] A second-order stochastic dominant shift in the distribution of the pretreatment health can be used to analyze the consequences of this kind of changing risk. The following definition provides the necessary and sufficient conditions for an increase in τ to induce a second-order stochastic dominant shift in the distribution of the pretreatment health.

[25] This result also holds for utility functions which are additively separable, if the absolute risk aversion with respect to c (i.e., the consumption good) is decreasing in c and if the net benefit of additional insurance exceeds some negative threshold value for all health states.

[26] The relevance of a mean-preserving spread to the Rothschild and Stiglitz (1976) screening model in this sense is suggested by Demers and Demers (1991).

[27] We are grateful to an anonymous referee for suggesting this example.

Definition 6 (Machina, 1983). *Comparing members of the family of distributions, which are close to each other, an increase in τ is said to represent a second-order stochastic dominant shift if*

1. $\int_0^1 F_\tau(s,\tau)\,ds \leq 0$, *and*
2. $T(y,\tau) = \int_0^y F_\tau(s,\tau)\,ds \leq 0,\ \forall\, 0 \leq y \leq 1.$

Integrating Equation (16) by parts twice yields

$$\frac{\partial\Psi}{\partial\tau} = \frac{p_m}{p}\frac{\partial m}{\partial s}\int_0^1 F_\tau(s,\tau)\,ds - \frac{p_m}{p}\int_S \frac{\partial^2 m}{\partial s^2}\left[\int_0^s F_\tau(y,\tau)\,dy\right]ds. \tag{22}$$

Using Definition (6) and assuming that $\frac{\partial|\varepsilon_{p_m m}|}{\partial s} \geq 0$, we get $\frac{\partial\Psi}{\partial\tau} \geq 0$, which means that the demand for insurance is decreasing since increases in the co-insurance rate represent decreases in demand.

The assumptions that are used to reach this result are the same as those employed in the mean-preserving spread case. This is not surprising, since a second-order stochastic dominant shift in the distribution of a random variable is a combination of first-order stochastically dominating shifts and mean-preserving reductions in risk. This can be clearly seen in Equation (22) where the effects of a second-order stochastically dominating shift are divided into two: the first part of the equation corresponding to the increase in the mean of the pretreatment health and the second part corresponding to the reduction in the spread of the distribution of this variable.

Changes in Risk Surrounding the Productivity of Health Care

When the health status is interpreted as the productivity of health care, $H_{12} > 0$ by the derivative assumptions on the health production function. Even with the quasi-linear utility function, there is no unambiguous relationship between health care demand and the productivity of health care. However, if the health production function is in a special class provided by Proposition 2, and if the demand for health care is own-price inelastic, then health care utilization is decreasing in health care productivity. Thus, all results related to the changes in risk surrounding the pretreatment health follow in this case as well.[28]

To understand the changes in the interpretation of the assumptions compared to the previous case, some comments about Assumption 2 would be useful. It can easily be

[28] The interpretation of a mean-preserving spread in the distribution of the health care productivity is that the individual keeps his/her original estimate of health care productivity but becomes less confident about it. An example of a second-order stochastic dominating shift in the distribution of the health care productivity is provided by Dardanoni and Wagstaff (1990). They argue that measures aimed at reducing the variation in the quality of care provided by general practitioners constitutes an example for two reasons. First, they reduce the uncertainty surrounding the productivity of care provided by general practitioners. Second, if successful, this results in an increase in the average quality of care.

shown[29] that

$$\varepsilon_{\pi H} = \varepsilon_{p_m m}\varepsilon_{mH},$$

where $\varepsilon_{\pi H}$ is the elasticity of demand for health with respect to its shadow price and $\varepsilon_{mH} = \frac{\partial H}{\partial m}\frac{m}{H}$ is the elasticity of demand for health with respect to health care. Assumption 2 implies $\frac{|\varepsilon_{\pi H}|}{\varepsilon_{mH}} < 1$, which means that the absolute value of the elasticity of demand for health with respect to its shadow price is smaller than its elasticity with respect to health care.

The implication of this inequality for Proposition 1 can be best understood by considering the two effects that an increase in the productivity of health care produces. First, the amount of health produced increases due to an upward shift in the health production function. Second, since the shadow price of health, $\frac{\sigma p_m}{H_1}$, decreases, the quantity of health demanded increases. When $|\varepsilon_{\pi H}| < \varepsilon_{mH}$ is satisfied, however, the increase in the amount of health produced due to the first effect is more than the desired increase in health (i.e., the second effect). Consequently, the demand for health care decreases.

RISK AVERSION AND THE DEMAND FOR HEALTH INSURANCE

In this section, we attempt to examine the changes in the demand for insurance when the individual becomes more risk averse, holding income constant. Empirical research shows that more risk-averse individuals exhibit more precautionary behavior, both in the form of additional health care expenditures (Picone et al., 1998) and in the form of more generous insurance coverage (Friedman, 1974). The intuition provided by the assumption of the below demonstration may help interpreting these empirical findings.

The effect on the demand for health insurance of changes in risk aversion is found by $\frac{\partial \sigma}{\partial \rho} = -\frac{\frac{\partial \Psi}{\partial \rho}}{\frac{\partial \Psi}{\partial \sigma}}$, and the sign of $\frac{\partial \sigma}{\partial \rho}$ agrees with the sign of $\frac{\partial \Psi}{\partial \rho}$. Comparative statics analysis of the optimality of the insurance decision problem yields

$$\frac{\partial \Psi}{\partial \rho} = \int_S \left[-\frac{p_m}{p}\frac{\partial m}{\partial \rho}\right] dF(s,\tau). \tag{23}$$

Comparative statics analysis of the optimality of the health care decision problem yields that $\frac{\partial m}{\partial \rho} = \frac{U_{2\rho}H_1}{-\frac{\partial \Gamma}{\partial m}}$. Assuming $U_{2\rho} \geq 0$ and noting that the sign of $\frac{\partial m}{\partial \rho}$ agrees with the sign of $U_{2\rho}$, one gets $\frac{\partial \sigma}{\partial \rho} \leq 0$, which demonstrates that the demand for health insurance is increasing in risk aversion since decreases in the co-insurance rate represent increases in demand.

The assumption that the marginal utility of health is increasing in risk aversion accords with intuition. This assumption suggests that to reach the same amount of marginal increase in utility, a more risk-averse individual would need more from the commodity health. This means that everything else remaining constant, a more

[29] $\varepsilon_{\pi H} = \frac{\partial H}{\partial \pi}\frac{\pi}{H}$, where $\pi = \frac{p_m}{H_1}$. Since $\frac{\partial H}{\partial \pi} = \frac{\partial m}{\partial p_m}(H_1)^2$, one gets $\frac{\varepsilon_{\pi H}}{\varepsilon_{p_m m}} = \frac{H_1 m}{H} = \varepsilon_{mH}$, and the result follows.

risk-averse individual would demand more health care. This increases the expenditure risk against which the individual insures and thus increases his/her demand for insurance. Thus, an increase in risk aversion is simply equivalent to an increase in the expenditure risk, or equivalent to a decrease in the expected health status.

CONCLUDING REMARKS

In this article, we analyze the effects of uncertainty and increases in risk aversion on the demand for health insurance. The modeling approach highlights the interdependence between health care and insurance demand decisions, and emphasizes the role of expectation formulation about future health status. Two types of risk faced by the individuals are examined. The first risk represents the uncertainty in the consumer's pretreatment health that occurs due to the randomness in the exogenous determinants of health. The second risk represents the uncertainty surrounding the productivity of health care. The common interpretation is that treatment outcomes are unpredictable.

A first-order stochastic dominant shift in the distribution of the pretreatment health induces a decrease in the demand for insurance. The intuition is that an improvement in the pretreatment health decreases the expenditure risk against which the individual insures and thus decreases his/her demand for insurance. This result supports the identification of low (high) risk consumers with smaller (greater) demand for health insurance in the literature of asymmetric information and signaling. To analyze the effects of uncertainty, the consequences of a mean-preserving spread and a second-order stochastic dominant shift in the distribution of the pretreatment health is examined. We find that the demand for insurance increases in response to a Rothschild–Stiglitz increase in risk provided that the health production function is in a special class and the price elasticity of health care is nondecreasing in the pretreatment health. Under the same restrictions, it is shown that an increase in the mean of the pretreatment health coupled with a reduction in the spread of its distribution lead to a decrease in the demand for insurance.

When the uncertainty is about the productivity of health care, we obtain the same conclusions, provided also that the demand for health care is own-price inelastic. With this additional restriction, the demand for health care unambiguously decreases as health care productivity increases. Finally, we find that an increase in risk aversion is equivalent to an increase in the expenditure risk, which leads the individual to buy more generous insurance coverage.

APPENDIX A

Proof of Proposition 2:

Part 1 (Sufficiency): Suppose that $\frac{mH_{11}}{H_1} = \frac{mH_{12}}{H_2} - 1$, which is equivalent to

$$\frac{H_{11}}{H_1} = \frac{H_{12}}{H_2} - \frac{1}{m}. \tag{A1}$$

Integrating Equation (A1) with respect to m yields

$$\ln(H_1) = \ln(H_2) - \ln(m) + \Theta(s), \tag{A2}$$

where Θ is an arbitrary function of health status. Rearranging the terms in Equation (A2) yields

$$mH_1 - H_2 \exp\left[\Theta(s)\right]. \tag{A3}$$

Suppose a solution to Equation (A3) is of the form $H(m, s) = \alpha(m)\beta(s)$. This yields

$$H_1 = \alpha'(m)\beta(s) \quad \text{and} \quad H_2 = \alpha(m)\beta'(s). \tag{A4}$$

Substituting Equation (A4) into Equation (A3), we obtain

$$m\alpha'(m)\beta(s) - \alpha(m)\beta'(s) \exp\left[\Theta(s)\right] = 0.$$

Thus, the variables are separated and we get[30]

$$\frac{m\alpha'(m)}{\alpha(m)} = \frac{\beta'(s)\exp\left[\Theta(s)\right]}{\beta(s)} = -C,$$

where C is a separation constant. The separation of variables yields two ordinary differential equations, one for $\alpha(m)$ and one for $\beta(s)$:

$$m\alpha'(m) + C\alpha(m) = 0, \tag{A5}$$

$$\exp\left[\Theta(s)\right]\beta'(s) + C\beta(s) = 0. \tag{A6}$$

We solve each of these equations for any value of C. The product of these solutions provides a solution for the partial differential equation in Equation (A3). Rewriting Equation (A5) as $\frac{\alpha'(m)}{\alpha(m)} = \frac{-C}{m}$ and integrating with respect to m, we obtain

$$\ln(\alpha(m)) = -C\ln(m) + c, \tag{A7}$$

where c is an arbitrary constant of integration. Rewriting Equation (A7) as

$$\alpha(m) = m^{-C}C_1,$$

where $C_1 = \exp(c)$ is also an arbitrary constant, yields a solution for $\alpha(m)$. Following the same steps a solution for Equation (A6) is

$$\beta(s) = \exp\left(\int \frac{-C}{\exp[\Theta(s)]}\,ds\right)C_2,$$

where C_2 is an arbitrary constant. Thus,

$$H(m, s) = m^{-C}\exp\left(\int \frac{-C}{\exp[\Theta(s)]}\,ds\right)C_3, \tag{A8}$$

[30] It is convenient to exhibit the negative sign explicitly.

where $C_3 = C_1C_2$. Finally note that for the assumptions on H_1, H_2, and H_{11} to be satisfied, $-1 \leq C < 0$ must hold, and for H_{12} to be positive, C_3 must take on positive values. This proves the sufficiency. Q.E.D.

(Necessity): Taking the partial derivatives of $H(m, s)$ and verifying Equation (A1). Straightforward. Q.E.D.

Part 2: Suppose $\frac{mH_{11}}{H_1} = -1$, which is equivalent to

$$\frac{H_{11}}{H_1} + \frac{1}{m} = 0. \tag{A9}$$

Since $H_{12} = 0$, let $H(m, s) = \tau\ (m) + \xi(s)$ with $\tau'(m) > 0$, $\tau''(m) \leq 0$, and $\xi'(s) > 0$. Integrating Equation (A9) with respect to m yields

$$\ln(H_1) + \ln(m) + C_4 = 0, \tag{A10}$$

where C_4 is a constant of integration. Note that C_4 is not a function of s since both H_1 and H_{11} are not functions of s. Rewriting Equation (A10) as $\tau'(m) = H_1 = \frac{\exp(-C_4)}{m}$ and solving for $\tau\ (m)$, we obtain

$$\tau(m) = \exp(-C_4)\ln(m) + C_5,$$

where C_5 is a constant of integration. Q.E.D.

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