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Fixed-Reimbursement Insurance: Basic Properties and Comparative Statics

Louis Eeckhoudt
Olivier Mahul
John Moran

Abstract

In a number of settings, insurance contracts specify a fixed reimbursement in the event of a loss which is not conditioned on the size of the realized loss. In this article, we explore the theoretical properties of this form of insurance and draw comparisons with other types of insurance policies, such as those based on coinsurance and deductibles. We also examine links between our results and those from the literature on precautionary saving.

Introduction

In a number of settings, insurance contracts specify a fixed reimbursement in the event of a loss which is not conditioned on the size of the actual loss. Examples of this type of insurance include term life insurance, flight insurance, dread disease insurance, jewelry insurance, and in some cases, postal insurance. Although in most instances insureds are permitted to choose the magnitude of the reimbursement from a premium-reimbursement schedule, such contracts do not provide for ex post conditioning of the indemnification on the realized loss. In the case of term life insurance, insureds (actually their estates) receive a lump-sum payment at the time of death which is not adjusted ex post to reflect changes in either the earnings capacity of the policyholder or the financial condition of the policyholder's dependents.[1] Likewise, in the case of dread disease insurance, an insured who is diagnosed with a specific "dread disease" is paid a fixed indemnity, irrespective of the magnitude of the actual medical

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Louis Eeckhoudt is with the Department of Economics, Catholic Faculties of Mons and Lille. Olivier Mahul is with the Institut National de la Recherche Agronomique, Department of Economics, Rennes. John Moran is with the Department of Economics and Center for Policy Research, Syracuse University. A portion of this research was conducted while Moran was a fellow in the Robert Wood Johnson Foundation's Scholars in Health Policy Research Program. We thank Keith Crocker, Christian Gollier, and Jeff Kubik for helpful conversations. Address all correspondence to John Moran, Center for Policy Research, Syracuse University, e-mail: jmoran@maxwell.syr.edu.

[1] Using data on household income and wealth from the Health and Retirement Study, Bernheim et al. (forthcoming) find evidence of substantial uncertainty on the part of consumers regarding the overall financial risk associated with a spouse's death.

expenses. In the case of jewelry insurance, changes in the underlying market value of the insured object are often not incorporated into the insurance payment.[2,3] This is in contrast to other types of insurance policies, such as automobile insurance, property-liability insurance, and health insurance, in which payments made in the loss state are explicitly conditioned on the magnitude of the realized loss, typically through the use of coinsurance or deductibles.

Given the results of Arrow (1965) and Raviv (1979) on the optimality of straight deductible policies, one may wonder what gives rise to insurance contracts that deviate from this form. In the present context, policies with fixed indemnities arise because they are the only policies that are feasible in environments where the occurrence of a loss is observable, but the magnitude of the loss is either unobservable or unverifiable at reasonable cost. Gollier (1996) studies a similar environment in which the insurer is only able to observe a noisy signal of the true loss. He demonstrates that in such a setting a deductible policy is optimal, as long as the signal is informative. However, when the signal becomes independent of the loss, which corresponds to the case we study, the optimal contract defaults to a constant reimbursement paid in all loss states.

In Gollier's model, the optimal (fixed) reimbursement is zero because he does not allow for a state in which no loss occurs. In our model, which incorporates a "no-loss" state, the optimal reimbursement will generally not be zero. The difference occurs because fixed-reimbursement contracts, by their nature, do not allow for wealth transfers across loss states. Since the reimbursement is the same in all loss states, in the absence of a no-loss state from which wealth can be transferred, the constant reimbursement must be equal to the premium for the insurer to break even. This obviously eliminates any insurance aspect to the contract. However, when a state of nature exists in which no loss occurs, as in our model, it is possible (and generally, optimal) for the insured to transfer wealth from the no-loss state to each of the loss states.

In this article, we examine the theoretical properties of insurance contracts that pay a fixed indemnity in the event of a loss, which we label *fixed-reimbursement insurance*.[4] Fixed-reimbursement insurance contracts have specific properties that distinguish

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[2] An exception is when the insured contracts to receive the replacement value of the item.

[3] Postal insurance pays a fixed reimbursement, independent of the realized loss, in those instances where the object's future value is unknown as the time of mailing, e.g., if a collectible of unknown value were being sent to an appraiser.

[4] Although many of the applications that motivate our work are most naturally cast in a state-dependent utility framework, doing so here would be problematic for several reasons. First, in order to obtain determinate comparative static results with state-dependent utility, we must assume that the marginal utility of wealth is higher in some states of the world than in others. For some applications, such as jewelry insurance, it is not unreasonable to assume that the marginal utility of wealth is lower in the loss state. However, for applications such as life insurance, where the purchase of an insurance policy is typically motivated by altruism toward surviving family members, it seems more difficult to assess a priori whether the marginal value of an additional dollar is higher in the "life" state or in the "death" state. Second, although some work has been done to derive measures of risk aversion in a state-dependent setting (see, for example, Karni [1983] or Kelsey and Nordquist [1991]), these measures are rather restrictive and have not served as a basis for many comparative static results in the insurance literature. Finally, no corresponding measures have been derived for the prudence concept, which, as we demonstrate below, is critical for characterizing the optimal

them from standard coinsurance or deductible contracts. For example, in contrast to standard contracts, the policyholder can realize an ex post profit in states of the world where the reimbursement exceeds the realized loss, an outcome that cannot occur with deductible or coinsurance contracts.[5] Further, it turns out that fixed-reimbursement insurance mimics, to a large extent, the role of precautionary saving in intertemporal models with income risk. This is why the prudence concept introduced by Kimball (1990a, 1990b) and assumptions about its behavior will be quite important for illustrating the main features of this type of policy.

The article proceeds as follows. In "Fixed-Reimbursement Insurance," we lay out the basic model. In "Basic Properties and Comparative Statics," we examine the optimal level of coverage and investigate how the fixed indemnification responds to changes in risk aversion, prudence, wealth, and various perturbations of the loss distribution. Our findings complement those obtained for other types of insurance policies, such as those based on coinsurance or deductibles, and bear interesting similarities to results from the literature on saving behavior in the presence of income risk. "Conclusions" contains concluding remarks and discusses some possible extensions.

FIXED-REIMBURSEMENT INSURANCE

Consider a risk-averse individual who, with probability $p \in (0,1)$, faces a risk of incurring a loss $\tilde{L}$. We assume that $\tilde{L}$ is a random variable with a support $\tilde{L} \in [\underline{L}, \bar{L}]$, with $0 < \underline{L} \leq \bar{L}$ and cumulative distribution function $F(\cdot)$. To protect against the loss, the agent may purchase an insurance contract that pays an indemnity to the policyholder in the event that a loss occurs but that is not conditioned on the size of the realized loss. Insurers are thus restricted to contracts of the form $C \equiv \{R, Z\}$, where R is a lump-sum reimbursement paid in the loss state and Z is the associated premium. Assume that insured individuals may choose any premium-reimbursement pair that satisfies the following zero-profit condition for the insurer:

$$Z = p(1 + \lambda)R, \tag{1}$$

where $\lambda \geq 0$ is a proportional loading factor. Given initial wealth W_0, the policyholder's expected utility is

$$V = (1 - p)U(W_0 - Z) + pEU(W_0 - Z - \tilde{L} + R), \tag{2}$$

where $U(\cdot)$ is a three-times differentiable von Neumann-Morgenstern utility function, with $U' > 0$ and $U'' < 0$, and E is the expectation operator conditional on the occur-

reimbursement schedule. For these reasons, we will not introduce this additional complication into our analysis.

[5] Some care of interpretation is required here. In many settings where fixed-reimbursement insurance is purchased, it seems likely that the insured would not even know the magnitude of the loss *after the fact* (e.g., a spouse's lost earnings over the remainder of his or her career, or the value of a piece of jewelry that had not been appraised recently). In these cases, there may be a de facto "profit" earned after a loss, but it is unclear that the insured would be aware of it. Nonetheless, the fact that the insurance payment may exceed the loss in some instances represents an interesting departure from other forms of insurance.

rence of a loss. Differentiating Equation (2) with respect to R and using Equation (1) yields the first-order condition (FOC)[6]

$$V_R \equiv \frac{\partial V}{\partial R} = -p(1-p)(1+\lambda)U'(W_0 - p(1+\lambda)R) + p(1-p(1+\lambda))EU'(W_0 - p(1+\lambda)R - \tilde{L} + R) = 0. \quad (3)$$

Observe that for the FOC to hold, $p\lambda$ must be sufficiently small that $(1 - p(1+\lambda)) > 0$; no coverage is purchased otherwise.

In what follows, we assume that the insurance market is perfectly competitive and use Equation (3) to characterize the equilibrium properties of fixed-reimbursement insurance contracts.

BASIC PROPERTIES AND COMPARATIVE STATICS

Optimal Level of Coverage

First, note that "full insurance," i.e., $R(L) = L$ for all L, is not possible since by assumption R cannot be conditioned on the realization of the loss, L. If $\lambda = 0$, so that coverage is provided at an actuarially fair price, the FOC of Equation (3) simplifies to

$$U'(W_0 - pR) = EU'\big(W_0 - pR - \tilde{L} + R\big). \quad (4)$$

Readers familiar with the literature on saving choices under income risk will immediately notice that the first-order condition of Equation (4) in our problem has the same structure as the first-order condition in the two-date saving problem (with additively separable preferences). In fact, fixed-reimbursement insurance sold at an actuarially fair price more closely mimics the role of precautionary savings in intertemporal saving models than the role of either coinsurance or deductibles in classical insurance models.[7] It is therefore not surprising that prudence and related concepts, which were initially developed to analyze saving decisions (see Kimball, 1990a, 1990b), will also play an important role in this article.

Using the precautionary (equivalent) premium proposed by Kimball (1990a), we can rewrite Equation (4) as

$$U'(W_0 - pR) = U'(W_0 - pR - E\tilde{L} + R - \Psi(c,\tilde{L})), \quad (5)$$

where $c \equiv W_0 - pR - E\tilde{L} + R$ is the expected wealth when a loss occurs. Given the assumption of strict concavity, $U'' < 0$, it follows that

$$R^* = E\tilde{L} + \Psi(c,\tilde{L}). \quad (6)$$

Note that the optimal reimbursement, R^*, is the solution to an implicit equation because it appears on both sides of the equality in Equation (6); on the right-hand side (RHS), it is embedded in the expected wealth term, c.

[6] It is straightforward to verify that the second-order condition will hold for all levels of R.

[7] It bears mentioning that, despite the similarities, the two concepts are not equivalent because precautionary saving does not involve cross-subsidization with other agents.

Kimball (1990a) shows that the precautionary premium is positive if and only if the policyholder exhibits prudence, i.e., the policyholder's marginal utility function is convex, $U''' > 0$. This leads to the following result.

Proposition 1: *Under actuarially fair pricing, the optimal reimbursement R^* satisfies the inequality $R^* > E\tilde{L}$ if and only if the policyholder is prudent ($U''' > 0$).*

Proposition 1 provides a new application of the prudence concept: a policyholder is prudent if and only if, under actuarially fair fixed-reimbursement insurance, the policyholder's optimal reimbursement is higher than the expected loss. The difference between the two, $(R^* - E\tilde{L})$, plays the same role as precautionary savings in the two-date saving model.

In the presence of a positive loading charge, $\lambda > 0$, a similar analysis based on Equation (3) leads to the inequality

$$R^* < E\tilde{L} + \Psi(c,\tilde{L}).$$

Thus, it is not possible to say whether the reimbursement will be above or below the expected loss when premiums are subject to positive loading. However, from the strict concavity of the utility function, it is easy to show that the FOC of Equation (3) evaluated at $R = \bar{L}$ satisfies $V_R|_{R=\bar{L}} < -p\lambda U'(W_0 - p(1+\lambda)\bar{L})$.

This implies that the optimal reimbursement R^* is strictly lower than the largest possible loss $\bar{L}$ for any loading factor $\lambda \geq 0$.

Increases in Risk Aversion and Increases in Prudence

For the case where insurance is actuarially fair, the following proposition provides a necessary and sufficient condition for a change in the policyholder's attitude toward risk to increase the optimal reimbursement.

Proposition 2: *Consider a change in risk preferences from U_1 to U_2. Under actuarially fair pricing, this increases the optimal reimbursement if and only if U_2 is more prudent than U_1.*

The Appendix shows the proof of this proposition. Once again, the fixed indemnity mimics the role of precautionary savings, and therefore, a more prudent policyholder will choose a larger reimbursement. Since greater risk aversion may lead to either higher or lower prudence (Eeckhoudt and Schlesinger, 1994), an increase in risk aversion in the sense of Arrow and Pratt may lead to either an increase or a decrease in the optimal reimbursement. This is in contrast to (unfair) policies based on coinsurance or deductibles, where a more risk-averse policyholder always selects a higher level of coverage (see Schlesinger, 1981, 2000). The only exception is when the utility function exhibits constant absolute risk aversion (CARA). In that case, the index of absolute prudence, $-u'''/u''$, is equal to the (constant) index of absolute risk aversion, $-u''/u'$. Thus, under CARA, an increase in risk aversion, which is equivalent to an increase in prudence, will lead to a larger reimbursement in the loss states. However, this result does not extend to the case where a positive loading charge is imposed.

Increases in Initial Wealth

Suppose first that fixed-reimbursement insurance is sold at an actuarially fair price. We can derive the effect of a change in initial wealth on the optimal reimbursement by totally differentiating the condition in Equation (6) with respect to R^* and W_0:

$$\frac{dR^*}{dW_0} = \frac{\Psi'\left(c^*,\tilde{L}\right)}{\left[1-(1-p)\,\Psi'\left(c^*,\tilde{L}\right)\right]}, \tag{7}$$

where $c^* \equiv W_0 - pR^* + R^* - E\tilde{L}$. Since $\Psi' < 1$, $dR^*/dW_0 < 0$ if and only if Ψ is decreasing in c for all c, or equivalently, from Kimball (1990a), if and only if the utility function U exhibits decreasing absolute prudence (DAP).[8,9] This leads to the following proposition:[10]

Proposition 3: *Under actuarially fair pricing, fixed-reimbursement insurance is an inferior good if and only if preferences satisfy decreasing absolute prudence.*

Gollier (2001, Proposition 21) derives an important link between risk aversion and prudence. He shows that if absolute prudence is uniformly decreasing in wealth, then absolute risk aversion must also be uniformly decreasing in wealth. As a consequence, decreasing absolute risk aversion (DARA) is only a necessary condition to induce a wealthier policyholder to select a lower fixed reimbursement under fair insurance. This differs from the standard result obtained in the two-date saving model, where the strict concavity of the utility function is a necessary and sufficient condition for an increase in first-period wealth to increase precautionary saving. This difference exists because in our model, changes in initial wealth affect both the loss and no-loss states. As a result, wealth appears in both terms of the first-order condition for our problem (Equation [3]), while future income only appears in one term of the first-order condition for the saving problem. This makes the comparative static analysis more difficult in the present case and necessitates our making more specific assumptions concerning the properties of the utility function.

Next, we reexamine this result when fixed reimbursement insurance is sold at an unfair price, i.e., when the insurance loading factor λ is positive. From the first-order condition in Equation (3) and the implicit function theorem, we have

$$\begin{aligned}\frac{dR^*}{dW_0} &= -\frac{V_{RW_0}}{V_{RR}}\\ &= -\frac{1}{V_{RR}}p(1-p)(1+\lambda)U'(A)\left[r(A) - \left(1-\Psi'(B,\tilde{L})\right) r\left(B-\Psi(B,\tilde{L})\right)\right],\end{aligned} \tag{8}$$

[8] The condition $\Psi' < 1$ follows from risk aversion and the definition of the precautionary premium. It is needed here to establish that DAP is a necessary condition for fixed-reimbursement insurance to be an inferior good.

[9] Eeckhoudt and Kimball (1992) argues that DAP is a reasonable assumption in most contexts. In particular, he offers the following thought experiment in support of the assumption: "Consider a college professor who has $10,000 in the bank and a Rockefeller who has a net worth of $10,000,000, who have the same preferences except for their differences in initial wealth. If each is forced to face a coin toss at the beginning of next year, with $5,000 to be gained or lost depending on the outcome, which one will do more extra saving to be ready for the possibility of losing? If one's answer is that the college professor will do more extra saving, it argues for decreasing absolute prudence" (1992, p. 241-242).

[10] Decreasing absolute prudence means that $U_2(w)$ is more prudent than $U_1(w) = U_2(w+\varepsilon)$ for all $\varepsilon > 0$. We can thus view Proposition 3 as a particular case of Proposition 2.

where $A \equiv W_0 - p(1+\lambda)R$, $B \equiv W_0 - p(1+\lambda)R - E\tilde{L} + R$, and $r(w) = -U''(w)/U'(w)$. Since $A > B - \Psi\left(B,\tilde{L}\right)$ under prudence, and $\Psi' < 0$ under DAP, the bracketed term on the RHS of Equation (8) is negative (because DAP implies DARA). Consequently, $dR^*/dW_0 < 0$ under DAP. Decreasing absolute prudence appears to be a sufficient, but not necessary, condition for a wealthier policyholder to choose a lower fixed reimbursement when insurance is not actuarially fair. Proposition 4 formally states this result.

Proposition 4: *With a positive insurance loading, fixed-reimbursement insurance is an inferior good if preferences satisfy decreasing absolute prudence.*

Under DAP, the optimal reimbursement is a decreasing function of wealth. This is in contrast to policies based on coinsurance or deductibles, where DARA is a sufficient condition for wealthier agents to purchase less insurance coverage (see, for example, Mossin, 1968; Schlesinger, 1981). Under CARA, which is equivalent to constant absolute prudence, a change in initial wealth affects neither the optimal fixed reimbursement nor the optimal deductible or coinsurance rate.

Effects of Changes in the Loss Distribution (First-Order Stochastic Dominance Shifts)

First-order stochastic dominance (FSD) shifts in the loss distribution illustrate another major difference between coinsurance and fixed-reimbursement insurance. With coinsurance, an FSD deterioration in the loss distribution always induces an increase in the premium paid by the insured for any (strictly positive) coverage level he or she selects.[11] This is no longer true with fixed-reimbursement insurance. Here, an FSD deterioration can take two forms: either an increase in p with $F(L)$ unchanged, or an FSD deterioration of $F(L)$ for a given p. These two changes have different implications for the premium paid by the insured. If p increases, the premium will increase at each R, while if $F(L)$ deteriorates (for a given p), the premium does not change since the reimbursement is fixed. Because this last change has no "premium effect," its comparative static analysis is much easier than that of a change in p, as we will see.

Changes in the Probability of Loss. Consider first the case where the fixed-reimbursement insurance policy is sold at an actuarially fair price. Totally differentiating Equation (6) with respect to R^* and p and rearranging terms yields

$$\frac{dR^*}{dp} = -\frac{R^*\Psi'\left(c^*,\tilde{L}\right)}{\left[1-\left(1-p\right)\Psi'\left(c^*,\tilde{L}\right)\right]}\,, \tag{9}$$

where $c^* \equiv W_0 - pR^* + R^* - E\tilde{L}$. Since $\Psi' < 1$, and $R^* > 0$ under fair insurance, the optimal reimbursement increases with the probability of loss if and only if $\Psi' < 0$ or, equivalently, U exhibits DAP.

Proposition 5: *Under actuarially fair pricing, the policyholder responds to an increase in the probability of loss by increasing his or her reimbursement if and only if his or her preferences satisfy DAP.*

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[11] Assuming, as we have throughout, that insurers and insureds have the same information about the distribution of losses.

Suppose that the insurance loading is positive, $\lambda > 0$. Using the first-order condition in Equation (3) and Equation (8), it is easy to show that

$$\frac{dR^*}{dp} = -\frac{V_{Rp}}{V_{RR}}$$
$$= -\frac{1}{V_{RR}}(1+\lambda)\left[U'(A) - U'\left(B - \Psi\left(B,\tilde{L}\right)\right)\right] - (1+\lambda)R^*\frac{dR^*}{dW_0}. \tag{10}$$

The first term on the RHS of Equation (10) is the direct effect; it is negative because $A > B - \Psi(B,\tilde{L})$ and the policyholder is strictly risk averse and prudent. The second term is the wealth effect. From Proposition 4, it is negative under DAP. As a consequence, the total effect is ambiguous, except when preferences satisfy CARA, since wealth effects are absent under CARA. In that case, as stated in the following proposition, an increase in the probability of loss *decreases* the optimal reimbursement.

Proposition 6: *With a positive insurance loading, the policyholder responds to an increase in the probability of loss by decreasing his reimbursement if his preferences satisfy CARA.*

Changes in the Conditional Loss Distribution. We next examine how a deterioration in the conditional loss distribution affects the optimal reimbursement. We define an increase in risk by a change in the cumulative distribution function from F_1 to F_2, with associated random variables $\tilde{L}_1$ and $\tilde{L}_2$. Let R_i^* denote the solution of the FOC in Equation (3) under the conditional loss distribution F_i, $i = 1,2$. It thus satisfies $V_R(R_i^*,F_i) = 0$ for $i = 1,2$. In order to compare R_1^* and R_2^*, we evaluate V_R at R_1^* under the cumulative distribution F_2. Using $V_R(R_1^*,F_1) = 0$, this yields

$$V_R\left(R_1^*,F_2\right) = p\left[1 - p(1+\lambda)\right]\left[EU'\left(W_0 - p(1+\lambda)R_1^* - \tilde{L}_2 + R_1^*\right)\right.$$
$$\left. - EU'\left(W_0 - p(1+\lambda)R_1^* - \tilde{L}_1 + R_1^*\right)\right], \tag{11}$$

which can be rewritten as

$$V_R\left(R_1^*,F_2\right) = p\left[1 - p(1+\lambda)\right]\int_{\underline{L}}^{\bar{L}} U'\left(W_0 - p(1+\lambda)R_1^* - L + R_1^*\right)dS(L), \tag{12}$$

where $S(L) \equiv F_2(L) - F_1(L)$.

Consider first the case where $\tilde{L}_2$ is dominated by $\tilde{L}_1$ in the sense of FSD, i.e., $S(L) \geq 0$ for all L, with at least one strict inequality. Since the random variable is a loss, the shift from F_1 to F_2 represents a favorable change in the distribution of potential losses. Integrating Equation (12) by parts yields

$$V_R\left(R_1^*,F_2\right) = p\left[1 - p(1+\lambda)\right]\int_{\underline{L}}^{\bar{L}} U''\left(W_0 - p(1+\lambda)R_1^* - L + R_1^*\right)S(L)dL. \tag{13}$$

This implies that $V_R(R_1^*,F_2)$ is negative under risk aversion and, because V is concave in R, $R_2^* < R_1^*$. We state this result formally below.

Proposition 7: *An FSD deterioration in the conditional loss distribution increases the optimal reimbursement as long as the policyholder is risk averse ($U'' < 0$).*

This result differs from what one finds in the case of deductible or coinsurance, where additional restrictions on either the utility function or the FSD change in risk are required to obtain a determinate result (see Gollier and Eeckhoudt, 2000, for a survey on the effects of changes in risk on risk-taking behavior). Matters are much simpler for fixed-reimbursement insurance because, as discussed earlier, a change in the conditional loss distribution leaves the premium unchanged. Note that a result similar to Proposition 7 arises in the two-period saving model, where the FSD change applies to the distribution of income in the second period.

A Mean-Preserving Increase in Risk. Consider a mean-preserving increase in the loss risk (MPIR) from $\tilde{L}_1$ to $\tilde{L}_2$, i.e., $\int_{\underline{L}}^{L} S(l)dl \geq 0$ for all L, with at least one strict inequality, and $\int_{\underline{L}}^{\bar{L}} S(l)dl = 0$. Integrating Equation (13) by parts yields

$$V_R\left(R_1^*,F_2\right) = p\left[1 - p(1+\lambda)\right] \int_{\underline{L}}^{\bar{L}} \left\{ U'''\left(W_0 - p(1+\lambda)R_1^* - L + R_1^*\right) \int_{\underline{L}}^{L} S(l)dl \right\} dL. \quad (14)$$

This implies that $V_R(R_1^*,F_2)$ is positive and therefore that $R_2^* > R_1^*$ for all prudent policyholders, $U''' > 0$. This result can be stated as follows.

Proposition 8: *A mean-preserving increase in the loss risk increases the demand for fixed-reimbursement insurance as long as the policyholder is prudent ($U''' > 0$).*

The simple contract structure associated with fixed-reimbursement insurance permits us to sign the effect of an MPIR using weaker assumptions than are required when assessing the effect of a change in risk for either deductible or coinsurance. In the case of deductible insurance, we must restrict the change in risk to various subsets of the loss distribution, i.e., to above or below the optimal deductible for the original distribution, to obtain determinate results (Eeckhoudt et al., 1991).

In the case of coinsurance, one can obtain results by imposing assumptions on either the utility function or the change in risk. For example, Gollier and Eeckhoudt (2000) show that a necessary and sufficient condition for the optimal level of coinsurance coverage to rise following an MPIR is that relative prudence should be positive and less than 2. Alternatively, Alarie et al. (1992) show that restricting attention to a *relatively strong increase in risk* (a special case of a mean-preserving spread)[12] is a sufficient condition for an increase in risk to yield greater insurance coverage. We obtain simpler restrictions than in the coinsurance problem because under fixed-reimbursement insurance the indemnity is not a function of the loss.

Interestingly, the same result holds for precautionary saving when utility is additively separable. To see why, consider the first-order condition for the two-period saving problem.

[12] See Black and Bulkley (1989) for a discussion of relatively strong increases in risk.

$$U'(c_1) = \int U'(y_1 - c_1 + y_2)dF(y_2). \tag{15}$$

If F undergoes a mean-preserving increase in risk, the RHS of Equation (15) will increase as long as U' is convex. To maintain the equality, $U'(c_1)$ must increase as well, implying a decrease in c_1 and an increase in saving. Hence, in this case, precautionary saving behaves exactly as fixed-reimbursement insurance.

CONCLUSIONS

Insurance contracts that specify a lump-sum payment in the event of a loss are relatively common; examples include term life insurance, flight insurance, jewelry insurance, and, in some cases, postal insurance. We have shown that such contracts have interesting properties that differ from those based on coinsurance or deductibles. Indeed, fixed-reimbursement insurance protects the insured against risks in the same way that precautionary saving protects the consumer against fluctuations in future income. As a result, the concept of prudence—initially developed by Kimball (1990a, 1990b) to analyze the saving decision—turns out to be quite important for studying this form of insurance. Because by its nature fixed-reimbursement insurance more closely resembles precautionary saving than deductible or coinsurance, it is not surprising that the comparative static properties of fixed-reimbursement insurance contracts are quite different from the analogous properties possessed by other types of insurance policies. These differences, which apply to such diverse factors as risk aversion, prudence, wealth, and changes in the distribution of losses, are the subject of this article.

Although our article is fundamentally concerned with insurance, it appears that some of our results might be relevant for the analysis of certain financial products, such as weather derivatives. Indeed, for these products the true damage is hard to monitor, and the payment is not as strongly correlated with the damage as it is under coinsurance or deductible contracts. For example, if a fixed payment was made to a farmer as soon as the temperature fell below some threshold, then weather derivatives would precisely mimic a fixed-reimbursement insurance contract. As such, the relationship between weather derivatives and fixed-reimbursement insurance is a natural topic for future research. Another natural extension would be disability insurance, which differs from other insurance arrangements based on fixed indemnities due to the explicit conditioning of insurance payments on a variable (past earnings) that is potentially correlated with the realized loss. Finally, many fixed-reimbursement insurance policies arise in settings that are naturally cast within a state-dependent utility framework. Although technically difficult, extending the present article in that direction is also a potential area for future study.

APPENDIX

Proof of Proposition 2: Consider the following function:

$$\Phi_u(R) = R - E\tilde{L} - \Psi_u(W_0 - pR - E\tilde{L} + R, \tilde{L}), \tag{A1}$$

where Ψ_u is the precautionary premium for a prudent policyholder with the utility function u. Its first derivative satisfies $\Phi'_u(R) = 1 - (1-p)\Psi'_u(W_0 - pR - E\tilde{L} + R, \tilde{L}) > 0$ because $\Psi'_u < 1$. From Equation (6), the optimal reimbursement R_1 for a policyholder with utility function U_1 satisfies $\Phi_{U_1}(R_1) = 0$. Consider a second agent with utility function U_2 who is more prudent than the first agent. The optimal reimbursement for the second policyholder satisfies $\Phi_{U_2}(R_2) = 0$. From Kimball (1990a), this means that the second policyholder's precautionary premium $\Psi_{U_2}(w,\tilde{L})$ is larger than $\Psi_{U_1}(w,\tilde{L})$ for all $(w,\tilde{L})$. The following four conditions are therefore equivalent:

1. $R_2 > R_1$,
2. $\Phi_{U_2}(R_1) < \Phi_{U_2}(R_2) = 0 = \Phi_{U_1}(R_1)$ (because Φ'_{U_2} is increasing),
3. $\Psi_{U_2}(W_0 - pR_1 - E\tilde{L} + R_1,\tilde{L}) > \Psi_{U_1}(W_0 - pR_1 - E\tilde{L} + R_1, \tilde{L})$, and
4. The agent with utility function U_2 is more prudent than the agent with utility function U_1.

References

Alarie, Y., G. Dionne, and L. Eeckhoudt, 1992, Increases in Risk and the Demand for Insurance, in: G. Dionne, ed., *Contributions to Insurance Economics*, Huebner International Series on Risk, Insurance and Economic Security (Norwell, Mass., and London: Kluwer Academic).

Arrow, K., 1965, Aspects of the Theory of Risk Bearing, Yrjo Jahnsson Lectures, Helsinki. Reprinted in 1971, *Essays in the Theory of Risk Bearing* (Chicago: Markham).

Bernheim, D., L. Forni, J. Gokhale, and L. Kotlikoff, The Mismatch Between Life Insurance Holdings and Financial Vulnerabilities: Evidence from the Health and Retirement Study, *American Economic Review*, forthcoming.

Black, J., and G. Bulkley, 1989, A Ratio Criterion for Signing the Effects of an Increase in Uncertainty, *International Economic Review*, 30: 119-130.

Briys, E., G. Dionne, and L. Eeckhoudt, 1989, More on Insurance as a Giffen Good, *Journal of Risk and Uncertainty*, 2: 415-420.

Eeckhoudt, L., and M. Kimball, 1992, Background Risk, Prudence, and the Demand for Insurance, in: G. Dionne, ed., *Contributions to Insurance Economics*, Huebner International Series on Risk, Insurance and Economic Security (Norwell, Mass., and London: Kluwer Academic).

Eeckhoudt, L., and H. Schlesinger, 1994, A Precautionary Tale of Risk Aversion and Prudence, in: B. Munier and M. Machina, eds., *Models and Experiments in Risk and Rationality* (The Netherlands: Kluwer Academic).

Eeckhoudt, L., C. Gollier, and H. Schlesinger, 1991, Increases in Risk and the Demand for Deductible Insurance, *Journal of Economic Theory*, 55: 435-440.

Gollier, C., 1996, Optimum Insurance of Approximate Losses, *Journal of Risk and Insurance*, 63: 369-380.

Gollier, C., 2001, *The Economics of Risk and Time* (Cambridge, Mass.: MIT Press).

Gollier, C., and L. Eeckhoudt, 2000, The Effects of Changes in Risk on Risk Taking: A Survey, in: G. Dionne, ed., *Handbook of Insurance*, Huebner International Series on Risk, Insurance and Economic Security (Norwell, Mass., and London: Kluwer Academic).

Karni, E., 1983, Risk Aversion for State-Dependent Utility Functions: Measurement and Applications, *International Economic Review*, 24: 637-647.

Kelsey, D., and G. Nordquist, 1991, A More General Measure of Risk Aversion When Utility is State-Dependent, *Oxford Economic Papers*, 43: 59-74.

Kimball, M., 1990a, Precautionary Saving in the Small and in the Large, *Econometrica*, 58: 53-73.

Kimball, M., 1990b, Precautionary Saving and the Marginal Propensity to Consume, Department of Economics, University of Michigan, Mimeo.

Mossin, J., 1968, Aspects of Rational Insurance Purchasing, *Journal of Political Economy*, 76: 553-568.

Raviv, A., 1979, The Design of an Optimal Insurance Policy, *American Economic Review*, 69: 84-96.

Schlesinger, H., 1981, The Optimal Level of Deductibility in Insurance Contracts, *Journal of Risk and Insurance*, 48: 465-481.

Schlesinger, H., 2000, The Theory of Insurance Demand, in: G. Dionne, ed., *Handbook of Insurance*, Huebner International Series on Risk, Insurance and Economic Security (Norwell, Mass., and London: Kluwer Academic).

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