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Mandatory Pensions and the Intensity of Adverse Selection in Life Insurance Markets

Bertrand Villeneuve

Abstract

This article examines the impact of varying mandatory pensions on saving, life insurance, and annuity markets in an adverse selection economy. Under reasonable restrictions, we find unambiguous effects on market size, participation rates, and equilibrium prices. The degree of adverse selection, whether a market is active or inactive, and social welfare are analyzed.

Introduction

Annuities and life insurance are praised for the flexibility they offer consumers as compared with ordinary saving: estates can be complemented, or, conversely involuntary or excessive bequests can be avoided.[1] Life insurance is widespread and the market is generally considered to work competitively. The picture is quite different for annuities. Authors have argued that if demand and participation (percentage of buyers in the population) is low, this is because annuities are expensive and they hardly yield a better interest rate than simple saving (Friedman and Warshawsky, 1990). This pessimistic view of the rate of return of annuities has been challenged by, e.g., James and Vittas (1999), who find a higher "money's worth ratio" in an international setting than previously assumed. However, annuities markets are less successful than we might expect them to be.

To explain this fact, we need more sophisticated theories than "transaction costs." First, pensions (or any other collective system) offer a mandatory level of annuities so that the private market is at least partially crowded out. Second, altruism creates a demand for money when the policyholder dies, a case for which annuities are of course not appropriate. The third candidate is adverse selection in the annuities market. But simple explanations are insufficient. Pensions levels, as compared to salaries, are generally low (at most 70 percent in France in the private sector, which is still quite large compared to other developed countries) and crowding-out is not total unless the future is strongly discounted. Altruism does not rule out the incentive to buy annuities for a policyholder's own consumption, and his portfolio should

Bertrand Villeneuve works at the University of Toulouse. The author thanks Arnold Chassagnon, Pierre-André Chiappori, and Ralph Winter for stimulating discussions.

[1] Yaari (1965) first stressed this idea, and illustrated the impact of such insurance policies on consumption paths. Fischer (1973) also extensively studied this class of models.

remain diversified unless he chooses to prioritize his bequest, but the proportion of negligible bequests is substantial and many others seem to be involuntary. Adverse selection makes annuities expensive, though it still remains surprising that the price is sufficiently distorted for demand to be so low.

The aim of this article is to propose a complete and tractable model of the impact of pensions on saving, (life) annuities, and life insurance markets. We connect the above arguments together to clarify intuition, to simplify comparative statics, and to give hints for policy. The model is reasonably general in terms of consumers' heterogeneity and preference specification. To simplify, pensions are modeled as mandatory government provided payments. We propose a comparative statics exercise in which the effect on varying pensions is characterized. In particular, we define the *intensity of adverse selection*, i.e., the extent to which the average insurance purchaser differs from the general population. We offer conditions under which higher pension rates intensify adverse selection: clearly, pensions crowd out voluntary annuities; we provide conditions ensuring that annuities demands by higher risk consumers are *relatively* less reduced, which makes annuitants even less representative of the population. Overall, the model provides predictions of how markets adapt to pension changes caused by demographic or political factors.

We shall now discuss the existing literature.[2] Not all the cited papers were mainly interested in explaining the functioning of life insurance markets; however, all put forward arguments that were useful for our work. We wish to stress that there are positive questions left unanswered, and insist on the fact that policy recommendations and evaluations were missing.

Kotlikoff and Spivak (1981) showed that household members can gain considerable efficiency by mutualizing theirs risks (they inherit from one another in case of death). Latent demands for insurance are thus proved to be at least partly satisfied. Costly insurance markets are bypassed, but their inefficiency is a mere assumption of the model. Browning (2000) proposes a noncooperative game of saving in a two-person household in which consumption is a public good and in which members contribute to the second period consumption taking into account their survival probabilities and the other's contribution. The model predicts that one member of the household does not purchase annuities. The article does not consider either asymmetric information in the annuity market, but could serve as a basis for other developments.

Friedman and Warshawsky (1990) showed that, *given* the high price of annuities, a typical consumer discounting the future by his survival probability, and being reasonably altruistic, has no demand for annuities after a certain age: the consumer's (constant) pension *becomes* larger than his free demand for annuities.[3] The role of adverse selection in causing high prices is not modeled by Friedman and Warshawsky. Moreover, their model suggests that although short term annuities demand regress as consumers age, average young retirees should still buy annuities. This often proves not to be the case. It seems, however, that the latent demand for annuities (the willingness-to-pay

[2] For a survey on life insurance, see Villeneuve (2000).

[3] Each period, only short term annuities are considered, which does not correspond exactly to the contracts actually offered in the market.

for them being proxied by the interest for the asset appearing in marketing surveys) increases with age; still, markets for the very old are curiously inactive. The origin of this disequilibrium remains to be understood. Yagi and Nigiyashi (1993) confirm that *constant* long-term annuities are inefficient, which discourages demand if they are the only available annuity contracts. Poterba and Wise (1996) question whether concerns that people may be reluctant to take annuities (when they are imposed flexible mandatory private saving accounts) are empirically valid. Their estimates confirm this view, which is encouraging for the core hypothesis of most models.

In Abel (1986), the effects of adverse selection on annuities prices are endogenous, and remarkable results on the effects of pensions on capital accumulation are derived. However the comparative statics is focused on the case where consumers take part in the annuities market, which requires various restrictions on the strength of the bequest motive, the support of types, and the maximum feasible level of pension. Life insurance markets are not considered and welfare implications of the sort we wish to emphasize are not given. Nevertheless, our analysis borrows many traits from this model.

The article is organized as follows. In the next section the model is described and the assumptions regarding the insurance market and the provision of public insurance are discussed. Then the structure of demand is examined in detail; a notion of intensity of adverse selection, which characterizes aggregate demand, is defined in the succeeding section. In "Existence and Activity" existence of the equilibrium is established, and we give conditions under which a market is active or dormant. The effects of changing the level of public pensions on the participation rates and the prices in insurance markets are determined in the next section. Finally, welfare implications of the model are offered in "Welfare Implications." Proofs, except very short ones, are relegated to the Appendix.

THE MODEL

Consumers

The life cycle lasts two periods. The consumer lives period 1—activity—with certainty, and survives in period 2—retirement—with probability p. He leaves a bequest to his children and spouse at his death, either at the end of period 1 (with probability $1 - p$) or at the end of period 2 (with probability p). The consumer's preferences are represented by the utility function:

$$u_1(c_1) + (1 - p)v(l) + pu_2(c_2), \tag{1}$$

where c_1 and c_2 are periods 1 and 2 consumptions, and l is the bequest left at the beginning of period 2. Except for c_1, "goods" are contingent: with probability $1-p$, the policyholder is dead at the end of the first period and l is given to the heirs, and with probability p, he survives and consumes c_2. *Our specification applies to the case in which c_2 is split into consumption strictly speaking and second period bequests*. This simplifies notations without loss of generality.

All felicities are strictly increasing, concave, and twice continuously differentiable. To simplify, Inada conditions ($u_i'(x)$, $v'(x) \to +\infty$ as $x \to 0$ and $u_i'(x)$, $v'(x) \to 0$ as $x \to +\infty$) are assumed to be fulfilled. Felicities are not necessarily identical. Indeed, the marginal utility of consumption depends on age, and choosing $u_2 = \beta u_1$, where β is a discount factor, would be too restrictive. Moreover, the role of bequests changes over the life cycle, e.g., because heirs become economically independent as they grow up.[4] In the following,

$$\tau_1(c_1) \equiv -\frac{u_1'(c_1)}{u_1''(c_1)}; \quad \tau_2(c_2) \equiv -\frac{u_2'(c_2)}{u_2''(c_2)}; \quad \epsilon(l) \equiv -\frac{v'(l)}{v''(l)} \tag{2}$$

will denote the risk tolerances (the inverse of absolute risk aversions) of the felicities.

Incomes Y_1 and Y_2 are, respectively, period 1 labor income net of taxes and social security contributions, and the (net) pension paid in case of survival in period 2. The important assumption here is that they are imposed on the consumer by a mandatory, uniform, public pension scheme. Financial markets and insurance enable the supplementation of second-period consumption with annuities, of bequest with life insurance, or of both with unconditional saving (hereafter "saving" is used *stricto sensu*), as we shall see.

The economy is composed of a continuum of consumers whose felicities u_1, u_2, v, and incomes Y_1 and Y_2 are identical. The survival probability p is distributed over $[\underline{p}, \overline{p}]$ included in [0, 1] with the CDF F. The unconditional average risk is denoted by $\hat{p}$. We consider discrete as well as continuous distributions. Risks and types are synonyms here, and notice that we refer to "high risk" when survival probability is high, to "low risk" in the other case. This convention is based on the fact that the treatment of the annuities market is brought to the fore in this discussion. Characteristic p is known to the policyholder but is not observable. This informational asymmetry is the sole origin of adverse selection in the economy.

Private Insurance

Insurance contracts are freely purchased at the beginning of period 1 to cover survival risk in period 2. *Annuities* are an arbitrary positive quantity of an asset that pays 1 if the policyholder is alive in period 2, else zero. *Life insurance* is an arbitrary positive quantity of an asset that pays 1 to the beneficiaries if the policyholder is dead in period 2, else zero. *Saving* works as usual: the proceeds are available to the consumer in period 2 if he lives, and to his beneficiaries if he dies. One unit of saving can be seen as one unit of annuities plus one unit of life insurance.

Without loss of generality, the interest rate is assumed to be zero (felicities being general, they can be normalized without affecting curvatures). *In a perfect world*, the competitive price of annuities (respectively, of life insurance) for a consumer with survival probability p is p (respectively, $1 - p$).[5]

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[4] See Abel and Warshawsky (1988) for a discussion of how altruism should be specified. Our reduced form is less suitable to the strategic bequest motive of Bernheim, Shleifer, and Summers (1985).

[5] Other authors generally use the gross rate $1/p$ (respectively, $1/(1 - p)$). It seems to us easier to recall that the fair price of annuities is proportional to the survival risk.

We assume in our analysis that (1) due to adverse selection,[6] prices are not related to the individual risk p, and (2) prices are linear.

Competition in price seems to be at odds with the approach of asymmetric information popularized by Rothschild and Stiglitz (1976). Our main argument in favor of linear prices stems from the fact that exclusive contracts are hard to implement (Pauly, 1974): if insurance firms cannot observe whether a customer purchases other insurance contracts, then nonlinear tariffs are likely to be bypassed.[7] Another argument is that saving is a simple (though imperfect) substitute for insurance (Eichenbaum and Peled, 1987). Even if insurance policies were exclusive, the power of revealing menus would be severely constrained by the possibility of unobserved saving.[8] Cawley and Philipson (1999) proved empirically that the implicit price of life insurance declines with respect to quantity. This finding challenges the conclusions of the Rothschild–Stiglitz approach, but it is consistent with tariffs composed of fixed fees plus linear pricing. To simplify, we neglect the fixed cost.

Though prices are linear, some information is revealed by the nature of demand: an annuitant may be expected to live longer than a life-insurance policyholder, and vice versa. In this view, the fact of buying life insurance *reveals* something qualitatively different from the fact of buying annuities.

Definition 1: *A competitive equilibrium is a pair of prices* (q_a^*, q_l^*), *respectively the price of positive quantities of annuities and the price of positive quantities of life insurance, such that:*

1. *An insurance company quoting* q_a^* *or* q_l^* *expects to earn nonnegative profits.*

2. *No price* $q_a' < q_a^*$ *or* $q'_l < q_l^*$ *guarantees a strictly higher expected profit.*

Totally linear prices would imply, by arbitrage arguments, that $q_a^* + q_l^* = 1$. The following proposition shows why this logic is no longer perfectly true with "short sales" constraints.

Proposition 1: *In equilibrium,* $q_a^* + q_l^* \geq 1$.

The proof is direct. If, indeed, the inequality were reversed, any policyholder could borrow 1, buy one unit of annuities and one unit of life insurance so that the lender is reimbursed 1 in any contingency. The positive profit is kept by the policyholder. The argument cannot force equality, since the arbitrage would entail negative life insurance and annuities, which is impossible. The following shows that in equilibrium, the inequality is strict (see "Existence and Activity").

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6 For a discussion on the role of selection in life insurance markets, see Hoy and Polborn (2000).

7 If the price increases with the quantity (as one may expect with adverse selection), then the marginal price is typically strictly larger than the fair price of the marginal type. It becomes preferable to buy insurance from different companies.

8 Our arguments insist that nonlinear prices cannot be an equilibrium. This does not prove that linear prices emerge from a well-built game. Though this theoretical debate is largely beyond the scope of the article, we suggest that linear prices should be seen as a restriction on strategies (imposed by the government) to stabilize the market.

Provision of Public Insurance

We assume that pensions, here public annuities, are financed only by first-period *mandatory* contributions. Returns depend only on the interest rate and on the average survival probability. The resulting income profile is parameterized by Y_1 and $R = Y_2/Y_1$, where R is the replacement rate in the terminology of public pensions. The budget constraint of social security reads $Y_1(1 + \hat{p}R) = \text{constant}$, where $\hat{p}$ is the *marginal* cost for the social security institution of the annuities it guarantees. Our approach remains valid if we assume that pensions are fully funded at the margin only, which includes the possibility that pensions are, actuarially speaking, more costly on *average* than private insurance, notably if demographic evolution is adverse.

In the following, we fix R and characterize the equilibrium; the comparative statics will consist in evaluating how markets are affected when the public authority changes the replacement rate R. We shall then discuss the optimal replacement rate.

THE STRUCTURE OF DEMAND

We denote by A (respectively, L) the demand for annuities (respectively, life insurance) with $A \geq 0$ and $L \geq 0$; s stands for saving from period 1 to period 2; q_a and q_l are not necessarily equilibrium prices for the moment but they are such that $q_a + q_l \geq 1$ (Proposition 1). The consumer's program is

$$\max_{c_1, c_2, l} \quad u_1(c_1) + (1 - p)v(l) + pu_2(c_2) \tag{3}$$

such that

$$\begin{cases} 0 \leq c_1 \leq Y_1 - q_a A - q_l L - s \\ 0 \leq c_2 \leq Y_2 + A + s \\ 0 \leq \ l \ \leq s + L \\ 0 \leq A \\ 0 \leq L. \end{cases} \tag{4}$$

Eliminating transfer variables A, L, s yields

$$\begin{cases} c_1 + q_a c_2 + (1 - q_a)l \leq Y_1 + q_a Y_2 \\ \text{if } c_2 \geq l + Y_2 \end{cases} \tag{5}$$

and

$$\begin{cases} c_1 + (1 - q_l)c_2 + q_l l \leq Y_1 + (1 - q_l)Y_2 \\ \text{if } c_2 \leq l + Y_2 \end{cases} \tag{6}$$

Indeed, either $q_a + q_l = 1$, and (5) and (6) are equivalent; or $q_a + q_l > 1$, and it cannot be optimal to possess strictly positive quantities of annuities and life insurance at the same time.

Proposition 2: *Fix incomes and prices with $q_a + q_l \geq 1$.*

1. Optimal consumptions (c_1, c_2, l) and transfers (A, L, s) are unique and continuous with respect to p and to prices.

2. The derivative of A (respectively, L) with respect to p is piecewise continuous and positive (respectively, negative).

3. The consumers form three groups: annuitants, life insurance policyholders, nonparticipants. There are a unique survival probability $\overline{p}_l$ and a unique strictly larger survival probability $\underline{p}_a$ such that for all distributions of types:

$$\begin{array}{lll} \text{if} & p \in [\underline{p}, \overline{p}_l] & \text{then the consumer holds life insurance} \\ \text{if} & p \in (\overline{p}_l, \underline{p}_a) & \text{then the consumer is not insured} \\ \text{if} & p \in [\underline{p}_a, \overline{p}] & \text{then the consumer holds annuities.} \end{array} \tag{7}$$

Point 2 characterizes adverse selection: among those who buy annuities, the highest risks are the biggest buyers; among those who buy life insurance, those with higher death probabilities (lower survival probabilities) purchase larger quantities. In both cases, the less desirable types are systematically overrepresented in clienteles.

For insurers, what matters is the expected type of a policyholder, conditionally on the fact that he buys, say, annuities. This conditional weighted expectation is called Average Clientele Risk (ACR), and is denoted by $\tilde{p}_a$ where

$$\tilde{p}_a(R, q_a, q_l) \equiv \frac{\int_{\underline{p}_a(R,q_a,q_l)}^{\overline{p}} pA(p, R, q_a, q_l)\, dF(p)}{\int_{\underline{p}_a(R,q_a,q_l)}^{\overline{p}} A(p, R, q_a, q_l)\, dF(p)}. \tag{8}$$

The lower bound of integration is endogenous. From the continuity of $A(p, R, q_a, q_l)$ and $\underline{p}_a(R, q_a, q_l)$, the ACR is continuous in q_a and R when aggregate demand is not null. When no consumer takes part in the annuities market, the ACR is set at the conservative $\overline{p}$, which ensures continuity with respect to q_a everywhere. As for the life insurance market, the ACR $\tilde{p}_l$ is similarly defined:

$$\tilde{p}_l(R, q_a, q_l) \equiv \frac{\int_{\underline{p}}^{\overline{p}_l(R,q_a,q_l)} pL(p, R, q_a, q_l)\, dF(p)}{\int_{\underline{p}}^{\overline{p}_l(R,q_a,q_l)} L(p, R, q_a, q_l)\, dF(p)}. \tag{9}$$

The ratio $\frac{\tilde{p}_a(R,q_a)}{\hat{p}}$ is called Intensity of Adverse Selection in the annuities market. It is always larger than 1. Empirically, it measures the disparity between national mortality tables and experience tables in the life insurance industry.

Proposition 3:

1. The ACR $\tilde{p}_a$ (respectively $\tilde{p}_l$) is larger (respectively, lower) than the average risk of annuitants (respectively, life insurance policyholders), a fortiori larger (respectively, lower) than the average risk in the population $\hat{p}$.

2. $\tilde{p}_a + \tilde{p}_l > 1$.

3. $\tilde{p}_a$ *(respectively,* $\tilde{p}_l$*) does not depend on* q_l *(respectively,* q_a*).*

The proof is direct. The first point derives from the fact that demand, which serves as a weight function in a weighted sum, is increasing with respect to p. As a result, $\tilde{p}_a$ is strictly greater than $\hat{p}$ and $\tilde{p}_l$ is strictly greater than $1 - \hat{p}$. This implies that $\tilde{p}_a + \tilde{p}_l > 1$. The last point derives from this and the market separation property in Proposition 2.

EXISTENCE AND ACTIVITY

Existence

The search for an equilibrium in the annuities market is greatly simplified by the independence of the two markets (Proposition 3, point 3). The equilibrium price $q_a^*(R)$ must be (1) admissible, namely, $q_a^*(R) \in Q_a(R) \equiv \{q_a \mid 1 \geq q_a \geq \tilde{p}_a(R, q_a)\}$, and (2) "minimal," i.e., {no lower price in the set $Q_a(R)$ can lead to strictly higher profits. By the same argument: (1) $q_l^*(R) \in Q_l(R) \equiv \{q_l \mid 1 \geq q_l \geq 1 - \tilde{p}_l(R, q_l)\}$, and (2) $q_l^*(R)$ is minimal.

Proposition 4:

1. There exists an equilibrium, and $q_a^* + q_l^* > 1$*.*

2. The expected profit of insurance companies is zero, i.e., $q_a^*(R) = \tilde{p}_a(R, q_a^*(R))$ *and* $q_l^*(R) = 1 - \tilde{p}_l(R, q_l^*(R))$.

Uniqueness is not warranted.

Active and Inactive Insurance Markets

Existence does not mean *activity*, i.e., individual and aggregate demand in one market may be zero at the equilibrium price. We prove now that one market only is active.

Theorem 1:

1. There is a maximum of one active insurance market.

2. There is a positive scalar $R^\bullet$ *such that: (1) If* $R < R^\bullet$*, then the annuities market opens; (2) if* $R = R^\bullet$*, then both markets are closed; (3) if* $R > R^\bullet$*, then the life insurance market opens.*

Which market is active depends on the background income stream (Y_1, Y_2) and on actions consumers would undertake under *symmetric information.* In other terms, active markets are exactly the same under symmetric and asymmetric information. Indeed, if there is no demand for life insurance under perfect conditions, then the fact that the annuities market suffers from adverse selection cannot create a demand of life insurance. The noncoexistence result is robust to the introduction of different wealth (keeping a uniform replacement rate) if preferences are isoelastic, and it is likely that at least roughly (that is for most R), an inactive market would be predicted even without strong separability.

Bernheim's (1991) analysis is orthogonal to ours since the only difference between policyholders is their replacement rates, not risk. The two markets are open: consumers with low R demand annuities and consumers with high R demand life insurance. In between, they do not purchase supplementary insurance. This proves that the activity of the two markets that we observe in reality is essentially explained by the differences in replacement rates.

To come back to the theoretical significance of the result, note that in Brugiavini (1993), the policyholders (who have no bequest motives) buy annuities before knowing their risks (i.e., p), in this sense they are identical at the preliminary stage and they can buy annuities at the adverse-selection-free price ($\hat{p}$ with our notations). Brugiavini proves that no transactions occur after consumers privately learn their types: annuities markets are inactive in equilibrium. This is a variety of Laffont's (1985) result that optimality *ex ante* results in a no reopening of *interim* markets when information is revealed. In the likely situation of incomplete markets at the preliminary stage, our theorem gives insight into the functioning of *interim* markets.

INTENSIFICATION OF ADVERSE SELECTION

The results on market independence and on inactivity are crucial for the methodology: by considering replacement rates below $R^\bullet$, we can study the annuities market in isolation since $L = 0$ for all, but $A > 0$ for a fraction of the types.

The following definition expresses in what sense mandatory pensions can be said to aggravate adverse selection in insurance markets when they increase.

Definition 2 (IAS situation)**:** *The social-planner faces Intensification of Adverse Selection in the annuities market ("IAS" hereafter) if and only if the equilibrium price* $q_a^*(R)$ *increases in* $[0, R^\bullet)$.

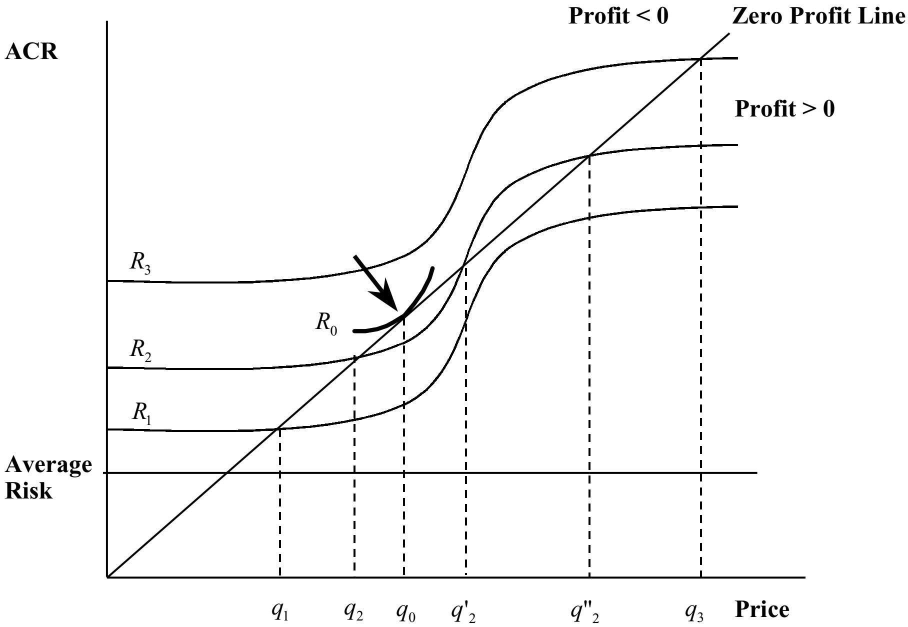
Lemma 1: *If for all* $q_a \in (\hat{p}, \overline{p}]$, $\tilde{p}_a(R, q_a)$ *is strictly increasing with respect to* R, *then the economy is in the situation of IAS.*

The equilibrium price correspondence being monotonic and bounded, the price is continuous with respect to R *almost everywhere*. Though discontinuities occur for a very small (possibly empty) set of replacement rates, changing R over an interval might imply that we come across a discontinuity.

Figure 1 shows where discontinuities of $q_a^*(R)$ take their origin, and how they are linked with multiple equilibria. We recall that we are under the hypothesis of Lemma 1 that the graphs $\{\tilde{p}_a(R, \cdot)\}_{R \in I}$ never cross. Around R_1 and R_3, the equilibrium price (respectively, q_1 and q_3) is continuous. One can see in the region marked by an arrow that when R_2 increases and reaches R_0, q_2 and q_2' converge to q_0. If the replacement rate further increases, then the equilibrium price jumps from $q_0 = \lim_{R_2 \to R_0} q_2 = \lim_{R_2 \to R_0} q_2'$ to $\lim_{R_2 \to R_0} q_2''$. This is a discontinuity.

More generally, $Q_a(R)$ is a finite or countable union of distinct closed intervals. Intervals can be *uniquely* ordered by increasing values. In the case where the first element in the sequence of ordered sets is a singleton—see R_0 in Figure 1—there are at least two equilibria: that isolated point, and the lower bound of the second interval in the order we defined. These equilibria are Pareto ordered: in both equilibria, prices correspond to zero profit (no incentive for deviation), but a lower price is preferable to consumers.

FIGURE 1
Discontinuity of the Equilibrium Price



Given that under IAS, uniqueness holds almost everywhere and that when it does not the most efficient equilibrium can be selected at no cost,[9] *we reason in the rest of the article on the efficient equilibrium only without loss of insight*.

To apply Lemma 1, we analyze the variations of q_a with respect to R for all types of consumers and we provide sufficient conditions for the economy to exhibit IAS.

Increasing the level of mandatory pensions has two effects on the individual demand for annuities $A = c_2 - l - Y_2$. Y_2 crowds out annuities: any increase of Y_2 with $Y_1 + q_a Y_2$ constant decreases A one-for-one. Second, there is a positive effect on consumption (income expansion) since the constraint imposed on social security is in fact $Y_1 + \hat{p} Y_2 =$ constant, with $\hat{p} < q_a$. This effect differs across types since their propensities to consume depend on their survival probabilities:

$$\frac{\partial A}{\partial R} = \underbrace{\frac{\partial (c_2 - l)}{\partial R}}_{\text{Differential Income Effect}} - \underbrace{\frac{\partial Y_2}{\partial R}}_{\text{Crowding Out}} . \tag{10}$$

Lemma 2: *If* $\frac{\partial A}{\partial R} < 0$ *and* $\frac{\partial^2 A}{\partial R \partial p} > 0$, *then for all* $q_a \in (\hat{p}, \overline{p}]$, $\tilde{p}_a(R, q_a)$ *is strictly increasing with respect to* R, *and Lemma 1 is applicable.*

[9] A simple public intervention (putting a price-cap for annuities) may be needed to restore efficiency and uniqueness.

Theorem 2 gives sufficient conditions for this lemma to be applicable. A definition is given first for keeping short statements in the theorem:

Definition 3: (K-tolerance) *The function* $x \to -\frac{u'(x)}{x^K u''(x)}$ *is the K-tolerance index of a utility function u. A function u is said to exhibit a constant K-tolerance t if and only if there are real numbers K and t such that*

$$\forall x, -\frac{u'(x)}{u''(x)} = tx^K$$

K is the elasticity of risk tolerance; t is the value of the constant K-tolerance.

Note that for $K = 0$, the notion corresponds to Constant Absolute Risk Aversion (CARA) and for $K = 1$, it corresponds to Constant Relative Risk Aversion (CRRA). This parametric class of function, which is broader than usual, is interesting since the marginal utility can be integrated in closed-form. Indeed, if u exhibits a constant K-tolerance equal to t, then there will be a U such that

$$\forall x, u'(x) = U \exp\left(-\frac{x^{1-K}}{t(1-K)}\right), \tag{11}$$

where $U = u'(0)$ if $0 \leq K < 1$. For $0 \leq K < 1$, these functions do not satisfy the Inada condition at zero (as for all CARA functions). When u_1 is a member of this family of functions, the condition can be replaced by the assumption that c_1 is interior ($c_1 > 0$), which is the only economically relevant case, and the argument still pertains.

Theorem 2: *Any of the following alternative conditions is sufficient for the fully funded increase of the replacement rate to cause intensification of adverse selection in the annuities market:*

1. *All felicities are DARA. The consumer has no bequest motive in the second period.*

2. u_1 *is CARA and* u_2, v *are DARA.*

3. u_1 *and* v *are CRRA with, respectively, elasticities* ρ_1 *and* σ *where* $\sigma \leq \rho_1$. *Moreover* u_2 *is DARA.*

4. *There is a K such that* u_1 *and* v *exhibit constant K-tolerances with* $\frac{u_1'(0)}{v'(0)} \geq \frac{1-p}{1-\hat{p}}$. *Moreover* u_2 *is DARA.*

5. *All felicities are DARA. For all x*

$$u_2'(x) \geq \frac{\underline{p}(1-\overline{p})}{\overline{p}(1-\underline{p})} v'(x) \tag{12}$$

$$\tau_2(x) \geq \frac{\overline{p}(1-\hat{p})}{\hat{p}(1-\overline{p})} \epsilon(x). \tag{13}$$

Interestingly, the parametric restrictions in conditions 1–4 are imposed on one or two felicities only. Case 5 shows that strict parametric restrictions are not indispensable, but we need that consumption in the second period be a priority compared to transmission. Remark that in conditions 1, 2, and 3, no restriction on the distribution of p is made. Conditions 4 and 5 give examples of weaker assumptions that can be made if we are prepared to reduce our parameters' range.

To summarize, risk aversion causes adverse selection (see "The Structure of Demand"), whereas decreasing risk aversion causes intensification of adverse selection. All our conditions indicate, by their diversity, that the likelihood that increasing pensions intensifies adverse selection is large, provided felicities are decreasingly risk averse.

Proposition 5 (participation rate)**:** *If the economy exhibits IAS, the equilibrium participation rate to the annuities market (weakly) decreases as the replacement rate goes up.*

The effects of increasing pensions on the average risk among annuitants are not of the same kind as those in the "Lemons" models. In Akerlof (1970), all the "intensification" comes from modification of the types of participants (best types leave first) since traded commodities are indivisible. In our model, intensification occurs even if no policyholder leaves the market. The phenomenon depends mainly on the structure of demand and is only reinforced by the support effect.

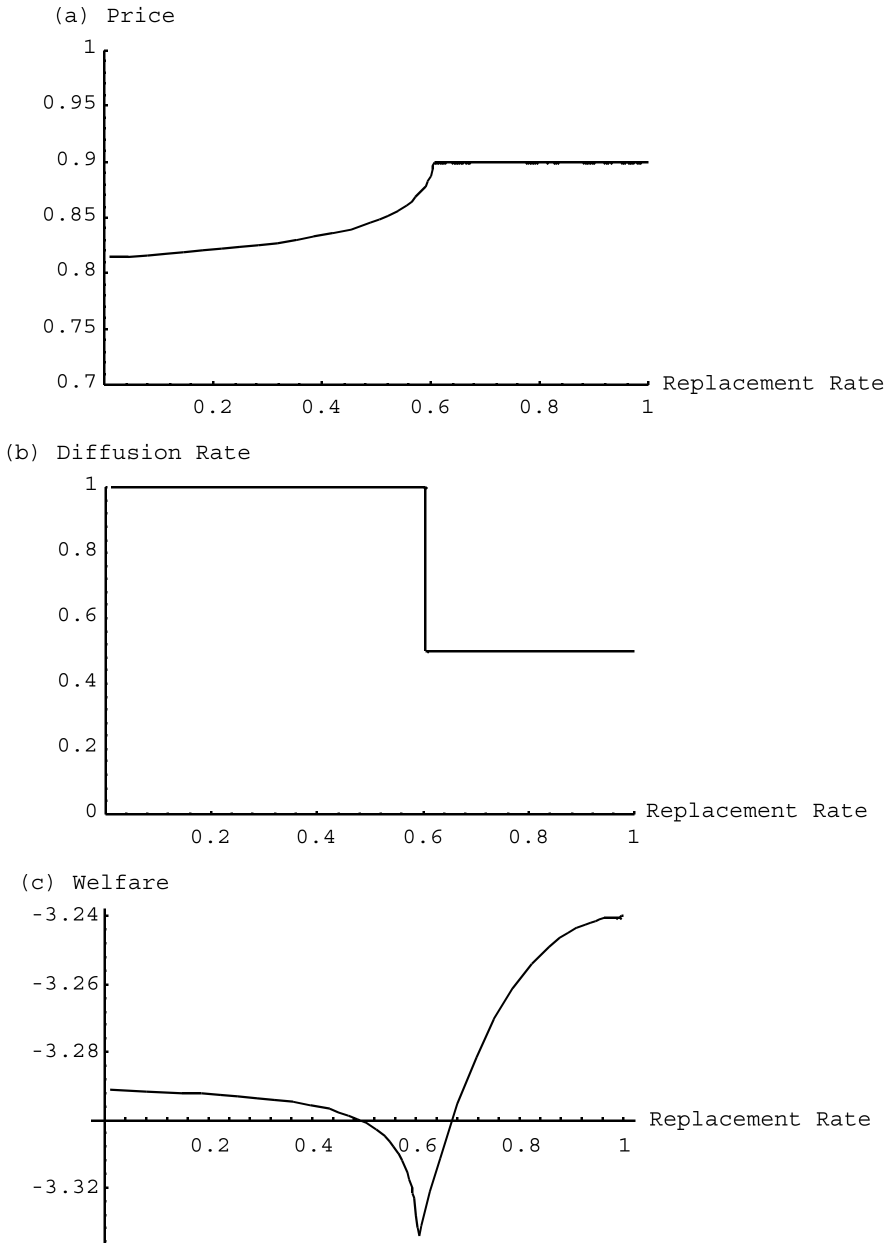
Life insurance. Life insurance markets are active when $R > R^{\bullet}$. In this case $A = 0$ for all and $L > 0$ for a fraction of the types. A symmetrical treatment of the proofs shows that under conditions similar to those given in Theorem 2, *decreasing R intensifies adverse selection in the life insurance market and discourages participation.*

WELFARE IMPLICATIONS

A few results on the welfare enhancing properties of mandatory insurance in an adverse selection economy must be mentioned. Dahlby (1981) suggested that, in the model of Rothschild and Stiglitz (1976), a uniform mandatory insurance scheme, completed by freely purchased private insurance, can enhance efficiency. More complete results on the possibility of reaching a second-best optimum are given in Crocker and Snow (1985; see also Henriet and Rochet, 1990).

In this model, the absence of exclusivity in insurance markets makes public policy a much harder task. Indeed, when only a fraction of consumers take part in the market in equilibrium, marginal public intervention (changing the replacement rate) has a redistributive role across types without Pareto improvement. To see this, take the simplified version without bequest motive, suppose a continuous distribution of p with positive density over [0, 1], and assume that R is strictly positive. (1) Very low risks (i.e., with no supplementary demand) lose when the level of pensions is raised, because $\hat{p}$ (price of pensions) is too high a price for them. (2) The risks whose MRS lie between $\hat{p}$ and q_a^* gain from higher pension, though they were not purchasing additional annuities. (3) The higher risks are advantaged by higher pension (they are subsidized by the other types), but this is mitigated by the subsequent intensification of adverse selection.

FIGURE 2
Example



Consider the *ex ante* (that is before consumers know their types) optimum.

Proposition 6: *If the pension is a fully funded annuity priced at the average price, an ex ante optimum is reached for replacement rate $R^{\bullet}$, a level at which all private insurance markets stay inactive.*

Due to the redistributive phenomena evoked above, the social welfare when the level of the replacement rate varies from zero to $R^{\bullet}$ is not monotonic in general. As an illustration, let us now assume that the equilibrium price is discontinuous at R_0 (with $0 < R_0 < R^{\bullet}$; see Figure 1). The left limit of welfare at R_0 is obviously larger than the right limit. This is a case where the social objective has several local optima: there is a local maximum on the left of R_0, and a global one at $R^{\bullet}$.

The optimal level of pensions may not be implementable for nonmodeled reasons, e.g., the retirement age is typically endogenous and has to be optimized. As usual, there is a trade-off between the quality of coverage and incentives to work. In that case, the social planner may not wish to select the maximal level. See our example in Figure 2:[10] if $R_0 = 0.4$ preserves better incentives (this aspect is not modeled) than $R^{\bullet} = 1$, then a best response to intense adverse selection when R lies between R_0 and $R^{\bullet}$ *may be* a reduction of mandatory insurance to R_0 rather than an increase to $R^{\bullet}$.

In the social objective, another weight function than the objective distribution of types could be preferred. But, whatever the weights, a discontinuity of the price with respect to the replacement rate automatically provokes a downward discontinuity of welfare. In other terms, discontinuities of the price correspond to strict Pareto efficiency losses and the conclusion remains true.

CONCLUSION

If pensions are below the ideal threshold, all the consumers, under symmetric information, would demand additional annuities at a fair price. But, because of adverse selection, an endogenously determined fraction of consumers abstain from these purchases. Abstention results from the position of one policyholder's risk relatively to the others' and cannot be ascribed to loadings or other financial imperfections. Adverse selection never causes market closure, but causes partial participation in the active market. As mandatory insurance increases, the intensification of adverse selection that occurs under mild assumptions does not come principally from the best risks leaving the market, but from an unambiguous change in the structure of demand: the worst risks buy *relatively* more annuities.

In most developed countries, the relative level of public pensions has decreased over the last decade or is expected to decrease. Our analysis predicts success for annuities markets in the future in terms of diffusion and quantities purchased by households. The good news is that the rates of return of annuities should improve: indeed, the intensification of adverse selection is reversible and as the replacement decreases, adverse selection is attenuated and the market grows. The bad news is that life insurance will experience more intense adverse selection. In the data, these effects may be hard to separate from the fact that economic growth causes the development of insurance markets. Still, with aging populations, the life insurance industry as a whole is likely to experience continued shifts (in terms of percentage) from the life insurance line toward the annuities line.

[10] Figure 2(a) shows the variations of the equilibrium price with respect to R, Figure 2(b) the variations of the participation rate. Figure 2(c) illustrates that the social objective is not single-peaked. Parameters of the model: (preferences) $u(c_1) + pu(c_2)$; $u(x) = \frac{x^{1-\gamma}}{1-\gamma}$; $\gamma = 2$; (distribution) $f(0.7) = 0.5\delta$; $f(0.9) = 0.5\delta$.

APPENDIX A: PROOFS

A.1 Proof of Proposition 2:

1. Define $Z_1 = c_2 - l - Y_2$ and $Z_2 = Y_1 - c_1 - l$. Then (5) and (6) can be rewritten as

$$q_a Z_1 \leq Z_2 \quad \text{if} \quad Z_1 \geq 0 \tag{A1}$$

$$(1 - q_l) Z_1 \leq Z_2 \quad \text{if} \quad Z_1 \leq 0. \tag{A2}$$

Clearly, this union of two-dimensional sets is convex if and only if $q_a \geq 1 - q_l$. In addition, the conditions that consumptions be positive are not problematic for convexity since the intersection of convex sets is convex. Consequently, the constraint set is convex if and only if $q_a + q_l \geq 1$. This implies 1.

2. We prove that if a certain type p purchases annuities (respectively, life insurance), then a higher (lower) type purchases a larger quantity of annuities (respectively, life insurance). The proof is given for those who buy annuities ($A > 0$) *and* save ($s > 0$). Similar reasonings are applicable to the other regimes ($A > 0$ and $s = 0$, or $L > 0$, with $s > 0$ or $s = 0$).

For alleviating the notations, we shall write u_1' rather than $u_1'(c_1)$ or c'_{1q_a} rather than $\frac{\partial c_1}{\partial q_a}$, with obvious similar conventions for the other felicities, other goods or other derivatives.

The FOC of the program are

$$u_1' = p u_2' + (1 - p) v' \tag{A3}$$

$$q_a u_1' = p u_2'. \tag{A4}$$

The relevant budget constraint is (5). Equations (5), (A3), and (A4) fully determine the solution (c_1, c_2, l).

Eliminating first and second derivatives of the felicity functions yields

$$c'_{1p} + q_a c'_{2p} + (1 - q_a) l'_p = 0 \tag{A5}$$

$$c'_{2p}/\tau_2 = c'_{1p}/\tau_1 + 1/p \tag{A6}$$

$$l'_p/\epsilon = c'_{1p}/\tau_1 - 1/(1 - p) \tag{A7}$$

where risk tolerances are computed for optimal consumption. This gives

$$c'_{1p} = \frac{-\tau_1((1-p) q_a \tau_2 - p(1 - q_a)\epsilon)}{p(1-p)(\tau_1 + q_a \tau_2 + (1 - q_a)\epsilon)} \tag{A8}$$

$$c'_{2p} = \frac{\tau_2((1-p)\tau_1 + (1 - q_a)\epsilon)}{p(1-p)(\tau_1 + q_a \tau_2 + (1 - q_a)\epsilon)} \tag{A9}$$

$$l'_p = \frac{-\epsilon(p\tau_1 + q_a \tau_2)}{p(1-p)(\tau_1 + q_a \tau_2 + (1 - q_a)\epsilon)}. \tag{A10}$$

Hence, the derivative of annuities demand, $A'_p = c'_{2p} - l'_p$, is

$$A'_p = \frac{\epsilon_2(p\tau_1 + \tau_2) + (1-p)\tau_1\tau_2}{p(1-p)(\tau_1 + q_a\tau_2 + (1-q_a)\epsilon)}. \tag{A11}$$

The sign is obviously positive, which proves our point.

3. The monotonicity of demand functions is the first ingredient of the proof. Uniqueness and continuity due to the regularity of the objective with respect to p ensures that risk supports of the two markets do not overlap. Q.E.D.

A.2 Proof of Proposition 4: We reason on the annuities market. The set of admissible prices is never empty since $q_a = \overline{p}$ guarantees positive profits whatever the distribution of types. The lower bound comes from adverse selection: $\tilde{p}_a(R, q_a)$ is strictly bounded below by $\hat{p}$. The expected profit per unit $q_a - \tilde{p}_a(R, q_a)$ is continuous in the price since the ACR is continuous in q_a, therefore the profit function has at least one zero in the admissible set, which is closed. Results concerning life insurance can be established by a symmetric argument. Any equilibrium pair of prices $q_a^*(R)$ and $q_l^*(R)$ satisfies $\hat{p} < q_a^*(R) \leq \overline{p}$ and $1 - \hat{p} < q_l^*(R) \leq 1 - \underline{p}$. In particular, $1 < q_a^* + q_l^*$.
Q.E.D.

A.3 Proof of Theorem 1: We have to determine the variations in consumption with respect to the price. The equations derived from the FOC are:

$$c'_{1q_a} + q_a c'_{2q_a} + (1-q_a)l'_{q_a} = Y_2 - c_2 + l = -A \tag{A12}$$

$$c'_{2q_a}/\tau_2 = c'_{1q_a}/\tau_1 - 1/q_a \tag{A13}$$

$$l'_{q_a}/\epsilon = c'_{1q_a}/\tau_1 + 1/(1-q_a). \tag{A14}$$

After a series of calculations, we get:

$$c'_{1q_a} = \frac{\tau_1(\tau_2 - \epsilon - A)}{\tau_1 + q_a\tau_2 + (1-q_a)\epsilon} \tag{A15}$$

$$c'_{2q_a} = \frac{-\tau_2(\tau_1 + \epsilon + q_a A)}{q_a(\tau_1 + q_a\tau_2 + (1-q_a)\epsilon)} \tag{A16}$$

$$l'_{q_a} = \frac{\epsilon(\tau_1 + \tau_2 - (1-q_a)A)}{(1-q_a)(\tau_1 + q_a\tau_2 + (1-q_a)\epsilon)}. \tag{A17}$$

As a result:

$$A_{q_a} = -\frac{\epsilon(q_a\tau_1 + \tau_2) + (1-q_a)\tau_1\tau_2 + q_a(1-q_a)(\tau_2 - \epsilon)A}{q_a(1-q_a)(\tau_1 + q_a\tau_2 + (1-q_a)\epsilon)}. \tag{A18}$$

1. Fix R, and assume that prices are fair for all ($q_a = p$ and $q_l = 1 - p$ for all p, which amounts to assuming symmetric information). We define A_f as $c_2 - l - Y_2$ with individually fair prices. We have

$$\frac{\partial A_f(p)}{\partial p} = \left.\frac{\partial A}{\partial p}\right|_{q_a=p} + \left.\frac{\partial A}{\partial q_a}\right|_{q_a=p}. \tag{A19}$$

Hence

$$\frac{\partial A_f(p)}{\partial p} = -\frac{(\tau_2 - \epsilon)A_f(p)}{\tau_1 + p\tau_2 + (1-p)\epsilon}. \tag{A20}$$

Qualitatively, a solution to such a differential equation is of one of the following three types: A_f is uniformly either (1) strictly positive, (2) equal to zero, or (3) strictly negative. Therefore, under symmetric information, $A_f > 0$ (respectively, < 0) for one type if and only if $A_f > 0$ (respectively, < 0) for any other type.

Now, assume that the annuities market is active at price q_a^*. This means that there is a buyer of minimal risk $\underline{p}_a$, such that $\underline{p}_a < q_a^*$. Monotonicity of demand with respect to p proves that a type of survival probability q_a^* would demand a positive quantity at price q_a^*; in other terms $A_f(q_a^*) > 0$. Therefore, we are in a situation where $A_f > 0$ for all types. Assume now that the life insurance market is also active. The same line of reasoning proves in turn that $A_f < 0$ for at least one type, therefore for all types. This is a contradiction. The two markets cannot be simultaneously open.

2. If the annuities market opens for R, then monotonicity of A_f with respect to R ensures that it opens at a lower level $R' < R$. All we then need to do is observe that, for the average consumer, there is then no income effect and only a crowding-out effect. Conversely, if the life insurance market opens for R, then it opens for $R'' > R$. This proves the uniqueness of $R^\bullet$. The Inada conditions on u_1 and u_2 guarantee that $R^\bullet < +\infty$. When $R = 0$, consumers may prefer, if the price is fair, to purchase life insurance to supplement their savings. Q.E.D.

A.4 Proof of Lemma 1: We are in the class of comparative statics problems addressed by Milgrom and Roberts (1994). We give a simple proof. If q_a is admissible for R', then $q_a \geq \tilde{p}_a(R', q_a)$; if $R' > R$, q_a is admissible for R since $\tilde{p}_a(R', q_a) > \tilde{p}_a(R, q_a)$. Consequently, $Q_a(R') \subset Q_a(R)$. The equilibrium price being a minimum of this set, $q_a^*(R)$ is necessarily increasing. Monotonicity is strict until $Q_a(R)$ becomes the singleton $\{\overline{p}\}$: $\tilde{p}_a(R', q_a^*(R')) = q_a^*(R')$, and $\tilde{p}_a(\cdot)$ is strictly increasing in R; therefore $\tilde{p}_a(R, q_a^*(R')) < q_a^*(R')$, which proves that $q_a^*(R')$ is not minimal in the set of admissible prices for the replacement rate R.

Q.E.D.

A.5 Proof of Lemma 2: If increasing R diminishes the demand of every consumer by the same proportion, then obviously the ACR remains unchanged and adverse selection keeps the same intensity, no more, no less. In contrast, if at every price q_a, the elasticity of demand with respect to R is decreasing with respect to p in absolute value (demand for insurance of higher risks is relatively more rigid), then the ACR $\tilde{p}_a(R, q_a)$ is a strictly increasing function of R.

Indeed, given that $A \geq 0$ and $\partial A/\partial p > 0$, we have

$$\operatorname{sgn}\left\{\frac{\partial}{\partial p}\left(\frac{R}{A}\frac{\partial A}{\partial R}\right)\right\} = \operatorname{sgn}\left\{A\frac{\partial^2 A}{\partial R\partial p} - \frac{\partial A}{\partial R}\frac{\partial A}{\partial p}\right\} \tag{A21}$$

which is positive under the assumptions. Given the remark above on the elasticity of demand, this implies that $\tilde{p}_a(R, q_a)$ is strictly increasing with respect to R. Q.E.D.

A.6 Proof of Theorem 2: Here, we give the methodology that we apply to the various sufficient conditions. Marginal propensities to consume the four goods are proportional to the risk tolerances of the felicity functions of the goods (see Equation (2)). Similar calculations appear in Wilson's (1968) study of optimal risk-sharing.

$$\frac{\partial c_i(p, q_a, W)}{\partial W} = \frac{\tau_i}{\tau_1 + q_a\tau_2 + (1 - q_a)\epsilon}, \quad i = 1, 2 \tag{A22}$$

$$\frac{\partial l(p, q_a, W)}{\partial W} = \frac{\epsilon}{\tau_1 + q_a\tau_2 + (1 - q_a)\epsilon} \tag{A23}$$

where $W = Y_1(1 + q_a R)$ is net present wealth. This allows us to write the propensity to buy annuities:

$$\frac{\partial A}{\partial W} = \frac{\tau_2 - \epsilon}{\tau_1 + q_a\tau_2 + (1 - q_a)\epsilon}. \tag{A24}$$

Now, remark that

$$\frac{\partial W}{\partial Y_2} = \frac{(q_a - \hat{p})Y_1}{1 + \hat{p}R} \quad \text{and} \quad \frac{\partial Y_2}{\partial R} = \frac{Y_1}{1 + \hat{p}R}. \tag{A25}$$

Hence

$$\begin{aligned}\frac{\partial A}{\partial R} &= \frac{Y_1}{1 + \hat{p}R}\left((q_a - \hat{p})\frac{\partial A}{\partial W} - 1\right)\\ &= \frac{-Y_1}{1 + \hat{p}R}\frac{\tau_1 + \hat{p}\tau_2 + (1 - \hat{p})\epsilon}{\tau_1 + q_a\tau_2 + (1 - q_a)\epsilon}.\end{aligned} \tag{A26}$$

Equation (A26) proves that $\frac{\partial}{\partial p}(\frac{\tau_2 - \epsilon}{\tau_1 + \epsilon}) > 0$ implies $\frac{\partial^2 A}{\partial R\partial p} > 0$.

We have to give the sign of $\frac{\partial}{\partial p}(\frac{\tau_2 - \epsilon}{\tau_1 + \epsilon})$. The idea is as follows: if (1) $\partial c_1/\partial p < 0$, $\partial c_2/\partial p > 0$, $\partial l/\partial p < 0$ and (2) the felicity functions u_1, u_2, v exhibit Decreasing Absolute Risk Aversion (DARA), then the fully funded increase of the replacement rate provokes an intensification of adverse selection in the annuities market.

The difficulty is to give the variations of $\tau_1 + \epsilon$. The ambiguity comes from the variations of c_1 (other derivatives are signed unambiguously): the intuition is that c_1 is what is left after saving takes its share of the life-cycle income and is allocated to the various contingencies. There are two antagonistic motives for saving: one is supplementing the consumer's consumption in case of survival and the other is supplementing the bequest. When p is increased, the former is reinforced and the latter is weakened: the net effect is ambiguous. Their relative importance does not come from marginal utilities (i.e., the weights attached to each felicity), but essentially from risk tolerances.

1. If $\tau_2 = 0$, then c_1 is decreasing with respect to p. The preceding method is then applicable.

2. Obvious since then the variations of c_1 are irrelevant for the variations of $\frac{\tau_2 - \epsilon}{\tau_1 + \epsilon}$.

3. The expressions of the variations in consumption with respect to p show that for all $\alpha \geq 1 - q_a$:

$$\frac{\partial(c_1 + \alpha l)}{\partial p} < 0. \tag{A27}$$

Since

$$\tau_1 + \epsilon = \frac{c_1}{\rho_1} + \frac{l}{\sigma} = \frac{1}{\rho_1}\left(c_1 + \frac{\rho_1}{\sigma} l\right) \tag{A28}$$

and since $1 - q_a < 1 - \hat{p}$, the result is obtained.

4. We have

$$\begin{aligned}\frac{\partial(\tau_1 + \epsilon)}{\partial p} &= c'_{1p}\tau'_1 + l'_p\epsilon' \\ &= \frac{p\tau_1\epsilon((1 - q_a)\tau'_1 - \epsilon') - q_a\tau_2((1 - p)\tau_1\tau'_1 + \epsilon\epsilon')}{p(1 - p)(\tau_1 + q_a\tau_2 + (1 - q_a)\epsilon)}.\end{aligned} \tag{A29}$$

The interest of this factorization is to show that $(1 - q_a)\tau'_1 - \epsilon' < 0$ is sufficient for the result. It turns out that this expression is tractable with felicities exhibiting constant K -tolerances (we denote by t_1 and t the K-tolerances of u_1 and v):

$$(1 - q_a)\frac{l^{1-K}}{t} \leq \frac{c_1^{1-K}}{t_1}. \tag{A30}$$

The first-order conditions give

$$\frac{u'_1(c_1)}{v'(l_2)} = \frac{u'_1(0)}{u'_2(0)} \exp\left(\frac{l^{1-K}}{t(1 - K)} - \frac{c_1^{1-K}}{t_1(1 - K)}\right) = \frac{1 - p}{1 - q_a}. \tag{A31}$$

Hence, a sufficient condition for ensuring (A30) is

$$\frac{u'_1(0)}{v'(0)} \geq \frac{1 - p}{1 - q_a}. \tag{A32}$$

But we want this to be true even if the RHS is the largest possible, i.e., if $p = \underline{p}$ and $q_a = \hat{p}$. This proves point 4.

5. Here, we prove that c_1 is decreasing with respect to p. Using (A15), we prove that the conditions are sufficient for

$$p(1-q_a)\epsilon - q_a(1-p)\tau_2 < 0. \tag{A33}$$

Rearranging the first-order conditions (A3) and (A4) yields

$$\frac{v'}{u_2'} = \frac{p(1-q_a)}{q_a(1-p)}. \tag{A34}$$

Associated with the condition on the marginal utilities (12), this ensures that $l \le c_2$. A sufficient condition for (A33) is

$$\frac{\overline{p}(1-\hat{p})}{\hat{p}(1-\overline{p})}\epsilon(l) < \tau_2(c_2). \tag{A35}$$

The fact that $l \le c_2$, associated with DARA and the condition on tolerances (13) establishes (A35), hence the point is proved. Q.E.D.

A.7 Proof of Proposition 5: The lower risk buyer has probability $\underline{p}_a(R, q_a^*(R))$ of surviving in the second period. The participation rate is, by definition, $1 - F(\underline{p}_a(R, q_a^*(R)))$. Since $\partial \underline{p}_a/\partial R > 0$ and $\partial \underline{p}_a/\partial q_a > 0$ (see in the Appendix the variations of $A(\cdot)$ for $A = 0$, where $A = 0$ defines $\underline{p}_a(\cdot)$), and since $q_a^*(R)$ is increasing by Proposition 2, then $\underline{p}_a$ increases with R. Q.E.D.

A.8 Proof of Proposition 6: The *ex ante* optimum is first calculated without informational constraints. Then we check that the allocation so determined can be implemented by mandatory insurance. The optimum solves

$$\max \int_{\underline{p}}^{\overline{p}} \{u_1(c_1(p)) + pu_2(c_2(p)) + (1-p)v(l(p))\}\, dF(p) \tag{A36}$$

with respect to functions c_1, c_2, and l under the resource constraint

$$\int_{\underline{p}}^{\overline{p}} \{c_1(p) + pc_2(p) + (1-p)l(p)\}\, dF(p) = Y_1 + \int_{\underline{p}}^{\overline{p}} p\, dF(p) Y_2. \tag{A37}$$

The first-order conditions yield

$$\forall p, p' : c_1(p) = c_1(p'), \quad c_2(p) = c_2(p'), \quad v(p) = v(p'). \tag{A38}$$

It is therefore sufficient to solve

$$\max_{c_1, c_2, l} u_1(c_1) + \hat{p}u_2(c_2) + (1-\hat{p})v(l) \tag{A39}$$

under the constraint

$$c_1 + \hat{p}c_2 + (1 - \hat{p})l = Y_1 + \hat{p}Y_2. \tag{A40}$$

Obviously, this program amounts to choosing a uniform replacement rate. Moreover, type $\hat{p}$ being necessarily optimally covered at a fair price, the conditions for closure of the markets are satisfied. The optimal level of pension is $R^{\bullet}$. Q.E.D.

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