

EQUITY RISK, CONVERSION RISK, AND THE DEMAND FOR INSURANCE

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ABSTRACT

Existing insurance theory fails when applied to real property because it does not account for variations in the economic environment. The article studies optimal property insurance in the presence of two sources of variation: equity risk and conversion risk. Equity risk is randomness of the value of a property. It tends to raise demand for conventional insurance. In contrast, conversion risk is randomness in the value the property would have if, after severe damage, it were converted to the highest-valued use. It is distinct from equity risk because the highest-valued use is typically not the current one. Under independent conversion risk, the optimum upper limit is a compromise among underlying conversion thresholds. Absent independence, the optimum can be quite different. Conversion risk can raise or lower the demand for property insurance. Insurance contracts that fail to address conversion tend to undermine the orderly disposition of obligations and reduce the gains from reallocation of risks through insurance.

INTRODUCTION

When real estate is destroyed it is typically not replaced by a replica of itself but by something different—something more up to date, more efficient, or more suitable. It is, in short, converted. The truth of this general observation is most apparent when the destruction is the result of a disaster and conversion extends over whole neighborhoods. For instance, each of the disastrous fires and earthquakes in San Francisco led to a significant rearrangement of business and residential districts. The area of the Oakland fire has been rebuilt in far grander residential structures than the ones destroyed. In some of the areas damaged by the Northridge earthquake, new building is an improvement on what was lost. Such examples show the prevalence of conversion.

A well-informed consumer knows about conversion and wants to factor it into her demand for insurance. Her difficulty is that the values governing the conversion decision depend upon the economic environment at the time of loss and hence are themselves random variables. The risks she confronts are, briefly, externality risks.

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Disaster is an obvious source of externality risk. After disaster, the neighborhood rebuilds. If it becomes a retail furniture district, the value of apartments is compromised. If it regentrifies, converting a property to low-income housing is unwise. If it is taken for a stadium, a parking lot is the best land use. Other sources contribute to externality risk because in the normal course of events, a neighborhood may upgrade, deteriorate, or simply change, altering the surroundings of a structure and thus altering the values behind the decision to replace or convert it. Values also change as a result of economy-wide fluctuations, again creating risks of the type discussed here.

Neighborhood conversion can help or hurt the individual land owner, depending on circumstances. Predominantly, it helps. The reason is a reduction in constraints. When only a single property is converted, choice is constrained by the age and character of the existing neighborhood. That constraint vanishes when all properties are converted together, and owners can attain higher values. Certainly failures of coordination and costs of bargaining may also affect conversion, but when change is triggered by disaster losses, it tends to soften their impact.¹

Externality risk has two components: equity risk and conversion risk. Equity risk is variability in the value of the currently existing property. It is evidenced by the continually reported sales of comparable properties. Conversion risk is uncertainty about the values of structures that could replace the current one if it were badly damaged. It is less obvious than equity risk because conversions are much less numerous than sales. The two forms of externality risk impact the demand for property insurance. The manner in which they do so and their effect on the demand for insurance of real property are the topics of this article.

This article is organized as follows. The next section defines the economic environment of the owner of real property. The succeeding section shows that contracts of insurance that have upper limits on coverage are natural in the context although technically difficult to handle. "Equity Risk" examines equity risk and shows that under suitable conditions it increases demand for insurance. The section "Conversion Risk" discusses why the conversion threshold is not a contractible variable. It shows that the optimum upper limit is a compromise when the conversion risk is independent of the risk of damage and that it can be something quite different when the risks are dependent. It shows further that an independent conversion risk may raise or lower demand for insurance and supplies some intuition for that result.

UPPER LIMITS AND CONVERSION

The model used in this article has two characteristics that are desirable for the purpose of studying externality risk. The insurance contract is written in terms of damage to property, and the choice variable for the consumer is the upper limit on the amount insured. As in other models, consumers demand insurance for the purpose of limiting variations in wealth. The difference here is that randomness of wealth arises from

¹ Dacy and Kunreuther (1969) support this claim in a long section that reads, in part "In fact, a disaster may turn out to be a blessing in disguise . . . there is an opportunity for commercial establishments and homeowners to improve their facilities" (p. 168). They illustrate the general principle with observations from disaster recovery in several cities.

randomness in the economic environment as well as from risk of damage. Conversion risk and equity risk are the new elements.

Look at the situation that exists when an event has caused damage. The realization of the random variable for damage is t . The same property in undamaged condition would have a value of v , which is also the realization of a random variable. When the owner of damaged property restores it, he attains wealth $v - t$. On the other hand, the best option for converting the property is the one that maximizes net value. Let the value of the land and improvements in the highest-valued use be v^* , and let the cost of building the best improvements be c . Among the conversion options, the greatest attainable net value is $v^* - c$, which is the value of the vacant land.

The decision to restore or convert is the decision to select the greater of $v - t$ and $v^* - c$. At the critical level of damage, $q = v - (v^* - c)$, the options are equally valuable. The point q is the conversion threshold.² In the absence of insurance, the consumer's wealth is $\max[v - t, v - q]$ or, equivalently,

$$v - \min[t, q]. \quad (1)$$

Equity risk is variation in equity value v , and conversion risk is variation in the conversion threshold q .

The conversion threshold limits the loss of wealth that can be caused by damage. When damage reaches or exceeds the threshold, the owner decides to raze the existing structure and build something more appropriate rather than restoring it to its previous use. Additional damage can occur beyond the threshold, but it causes no further loss of wealth. Thus, slight damage leads to restoration and reduces wealth by an amount equal to the damage, but heavy damage leads to conversion with loss of wealth equal to the threshold.

In this article, the upper limit is the variable of interest. Thus, in choosing an insurance contract, the consumer selects the upper limit, which is denoted by b and thought of as "bound." There is no coinsurance or deductible. Unconstrained choice of upper limit, coinsurance, and deductible are studied in Garratt and Marshall (2001) in the absence of externality risk. Under the present conditions, however, the indemnity is $\min[b, t]$, and its expected value is the fair premium which is, given a probability density function of damage $h(t)$,

$$P(b) = \int_0^b th(t) dt + b \int_b^\infty h(t) dt. \quad (2)$$

Combining the preinsurance wealth with the insurance variables, insured wealth is

$$v - \min[q, t] + \min[b, t] - P(b). \quad (3)$$

² A graphical exposition of the conversion threshold appears in Garratt and Marshall (1996). See Garratt and Marshall (2001) for a more detailed derivation of the conversion threshold.

The expression captures two key features of property insurance: the conversion threshold appears in the second term, and the indemnity with upper limit is in the third term.

The consumer of insurance is a risk averse maximizer of expected utility with utility function u . Loading is given by the parameter λ . Let E be the expectation operator. Then the consumer's goal is to choose b to maximize

$$T(b) = Eu(v - \min[q, t] + \min[b, t] - (1 + \lambda)P(b)). \quad (4)$$

The function being optimized has some unusual features. The presence of the convex function $-P(b)$ in the objective suggests that the objective might not be globally concave in b , and that is in fact correct. Lack of global concavity is characteristic of insurance having upper limits. The prevalence of upper limits in real contracts suggests that markets are unconcerned about nonconcavity and that insurance theory should deal with it, as is done in the following sections.

Equation (3) raises the issue of incentives. Arson is theoretically profitable because the client might recover the contractual upper limit when it is greater than the threshold. Contracts that are conditioned on the threshold can eliminate the incentive problem, but the threshold is not contractible in contracts insuring real estate. Reasons for noncontractibility are discussed in "Conversion Risk." Consumers in this article lack the ability to recognize and profit from opportunities to commit arson.

OPTIMUM UPPER LIMITS WITHOUT EXTERNALITY RISKS

As a starting point consider fair pricing and no externality risk, that is, $\lambda = 0$, $q = \bar{q}$, and $v = \bar{v}$. Expected utility is

$$T(b; 0, \bar{v}, \bar{q}) = \int_{t=0}^{\infty} u(\bar{v} - \min[\bar{q}, t] + \min[b, t] - P(b))h(t) dt. \quad (5)$$

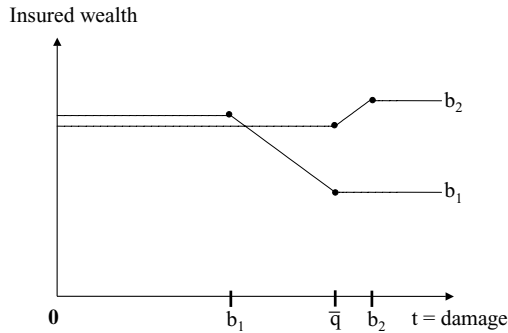
After some manipulation or by using Lemma A1 in the Appendix, the first derivative is written as

$$T_b(b; 0, \bar{v}, \bar{q}) = P'(b)Eu' \left[\frac{E[u' | t > b]}{Eu'} - 1 \right]. \quad (6)$$

The terms in square brackets need explanation. The term Eu' is the marginal utility of certainty wealth spread across all states. The term $E[u' | t > b]$ is the marginal utility of the same unit of certainty wealth when it purchases, at fair prices, a uniform increase in wealth in states in which damage exceeds the upper limit of coverage. The ratio is the marginal rate of substitution between wealths used in the two ways. The consumer can make the exchange at a price of unity and does so in the direction required to equate the marginal rate of substitution to the exchange ratio.

The relation between these quantities is shown in Figure 1. Two values of the upper limit are illustrated, b_1 and b_2 , which bracket the threshold q . Consider b_1 . The marginal utilities in $E[u' | t > b_1]$ are integrated over the two segments to the right of $t = b_1$, and those in Eu' are integrated over all three segments. The former average is higher than

FIGURE 1
Insured Wealth for Two Upper Limits



the latter, implying that the marginal rate of substitution $E[u' | t > b_1]/Eu'$ is greater than unity. On the other hand, at b_2 the marginal utilities in $E[u' | t > b_2]$ are integrated over only the segment furthest to the right. Again, Eu' aggregates marginal utilities over all three segments. Now the marginal rate of substitution $E[u' | t > b_2]/Eu'$ is less than unity.

Optimum b requires that the first derivative in Equation (6) is zero. From the diagram, the condition is satisfied at the points $b = 0$ and $b = \bar{q}$. By inspection of the diagram, the slope of the objective is strictly positive at points of $(0, \bar{q})$. When b is above $\bar{q}$, the relations in the figure are reversed and the objective is falling. Therefore $b = \bar{q}$ is the global maximum.

The objective is concave in a neighborhood of the maximum, but concavity is not global. In particular, the other critical point is the minimum at $b = 0$, which is a point of convexity. However, by the arguments of the previous paragraph, the objective is strictly increasing on $[0, \bar{q}]$ and strictly decreasing on $[\bar{q}, \infty)$, and therefore strictly quasi-concave on $[0, \infty)$.

Loading. Consider the case of a loaded premium. Differentiating (4) yields

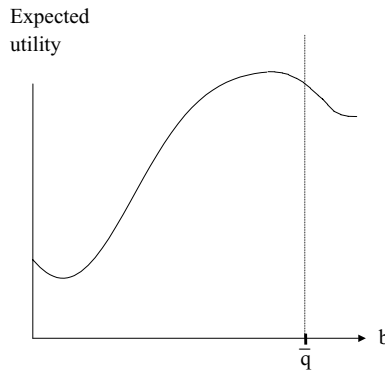
$$T_b(b; \lambda, \bar{v}, \bar{q}) = P'(b)Eu' \left[\frac{E[u' | t > b]}{Eu'} - (1 + \lambda) \right]. \quad (7)$$

In this expression, the marginal rate of substitution is the same as above in the unloaded case, but the marginal rate of transformation is higher because of the loading.

The objective has the shape indicated in Figure 2. The local minimum at low values of the upper limit is a characteristic of this type of insurance. A minimal coverage is worse than none at all. Moreover, when the upper limit is near zero, the marginal value of increased insurance is negative. At such low upper limits, the marginal addition to insurance from raising the upper limit does more harm by raising premiums than it does good by reducing risk.

FIGURE 2

Utility versus Upper Limit with Loading



In the loaded case, the optimum upper limit is less than the threshold. To see that, consider the slope of the objective, given in Equation (7) at points $b \geq \bar{q}$. The observations in the paragraphs on the case of fair pricing assure that

$$\frac{E[u' | t > b]}{Eu'} \leq 1. \quad (8)$$

It follows that $T_b(b; \lambda, \bar{v}, \bar{q}) < 0$ for $b \geq \bar{q}$. The slope of the objective is never positive in this domain, although it does approach zero from below as b becomes large. Therefore no local maximum can exist for $b > \bar{q}$. The interval $[0, \bar{q}]$ is closed and as a consequence the objective, which is a continuous function, must assume a maximum at some point of it. That global maximum is located somewhere to the left, at a value $b < \bar{q}$. Therefore optimum upper-limit insurance is less than full.

The contracts studied here are realistic and conceptually simple. Mathematically, they cause difficulties centering on the question of whether a second, local maximum might exist. The problem arises because the objective function is not concave in the choice variable. As illustrated in Figure 2, the objective function typically has three inflection points. Intuition and numerous calculations suggest that the optimum is unique under reasonable conditions, but we are unable to give a proof of that impression. Absent (the assurance of) unique maximizers, all proofs become much more complex. As a consequence, the article examines the two forms of externality risk separately. Separation is necessary because of the curvature conditions, and it is realistic in many situations.

EQUITY RISK

Consider a neighborhood of single-story homes for which the highest-valued conversion is to two-story homes. Either type of home becomes more valuable as its neighborhood is increasingly made up of two-story homes and construction costs are constant. Suppose in addition that the increase in value as the neighborhood improves is exactly the same for both types of homes. Or, put more formally, the difference between the value of an existing, one-story home and that of a new two-story home,

$v - v^*$, is invariant as both values vary randomly. Then the conversion threshold $q = v - (v^* - c)$ is constant, meaning there is no conversion risk.

Consider the consumer who purchased full insurance when $q = \bar{q}$ and $v = \bar{v}$ were deterministic. Now in addition to the insurable risk of physical damage, she faces an uninsurable risk in the random variable v , the equity in her home. Her uninsured wealth is

$$v - \min[\bar{q}, t]. \quad (9)$$

The substitution of the random v for the fixed $\bar{v}$ is very much like the addition of a second, uninsurable property. Wealth is the sum of two random components, of which one is insurable and the other is an uninsurable background risk.

Such situations are studied by Mayers and Smith (1983), Doherty and Schlesinger (1983), Kimball (1990), and Kimball and Eeckhoudt (1992). The literature creates expectations for the present situation. Doherty and Schlesinger find that at loaded prices, a background risk that is independent of the insurable risk boosts demand for insurance. Kimball and Eeckhoudt extend the finding to a more general setting and show that the optimum deductible is also lowered by an independent background risk.

The task here is to show that similar results hold in the upper-limit contract, given independence of the equity risk. To that end the following notation is employed. The probability density function of value v is $g(v)$, which is independent of t . The consumer has utility function u . The fair premium is $P(b)$, as specified in (2) and it is loaded by a factor $(1 + \lambda)$. State-contingent wealth of the consumer after choosing insurance is

$$v - \min[\bar{q}, t] + \min[b, t] - (1 + \lambda)P(b). \quad (10)$$

In the case of fair prices, the consumer insures fully. That is, she selects an upper limit equal to the threshold, which is intuitive and easy to prove. The interesting case involves loading.

Because of the loading, the optimum upper limit is less than the threshold, which was demonstrated in the previous section for the case of deterministic equity. The following derivations show that the variability in equity raises the amount of insurance demanded.

The objective of the consumer is to choose b that maximizes

$$\int_{t=0}^{\infty} \left[\int_{v=v_0}^{v_1} u(v - \min[\bar{q}, t] + \min[b, t] - (1 + \lambda)P(b))g(v)dv \right] h(t)dt. \quad (11)$$

The key to the analysis is a surrogate consumer who behaves in the absence of equity risk just as the original consumer would in its presence. The surrogate has utility

$$\hat{U}(w) = \int_{v=v_0}^{v_1} u(w + (v - \bar{v}))g(v)dv. \quad (12)$$

It is clear that $\hat{U}(w)$ is increasing and concave in w . Substitute the expression

$$w = \bar{v} - \min[\bar{q}, t] + \min[b, t] - (1 + \lambda)P(b)$$

into Equation (12). The result is

$$\begin{aligned} \hat{U}(\bar{v} - \min[\bar{q}, t] + \min[b, t] - (1 + \lambda)P(b)) &= \int_{v=v_0}^{v_1} u(v - \min[\bar{q}, t] \\ &\quad + \min[b, t] - (1 + \lambda)P(b))g(v) dv. \end{aligned} \quad (13)$$

The right-hand side is the same as the inner term in the objective function in Equation (11). Therefore Equation (11) can be rewritten as

$$\hat{T}(b; \lambda, \bar{v}, \bar{q}) = \int_0^\infty \hat{U}(\bar{v} - \min[\bar{q}, t] + \min[b, t] - (1 + \lambda)P(b))h(t) dt. \quad (14)$$

In other words, when fixed equity is replaced by risky equity, the effect is the same as if the consumer were replaced by another, slightly different consumer, who still solves a fixed-equity problem.

The question is whether the surrogate consumer chooses more insurance. It is answered through the use of a lemma by Kimball showing the consumer represented by $\hat{U}(w)$ is, under reasonable conditions, more risk averse than the one who had $u(w)$.

In posing the lemma, risk premium $\pi(v, w)$ and precautionary premium $\psi(v, w)$ are needed. They are defined by

$$u(w - \pi(v, w)) = E[u(w + (v - \bar{v}))] \quad (15)$$

and

$$u'(w - \psi(v, w)) = E[u'(w + (v - \bar{v}))]. \quad (16)$$

The assumption of decreasing absolute risk aversion is equivalent to $\partial\pi(v, w)/\partial w < 0$, and the assumption of decreasing absolute prudence is equivalent to $\partial\psi(v, w)/\partial w < 0$. See Kimball (1990) and Kimball and Eeckhoudt (1992) for discussion of the equivalence.

Lemma 1 (Kimball): *Suppose the utility function u exhibits decreasing absolute risk aversion and decreasing absolute prudence. Then $\hat{U}$ is absolutely more risk averse than u at all points w .*

The proof given in the Appendix is essentially the same as Kimball's and is repeated purely for reasons of continuity.

The lemma is used to prove that the introduction of equity risk raises the optimum upper limit. The logical sequence is clear: Introducing equity risk has the same effect as putting in a consumer who is more risk averse, and a consumer who is more

risk averse demands a higher upper limit. The formal proposition applies when the consumer buys insurance even in the absence of the risk. It can also happen that the equity risk induces the consumer to purchase insurance when she otherwise would go without.

Because the objective lacks global concavity, the proposition is limited to a mathematical neighborhood in which loading is not too large and the optimum upper limit is not too far from $\bar{q}$. The limitation is unimportant as long as the objective function has a unique local maximum, as does the one in Figure 2 and in all numerical examples we have tried. The presence of more local maxima would create problems of a well-understood type. The neighborhood of applicability of the proposition might be small if loading is large, if the consumer is risk-tolerant, or if the conversion threshold is near zero. In such cases the consumer tends to stay out of the market. The proposition is stated here somewhat informally. The Appendix gives a precise statement and proof.

Proposition 1: *Suppose the risk-averse utility function u exhibits decreasing absolute risk aversion and decreasing absolute prudence. Then the introduction of an independent equity risk raises the demand for insurance in a neighborhood of the fair-priced, full-insurance optimum.*

CONVERSION RISK

For purposes of this section, regard the conversion threshold as random and treat the value of the existing property as fixed. Such situations arise under plausible, interesting conditions. In one scenario, the consumer who has both types of risk has hedged the equity risk in some manner, perhaps in a market like the one proposed by Case, Shiller, and Weiss.³ That done, she confronts the conversion risk.

Equity risk may be absent for other reasons. Think of the cement plants located near the waterfront in Santa Barbara. No single factory owner can gain by building housing among the other factories, but if an earthquake devastates them all, the whole area will be rebuilt as residences. The value of a cement factory is the same as before, but the value of converting to housing is higher. The general description of this situation is that the neighborhood affects the value of the highest-valued conversion but not the value of the incumbent one.

Cost is another source of conversion risk. The conversion threshold depends on the cost of building the alternative structure, which varies for numerous reasons. Variations in the cost of conversion affect the threshold while the value of the incumbent structure remains fixed.

Noncontractibility of Conversion Thresholds

There are two objections a person might make to studying conversion risk. The first is that conversion thresholds are not explicitly treated in contracts for insurance of real property and are therefore unimportant. The other is that conversion thresholds are implicit in the principle of indemnity and are therefore insured.

The response to both objections is that conversion thresholds are not contractible. Why is the threshold not contractible? First consider damage, the variable that is

³ See Shiller and Weiss (1999) and Case, Shiller, and Weiss (1993).

contractible. Damage is the cost to restore property to its predamage condition. It is contractible because it is cheaply and accurately measurable by methods that stay the same from one instance to another. Uniformity is important because insurance is a contract of adhesion—a standardized contract that the client accepts after filling in a few blanks. The contracts are uniform. In contrast, determination of the conversion threshold differs from one property to the next because the best conversion differs. The threshold is not a fit subject for a contract of adhesion.

Insurance texts recognize the concept of conversion threshold under the headings of actual cash value and the principle of indemnity. They do not always recognize that it might not be operational. The problem is that an impartial court cannot, at reasonable expense, determine the dollar value of the threshold. In principle the threshold is total value minus land value, but even real-estate assessors cannot supply good evidence on land value because markets for urban land are thin. The best conversion and hence the conversion threshold are guessed at by professional real estate investors, but their estimates are not the type of evidence that is cheaply decisive in court. Thus lack of standardization and lack of suitable evidence explain why the conversion threshold is not contractible in insurance of real property.

In some lines of insurance the contract takes good account of the conversion threshold. Automobile collision insurance is an example. There the client does not subscribe to a fixed upper limit but agrees, in effect, to be limited by the future blue-book value. The blue-book value is the threshold of conversion. When a \$5,000 car has damage exceeding \$5,000, it is not repaired but is junked instead, and the indemnity is \$5,000. The blue-book value is based on numerous transactions and is widely published. It is credible and cheaply accessible by a court. It is therefore a contractible quantity. No comparable resource exists for real property, and those thresholds are therefore not contractible. Instead, the contracts are written for a fixed upper limit.

Bourgeon and Picard (2001) describe a situation in which the threshold is partially reflected in the contract. Under their contracts the conversion threshold is private information of the client that is signaled through the decision either to restore or convert. The indemnity is less in the case of conversion because when conversion is more attractive, the loss of wealth from the damage is less. The bivalent signal approximates roughly the continuously distributed threshold, and as Bourgeon and Picard stress, the resulting contracts are far from imitating contracts written in the threshold itself.

Optimum Upper Limits with Conversion Risk

Insurance contracts that pay different levels of indemnity for different values of the threshold are uncommon. The usual case for real property is a single upper limit that applies regardless of where the threshold stands when damage occurs. In response to the conversion risk, the consumer must somehow choose an upper limit. To examine the random conversion threshold, assume that the variation of q is supported on a finite interval $[q_0, q_1]$. The variation in v is nil and thus $v = \bar{v}$. The probability density function of insurable loss t is $h(t)$, and the probability density function of conversion risk is denoted $g(q | t)$. Other notation is as before.

For any fixed threshold, there is an optimum upper limit. It is natural to suppose that with threshold risk the optimum upper limit would be a compromise among the optima for all of the values of the threshold. Surprisingly, the supposition can fail under some types of joint distribution of damage and threshold. Positive correlation between the two distributions can lead to exceptions, as shown by a counterexample below. Independence between damage and threshold is sufficient to assure the expected relation. Assume therefore that the distributions are independent, that is, that $g(q | t) = g(q)$.

The consumer's wealth is

$$\bar{v} - \min[t, q] + \min[b, t] - (1 + \lambda)P(b)$$

and the goal of the consumer is to choose b to maximize the objective

$$\int_{q=q_0}^{q_1} \left[\int_{t=0}^{\infty} u(\bar{v} - \min[t, q] + \min[b, t] - (1 + \lambda)P(b))h(t) dt \right] g(q) dq. \quad (17)$$

The term in square brackets is the same as the $T(b; \lambda, q, \bar{v})$ defined previously. For each threshold q , there is an optimum upper limit $b(\lambda, q)$. The desired result is that the global optimizer of the expression above, denoted b^* should satisfy

$$b(\lambda, q_0) \leq b^* \leq b(\lambda, q_1). \quad (18)$$

The result would be easy to demonstrate if the objective functions possessed global concavity. Since concavity is lacking, the proof involves checking the slope of the objective throughout the domain of b .

Proposition 2: *Let u be risk averse and $\bar{v}$ deterministic. Assume that q is distributed independently of t . Let the probability density function $g(q)$ of q have support contained in $[q_0, q_1]$ and mean contained in (q_0, q_1) . Let $b(\lambda, q)$ be the optimum upper limit when the loading is λ and the conversion threshold is q . Then the optimum upper limit on insurance, b^* , satisfies $b(\lambda, q_0) \leq b^* \leq b(\lambda, q_1)$.*

Proof is given in the Appendix.

The following example shows that the proposition can fail if the distributions of damage and conversion risk are dependent. Price of insurance is fair, i.e., $\lambda = 0$. Utility is $u(\cdot) = \ln(\cdot)$. Let $\bar{v} = 4$. The distribution of damage is uniform on the interval $[0, 4]$. Then $P(b) = b(1 - \frac{1}{8}b)$. The conversion threshold q takes on one of two values, $q_0 = 0$ and $q_1 = 2$, with

$$\text{Prob}(q = q_1) = \begin{cases} 0 & \text{if } t < 2 \\ 1 & \text{if } t \geq 2. \end{cases}$$

Optimum upper limits for the two values are $b(0, q_0) = q_0 = 0$ and $b(0, q_1) = q_1 = 2$. However, the solution for the case of conversion risk is, from the Appendix, $b^* = 2.7256$ which is well outside the interval $[b(0, q_0), b(0, q_1)] = [0, 2]$.

The example is intuitive. An upper limit of at least $q_1 = 2$ is needed to insure the risk of high damage and high threshold. The limit produces windfall gains when damage is at lower levels. The marginal utility of wealth is less in the windfall states than in high-damage states, prompting a demand for further insurance. Thus positive dependence between damage and the conversion threshold can produce an optimum upper limit lying outside the support of the conversion risk.

Underinsurance in Disasters

Proposition 2 has practical significance. Consider the investigations often made after a disaster for the purpose of determining whether the victims were properly insured. The usual finding is that most of them were not, and the present model can partly explain that finding.

Disaster normally encourages conversion of the neighborhood, often making it more desirable, and that means that the conversion threshold is lower in a disaster than otherwise. Suppose the conversion threshold q takes on only the two extreme values q_0 and q_1 . The low value of q applies to disaster losses and the high value corresponds to nondisaster losses. At zero loading, q_0 and q_1 are the optimum upper limits. By Proposition 2, the optimum upper limit lies above the disaster value and below the nondisaster value.

When the disaster occurs, the investigator observes the compromise upper limit. He employs the conventional basis for judging the adequacy of insurance, which is the nondisaster state in which the conversion threshold takes the high value. His conclusion is that the consumer is underinsured. The conclusion is almost unavoidable because all available market data are from before the disaster, and data on disaster-induced values of land and improvements are available only from transactions made in the future. Thus the optimally-insured consumer is judged to have inadequate insurance when, in fact, she is overinsured for the disaster state. Of course, she really is underinsured for nondisaster losses.

Does Demand Increase?

An independent, mean-zero conversion risk can either raise or lower demand for insurance, depending upon the probabilistic distribution of damage. The finding is surprising because in the literature an independent, mean-zero background risk must raise demand for insurance of the insurable risk. The contrary facts about conversion risk are illustrated through two examples.

The first example confirms the possibility of increased demand. Damage is distributed as a negative exponential $h(t) = e^{-t}$. Utility is the time-honored $u(\cdot) = \ln(\cdot)$, and $\bar{v} = 4$. When the conversion point is deterministic at $\bar{q} = 1$, the optimum upper limit is $b = \bar{q} = 1$. Compare that to the situation in which the conversion threshold is $q = 0$ or $q = 2$ with equal probability, a distribution having a mean of 1 as in the initial situation. The goal of the consumer in the second case is to choose b to maximize her objective described in Equation (17). While the upper limit is unity in the case of no

conversion risk, it rises to $b = 1.401$ when conversion risk is added. That demonstrates the possibility that an independent mean-zero conversion risk can raise the demand for insurance.

Alternatively, an example of decreased demand emerges from the same situation when the distribution of damage is $h(t) = 4e^{-4t}$. The change has the effect of shifting probability weight from higher to lower levels of damage. As before, the optimum upper limit for fixed conversions value is $b = \bar{q} = 1$. In the presence of conversion risk the optimum upper limit is $b = 0.515$, a decrease in demand. Thus the independent conversion risk can either raise or lower demand for the underlying insurance.

Intuitively, the result depends upon variation in the probability that damage reaches the upper conversion threshold. In the second case the probability of damage reaching the higher conversion threshold is much lower—0.00134 versus 0.13533 in the first case. The consumer places less weight on the upper conversion threshold as the probability of reaching it becomes more remote.

The usual background risk problem adds risk in a way that leaves expected wealth constant, but here expected wealth is not constant. The addition of a mean-zero conversion risk raises expected wealth in both cases. Since the classic theory of background risk does not fit conversion risk, a difference in conclusions is expected.

CONCLUDING REMARKS

The result in Proposition 1 assures that an exogenous increase in equity risk increases demand for insurance. It should be interpreted with caution. The reason is that consumers might actively choose the level of equity risk instead of accepting it as an unavoidable nuisance. A property owner can choose less equity risk by owning less property, building less expensive structures, moving to a different location, engaging in another type of land use, or participating in markets that hedge the equity risk in the chosen structure, location, and use. The choice of equity risk is part of a larger portfolio decision. From the proposition above, it is apparent that the portfolio decision interacts in some way with the insurance decision. In fact, Mayers and Smith (1983) first developed conditions for independence between the decisions. Those conditions include lack of covariance between the insurable risk and portfolio risks—as is natural in the context of the mean-variance utility model they used. In case of disaster it is typical that equity covaries positively with damage, meaning that equity risk is, to some extent, a natural insurance. Thus the decision to accept equity risk and the decision to buy insurance are not wholly separable.

Equity risk is seen continually in the fluctuations of real estate markets, but conversion risk is hidden. In theory conversion risk is observable as a fluctuation in the difference between property value and land value, but in practice markets in vacant land are thin and prices in them are not publicized. Moreover, conversion issues are not part of the informal insurance education that consumers absorb.

Conversion is an issue when losses are large, and large losses account for a substantial fraction of indemnities paid in most lines of property insurance. It would seem to follow that conversion is central to insurance decisions, but property insurance contracts do not address it. Because conversion is not treated in the contract, issues that should be part of the agreement are left to be negotiated after the event. Certainly every

major loss will end with some negotiation between the insurer and the client. Such post-contractual negotiation is at best costly and at worst an invitation to abuse. Each party has a motive to distort, leading to a conflict of the sort that contractual relations are supposed to prevent. How contracts might be changed to address conversion value is unclear and worthy of further study and innovation.⁴

Conversions are typically more marked in disaster losses. Thus disasters are an extreme case in which the contracts are inappropriate, and it is expected that post-event negotiation will be common. The Oakland fire is a highly publicized example. In that case the outcome favored the clients. Although insurers could have argued that losses were less than they seemed, they did not do so, perhaps because the absence of any mention of conversion in the underlying contracts gave them no grounds.

Insurers sometimes offer a homeowner's policy that pays "replacement cost." The idea is that the cost to restore the property might turn out to be higher than the upper limit cited in the contract, and the insurer promises in that case to pay the higher cost. Such policies have limited value to the client when conversion occurs, since the replacement cost becomes a debatable quantity.

Recently homeowner's contracts offer a rider to cover the extra cost of upgrades that are required to meet current building codes, a cost that is reported to be substantial. The rider is an explicit recognition that conversion of at least one type is typical. At the same time, the rider is retrograde in assuming that the conversion will consist solely of code upgrades needed to replace the existing structure. Nevertheless, it illustrates the possibility of gains for consumers and insurers that are available from explicitly recognizing conversion.

APPENDIX

The following lemmas are needed.

Lemma A1: Let $E[u' | t \leq b]$ and $E[u' | t > b]$ be the conditional expectations of marginal utility of wealth in the intervals $[0, b]$ and $[b, \infty]$, respectively. Let Eu' be the unconditional expectation of marginal utility. Let $T(b; \lambda)$ be expected utility and assume that $\int_b^\infty h(t) dt < 1/(1 + \lambda)$. Then

$$T_b(b; \lambda) = P'(b)Eu' \left(\frac{E[u' | t > b]}{Eu'} - (1 + \lambda) \right) \quad (A1)$$

and

$$T_b(b; \lambda) = P'(b)(1 - (1 + \lambda)P'(b)) \cdot \left[E[u' | t > b] - \left(1 + \frac{\lambda}{1 - (1 + \lambda)P'(b)} \right) E[u' | t \leq b] \right]. \quad (A2)$$

⁴ For more on the bargaining game that arises between the insurer and the insured following damage to property see Marshall (2001).

Before the proof, some notation is needed. The unconditional probability density of damage is $h(t)$. Given t , the conditional probability density of v and q is $g(v, q | t)$, the support of which is contained in $[v_0, v_1] \times [q_0, q_1]$. Because damage is the only random variable in the insurance contract, the premium function is independent of q and v and is still given by Equation (2). The joint density of v, q , and t is $g(v, q | t)h(t)$. If there is loading, it is represented by the parameter $\lambda \geq 0$ and the whole premium is $(1 + \lambda)P(b)$. Insured wealth is then

$$v - \min[q, t] + \min[b, t] - (1 + \lambda)P(b). \quad (\text{A3})$$

Note that

$$\int_b^\infty h(s) ds = P'(b). \quad (\text{A4})$$

The goal of the consumer is to choose b to maximize the objective

$$\begin{aligned} T(b; \lambda) = \int_{t=0}^\infty \left[\int_{v=v_0}^{v_1} \int_{q=q_0}^{q_1} u(v - \min[t, q] + \min[b, t] \right. \\ \left. - (1 + \lambda)P(b))g(v, q | t) dv dq \right] h(t) dt. \end{aligned} \quad (\text{A5})$$

Expected marginal utility is

$$\begin{aligned} Eu' = \int_{t=0}^\infty \left[\int_{v=v_0}^{v_1} \int_{q=q_0}^{q_1} u'(v - \min[t, q] + \min[b, t] \right. \\ \left. - (1 + \lambda)P(b))g(v, q | t) dv dq \right] h(t) dt. \end{aligned} \quad (\text{A6})$$

The conditional expected marginal utilities are

$$\begin{aligned} E[u' | t \leq b] = \int_{t=0}^b \left[\int_{v=v_0}^{v_1} \int_{q=q_0}^{q_1} u'(v - \min[t, q] + \min[b, t] \right. \\ \left. - (1 + \lambda)P(b))g(v, q | t) dv dq \right] \frac{h(t)}{1 - P'(b)} dt \end{aligned} \quad (\text{A7})$$

and

$$\begin{aligned} E[u' | t > b] = \int_{t=b}^\infty \left[\int_{v=v_0}^{v_1} \int_{q=q_0}^{q_1} u'(v - \min[t, q] + \min[b, t] \right. \\ \left. - (1 + \lambda)P(b))g(v, q | t) dv dq \right] \frac{h(t)}{P'(b)} dt. \end{aligned} \quad (\text{A8})$$

Proof: The derivative is

$$\begin{aligned}
 T_b(b; \lambda) = & \int_{t=b}^{\infty} \left[\int_{v=v_0}^{v_1} \int_{q=q_0}^{q_1} u'(v - \min[t, q] + \min[b, t]) \right. \\
 & \left. - (1 + \lambda)P(b))g(q | t) dv dq \right] h(t) dt - (1 + \lambda)P'(b) \\
 & \times \int_{t=0}^{\infty} \left[\int_{v=v_0}^{v_1} \int_{q=q_0}^{q_1} u'(v - \min[t, q] + \min[b, t]) \right. \\
 & \left. - (1 + \lambda)P(b))g(q | t) dv dq \right] h(t) dt.
 \end{aligned} \tag{A9}$$

The result is that

$$T_b(b; \lambda) = P'(b)[E[u' | t > b] - (1 + \lambda)P'(b)Eu'] \tag{A10}$$

leading to Equation (A1). Now substitute for Eu' using

$$Eu' = P'(b)E[u' | t > b] + (1 - P'(b))E[u' | t \leq b]. \tag{A11}$$

After several rearrangements, the result is Equation (A2).

Q.E.D.

Corollary A1: Assume that q is fixed at $\bar{q}$ and that $b > \bar{q}$. Then $T_b(b; \lambda, \bar{q}, v) < 0$.

Proof: Referring to Equation (A1), notice that for $b > \bar{q}$, $\frac{E[u' | t > b]}{Eu'}$ is less than unity, because insured wealth is greatest on (b, ∞) . The result is immediate. Q.E.D.

Lemma 1 (Kimball): Suppose the utility function u exhibits decreasing absolute risk aversion and decreasing absolute prudence. Then $\hat{U}$ is absolutely more risk averse than u at all points w .

Proof: (From Kimball and Eeckhoudt (1992), who credit Christian Gollier for improvements in the proof.) The risk premium is positive because the random variable $v - \bar{v}$ has mean zero and utility is risk averse. The precautionary premium is also positive because given decreasing absolute risk aversion, it is greater than the risk premium (Kimball and Eeckhoudt, 1992). Write

$$\hat{U}'(w) = E[u'(w + (v - \bar{v}))] = u'(w - \psi(v, w)). \tag{A12}$$

It follows that

$$\hat{U}''(w) = \left(1 - \frac{\partial \psi(v, w)}{\partial w}\right) u''(w - \psi(v, w)), \tag{A13}$$

and hence,

$$-\frac{\hat{U}''(w)}{\hat{U}'(w)} = \left(1 - \frac{\partial \psi(v, w)}{\partial w}\right) \left[-\frac{u''(w - \psi(v, w))}{u'(w - \psi(v, w))}\right]. \quad (\text{A14})$$

The term in round parentheses is greater than unity because absolute prudence is decreasing. Absolute risk aversion falls when wealth rises by the amount $\psi(v, w)$. Hence, for all w

$$-\frac{\hat{U}''(w)}{\hat{U}'(w)} \geq -\frac{u''(w)}{u'(w)}. \quad (\text{A15})$$

Q.E.D.

Proposition 1 (restated): Suppose the risk-averse utility function u exhibits decreasing absolute risk aversion and decreasing absolute prudence. The conversion point $\bar{q}$ is deterministic. The premium is loaded by a factor $(1 + \lambda)$. Equity risk v is independent of the insurable risk t . Let the optimum upper limit under equity risk be $\hat{b}(\lambda)$. When the equity risk is replaced by a certainty value $\bar{v} = Ev$, the optimum upper limit is $b(\lambda)$. Then there is a neighborhood in b, λ -space containing the point $(b, \lambda) = (\bar{q}, 0)$ such that for λ, b in that neighborhood, the functions $b(\lambda)$ and $\hat{b}(\lambda)$ exist, and $b(\lambda) < \hat{b}(\lambda) < \bar{q}$.

Proof: Lemma 1 assures that adding equity risk is equivalent to considering a surrogate consumer who is more risk averse than the original one. For a fixed value of equity $\bar{v}$, the surrogate chooses as the original consumer would in the presence of a risky equity.

The objective function for the fixed equity case is that of Equation (5). It is shown in the body of the article that with $\lambda = 0$ the globally optimum b is $\bar{q}$. Straightforward calculation shows that

$$T_{bb}(\bar{q}; 0, \bar{v}, \bar{q}) < 0. \quad (\text{A16})$$

By continuity, there is a largest neighborhood N_1 in λ, b -space of the point $(0, \bar{q})$ in which

$$T_{bb}(b; \lambda, \bar{v}, \bar{q}) < 0. \quad (\text{A17})$$

Because the solution $b(0)$ to the $\lambda = 0$ case is unique, there is a λ -neighborhood N_2 of the point $\lambda = 0$ in which the solution $b(\lambda)$ to

$$T_b(b; \lambda, \bar{v}, \bar{q}) = 0 \quad (\text{A18})$$

is unique and is the global maximizer of T . Define the λ -neighborhood $N = N_2 \cap \{\lambda \mid \exists b \text{ such that } (\lambda, b) \in N_1\}$. The solution function $b(\lambda)$ is well-defined in N . In N

$$b'(\lambda) = -T_{\lambda b}(b; \lambda, \bar{v}, \bar{q}) / T_{bb}(b; \lambda, \bar{v}, \bar{q}) < 0 \quad (\text{A19})$$

implying $b(\lambda) < \bar{q}$ for $\lambda > 0$.

For the case of equity risk, $\hat{U}$ replaces u and the objective becomes

$$\hat{T}(b; \lambda, \bar{v}, \bar{q}) = \int_{t=0}^{\infty} [\hat{U}(\bar{v} - \min[\bar{q}, t] + \min[b, t] - (1 + \lambda)P(b))]h(t) dt. \quad (\text{A20})$$

Again, there exists a neighborhood $\hat{N}$ of $\lambda = 0$ in which for each λ , $\hat{b}(\lambda)$ is a unique global maximizer over b of the objective function $\hat{T}(b; \lambda, \bar{v}, \bar{q})$ found in Equation (A20). Both $\hat{b}(\lambda)$ and $b(\lambda)$ are defined and optimum in the neighborhood $\tilde{N} = N \cap \hat{N}$, and in that neighborhood they can be compared.

The first-order condition for the optimum upper limit for the original consumer is

$$E[u'(\cdot; \bar{v}, \bar{q}) | t > b(\lambda)] - \left(1 + \frac{\lambda}{1 - (1 + \lambda)P'(b(\lambda))}\right) E[u'(\cdot; \bar{v}, \bar{q}) | t \leq b(\lambda)] = 0. \quad (\text{A21})$$

The task is to show that the derivative of $\hat{T}(b; \lambda, \bar{v}, \bar{q})$ evaluated at the same $b(\lambda)$ is still positive. That is, show that

$$E[\hat{U}'(\cdot; \bar{v}, \bar{q}) | t > b(\lambda)] - \left(1 + \frac{\lambda}{1 - (1 + \lambda)P'(b(\lambda))}\right) E[\hat{U}'(\cdot; \bar{v}, \bar{q}) | t \leq b(\lambda)] > 0. \quad (\text{A22})$$

The first step is to make u and $\hat{U}$ agree in value and in slope at the point where $t = b(\lambda)$. At that point, wealth is $\bar{v} - \min[\bar{q}, t] + \min[b(\lambda), t] - (1 + \lambda)P(b(\lambda)) = \bar{v} - (1 + \lambda)P(b(\lambda))$. Without loss of generality, $\hat{U}$ may be multiplied by an appropriate positive number so that

$$\hat{U}'[\bar{v} - (1 + \lambda)P(b(\lambda))] = u'[\bar{v} - (1 + \lambda)P(b(\lambda))] \quad (\text{A23})$$

and an appropriate constant can also be added so that

$$\hat{U}[\bar{v} - (1 + \lambda)P(b(\lambda))] = u[\bar{v} - (1 + \lambda)P(b(\lambda))] \quad (\text{A24})$$

holds true. Now the more risk averse function $\hat{U}$ lies everywhere below the less risk averse u , with a tangency at the point $\bar{v} - (1 + \lambda)P(b(\lambda))$.

Looking at the tangency point and using the fact that $\hat{U}$ is absolutely more risk averse than u , it is graphically apparent and easy to prove that for any $w < \bar{v} - (1 + \lambda)P(b(\lambda))$ —at a point below the tangency— $\hat{U}'(w) > u'(w)$. For all t satisfying $\bar{q} > t > b(\lambda)$, damage is more than the upper limit of insurance and consequently

$$\bar{v} - b(\lambda) - (1 + \lambda)P(b(\lambda)) < \bar{v} - (1 + \lambda)P(b(\lambda)). \quad (\text{A25})$$

It follows that the first terms in Equations (A21) and (A22) satisfy

$$E[\hat{U}'(\cdot; \bar{v}, \bar{q}) | t > b(\lambda)] > E[u'(\cdot; \bar{v}, \bar{q}) | t > b(\lambda)]. \quad (\text{A26})$$

That takes care of $t \in (b(\lambda), \infty)$.

Look at $t \in [0, b(\lambda)]$. When $t \leq b(\lambda)$, damage is less than the conversion point and also less than the upper limit. Thus wealth reduces to $\bar{v} - (1 + \lambda)P(b(\lambda))$, the wealth at the tangency point. Then the second terms in Equations (A21) and (A22) bear the relation

$$E[\hat{U}'(\cdot; \bar{v}, \bar{q}) | t \leq b(\lambda)] = E[u'(\cdot; \bar{v}, \bar{q}) | t \leq b(\lambda)]. \quad (\text{A27})$$

Use the results from Equations (A26) and (A27) to compare Equations (A22) and (A21). Clearly the expression in Equation (A22) is positive. By Lemma A1 the objective is increasing, and hence the optimum upper limit is greater for the consumer who is more risk averse. This holds for each value of λ in $\tilde{N}$. Therefore, $b(\lambda) < \bar{b}(\lambda) < \bar{q}$. Q.E.D.

Proposition 2: Let u be risk averse and $\bar{v}$ deterministic. Assume that q is distributed independently of t . Let the probability density function $g(q)$ of q have support contained in $[q_0, q_1]$ and mean contained in (q_0, q_1) . Let $b(\lambda, q)$ be the optimum upper limit when the loading is λ and the conversion threshold is q . Then the optimum upper limit on insurance, b^* , satisfies $b(\lambda, q_0) \leq b^* \leq b(\lambda, q_1)$.

The objective here is $\int_{q_0}^{q_1} T(b; \lambda, q, \bar{v})g(q) dq$. The proof needs a lemma:

Lemma A2: $q_1 > q_0$ implies for $b \in (0, q_1]$, $T_b(b; \lambda, q_1, \bar{v}) > T_b(b; \lambda, q_0, \bar{v})$.

Proof: The proof applies Lemma A1. First take any $b \in (0, q_0)$. In this domain

$$E[u' | t < b; q = q_0] = E[u' | t < b; q = q_1]. \quad (\text{A28})$$

Because the threshold is higher when $q = q_1$, more wealth can be lost in that case. Therefore,

$$E[u' | t > b; q = q_0] < E[u' | t > b; q = q_1]. \quad (\text{A29})$$

The result follows from Lemma A1 in the form of Equation (A2).

Next consider $b \in (q_0, q_1]$. In this domain

$$E[u' | t < b; q = q_0] < E[u' | t < b; q = q_1]. \quad (\text{A30})$$

Without loss of generality, substitute in the $q = q_1$ case a utility function $\tilde{u}(w) = \gamma u(w)$, where γ is chosen to satisfy

$$E[u' | t < b; q = q_0] = E[\tilde{u}' | t < b; q = q_1]. \quad (\text{A31})$$

For $t > b$, it is still true that

$$E[u' | t > b; q = q_0] < E[\tilde{u}' | t > b; q = q_1] \quad (\text{A32})$$

as is apparent diagrammatically. Therefore, again from Lemma A1, the result follows. Q.E.D.

Now the proof of Proposition 2 is as follows:

Proof: It is sufficient to prove the result for the case in which

$$\begin{aligned}\Pr\{q = q_0\} &= 0.5 \\ \Pr\{q = q_1\} &= 0.5.\end{aligned}\tag{A33}$$

Obvious extensions go from this case to the full proposition. The objective here is

$$0.5T(b; \lambda, q_0, \bar{v}) + 0.5T(b; \lambda, q_1, \bar{v}).\tag{A34}$$

First consider any $b < b(\lambda, q_0) < q_0$. From Lemma A2

$$T(b(\lambda, q_0); \lambda, q_1, \bar{v}) - T(b; \lambda, q_1, \bar{v}) > T(b(\lambda, q_0); \lambda, q_0, \bar{v}) - T(b; \lambda, q_0, \bar{v}).\tag{A35}$$

Because $b(\lambda, q_0)$ is the global maximizer, the right-hand side of the expression is positive. It follows that the left-hand side is positive and that

$$0.5T(b(\lambda, q_0); \lambda, q_1, \bar{v}) + 0.5T(b(\lambda, q_0); \lambda, q_0, \bar{v}) > 0.5T(b; \lambda, q_0, \bar{v}) + 0.5T(b; \lambda, q_1, \bar{v}).\tag{A36}$$

Thus $b < b(\lambda, q_0)$ is never a global optimum.

Now consider $b > b(\lambda, q_1)$. Several cases can occur. In case $b(\lambda, q_1) > q_0$, it is clear from Corollary A1 of Lemma A1 that $T(b; \lambda, q_0, \bar{v})$ is decreasing for all $b > q_0$ and thus for all $b > b(\lambda, q_1)$. On the other hand, $b(\lambda, q_1)$ is the global maximizer of $T(b; \lambda, q_1, \bar{v})$. Therefore,

$$0.5T(b(\lambda, q_1); \lambda, q_1, \bar{v}) + 0.5T(b(\lambda, q_1); \lambda, q_0, \bar{v}) > 0.5T(b; \lambda, q_0, \bar{v}) + 0.5T(b; \lambda, q_1, \bar{v}).\tag{A37}$$

In case $b(\lambda, q_1) < q_0$, b must be examined in two subdomains. If $b(\lambda, q_1) < b \leq q_1$, by Lemma A2, $T_b(b; \lambda, q_1, \bar{v}) > T_b(b; \lambda, q_0, \bar{v})$. Therefore,

$$T(b; \lambda, q_1, \bar{v}) - T(b(\lambda, q_1); \lambda, q_1, \bar{v}) > T(b; \lambda, q_0, \bar{v}) - T(b(\lambda, q_1); \lambda, q_0, \bar{v})\tag{A38}$$

which is equivalent to

$$T(b(\lambda, q_1); \lambda, q_0, \bar{v}) - T(b; \lambda, q_0, \bar{v}) > T(b(\lambda, q_1); \lambda, q_1, \bar{v}) - T(b; \lambda, q_1, \bar{v}).\tag{A39}$$

The right-hand side is positive because $b(\lambda, q_1)$ is the global maximizer and consequently so is the left-hand side. Therefore (A37) holds for the domain $b(\lambda, q_1) < b \leq q_1$.

In the domain of b in which $b > q_1$, both $T(b; \lambda, q_1, \bar{v})$ and $T(b; \lambda, q_0, \bar{v})$ are decreasing, from Corollary A1 after Lemma A1. Therefore,

$$0.5T(q_1; \lambda, q_1, \bar{v}) + 0.5T(q_1; \lambda, q_0, \bar{v}) > 0.5T(b; \lambda, q_0, \bar{v}) + 0.5T(b; \lambda, q_1, \bar{v}). \quad (\text{A40})$$

Therefore $b > q_1$ cannot be the global optimizer. The global optimizer belongs to $[b(\lambda, q_0), (b(\lambda, q_1))]$. Q.E.D.

Example of dependent conversion risk: The objective over the range $b \in [0, 2]$ is given by

$$\begin{aligned} & \int_0^b \ln \left[4 + t - b \left(1 - \frac{1}{8}b \right) \right] \frac{1}{4} dt + \int_b^2 \ln \left[4 + b - b \left(1 - \frac{1}{8}b \right) \right] \frac{1}{4} dt \\ & + \int_2^4 \ln \left[2 + b - b \left(1 - \frac{1}{8}b \right) \right] \frac{1}{4} dt. \end{aligned}$$

Over the range $b \in [2, 4]$ it is

$$\begin{aligned} & \int_0^2 \ln \left[4 + t - b \left(1 - \frac{1}{8}b \right) \right] \frac{1}{4} dt + \int_2^b \ln \left[2 + t - b \left(1 - \frac{1}{8}b \right) \right] \frac{1}{4} dt \\ & + \int_b^4 \ln \left[2 + b - b \left(1 - \frac{1}{8}b \right) \right] \frac{1}{4} dt. \end{aligned}$$

The first part of the objective is increasing over the entire range $b \in [0, 2]$ and reaches its maximum value at $b = 2$. The second part of the objective has a global maximum on the interval $b \in [2, 4]$. The maximum is a critical point of the function, which is found using Lemma A1 of the Appendix. In this case it is

$$\int_0^2 \frac{1}{4 + t - b(1 - \frac{1}{8}b)} \frac{1}{4} dt + \int_2^b \frac{1}{2 + t - b(1 - \frac{1}{8}b)} \frac{1}{4} dt = \int_b^4 \frac{1}{2 + b - b(1 - \frac{1}{8}b)} \frac{1}{4} dt.$$

The equation is solved at $b^* = 2.7256$, which is well outside the interval $[q_0, q_1] = [0, 2]$. A plot of both parts of the objective confirms that no other local maxima exist.

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