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Incomplete Markets and Hyperbolic Discounting

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Abstract

A growing number of empirical researchers are finding evidence of hyperbolic discounting in their investigations on the nature of preferences for distributing consumption over time. This article contributes to the literature by exposing a large class of models in which hyperbolic and exponential discounting are observationally equivalent. The results of the modeling approach simultaneously resolve serious concerns raised by other models in the literature that have been used to explain the empirical findings and answer other questions raised by the phenomenon that are unexplained by earlier contributions. By analyzing an intertemporal general equilibrium model with incomplete insurance markets, this article demonstrates that for sufficiently short time horizons, values implied by a hyperbolic discount function fall within incomplete market valuation bounds.

Introduction

A growing number of empirical researchers are finding evidence of hyperbolic discounting in their investigations on the nature of preferences for distributing consumption over time. This article contributes to the literature by exposing a large class of models in which hyperbolic and exponential discounting are observationally equivalent. The results of the modeling approach simultaneously resolve serious concerns raised by other models in the literature that have been used to explain the empirical findings and answer other questions raised by the phenomenon that are unexplained by earlier contributions. By analyzing an intertemporal general equilibrium model with incomplete insurance markets (hereafter, referred to simply as *incomplete markets*), this article demonstrates that for sufficiently short time horizons, values implied by a hyperbolic discount function fall within the incomplete market valuation bounds.

Most economic models that represent impatience as a constant in the utility function assume that agents interact through complete insurance markets. These models predict that agents' personal discount factor will be equal to the equilibrium discount rate.[1] Empirical investigations have attempted to test this prediction with experiments,

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[1] For example, the steady-state of the well-known 1928 Ramsey model.

surveys, and revealed preference methods (Lowenstein and Thaler, 1989). The common finding of these empirical investigations is that individuals appear to discount the future hyperbolically, which means they place more weight on the present and distant future than on an exponential discount function. The literature indicates that these results may be influenced by the absence of markets for insurance (Benzion et al., 1989; Lowenstein and Prelec, 1992). Despite these indications, the literature does not specify whether the results are an attempt to find individuals' personal or equilibrium rates of discount.[2] In addition, no formal analysis exists of how the two rates may be related in the presence of incomplete markets. By contrast, theoretical investigations into the implications of this empirical phenomenon have assumed that the subjects in the empirical findings are revealing personal preferences for discounting.

To motivate the issue of incomplete markets and connect the current article with recent interest in the use of hyperbolic discounting for public goods provision, I examine a public goods model with incomplete markets. Because of the free-rider problem inherent in questions of public goods provision and the associated unobservability of demand for providing the good, one often assumes that markets in contingent claims for events affecting the value of a public good are incomplete (Arrow and Lind, 1970). However, note that the results in no way explicitly depend on the presence of a public good, other than as a device to motivate the presence of market incompleteness in the model.

The analysis reveals that in the presence of incomplete markets, the relationship between the equilibrium discount rate and agents' personal impatience diverges sufficiently to allow for the possibility of hyperbolic discount paths. In this sense, the hyperbolic discount functions presumed by some authors are observationally equivalent to an incomplete market equilibrium with constant (i.e. exponential) discounting. This result is similar to findings that declining rates of discount can be generated by discount rate uncertainty (Azfar, 1999; Weitzman, 1998), but this article is the first to establish a connection to incomplete markets.

The incomplete markets hypothesis illuminates how agents formulate discount rates in different situations. Incomplete markets allow for a systematic separation between "market" and "nonmarket" activities. This separation is important because it helps explain how an agent may report a discount rate in a survey situation that contradicts the discount rate that the same agent uses for borrowing or lending in the market. Specifically, the model predicts that within the market, discount rates should behave in accordance with traditional theory. However, outside the market, discount rates may appear to be hyperbolic.

After a brief review of the related literature, "Bounds on Hyperbolic Discounting" shows that violating the complete markets assumption of the discounted utility model can reconcile the empirical evidence. The section introduces a two-date public goods model with incomplete markets. The model generates bounds on the present value of the benefits for public goods provision at date two. The model extends to multiple dates, generating valuation bounds for the public good at every date. Finally, the

[2] Benzion et al. (1989) is a notable exception: "[O]ur assumption that the subjects' responses in the present study reflected individual utilities rather than merely an interest rate (which is uniform)" (p. 283).

model shows that a hyperbolic discount path for the public good may fall within the valuation bounds for some finite period of time. The article ends with an evaluation of the results and a summary of possible extensions.

The related literature can be divided into three categories: empirical reports, implications of dynamic inconsistency, and the role of uncertainty. Many empirical investigations examine the nature of how individuals discount future consumption. Lowenstein and Thaler (1989) present a good recent summary. Most of the more recent investigations rely on surveys about hypothetical intertemporal trade-offs. In particular, Benzion et al. (1989) and Cropper et al. (1994) offer thorough analysis of their survey results. Benzion et al. investigates how students trade off hypothetical sums of money received at different times. Cropper et al. asks survey respondents to choose between different government life saving programs implemented at various times. Benzion et al. seems particularly aware of how incomplete markets complicate interpreting results:

> [W]e cannot determine whether our subjects actually considered the existence of financial markets in determining their implicit rate of substitution (1989, p. 271).

The second category of literature is concerned with interpreting the empirical findings. The conclusion of these authors is uniform acceptance of personal hyperbolic discount functions (Chichilnisky, 1997; Cropper and Laibson, 1997; Laibson, 1997). This literature is primarily concerned with ameliorating the negative effects of dynamic inconsistency caused by hyperbolic time preferences. The literature analyzes possible strategies for addressing dynamic inconsistency such as subsidizing investment and preserving market illiquidity.

The third category of literature contributes to an alternative explanation for hyperbolic discounting: interest rate uncertainty (Azfar, 1999; Weitzman, 1998). The results are similar in spirit to those presented here. However, the authors do not deal with the issue of incomplete markets. More important, they do not clearly address the multiplicity of interest rates implied by the empirical findings.

The Model

Optimization

The model is a two-date exchange economy with two states of nature at the second date. A benevolent principal uses a technology that requires at least one period to produce a public good. The two states of nature at the second date are u for "up" and d for "down." In the "up" state, both consumption and the public good are assumed to be more dear.[3] This means that aggregate risk in the economy is correlated with the value of the public good, though the correlation does not imply causation. For

[3] Any correlation between aggregate risk in the economy makes discounted net benefit calculations using the unweighted risk-free rate biased. The present analysis chooses to examine the case of a project that is negatively correlated with aggregate risk. However, none of the results rely on this assumption. The case of positive correlation is symmetric. Assume that correlation merely serves to highlight the contribution of the article, because it has traditionally been a difficult issue to address (e.g., Arrow and Lind, 1970).

example, consider a public project to build a dike in the Netherlands to protect against flood damage in the event of climate change. The payoff to having the dike in place is proportional to the sea level (i.e., higher oceans caused by climate change). In the "up" state of this example, the sea level is high and the dike is more valuable because it prevents a lot of damage. In the "down" state, the sea level is low and the dike is less valuable because it does not prevent much damage. In addition, other effects of climate change such as crop damage would lead one to conclude that the overall economic impact on the Netherlands from climate change would probably be negative. This means that the dike's value would be greater than its expected net present value because it functions as an insurance contract against the damages caused by climate change.[4]

In each state of nature (s), agent i derives utility from a single consumption good, x_s^i, and the benefits from a public good, h. For convenience in notation, I label activities occurring at the first date, t_0 , with a zero. The analysis proceeds by considering a project to increase the level of the public good.

The N agents in the model have additively separable utility functions. Thus, in each state agent i has a state-dependent utility function, u_s^i. Following the expected utility representation of Arrow and Lind (1970), agent i's utility function is linear in the state probabilities, π_s^i, and has a constant personal discount factor ρ^i:

$$\begin{aligned} &U^i(x_0^i, x_u^i, x_d^i; h, \pi_u^i, \pi_d^i, \rho^i) \\ &\quad = u_0^i\left(x_0^i\right) + \pi_u^i \rho^i u_u^i\left(g_u^i(h) + x_u^i\right) + \pi_d^i \rho^i u_d^i\left(g_d^i(h) + x_d^i\right). \end{aligned} \tag{1}$$

Agent i's optimization problem takes the following form:

$$\max_{(x_0^i, x_u^i, x_d^i, b^i)} U^i(x_0^i, x_u^i, x_d^i; h, \pi_u^i, \pi_d^i, \rho^i) \tag{2a}$$

subject to

$$x_0^i = \omega_0^i - \frac{1}{R} b^i, \tag{2b}$$

$$x_u^i = \omega_u^i + b^i, \text{ and} \tag{2c}$$

$$x_d^i = \omega_d^i + b^i, \tag{2d}$$

where b^i is agent i's holding of the bond.

The equilibrium condition for the model is

$$\sum_{i=1}^{N} b^i = 0 \tag{3}$$

[4] Though the example gives the impression that the subsequent analysis may involve issues of "self-protection" and "self-insurance," it does not.

Present Value and the Equilibrium Interest Rate. This subsection uses the model to develop expressions for the present value of a public project and the interest rate on a bond. The expressions show how incomplete markets obscure agent i's personal discount factor in revealed preference settings where the present value of a public project is being sought. The first-order conditions from the model reveal a relationship between the interest rate and each agent's state-dependent intertemporal marginal rate of substitution (IMRS_s).

We can write agent i's problem in the unconstrained form:

$$\max_{b^i} V\left(b^i; h, \pi_u^i, \pi_d^i, \rho^i, \omega_0^i, \omega_u^i, \omega_d^i, R\right) \equiv u_0^i\left(\omega_0^i - \left(\frac{1}{R}\right)b^i\right) \tag{4a}$$

$$+\,\pi_u^i \rho^i u_u^i\left(\omega_u^i + b^i + g_u^i(h)\right) \tag{4b}$$

$$+\,\pi_d^i \rho^i u_d^i\left(\omega_d^i + b^i + g_d^i(h)\right). \tag{4c}$$

The first-order condition is the following:

$$\frac{\partial V\left(b^i\right)}{\partial b^i} = -u_0^{i\prime}\left(\omega_0^i - \left(\frac{1}{R}\right)b^i\right)\frac{1}{R} + \pi_u^i \rho^i u_u^{i\prime}\left(\omega_u^i + b^i + g_u^i(h)\right)\pi_d^i \rho^i u_d^{i\prime}\left(\omega_d^i + b^i + g_d^i(h)\right) = 0. \tag{5}$$

In equilibrium, Equation (5) implies:

$$\frac{1}{R} = \frac{\pi_u^i \rho^i u_u^{i\prime}\left(\omega_u^i + b^i + g_u^i(h)\right) + \pi_d^i \rho^i u_d^{i\prime}\left(\omega_d^i + b^i + g_d^i(h)\right)}{u_0^{i\prime}\left(\omega_0^i - \left(\frac{1}{R}\right)b^i\right)}. \tag{6}$$

Let

$$\mu_s^i = \frac{\pi_s^i \rho^i u_s^{i\prime}\left(\omega_s^i + b^i + g_s^i(h)\right)}{u_0^{i\prime}\left(\omega_0^i - \left(\frac{1}{R}\right)b^i\right)} \quad \text{for } s = u, d. \tag{7}$$

Then write Equation (6) as:

$$\frac{1}{R} = \mu_u^i + \mu_d^i \tag{8}$$

Note that R places a condition on every agent's IMRS_s, μ_s^i for $s = u, d$.

Define the public good's present value as the amount of consumption in t_0 an agent would forego to be given an increase in the public good, keeping utility constant. Using the agent's utility function from Equation (1) and differentiating gives the following:

$$dU^i(x_0^i, x_u^i, x_d^i; h, \rho^i, \pi_u^i, \pi_d^i) = 0 \tag{9a}$$

$$\frac{\partial U^i}{\partial x_0^i}dx_0^i + \frac{\partial U^i}{\partial x_u^i}dx_u^i + \frac{\partial U^i}{\partial x_d^i}dx_d^i + \frac{\partial U^i}{\partial h}dh = 0, \tag{9b}$$

by the definition of present value $dx_u^i = dx_d^i = 0$;

$$\frac{\partial U^i}{\partial x_0^i}dx_0^i + \frac{\partial U^i}{\partial h}dh = 0 \tag{9c}$$

$$\Rightarrow u_0^{i\prime}\left(x_0^i\right) dx_0^i \tag{9d}$$

$$+\pi_u^i \rho^i u_u^{i\prime}\left(x_u^i + g_u^i(h)\right) \frac{\partial g_u^i}{\partial h} dh + \pi_d^i \rho^i u_d^{i\prime}\left(x_d^i + g_u^i(h)\right) \frac{\partial g_d^i}{\partial h} dh = 0, \tag{9e}$$

rearranging and using Equation (7)

$$\Rightarrow -\frac{dx_0^i}{dh} = \mu_u^i \left(\frac{\partial g_u^i}{\partial h}\right) + \mu_d^i \left(\frac{\partial g_d^i}{\partial h}\right). \tag{9f}$$

Equation (9f) gives the marginal rate of substitution between an increase in the public good and consumption at t_0. From this formulation, we easily see that μ_s^i is the marginal rate of substitution between wealth in date t_0 and wealth in state s at date t_1.

The public project is a proposal to increase the level of the public good from the status quo, $\bar{h}$, to $\bar{h}+\varepsilon$. The difference between these two levels is some finite amount: $\Delta h = \varepsilon$.

Following Arrow and Lind (1970),[5] the public project considered in this context is a small part of the economy. Thus, we assume that the project has a negligible impact on the equilibrium marginal rate of substitution, μ_s^i, where $s = u,d$. In the case of complete markets, the assumption is identical to Arrow and Lind's (1970), because market prices are defined by the agents equalized, IMRS$_s$. When markets are incomplete for the μ_s^i to be independent of the public project for every individual, one must consider that the μ_s^i is the marginal rate of substitution for state-dependent consumption. If the agent's state-dependent value for the benefits of providing an increase in the public good is a relatively small part of the total state-dependent consumption, then changing the level of the public project should have a very small affect on the IMRS$_s$. Hence, an agent's present value for the public good treats the μ_s^i as a constant. Taking this into consideration, we can formulate an expression for the present value of the public project, ϕ_0^i, integrating the marginal rate of substitution derived in Equation (9f):

$$\phi_0^i \equiv -\int_{\bar{h}}^{\bar{h}+\varepsilon} \frac{dx_0^i}{dh} dh = \mu_u^i \int_{\bar{h}}^{\bar{h}+\varepsilon} \left(\frac{\partial g_u^i}{\partial h}\right) dh + \mu_d^i \int_{\bar{h}}^{\bar{h}+\varepsilon} \left(\frac{\partial g_d^i}{\partial h}\right) dh. \tag{10}$$

To allow for a succinct exposition, let

$$\theta_s^i \equiv \int_{\bar{h}}^{\bar{h}+\varepsilon} \left(\frac{\partial g_s^i}{\partial h}\right) dh \tag{11}$$

$$\phi_0^i \equiv \mu_u^i \theta_u^i + \mu_d^i \theta_d^i. \tag{12}$$

Bounding the Present Value of Public Goods Provision. The expressions derived in the previous section have prepared the way for analyzing this article's primary issues. Specifically, when markets are incomplete, what are the relationships between an agent's

[5] "Consider an investment where the overall effect on market prices can be assumed to be negligible…" (Arrow and Lind, 1970, p. 368).

personal discount factor, the equilibrium discount rate, and the agent's present value for a public project?

We can think of the parameter θ_s^i in Equation (12) as a state-dependent consumption bundle of equal value to the increase in public good represented by Δh. If we achieve an arbitrary consumption pattern by trading assets on financial markets, then in equilibrium we can calculate an agents' present value for the public project at t_0, ϕ_0^i, by finding the price of a financial portfolio that delivers the consumption bundle $\left(\theta_u^i, \theta_d^i\right)$ represented by the proposed project. This implies that if financial markets were complete, then the present value for the public project at t_0 for each individual would be revealed by equilibrium prices. Because financial markets are incomplete, equilibrium prices merely restrict the present value of the project for any individual to lie within a bounded interval on the real line.

Theorem 1. *If* $\theta_u^i > \theta_d^i$, *then*

$$\frac{\theta_d^i}{R} \leq \phi_0^i \leq \frac{\theta_u^i}{R}. \tag{13}$$

Proof of Theorem 1: According to the financial market equilibrium,

$$\frac{1}{R} = \mu_u^i + \mu_d^i \tag{14}$$

and

$$\phi_0^i = \mu_u^i \theta_u^i + \mu_d^i \theta_d^i \tag{15}$$

$$\Rightarrow \left(\mu_u^i \theta_d^i + \mu_d^i \theta_d^i\right) \leq \phi_0^i \leq \left(\mu_u^i \theta_u^i + \mu_d^i \theta_u^i\right), \forall_i. \tag{16}$$

The intuition behind this result is straightforward. It says that every agent will value a riskless payoff that guarantees the highest state-contingent project benefit at least as much as the uncertain prospect presented by implementing the public project. Likewise, every agent will value a riskless payoff that guarantees that the lowest state-contingent project benefit less than the project. Thus, the values of these two riskless payoffs bound each agent's present value for the project. For our purposes, it suffices to acknowledge that these bounds make the dependence between the equilibrium interest rate and the present value of the public good ambiguous.

We can write the bounds in Equation (16) in terms of the certainty equivalent benefit at date t_1 from the public good, $ce_{t_1}[\theta^i]$:

$$\frac{\theta_d^i}{R} \leq \frac{ce_{t_1}[\theta^i]}{R} = \phi_0^i \leq \frac{\theta_u^i}{R}. \tag{17}$$

What Equation (17) reveals is that in a circumstance where an individual is asked about the present value for nontraded benefits, the answer can depend on risk factors influencing the certainty equivalent value for benefits as well as the equilibrium rate of time preference.

The Multi-Date Model. This subsection extends the model of the last section to T dates. The expression in Equation (17) details the relationship between the equilibrium discount rate, R, and the agent's present value for providing the public good, ϕ_0^i. We use the multi-date analog of Equation (17) to demonstrate that hyperbolic discount paths for the public good are admissible.

The length of a time period in the previous model is arbitrary. Assume that the time lapse between date $t = 0$ and the receipt of benefits is T periods in the future. If the one period, nonstochastic interest rate is given by R , then the present value of benefits received in date T will be bounded by:

$$\frac{\theta_d^i}{R^T} \leq \frac{ce_T[\theta^i]}{R^T} = \phi_0^i \leq \frac{\theta_u^i}{R^T}. \tag{18}$$

From this formulation, it is easy to see that as long as the one-period interest rate is positive, then the present value bounds of the benefits will go to zero exponentially. This prediction clashes with a discounting function that is hyperbolic. However, the next section will show that it could be very long time before the present value given by a hyperbolic discount function violates these bounds.

Bounds on Hyperbolic Discounting. The bounds in the last section introduce a considerable amount of freedom to the valuation of nontraded benefits. Agents can respond with a wide variety of values for the public good before their announcements conflict with values inferred by equilibrium prices. This observation helps reconcile reported empirical anomalies, such as like agents conducting market transactions at different prices than the ones implied by survey data.

Next, we demonstrate that the valuation bounds allow agents to report present values consistent with hyperbolic discount functions for some finite period of time without contradicting equilibrium prices. In particular, we assume that agents report present values consistent with a generalized hyperbolic discount function, $(1 + \alpha t)^{-\gamma/\alpha}$.

Before proceeding, we must confront the following question: When markets are incomplete, how does one separate the effects of risk on valuation from those of time? In an incomplete market environment, a dearth of information is available for separating these effects. Incomplete markets require a conjecture about the individual's behavior to calculate the present value of a risky project even if preferences for trading consumption across time are known. Here, we assume that the agent follows a simple expected net present value rule using subjective probabilities. Specifically, the agent calculates the present value for the public good by taking the expected value of future benefits with respect to subjective beliefs about the likelihood of each outcome and then discounts the expected value according to a hyperbolic discount function.[6] Thus, in tracing out the hyperbolic valuation function, we use the following functional form:

[6] We base this assumption on the implied method for calculating a respondent's discount rate, as used in the empirical literature (Benzion et al., 1989; Cropper et al., 1994). The method assumes that agents are risk-neutral.

$$\phi_0^i = \frac{E_\pi(\theta)}{(1+\alpha t)^{\frac{\gamma}{\alpha}}} \tag{19}$$

where we calculate $E_\pi(\cdot)$ using the individual's subjective probabilities.

Since only two states of nature are in the model, for each agent the project is completely determined by the benefits received in the two states (θ_u^i, θ_d^i).[7] Given this information, three scenarios can possibly be explored assuming $\theta_d^i < \theta_u^i$, where c^i is the cost share of agent i in present value terms.

$$(1) \quad Rc^i < \theta_d^i < \theta_u^i, \tag{20}$$

$$(2) \quad \theta_d^i < Rc^i < \theta_u^i, \quad \text{or} \tag{21}$$

$$(3) \quad \theta_d^i < \theta_u^i < Rc^i. \tag{22}$$

The second scenario seems the most interesting, so it will be the focus of the remainder of the article. However, all the results easily extend to the other cases. For simplicity, assume that the project is costless and the agent's cost share $c^i = 0$.

Theorem 2: *Let $t^* \in [0,\infty)$ be defined by*

$$\frac{\theta_u^i}{R^{t^*}} = \frac{E_\pi\left(\theta^i\right)}{(1+\alpha t^*)^{\frac{\gamma}{\alpha}}}. \tag{23}$$

If $\theta_d^i < 0 < \theta_u^i$ and $E_\pi(\theta^i) > 0$, then for all $t > t^*$, the present value defined by the hyperbolic discount function in Equation (19) is inconsistent with the equilibrium prices given by Equations (2a) and (3).

Proof of Theorem 2: First, show that t^* exists. We cannot solve for t^* directly because R^{t^*} is a transcendental function. So, we first recognize that $E_\pi(\theta^i) \equiv \pi_u^i\theta_u^i + \pi_d^i\theta_d^i \leq \theta_u^i$. If these quantities are equal, then the first part of the theorem is proved with $t^* = 0$. Assuming that this is not the case, then $E_\pi(\theta^i) < \theta_u^i$. Then, we know that

$$\frac{\theta_u^i}{R^t} > \frac{E_\pi\left(\theta^i\right)}{(1+\alpha t)^{\frac{\gamma}{\alpha}}}, \quad \text{for } t = 0. \tag{24}$$

In order for these functions to be equal at some point, it is sufficient that they cross on $\mathbb{R}^+$. Since the functions are continuous, for them to cross it must be the case that for some $\widetilde{t} > 0$,

$$\frac{\theta_u^i}{R^t} < \frac{E_\pi\left(\theta^i\right)}{(1+\alpha t)^{\frac{\gamma}{\alpha}}}, \quad \text{for } t = \widetilde{t}. \tag{25}$$

For this to occur, it must be the case that

$$\frac{\theta_u^i}{E_\pi\left(\theta^i\right)} < \frac{R^t}{(1+\alpha t)^{\frac{\gamma}{\alpha}}}. \tag{26}$$

[7] The results encompass any discrete state space model with a finite number of outcomes.

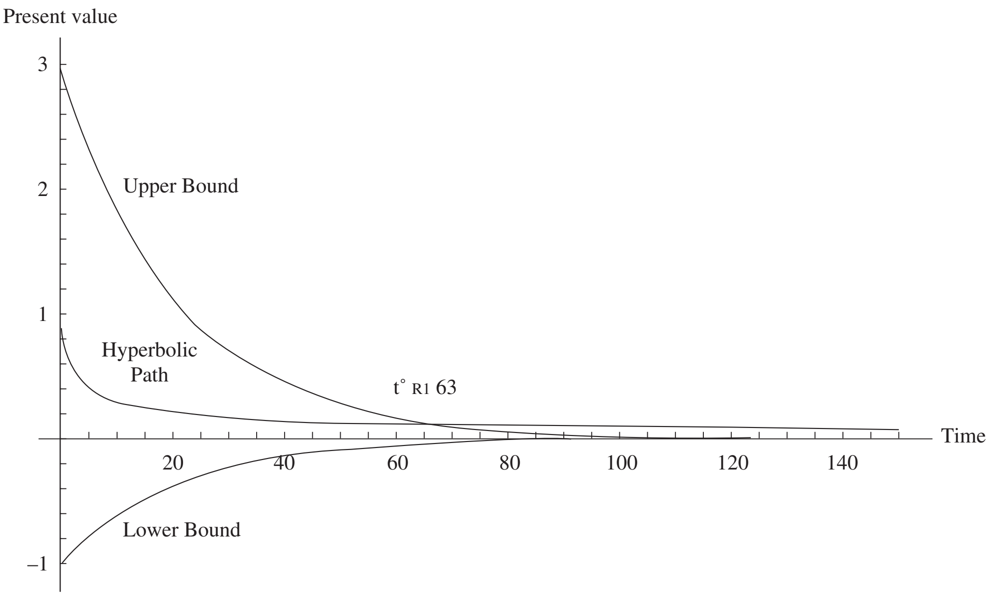
This condition will certainly be satisfied for any fixed $\frac{\theta_u^i}{E_\pi(\theta^i)}$, because in the limit as $t \to \infty$, the exponential function dominates all polynomials. Thus, $\lim_{t\to\infty} \left[\frac{R^t}{(1+\alpha t)^{\frac{\gamma}{\alpha}}}\right] = \infty$. This also proves the second half of the theorem.

Theorem 2 says that when markets are incomplete, it is possible for agents to report values for a public project that follow a hyperbolic discount path until time t^*. In what sense does the hyperbolic discount path conflict with equilibrium prices after date t^*? To answer this question, recall that the benefits of providing the public good are measured in terms of a state-contingent consumption bundle at each date. Equilibrium prices, as reflected in the bond price in this model, assign a present value to a set of consumption bundles that envelop the consumption possibilities offered by the public good. If the present value assigned to the public good by hyperbolic discounting is not a convex combination of the values assigned by equilibrium prices to the enveloping set, then individuals are assigning multiple values to the same consumption bundle. It is interesting to note that for a fixed set of parameters in the hyperbolic discount function, date t^* grows with the ratio $\theta_u^i / E_\pi\left(\theta^i\right)$. Next, we provide an example based on the parameter values used in Laibson (1997, p. 450, Figure 1) to demonstrate the results of Theorem 2.

Example 1: *Let* $R = 1.03$, $(\theta_u^i, \theta_d^i) = (3, -1)$, $\pi_u^i = \pi_d^i = \frac{1}{2}$, $\gamma = 5 \times 10^3$, $\alpha = 10^5$. *Thus,* $E_\pi[\theta] = 1$. *The graph in Figure 1 illustrates the valuation bounds and the hyperbolic discounting path defined by Equation (19).*

FIGURE 1

Equilibrium Valuation Bounds Enveloping Hyperbolic Disount Path



This example shows that individuals may report a present value for nontraded consumption bundles (e.g., a public good) that declines hyperbolically and is consistent with an equilibrium discount rate for traded consumption that declines exponentially. Next, we use a hypothetical example to show how the results of this article connect to the empirical literature on hyperbolic discounting. The following example is a stylized version of the survey conducted in Equation (5).

Example 2: *Suppose a respondent to a survey testing for the presence of hyperbolic discounting preferences is faced with the following question:*

The government is deciding whether to implement an air pollution control program and must choose between the following equal cost programs:

(1) *The government could implement a short-term program immediately, which would save the lives of X people this year.*
(2) *The government could implement a long-term program, which would save Y lives in year T.*

How large does Y have to be before you would be indifferent to either the short-term or the long-term program?

The calculation used in Benzion (1989) and Cropper (1994) to infer the respondent's discount rate in period T is given by:

$$R = \left(\frac{Y}{X}\right)^{1/T}. \tag{27}$$

It is clear that this calculation assumes that the respondent believes the government has the ability to save exactly X and Y lives with each of the programs listed. If the respondent believes that X and Y represent the average number of lives that would be saved by the proposed programs, then the response will depend on other things than just the interest rate, such as whether the respondent's wealth is correlated with the outcome of these programs and market completeness. For example, a respondent who is an asthmatic or a smoker might believe that reducing pollution has benefits that are correlated with income. If X and Y are random and correlated with the respondent's wealth, the proper relationship between the response and the interest rate utilizes the agent's certainty equivalent value for the benefits of the program:

$$R = \left(\frac{ce_T(Y)}{ce_0(X)}\right)^{1/T}. \tag{28}$$

In Equation (28), the respondent is picking the risk-free interest rate needed to give a present value to the benefits received at date T:

$$\phi_0^i = \left(\frac{ce_T\,(Y)}{R^T}\right). \tag{29}$$

The expression in Equation (29) is identical to the central part of the inequality in Equation (18).

EVALUATION OF RESULTS AND EXTENSIONS

Do these results contribute to a better understanding of the empirical anomalies reported? The model shows that hyperbolic discount paths can be supported by an equilibrium with a constant discount rate. This conclusion relies on two assumptions: uncertainty and market incompleteness. Most of the empirical findings fit this profile, even though they may not explicitly mention risk or trading opportunities.[8] Where uncertainty exists, the model usefully details how an agent's parametric impatience and the market rate of interest are related through the available market structure.

It is important to interpret the results carefully. The model does not predict that agents will discount hyperbolically when markets are incomplete. The result is that when markets are incomplete, hyperbolic discount paths are a possibility, for sufficiently short-time horizons. The likelihood of this possibility is a different question that is addressed by the empirical literature. Commonplace reports of hyperbolic discounting may make the model seem somewhat less relevant since it only admits that they are a possibility. However, even in the literature, which uses relatively sophisticated statistical techniques to gauge how well a hyperbolic function fits the data, unusual cases make the estimation results less convincing. For example, Cropper et al. (1994) finds an unusual persistence in the responses of a significant percentage of respondents:

> 8.7% of persons faced with a five-year horizon and 12% of persons faced with a ten-year horizon said that they would never choose the future-oriented program (1994, p. 263).

These sorts of responses are inconsistent with both hyperbolic and exponential discount paths but can be reconciled with the incomplete market valuation bounds. Depending on the point of view, the flexibility of the incomplete markets model may be a blessing or a curse. It is important not to confuse this flexibility with a nontestable hypothesis. The model does place testable restrictions on admissible discount paths. The model predicts that when markets are incomplete, the extent to which hyperbolic discounting can be reconciled with equilibrium prices depends critically on the time horizon. For any given situation, eventually hyperbolic discount paths will violate the law of one price. The model matches the data better than either the exponential or hyperbolic models because it encompasses both. However, it confounds efforts to make solid predictions about the nature of intertemporal preference with the data currently available. With all of this taken into consideration, the apparent systematic nature of hyperbolic discounting is still a mystery.

The policy implications of these results for public goods provision are straightforward. If market incompleteness has caused an agent's valuation of a public good to decline hyperbolically over time, then a discount rate that declines hyperbolically is best for public investment decisions concerning the good in question. Ultimately, the only

[8] Consider the alternative: To assume that respondents do not view hypothetical prizes as risky prospects is to presume that respondents treat the hypothetical as if it is real, trust the information contained in the experiment completely, and find the interviewer's motives unbiased. To assume that markets are complete, with respect to the opportunities presented in any particular experiment, implies that the information being sought in the experiment is readily observable from transactions in the market.

issue is to correctly measure individuals' willingness to pay for the public good. Finding an appropriate discount rate is tangential.

The preceding discussion suggests two useful extensions to the current literature. First, new experiments that attempt to test for the incomplete market and uncertainty effects that may be biasing respondents' answers would be useful. One can test for incomplete market effects by embedding a synthetic market for the prizes within the experiment. Second, estimating the parameters for a hyperbolic discount function from a particular public good valuation study would be a fruitful exercise for beginning to apply these results. One could use the parameters with interest rate data to estimate the date t^* when the hyperbolic path violates the equilibrium present value bounds implied by market prices. For a particular public project, policymakers could use this information to determine when it is appropriate to use a hyperbolic discount factor for calculating the present value of policy benefits.

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