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On the Possibility of a Private Crop Insurance Market: A Spatial Statistics Approach

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Abstract

Risk theory tells us if an insurer can effectively pool a large number of individuals to reduce total risk, the insurer can then provide insurance by charging a premium close to the actuarially fair rate. However, a common belief exists that risk can be effectively pooled only when random loss is independent. Therefore crop insurance markets cannot survive without government subsidy because crop yields are not independent among growers. In this article, we take a spatial statistics approach to examine the effectiveness of risk pooling for crop insurance under correlation. We develop a method for evaluating the effectiveness of risk pooling under correlation and apply the method to three major crops in the United States: wheat, soybeans, and corn. The empirical study shows that yields for the three crops present zero or negative correlation when two counties are far apart, which complies with a weaker condition than independence, finite-range positive dependency. The results show that effective risk pooling is possible and reveal a high possibility of a private crop insurance market in the United States.

Crop insurance has been an important instrument for protecting farmers' income against low yields resulting from adverse weather and other natural disasters. Except for a few perils such as hail and fire, multiple peril crop insurance (MPCI) has only been offered by the U.S. government with a huge subsidy. MPCI was first introduced in 1938 on a trial basis and was extended in 1980 to most crops in the United States. MPCI pays an indemnity to a farmer based on the difference between a preselected coverage yield level and the farmer's realized yield level. For years, billions of dollars in financial deficits have been accumulated by the government to provide MPCI, and farmers' participation rate is still low. These problems have been attributed to moral hazard, adverse selection, and high administrative costs (Knight and Coble, 1997; Skees et al., 1997; Goodwin and Smith, 1995). To deal with these problems,

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a group risk plan (GRP) was introduced in 1994 that pays a farmer an indemnity only when the realized average yield of the county falls below the preselected coverage level. Evaluation of county yields instead of individual farm yields for indemnification greatly reduces the insurer's administrative costs. The disadvantage of GRP is that it does not protect farmers as effectively as MPCI when the farm yield and county yield are not highly correlated. In the late 1990s, revenue insurance programs were piloted that could protect farmers from both price and yield risks, including those that were based on farm yields such as income protection, crop revenue coverage, and revenue assurance, and those based on county average yields such as group revenue insurance programs.

To cope with the actuarial problems with federal crop insurance programs, economists currently advance three lines of study in the area of agricultural risk protection. The first line is to investigate the theory of area-based insurance and the risk protection effectiveness of GRP (Miranda, 1991; Wang et al., 1998). The second line is to study revenue insurance, which takes advantage of usually negatively correlated price and yield risks and focuses on stabilizing overall income (Hennessy et al., 1997; Skees et al., 1998). The third line is to study market instruments that are based on natural conditions, such as weather derivatives, which also deal with the moral hazard problem (Turvey, 1999). The first two lines focus on current government programs, and the last line fails to address the question of whether it is feasible for private insurers to provide agricultural insurance.

Although positive correlation among farm yields is the basis for GRP and weather derivatives, it is perhaps the major factor that has discouraged the consideration of private crop insurance. It is a common belief that effective risk pooling is built upon independence between risk exposure units and that a private market for crop insurance is doomed to fail because of the systemic risk existing in crop yield (Miranda and Glauber, 1997). However, this belief has not been thoroughly studied. Therefore, it is important to carefully investigate whether private agricultural insurance and reinsurance markets can exist with or without a minimum government subsidy, what conditions are required, and whether these conditions are present in the current situation. The objectives of this research are to (1) explore necessary statistical conditions for effective risk pooling, (2) investigate the pattern of U.S. crop yields' correlation, and (3) develop a method to evaluate the effectiveness of risk pooling under correlation and apply it to U.S. crops. We pursue each of the objectives in one of the following three sections, and the last section consists of a summary and discussion.

STATISTICAL FOUNDATIONS OF INSURANCE

The primary function of insurance is risk pooling. Mehr et al. (1985) offer the following definition, "Insurance may be defined as a device for reducing risk by combining a sufficient number of exposure units to make their individual losses collectively predictable" (1985, p. 32). In the insurance literature, the occurrence of an aggregate loss that is so large as to deplete the insurance fund is captured by the concept of ruin. Researchers have suggested that a possible objective criterion for the management of an insurance pool is to minimize the probability of ruin in a given time period or perhaps maximize returns subject to maintaining a specified probability of ruin (Bühlmann, 1970). These are the Safety-First criteria developed by Roy (1952) and Telser (1956). The aggregated premium surplus above the expected value of the aggregate loss required to maintain a particular probability of ruin is referred to as the buffer fund. In what follows, we will consider the statistical foundations of insurance based upon the framework provided by Cummins (1991) while incorporating the spatial characteristics of crop insurance.

We now review some basic concepts regarding premium rates. Premium rate setting generally begins with the concept of pure or net premiums (Hogg and Klugman, 1984; Borch, 1974; Goodwin and Smith, 1995). The net premium is simply the expected indemnity per exposure unit. The gross premium, the amount paid by the insured per exposure unit in order to be eligible for coverage, is larger than the net premium by an amount referred to as the premium loading factor. We can examine the components of the gross premium by decomposing the gross premium into three parts, $P = P_N + A + L$, where P is the gross premium, P_N is the net premium, A is the administrative cost load, L is the buffer load, and $A + L$ is the premium loading factor.

Let $Y(\boldsymbol{s},t)$ be the yield at the tth year of the exposure unit with a two-dimensional spatial index $\boldsymbol{s}$ representing its location, and $c(\boldsymbol{s},t)$ is the prespecified coverage level. The exposure unit will experience a production loss in the amount of $X(\boldsymbol{s},t) = \max\{0, c(\boldsymbol{s},t) - Y(\boldsymbol{s},t)\}$, which is also the indemnity payment for this unit at year t if we assume that price is normalized to be unitary. Once a model on the yield $Y(\boldsymbol{s},t)$ is built, the distribution of $Y(\boldsymbol{s},t)$ is then specified, from which we can obtain the distribution of $X(\boldsymbol{s},t)$ and hence the mean loss $E(X(\boldsymbol{s},t))$ for any given prespecified value $c(\boldsymbol{s},t)$. The total loss of N exposure units at year t is therefore

$$S_N(t) = \sum_{i=1}^{N} X(\boldsymbol{s}_i,t), \tag{1}$$

where i denotes the ith exposure unit and $\boldsymbol{s}_i$ is the spatial location of the ith unit. The model given by Equation (1) conceptualizes each individual loss as a random variable, and the total loss experienced by the pool is random as well. The expected total loss of the pool at year t is: $E(S_N(t)) = \sum_{i=1}^{N} E(X(\boldsymbol{s}_i,t))$. We need to obtain the variance of the total loss to know how much the actual total loss, $S_N(t)$, can differ from the expected total loss. The variance of the total loss at year t is:

$$Var(S_N(t)) = \sum_{i=1}^{N} Var(X(\boldsymbol{s}_i,t)) + 2\sum_{i=1}^{N}\sum_{j=i+1}^{N} Cov(X(\boldsymbol{s}_i,t), X(\boldsymbol{s}_j,t)). \tag{2}$$

Under some regularity conditions, the following central limit theorem holds for a spatial process

$$\frac{S_N(t) - E(S_N(t))}{\sqrt{Var(S_N(t))}} \sim N(0,\, 1). \tag{3}$$

See, for example, Bolthausen (1982) for stationary random fields and Guyon (1995, p. 112, Theorem 3.3.1) for nonstationary random fields. Therefore,

$$P\left(|S_N(t) - E(S_N(t))| < 1.96\sqrt{Var(S_N(t))} \right) \approx 0.95. \tag{4}$$

In this case, the buffer fund needs to be $E(S_N(t)) + 2\sqrt{Var(S_N(t))}$ in order to maintain a 0.025 probability of ruin for year t.

As mentioned previously, we can compute $E(S_N(t))$ once we model the yield $Y(\mathbf{s},t)$ and know and all the prespecified coverage levels. We can also calculate $E(S_N(t))$ if the prespecified levels are unknown but have some known probability distribution. Calculating $Var(S_N(t))$ is more cumbersome unless individual losses are independent. In addition, the losses are truncated and spatially dependent random variables. Even though some methods in spatial statistics may seem appropriate and applicable here, it is difficult to directly apply the methods because losses are generally nonstationary unless we put some unrealistic assumptions of the prespecified levels $c(\mathbf{s},t)$. In this article, we propose an approach to overcome this difficulty.

In order to gain some insight to the risk pooling theory, we first consider the simplest case, when losses are independently and identically distributed.

An Example—The i.i.d. Case

Even though the assumption of i.i.d. losses is unnecessarily strong and generally not realistic (Bühlmann, 1970), results are much simplified under these assumptions and can help us see some key issues in risk pooling. When the losses are i.i.d,

$$E(S_N(t)) = N\mu, \quad Var(S_N(t)) = N\sigma^2, \tag{5}$$

where μ and σ^2 are the mean and variance of loss from each exposure unit, respectively. The central limit theorem implies that, when N is large, $S_N(t)$ is approximately normally distributed and thus, for any $0 < \alpha < 1$,

$$P(|S_N(t) - N\mu| < z_{\alpha/2}\sqrt{N}\sigma) \approx 1 - \alpha,$$

where $z_{\alpha/2}$ is the positive value that the standard normal random variable exceeds with a probability of $\alpha/2$. For $\alpha = 0.05$, the corresponding value of $z_{\alpha/2}$ is 1.96. Hence, to maintain a probability of ruin of no more than 0.025, the liquid buffer fund needs to be approximately $1.96\sigma\sqrt{N}$ for N individuals. The buffer load L for each individual unit is $1.96\sigma/\sqrt{N}$, which decreases as the number of exposure units N increases.

If we assume economies of scale in the administration function, then the administrative cost per exposure unit, A, will decline as the size of the insurance pool grows. Thus, for a sufficiently large insurance pool, the premium loading $A + L$ will not be large, i.e., the risk premium that a risk-averse insured must pay to obtain coverage will be small.

From this example, we see that a critical ingredient for risk pooling is that the standard deviation of the average loss, i.e., $\sigma/\sqrt{N}$ in the i.i.d. case, diminishes as the size of the insurance pool increases. Under this condition, the buffer load also declines as the pool grows larger, thus ensuring (assuming economies of scale in administration) that the premium loading is relatively small. As we will see in the following, these properties may be retained when losses are not independent.

Finite-Range Positive Dependency of Spatial Variables

We next explicitly relax the assumption of independence. One can generalize many statistical results that hold under the i.i.d assumption into the dependence case. For example, under some mixing and regularity conditions, the central limit theorem of

Equation (3) holds for the spatial process $X(\boldsymbol{s},t)$ (see, e.g., Guyon, 1995 p. 112 Theorem 3.3.1), where the mixing conditions ensure that the dependency dies off sufficiently quickly as the lag distance increases for spatial process $X(\boldsymbol{s},t)$.

In the context of crop insurance, although an established insurance company can use reserve funds from earlier years' premium surplus to pay for a particular year's indemnity, the discussion of effective risk pooling still focuses on the loss in any particular year. Therefore, it is particularly useful to investigate how yields or yield losses are spatially correlated for a given year. For example, how likely is it that the majority of the insured experience a loss in a particular year, in which case the insurer might be jeopardized? It seems that the spatial statistics approach has not been employed in the study of crop insurance, although spatial statistics has been successfully used in agriculture for other purposes (e.g., Mercer and Hall, 1911; Besag and Higdon, 1999, both for uniformity trials; Roberts et al., 2000, for precision agriculture; and Zhang, 2002, for an application in plant pathology).

We first review some terminology and theories in spatial statistics. We refer readers to Cressie (1993, Chapter 2) for more details on spatial statistics. A spatial process, $X(\boldsymbol{s})$, is second-order stationary if the mean is a constant and for any vectors $\boldsymbol{s},\boldsymbol{h}$,

$$Cov(X(\boldsymbol{s}),X(\boldsymbol{s}+\boldsymbol{h})) = C(\boldsymbol{h}),$$

where $C(\cdot)$ is called the covariogram and $\boldsymbol{h}$ is called the lag. Therefore, the covariogram measures the correlation structure of the spatial process. Here $X(\boldsymbol{s})$ refers to a general spatial process and should not be confused with $X(\boldsymbol{s},t)$, the yield loss. Using data $X(\boldsymbol{s}_i), i = 1,\ldots,N$, we can estimate the covariogram by:

$$\hat{C}(\boldsymbol{h}) = (1/|N(\boldsymbol{h})|) \sum_{(i,j)\in N(\boldsymbol{h})} (X(\boldsymbol{s}_i) - \hat{\mu}_i)(X(\boldsymbol{s}_j) - \hat{\mu}_j) \tag{6}$$

where $N(\boldsymbol{h})$ is a set containing all pairs (i,j) such that $\boldsymbol{h} = \boldsymbol{s}_j - \boldsymbol{s}_i$ for a particular level of $\boldsymbol{h}$, $|N(\boldsymbol{h})|$ is the number of pairs in the set $N(\boldsymbol{h})$, and $\hat{\mu}_i$ and $\hat{\mu}_j$ are the estimated means of $X(\boldsymbol{s}_i)$ and $X(\boldsymbol{s}_j)$, respectively.

$X(\boldsymbol{s})$ is said to be isotropic if its covariogram $C(\boldsymbol{h})$ depends on distance $\|\boldsymbol{h}\|$ only but not on the direction, i.e., the covariogram is nondirectional. The covariogram is then a function of distance h and, following a conventional notation, we use $C(h), h > 0$ to denote the isotropic covariogram. Then we can estimate $C(h)$ by

$$\hat{C}(h) = (1/|N(h)|) \sum_{(i,j)\in N(h)} (X(\boldsymbol{s}_i) - \hat{\mu}_i)(X(\boldsymbol{s}_j) - \hat{\mu}_j) \tag{7}$$

where $N(h)$ is the set of all pairs $(\boldsymbol{s}_i,\boldsymbol{s}_j)$ such that the distance $\|\boldsymbol{s}_i - \boldsymbol{s}_j\|$ is h and $|N(h)|$ is the number of elements in $N(h)$. In reality, it is possible that $X(\boldsymbol{s}_i)$s have different means but still possess a stationary covariogram. In this case, we first remove the mean and then estimate the covariogram. Note that in practice, only a few pairs are exactly the same distance apart. Therefore, $N(h)$ is a set that contains all pairs $(\boldsymbol{s}_i,\boldsymbol{s}_j)$ so that the distance $\|\boldsymbol{s}_i - \boldsymbol{s}_j\|$ is approximately h. We refer to Journel and Huijbregts (1978, p. 194) and Cressie (1993, p. 70) for recommendation of the "tolerance" region.

Another measure of correlation for a spatial process is the correlogram, defined as the covariogram divided by the variance:

$$\rho(\boldsymbol{h}) = \frac{C(\boldsymbol{h})}{\sigma^2}, \tag{8}$$

where σ^2 is the common variance of each $X(\boldsymbol{s}_i)$. In almost all interesting situations, the correlation between any two points becomes weaker when the lag distance increases. A process is said to be finite-range dependent, or m-dependent (Davis and Borgman, 1982), if any two points, $X(\boldsymbol{s}_i)$ and $X(\boldsymbol{s}_j)$, are independent when the lag distance $\|\boldsymbol{s}_i - \boldsymbol{s}_j\| > m$ for some real positive number m. Finite-range dependence implies the mixing conditions that ensure the central limit theorem in Equation (3).

Here we define a spatial process, $X(\boldsymbol{s})$, to be finite-range positive dependent (f.r.p.d.) if the correlation between any two points, $X(\boldsymbol{s}_i)$ and $X(\boldsymbol{s}_j)$, is positive when the lag distance $\|\boldsymbol{s}_i - \boldsymbol{s}_j\| < d$ for some d and is zero or negative otherwise. This f.r.p.d. assumption seems quite sensible in light of the fact that the correlation between the yield losses of two fields caused by adverse weather will eventually vanish when the two fields are very far away from each other. We can give sufficient conditions under which the central limit theorem in Equation (3) holds for f.r.p.d. processes. These conditions must ensure that dependency quickly dies off (as does the correlation) when the lag distance increases. Discussion of these conditions is beyond the scope of this article. Note that in the definition of f.r.p.d, the spatial process does not need to be stationary.

For any location $\boldsymbol{s}_i$, denote by O_i the set of all locations whose distances from $\boldsymbol{s}_i$ are at most d, i.e., $O_i = \{\boldsymbol{s}_j : 0 \le \|\boldsymbol{s}_j - \boldsymbol{s}_i\| \le d\}$. If the process $X(\boldsymbol{s})$ is f.r.p.d. and the variance of $X(\boldsymbol{s})$ is bounded by some constant σ_m^2, we can place the following bound on the variance:

$$Var(\bar{X}_N) = (1/N^2)\left[\sum_{i=1}^{N} Var(X(\boldsymbol{s}_i) + \sum_{i=1}^{N}\sum_{j=1, j\neq i}^{N} C(\boldsymbol{s}_j - \boldsymbol{s}_i)\right] \tag{9}$$

$$\le (1/N^2)\left[N\sigma_m^2 + \sigma_m^2 \sum_{i=1}^{N}\sum_{j=1, j\neq i}^{N} \rho(X(\boldsymbol{s}_i), X(\boldsymbol{s}_j))\right]$$

$$\le \sigma_m^2/N + (\sigma_m^2/N^2)\sum_{i=1}^{N} |O_i| \tag{10}$$

$$\le (\sigma_m^2/N)\left(1 + \max_i(|O_i|)\right), \tag{11}$$

where $|O_i|$ denotes the number of elements in O_i. The inequality in Equation (10) holds because each correlation coefficient is no greater than one, the sum over j is no larger than the number of elements in set O_i under f.r.p.d., and the inequality in Equation (11) holds because each set cardinality $|O_i|$ is no larger than the largest set size.

If the spatial process $X(\boldsymbol{s})$ is f.r.p.d. with a finite-range d, then for any site $\boldsymbol{s}_i$ the corresponding set O_i has at most a finite number of elements. Indeed, if the exposure units (counties, farms, or plots) have a minimum area, a, then $\max_i(|O_i|) \le \pi d^2/a$, i.e., it is bounded by a constant that depends on d but not on N or i. We then have, in view of

Equation (10), $Var(\bar{X}_N) \rightarrow 0$, as N goes to infinity, i.e., the variance of $\bar{X}_N$ diminishes as N gets larger. If $X(\boldsymbol{s})$ represents the yield loss, this diminishment means that the buffer load per unit is very small for large N. Therefore, if a reasonably large number of units from a geographically large area participate in the insurance program, the premium charged to each participant can be very close to the actuarially fair level. As long as the farmer is risk-averse, he or she should be willing to pay for such insurance.

We have just shown that $Var(\bar{X}_N)$ vanishes as $N \rightarrow \infty$ for a f.r.p.d. process. This is an asymptotic result that assures us that $Var(\bar{X}_N)$ can be arbitrarily small if the sample size is sufficiently large. In reality, only a limited number of sample data are available. For a particular sample size, we may be able to evaluate $Var(\bar{X}_N)$ by directly applying Equation (9) to get a better bound than Equation (11).

If $X(\boldsymbol{s})$ represents the yield loss, the effectiveness of risk pooling depends much on how fast the variance of average loss diminishes. Several factors may affect this speed of diminishment: how fast the correlation of yields dies off when the lag distance increases, specified coverage levels, and range of dependence. Using some spatial statistics techniques, we show next how to calculate the variance of the average loss, as illustrated by U.S. crop yield data.

The Pattern of U.S. Crop Yield Dependence

Because crop yields are correlated due to weather effects, researchers believe that private insurance companies cannot effectively pool risks without significant subsidies from the government. We intend to show here that such a belief is not well grounded. We explore the spatial dependence pattern of U.S. crop yields for a given year to find out if the correlation between exposure units quickly dies off as two units become far apart, leaving enough room for risk pooling.

Data and Detrending

This analysis uses county-level crop yields from 1972 to 1997, provided by the National Agricultural Statistics Service, U.S. Department of Agriculture (USDA). This is because farm-level yield data are not available. We study the three major crops of corn, soybeans, and wheat. There are 2,591 counties for corn, 2,000 counties for soybeans, and 2,641 counties for wheat in the data set (we drop counties with less than five years of observations). We have obtained the centroid latitude and longitude of each county from the U.S. Census Bureau, U.S. Department of Commerce. We then transform these spherical references into plane coordinates using ArcInfo to apply them in the statistical software S-Plus (Mathsoft, Inc., Seattle), which has a spatial module. We remove the temporal trend first by location. For each location, we detrend yields by a log quadratic trend. Log quadratic trends are generally used for crop yields (Miranda and Glauber, 1997; Wang et al., 1998). Let $\xi(\boldsymbol{s},t)$ be the detrended yield, where $\boldsymbol{s}$ is the county centroid and t is the year. That is, $\xi(\boldsymbol{s},t)$ is the deviation of the yield from its mean yield at year t. The yield is modeled as

$$Y(\boldsymbol{s},t) = \exp\{\alpha(\boldsymbol{s}) + \beta(\boldsymbol{s})t + \gamma(\boldsymbol{s})t^2\} + \xi(\boldsymbol{s},t),$$

where α, β, and γ are location-specific parameters and the mean of the yield is $m(\boldsymbol{s},t) = E(Y(\boldsymbol{s},t)) = \exp(\alpha(\boldsymbol{s}) + \beta(\boldsymbol{s})t + \gamma(\boldsymbol{s})t^2)$, which is a log quadratic trend in the sense that $\ln(m(\boldsymbol{s},t))$ is quadratic in time. We fit the trend model for each location. It is very important for crop insurance to see how these $\xi(\boldsymbol{s},t)$s are correlated for a given year. We assume that the $\xi(\boldsymbol{s},t)$ is second-order stationary at each year with a variance, σ_t^2, across space.

Correlograms

Based on the detrended yields, $\xi(\boldsymbol{s},t)$, we calculate the covariogram and correlogram for each crop at each year from 1972 to 1997 using Equations (6) and (8). We calculate different directions, and the results suggest that it is reasonable to assume isotropy. Hence, we report the isotropic correlogram, which is a function of the distance, calculated as follows:

$$\hat{\rho}_t(h) = \frac{1}{|N(h)|\hat{\sigma}_t^2} \sum_{(i,j)\in N(h)} (\xi(\boldsymbol{s}_i,t) - \bar{\xi}_t)(\xi(\boldsymbol{s}_j,t) - \bar{\xi}_t),\ h > 0,$$

where $N(h)$ is the set of all pairs $(\boldsymbol{s}_i,\boldsymbol{s}_j)$ such that the distance $\|\boldsymbol{s}_i - \boldsymbol{s}_j\|$ is h, $|N(h)|$ is the number of elements in $N(h)$, $\bar{\xi}_t$ is the sample mean of $\xi(\boldsymbol{s}_i,t)$ for year t, and $\hat{\sigma}_t^2$ is the estimated variance of $\xi(\boldsymbol{s},t)$ for year t.

There are altogether 78 correlograms for the three crops and 26 years. For each crop, the correlograms of the 26 years show a consistent pattern. From these correlograms, we see that the positive dependency quickly dies off when the distance increases, i.e., the crop yields show a clear pattern of f.r.p.d. The distance for the positive dependency is at most 3×10^6 feet, or 570 miles, and in many instances, the range is much smaller than 570 miles. We only present the correlograms for 1981, 1991, and the average of 26 years for each of the three crops in Figure 1. Do the correlations die off sufficiently quickly to allow for effective risking pooling? We next propose a measure of effectiveness of risk pooling.

THE EFFECTIVENESS OF RISK POOLING

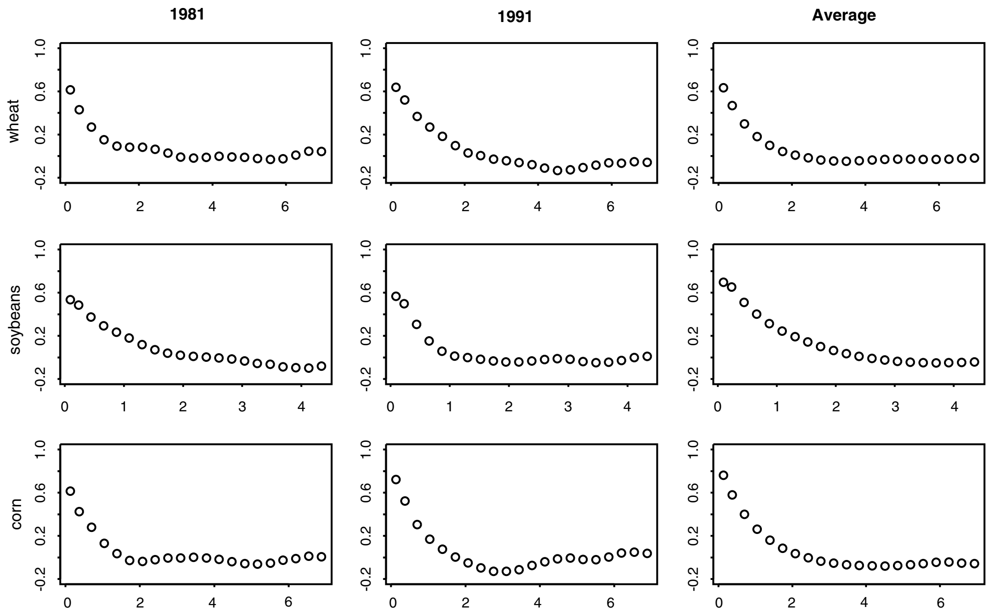
In this section, we introduce a measure of effectiveness for risk pooling and then calculate the premium loading factor for each of the three U.S. crops: wheat, soybeans, and corn.

A Measure of Risk Pooling Effectiveness

As we have pointed out, a critical ingredient for risk pooling is that the sample variance decreases as N increases. One way to measure the effectiveness of risk pooling is to see how small the variance of $\bar{X}_t$ is compared with the average variance of each individual $X(\boldsymbol{s},t)$, i.e., $Var(\bar{X}_t)/(\sum_{i=1}^N Var(X(\boldsymbol{s}_i,t))/N)$, where $\bar{X}_t = (1/N)\sum_{i=1}^N X(\boldsymbol{s}_i,t)$. We call the ratio, denoted by ϕ_t for year t, the coefficient of effectiveness, which can be expressed in terms of the covariances or correlations:

$$\begin{aligned}\phi_t &= \frac{Var(\bar{X}_t)}{\sum_{i=1}^N Var(X(\boldsymbol{s}_i,t))/N} \\ &= \frac{1}{N\sum_{i=1}^N Var(X(\boldsymbol{s}_i,t))}\left(\sum_{i=1}^N Var(X(\boldsymbol{s}_i,t)) + \sum_{i=1}^N\sum_{j\neq i}^N Cov(X(\boldsymbol{s}_i,t),\ X(\boldsymbol{s}_j,t))\right).\end{aligned}$$

FIGURE 1
Correlograms of Yields of Wheat, Soybeans, and Corn for 1981, 1991, and the Average of 26 Years From 1972 to 1997



Clearly, if the losses are independent, then $\phi_t = 1/N$. When the losses are positively correlated, ϕ_t will be bigger than $1/N$. In addition, if the $X(\mathbf{s}_i,t)$s have the same variance, then

$$\phi_t = \frac{1}{N^2}\left(N + \sum_{i=1}^{N}\sum_{j\neq i}^{N} Corr(X(\mathbf{s}_i,t),\ X(\mathbf{s}_j,t))\right). \tag{12}$$

The correlation coefficient $Corr(X(\mathbf{s}_i,t),\ X(\mathbf{s}_j,t))$ obviously depends on the prespecified levels as well as the joint distribution of yields $Y(\mathbf{s}_i,t)$ and $Y(\mathbf{s}_j,t)$. So far we have made no distributional assumptions on yields $Y(\mathbf{s},t)$ other than stationarity. To get a feeling of how the losses $X(\mathbf{s},t) = \max(0,\ c(\mathbf{s},t) - Y(\mathbf{s},t))$ are correlated, we now assume that yields $Y(\mathbf{s},t)$ are normal and that each of the prespecified levels $c(\mathbf{s},t)$ is one standard deviation below the the mean of $Y(\mathbf{s},t)$, i.e., $c(\mathbf{s},t) = \mu(\mathbf{s},t) - \sigma(\mathbf{s},t)$. Then

$$\begin{aligned} X(\mathbf{s},t) &= c(\mathbf{s},t) - \mu(\mathbf{s},t) - \sigma(\mathbf{s},t)\min\left(\frac{c(\mathbf{s},t)-\mu(\mathbf{s},t)}{\sigma(\mathbf{s},t)},\ \frac{Y(\mathbf{s},t)-\mu(\mathbf{s},t)}{\sigma(\mathbf{s},t)}\right) \\ &= c(\mathbf{s},t) - \mu(\mathbf{s},t) - \sigma(\mathbf{s},t)\min\big(-1, Z(\mathbf{s},t)\big), \end{aligned}$$

where $Z(\mathbf{s},t) = (Y(\mathbf{s},t) - \mu(\mathbf{s},t))/\sigma(\mathbf{s},t)$ is a standard normal random variable. Hence, we can express the correlation between the truncated losses as the correlation between truncated standard normal random variables, i.e., $Corr(X(\mathbf{s}_i,t),\ X(\mathbf{s}_j,t)) = Corr(\min(-1, Z(\mathbf{s}_i,t)), \min(-1, Z(\mathbf{s}_j,t))$. This correlation is clearly a function of

$$Corr(Z(\mathbf{s}_i,t),\ Z(\mathbf{s}_j,t)) = Corr(Y(\mathbf{s}_i,t),\ Y(\mathbf{s}_j,t)) = \rho_t(\|\mathbf{s}_i - \mathbf{s}_j\|).$$

TABLE 1
The Correlation Coefficient of Truncated Standard Normal Random Variables $g(c_1, c_2, \rho)$, as Defined by Equation (13)

ρ	−0.2	−0.1	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$c_1 = c_2 = -0.75$	−0.077	−0.04	0.053	0.11	0.18	0.26	0.34	0.45	0.56	0.68	0.83
$c_1 = c_2 = -1$	−0.059	−0.032	0.04	0.091	0.15	0.22	0.31	0.4	0.52	0.66	0.81
$c_1 = c_2 = -1.5$	−0.027	−0.017	0.022	0.058	0.1	0.17	0.23	0.33	0.44	0.58	0.77

We write $g(c_1, c_2, \rho) = Corr(\min(c_1, Z_1), \ \min(c_2, Z_2))$ for two standard normal variables, Z_1 and Z_2, that are also bivariate-normal with a correlation coefficient ρ. Then

$$\begin{aligned} g(c_1, c_2, \rho) &= Corr(\min(c_1, Z_1) - c_1, \ \min(c_2, Z_2) - c_2) \\ &= \frac{1}{2\pi(1-\rho^2)^{1/2}} \int_{-\infty}^{c_1} \int_{-\infty}^{c_2} (x_1 - c_1)(x_2 - c_2) \exp\left(-\frac{x_1^2 - 2\rho x_1 x_2 + x_2^2}{2(1-\rho^2)} \right) dx_1 dx_2 \\ &\quad - \frac{1}{2\pi} \int_{-\infty}^{c_1} (x_1 - c_1) \exp(-x_1^2/2) dx_1 \int_{-\infty}^{c_2} (x_2 - c_2) \exp(-x_2^2/2) dx_2. \end{aligned} \tag{13}$$

We cannot evaluate the integrals in closed form but can approximate them by Monte Carlo methods. We provide in Table 1 the correlation coefficients between the truncated normals corresponding to different ρs where the standard normal variables are truncated at −0.75, −1, and −1.5. Our results are based on 500,000 simulations. The truncated variables are significantly less correlated than corresponding untruncated variables.

Intuitively, the truncated variables should be less correlated. Above the truncating values, the two truncated variables are constants and hence are uncorrelated. Indeed, if the truncated values are far below the mean, the two truncated variables are constants with a probability close to one and hence are nearly uncorrelated. Note that we obtain the data in Table 1 for two standard normals. If the two variables are not normal, we might expect the truncated variables to be still less correlated. But the exact correlation coefficient of the truncated variables will depend on the joint distribution of the two untruncated variables.

Recently, the Risk Management Agency of the USDA allowed farmers in the MPCI program to specify a coverage level $c(\mathbf{s},t)$ for up to 75 percent of the mean yield $\mu(\mathbf{s},t)$, which is more than one standard deviation below the mean because the coefficient of variation for crops is usually less than 1/4. Although farmers can select their own coverage levels, we assume nonetheless that all farmers select their coverage levels to be one standard deviation below the expected yields, i.e., $c(\mathbf{s},t) = \mu(\mathbf{s},t) - \sigma(\mathbf{s},t)$, and see how effective the risk pooling is. In this case, the losses have the same variance, and Equation (12) is applicable. If some farmers select coverage levels that are more than one standard deviation below the means, the losses only become less correlated, according to Table 1. We continue to assume that yields are normal. Since

$$\begin{aligned}&Corr(X(\boldsymbol{s}_i,t),\ X(\boldsymbol{s}_j,t))\\&\quad = Corr(\min c(-1,Z(\boldsymbol{s}_i,t)),\ \min(-1,Z(\boldsymbol{s}_j,t)))\\&\quad = g(-1,-1,\rho_t(\|\boldsymbol{s}_i-\boldsymbol{s}_j\|)),\end{aligned}$$

then by Equation (12),

$$\phi_t = \frac{1}{N^2}\left(N + \sum_{i=1}^{N}\sum_{j\neq i}^{N} g(-1,\ -1,\ \rho_t(\|\boldsymbol{s}_i-\boldsymbol{s}_j\|))\right), \tag{14}$$

where $\rho_t(\cdot)$ is the covariogram of $Y(\boldsymbol{s},t)$ and can be estimated from the data. Replacing $\rho(\|\boldsymbol{s}_i-\boldsymbol{s}_j\|)$ by the estimated covariogram $\hat{\rho}(\|\boldsymbol{s}_i-\boldsymbol{s}_j\|)$, we get an estimate for ϕ_t.

Using the average correlogram of 26 years, we calculate the coefficient of effectiveness ϕ for wheat, soybeans, and corn by applying Equation (14) with ρ_t being replaced by the average correlogram. The coefficients of effectiveness are 0.00188, 0.00479, and 0.00320 for wheat, soybeans, and corn, respectively. For each crop, we use all of the counties in our database. Hence N = 2,641, 2,000, and 2,591 for wheat, soybeans and corn. These coefficients mean that if an insurer can pool all the growing areas in the United States and if each unit specifies the coverage level to be one standard deviation below its mean yield, over the 26 years the variance of average loss is only 0.188, 0.479, and 0.32 percent of the variance of each individual loss. Or equivalently, to make the risk pooling for the correlated losses to be as effective as independent losses, the number of correlated units should be approximately 4.97, 9.58, and 8.3 times as large as the number of uncorrelated units for wheat, soybeans, and corn, respectively, when all units select coverage levels to be one standard deviation below the mean yields. Choosing the coverage level to be one standard deviation below the mean would mean to insure against adverse events that occur 15.9 percent of the time. If the units select a lower coverage level, the losses will be less correlated and these factors will become smaller. These results show that, if a large number of counties buy the insurance, effective risk pooling is highly possible, at least when yields are normal. Moderate departure from normal distributions should not change the coefficients of effectiveness very much.

Premium Loading

Due to the weak correlation between the losses, we can apply the central limit theorem here. Therefore, if the insurer's probability of ruin is set at α, the premium loading for each insured at year t is $L_t = z_\alpha(Var(\bar{X}_N))^{1/2} = z_\alpha\sqrt{\phi_t}\,(\sum_{i=1}^{N} Var(X(\boldsymbol{s}_i,t))/N)^{1/2}$, assuming that the administrative cost load is zero. The variance of a loss is bounded by the variance of yield regardless of the distribution of yield, as shown in the Appendix. Hence $L_t \leq z_\alpha\sigma_t\sqrt{\phi_t}$.

We estimate the 26-year average variance, $\hat{\sigma}^2$, from the previous section as 52, 20, and 160 for wheat, soybeans, and corn, respectively. Using $z_{0.025}$ = 1.96 and the ϕs calculated in the previous subsection, we obtain the premium loading factor to be at most 0.6128, 0.6066, and 1.4024 bushels per acre for the three crops, respectively. In terms of dollars, if the prices are \$3.5, \$4.5, and \$3 per bushel for wheat, soybeans, and corn, respectively, the corresponding premium loadings are \$2.14, \$2.72, and \$4.20 per acre, which are quite moderate.

SUMMARY AND DISCUSSION

In this article, we have reviewed the statistical foundations for crop insurance under dependence and have looked closely at the correlation structure between crop yields at the county level in the United States. Using some techniques in spatial statistics, we have shown that the positive correlation between any two counties dies off quickly when the lag distance increases, a phenomenon of f.r.p.d. The yield losses, as truncated variables, are also f.r.p.d., and the correlation of losses dies off much faster. We have provided a general method for investigating the effectiveness of risk pooling under dependence. Our method is appropriate for crop insurance. The results indicate that risk pooling can be effective for the three crops (wheat, corn, and soybeans) if the coverage levels are at least one standard deviation below the mean yields, hence suggesting a high possibility of a viable private agricultural insurance market, if the insurance/reinsurance markets can cover most growing areas in the Unites States. These results may help alleviate the pressure on government to assume total responsibility of providing crop insurance.

Although in this work we have applied this method to county-level yields, it is also applicable to individual farms if the farms' geographical locations and yields are known. With the availability of global positioning systems, one can obtain farm locations easily. Insurance agencies should also have historical farm-level yield records gathered from farmers' years of enrollment in MPCI, and these farm yield data can be analyzed in the same way as the county yields. Without individual farm data, it is impossible to examine the effectiveness of crop insurance at the individual farm level. However, we can show that if growers are scattered nearly uniformly throughout counties, the correlogram of individual farm yields is less than the correlogram of county average yields and hence dies off faster. Further theoretical comparison of the coefficients of effectiveness for individual farms and for counties is beyond the scope of this article.

Several problems remain along this line for future research, one of which is the temporal effect on risk pooling. If we adopt the common belief that weather is independent over time, yield will also be independent. Pooling risks over time may further reduce premium loading.

Note that the buffer fund given as $E(S_N(t)) + 2(Var(S_N(t))^{1/2}$ is the upper bound. When the yield loss is finite-range dependent (or f.r.p.d. with additional conditions to ensure mixing conditions), the actual total loss $S_N(t)$ is asymptotically normal. This implies that when N is sufficiently large, $S_N(t)$ will be symmetric. Hence $S_N(t)$ has approximately a 50 percent chance to be below its mean $E(S_N(t))$. In the case that the actual total loss is below its mean, a surplus will be in the buffer fund. Accumulation of such surpluses will reduce the risk of pooling, and its effect on risk pooling warrants further studies. Private insurers can pursue after methods to reduce pooled risk, such as trading in international reinsurance markets and diversifying crops.

Due to spatial dependence, more exposure units/farms are needed to equate pooling of dependent risks to independent risks. The larger pool size could have a deterrent effect on a private crop insurer seeking to enter the private crop insurance market. A study on the profitability of private crop insurance will help determine how strong the deterrent effect may be.

APPENDIX

Here we show the following theorem, which implies that variance of loss is bounded by the variance of the yield.

Theorem 1: *Let Y be any random variable with mean μ and variance σ^2, and $X = \max(0, c - Y)$ for some constant $c < \mu$. Then $Var(X) \leq \sigma^2$.*

Proof of Theorem 1: First note that

$$X = c - \mu - \sigma \min\left(\frac{c-\mu}{\sigma}, \frac{Y-\mu}{\sigma}\right).$$

Then

$$Var(X) = \sigma^2 Var\left(\min\left(\frac{c-\mu}{\sigma}, \frac{Y-\mu}{\sigma}\right)\right) = \sigma^2 Var(\min(d,Z)),$$

where $d = (c - \mu)/\sigma < 0$ and $Z = (Y - \mu)/\sigma$. Then $Var(Z) = 1$, and if $f_Z(z)$ denotes the density function of Z, then

$$\begin{aligned} Var(\min(d, Z)) &= Var(\min(d, Z) - d) \\ &\leq E[(\min(d, Z) - d)^2] = \int_{-\infty}^{d} (z - d)^2 f_Z(z)\, dz. \end{aligned}$$

Since $z < d < 0$, then $z < z - d < 0$ and $z^2 > (z - d)^2$. The integral is bounded by

$$\int_{-\infty}^{d} z^2 f_Z(z)\, dz \leq \int_{-\infty}^{\infty} z^2 f_Z(z)\, dz = Var(Z) = 1.$$

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