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Optimal Loss Mitigation and Contract Design

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Abstract

This work examines the interaction between the premium rates set by an insurer and the incentives of an individual to purchase market insurance and undertake mitigation to reduce the size of a potential loss. A risk-neutral monopolistic insurer prices insurance according to the price-elasticity of demand for coverage. The elasticity of demand is affected by the presence of both mitigation and government intervention. The availability of loss reduction activities increases the consumer's elasticity of demand and lowers the optimal rate charged by the monopolist. Government intervention reduces both expenditures on mitigation and the rate charged by the monopolistic insurer.

Introduction

Natural disasters pose a significant threat to the profitability and viability of the property-liability insurance industry. Due to the real economic loss that could result from a catastrophic event, the past decade has seen a much greater emphasis on developing and maintaining a comprehensive strategy that would result in the adoption of cost-effective mitigation measures by individuals, businesses, and governments.

A number of bills that have come before the U.S. Senate focus on the importance of encouraging more loss mitigation to reduce expected losses from natural disasters. The implication of these bills is that the existing level of mitigation is below the optimal level for society. The purpose of the Natural Disaster Protection and Insurance Act of 1999 (U.S. Senate, 1999) is to reduce losses from natural disasters by a comprehensive and coordinated hazard mitigation program. If increased investment in mitigation is deemed to be necessary and beneficial to society and if insurance is deemed to be one way to encourage more loss mitigation, then it is necessary to examine how price, coverage, and mitigation interact.

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To this end, this work examines the interaction between the insurer's choice of price and the insured's choice of the amount of insurance to buy as well as the amount to spend on mitigation, or loss reduction activities.[1] The impact of the insured's incentive to mitigate on the contract structure offered by insurance companies is of interest. This research differs from earlier studies that examine optimal investment in mitigation and insurance coverage in which the insurance company acts passively and there is no strategic pricing of policies. Here we examine a consumer's optimal choice of both the portion of loss to insure and the amount spent on mitigation together with the optimal price charged by the insurer. To focus more directly on the interactions between price, the amount of insurance purchased, and the amount spent on mitigation, we assume informational frictions, such as adverse selection or moral hazard, to be nonexistent.

The models build on two lines of research: the construction of optimal contracts in a game theoretic framework and the effect of contract design on individuals' incentive to invest in mitigation. A detailed discussion of previous research is given in the literature review. One assumption that varies in the current literature is whether the insurer is assumed to be risk-averse or risk-neutral. Kunreuther (2000) notes that, in the context of natural disaster insurance, insurers may act in a "safety first" manner, putting limits on the amount of insurance they are willing to sell in order to reduce the probability of insolvency. In fact, these restrictions are evident in insurance companies' underwriting guidelines.[2] Further, with catastrophic insurance, insurers may be unable to adequately diversify this risk due to the dependence across exposure units and may face a constraint on the supply of reinsurance, thereby resulting in what appears to be risk-averse behavior. Cautious behavior also arises from insurers' concerns over regulatory restrictions on the entry into, and exit from, insurance markets in disaster-prone regions.[3]

In the model presented here, the monopolistic insurer can diversify adequately, has sufficient access to the reinsurance market, and thereby acts as if it is risk-neutral, concerned only with maximizing expected profits. The insurer chooses the price of insurance based on the consumer's elasticity of demand for insurance. Using an expected utility framework, we examine the impact of insurance price on individuals' incentive to mitigate potential losses. More specifically, we consider the optimal level of mitigation and optimal level of insurance that would be chosen by a rational, utility-maximizing individual under the following conditions:

- insurance when no mitigation is available
- insurance when mitigation is available

[1] Mitigation in this context refers to activities that reduce the size of loss, denoted as self-insurance by Ehrlich and Becker (1972).

[2] For example, earthquake underwriting guidelines for a large U.S. property-liability insurer state, "Accumulation of liability must be controlled in order to protect company capital and surplus. In some areas, it is necessary to establish aggregate dollar limits of earthquake liability."

[3] After Hurricane Andrew, for example, Florida regulators prohibited insurers from canceling more than 5 percent of homeowner policies statewide in a given year.

- insurance in a world where individuals correctly anticipate ex post government disaster relief
- insurance in a world where the mitigation is subsidized

After developing these four scenarios, we compare the results to the first best solution in which consumers purchase insurance from perfectly competitive insurers. Required proofs are in Appendix A. The following section of the article discusses the implications of a risk-neutral insurer, and the article concludes with a numerical example that illustrates the results of each scenario.

The numeric examples show that the presence of mitigation decreases the premiums charged by insurers. They also show that the presence of government assistance decreases the amount of insurance that consumers wish to purchase, reduces the amount spent on mitigation, and lowers the price that the insurer charges. As expected, if mitigation is subsidized, consumers spend more on loss reduction activities. In all models, the proportion of the maximum possible loss insured declines as the probability of loss increases. If mitigation is subsidized, consumers protect the largest proportion of their property through mitigation and insurance. Consumers purchase the least amount of protection if the government provides ex post disaster assistance.

The main results of the article demonstrate that in the presence of mitigation the insurance company lowers the price of insurance, but this is not a reward for undertaking mitigation. Rather, a lower price is the rational response of the monopolist to an increase in the elasticity of demand for insurance. In fact, the monopolistic insurer does not have an incentive to encourage mitigation through lower premiums because mitigation results in lower expected profits.

If insurance were offered at its actuarially fair price, then in all scenarios, consumers would purchase more insurance and undertake fewer loss-reducing activities. Consumers would insure up to the maximum possible loss for all scenarios except if ex post disaster assistance were rationally anticipated. If insurers operated in an imperfectly competitive market, then they would charge a rate between the actuarially fair premium and the monopolist's price. In this framework, consumers would insure more than in the monopoly framework but would not purchase full insurance. They would mitigate less than under monopoly pricing but more than if the market was perfectly competitive.

The main effect of risk aversion of the insurer is to increase the price of insurance, thus increasing the amount spent on mitigation and decreasing the amount of insurance purchased by consumers. Insurers will charge higher prices the higher the correlation between losses and the more insured homes that are at risk.

Previous Research

The mitigation model presented here is built upon two strands of research that are reviewed in this section. The first body of literature discussed deals with the consumer's decision on how much to invest in mitigation and how much insurance to purchase given various market conditions. This discussion is followed by a review of strategic pricing issues in insurance and a brief discussion of the demand for catastrophic insurance as it relates to the model and the results of this research.

Numerous authors have looked at the optimal level of loss mitigation under various scenarios. Previous studies have demonstrated how the amount spent to reduce the size of loss varies depending on the availability of insurance (Ehrlich and Becker, 1972), the degree of risk aversion (Dionne and Eeckhoudt, 1985), the certainty of the effectiveness of the loss-reduction activity (Hiebert, 1989; Briys et al., 1991), the presence of a full-insurance requirement (Kunreuther and Kleffner, 1992), whether premiums are risk-based (Kleffner and Kelly, 2001), and the presence of government disaster relief (Kaplow, 1991; Lewis and Nickerson, 1989).

Ehrlich and Becker (1972) show that market insurance and self-insurance are substitutes if the price of market insurance is independent of the amount spent on self-insurance. Hence an increase in the unit price of market insurance decreases the demand for market insurance and increases the demand for self-insurance. Dionne and Eeckhoudt (1985) show that increases in risk aversion result in an increased expenditure in loss-reduction activity. In a related article, Hiebert (1989) shows that increased risk aversion will lead to increases in self-insurance only if the effectiveness of the loss reduction is nonrandom; otherwise, the amount invested in mitigation will depend upon both the mean and variance of the loss. Briys et al. (1991) examine the interaction between market and self-insurance when nonreliability risk exists. In this case, market insurance and self-insurance may also be complements. Kunreuther and Kleffner (1992) show that the incentive to voluntarily adopt self-insurance is reduced if insureds are required to purchase full insurance and that individuals tend not to use the maximization of expected utility to make their decisions but rather use simple decision rules that result in their underinvesting in self-insurance. Kleffner and Kelly (2001) show that if premiums are not risk-based, insureds will invest less in self-insurance. Kaplow (1991) explores the effect of government relief on individual incentive to purchase insurance and to mitigate, and concludes that relief distorts incentives because individuals no longer bear the full cost of their actions. Lewis and Nickerson (1989) show that when private insurance is not available but disaster relief is, the optimal private expenditures on self-insurance will be excessive or insufficient depending on the nature of the efforts employed. When investment in loss mitigation is risk-reducing (risky), one finds that optimal private expenditures on self-insurance are inadequate (excessive) relative to the cost-minimizing level.

The previously cited studies do not address the strategic behavior of the insurer in response to the insured's choices. Miller (1972), Raviv (1979), and Schlesinger (1984) examine strategic firm behavior. They develop optimal contracts in a world without mitigation, given the preferences of both the consumer and the insurance company. Miller (1972) presents the contract design problem as a two-person, zero-sum game. Building on the work of Arrow (1973), who shows that the optimal contract for a buyer maximizing the expected utility of net loss is a stop-loss contract, Miller shows that the optimal contract, based on a risk-averse seller maximizing the expected utility of gain, is a proportional contract. Raviv (1979), also building on the optimal contracting work of Arrow (1971, 1973), derives the optimal contract as a function of risk preferences of both the consumer and the risk-averse insurance company. He finds conditions necessary to result in contract characteristics such as deductibles and coinsurance. Specifically, he shows that the Pareto optimal insurance policy involves a deductible and coinsurance of losses above the deductible. Schlesinger (1984) develops a two-person bargaining game between the risk-averse consumer and the risk-neutral insurance company and finds that an insurer's expected profits increase as the risk aversion of the consumer increases.

Schlesinger and Venezian (1986) examine the joint production of insurance protection and loss prevention activity. The insurer maximizes profit by choosing the optimal probability of loss and charges a price that reflects an individual's risk premium. In this setting, the insurer will invest in loss prevention as long as the marginal cost of reducing the loss probability does not exceed the reduction in premium that an individual will pay.

Catastrophic risks present some unique considerations in the context of the demand for insurance. Arvan and Nickerson (2000) examine the impact of eligibility for public compensation on the demand for private insurance. In a Nash equilibrium, they show that private underinsurance may in fact be an optimal solution. Grace et al. (2000) estimate the demand for catastrophe insurance using data for the Florida homeowners market. They find that the demand for catastrophe coverage is more price-elastic than the demand for noncatastrophe coverage. This finding is especially relevant to the results of this article, as discussed in the conclusion. Finally, Harrington and Niehaus (2001) show that due to the tax costs associated with equity finance, premiums for catastrophe insurance coverage have a very high loading and, as a result, property owners will often retain the risk of catastrophic losses.

This work extends the previous literature by combining the optimal contracting problem with the individual's decisions regarding the optimal level of insurance and the optimal expenditure on mitigation.

No Loss Mitigation

Consider a world in which one insurance company exists[4] that issues policies to N identical consumers in an earthquake zone. In the risk-neutral framework, firms want to insure as many homes as possible. In practice, the Office of the Superintendent of Financial Institutions (OSFI, the regulatory body for insurers in Canada) limits the maximum amount that insurers can endanger in case of an earthquake to 10 percent of capital and surplus (OSFI, 2001). Regulations in the United States are similar. However, most insurers in both countries limit their exposure to only 3 to 5 percent of their surplus. For the risk-neutral insurer, no relation exists between the premium charged and the number of homes insured.

The assumption of a monopolistic insurer implies, of course, that the consumer cannot purchase protection against catastrophes through market-based securities. This lack of market protection may arise because financial markets are incomplete—the consumer cannot easily create a linear combination of readily accessible securities that provide the same payout as the insurance policy. Although a market now exists for catastrophe-based options, the construction of these options is such that the payoff

[4] The use of a monopolistic insurer implies simply that for some reason, the insurer possesses market power and can exploit this power. In practice, market power can arise through consumer search costs, niche marketing, or government intervention. The assumption of a monopolist is in contrast to Ehrlich and Becker's (1972) model, which features no price-setting behavior. The imperfectly competitive framework, in which insurers have some price-setting power, is beyond the scope of this article.

would not be perfectly correlated with an individual's potential losses. And finally, even if such securities existed, high transaction costs or informational frictions may make this protection prohibitively expensive.

Risk-averse consumers are endowed with an initial wealth W and face a deterministic loss of size V with probability p. Policyholders cannot affect either the probability or severity of loss, and losses are perfectly correlated.[5] The monopolist sells insurance at a rate π (per dollar of coverage) and is assumed to maximize profits on a per contract basis. Therefore, the monopolist chooses the rate per dollar of coverage, π, to

$$\underset{\pi}{\text{Max}}\, N \cdot (\pi - p)\alpha V, \tag{1}$$

where $\pi - p$ is the expected profit of providing one unit (or dollar) of insurance to a consumer and α is the proportion of the building insured by the consumer. The total premium charged by the insurer is $\text{P} = \pi\alpha V$.

If it could, the monopolistic insurer would enforce $\alpha = 1$ (full insurance) and then set the rate such that individuals are indifferent to participating. The profit maximizing rate, π^m, satisfies the constraint that the utility to the consumer from purchasing full insurance equals the expected utility from not purchasing insurance. Then π^m would solve $U(W - \pi^m V) = (1 - p)U(W) + pU(W - V)$, where $U(\cdot)$, the consumer's utility function, is a von Neumann-Morgenstern utility function such that $U'(\cdot) > 0$, $U''(\cdot) < 0$. Mossin (1968) and Schlesinger and Venezian (1986) have discussed this case.

Suppose instead that the insured chooses the amount purchased, αV, in response to the price charged by the insurance company. Then α is the insured's demand for insurance and as π increases, insureds respond by decreasing α. One can derive the specific functional form of $\alpha(\pi)$ from each individual's utility function.[6] Specifically, an individual will choose $\alpha \in [0,1]$ to

$$\underset{\alpha}{\text{Max}}\, E[U] = (1 - p)U(W - \alpha\pi V) + pU(W - \alpha\pi V + \alpha V - V), \tag{2}$$

taking the premium rating structure as given. Implicit in this assumption is a market structure in which insurers act as Stackelberg leaders (von Stackelberg, 1934) whereby the insurer sets the optimal rate, correctly anticipating the consumer's reaction to that rate, and the consumer responds by taking the premium rate as given.

Lemma 1: *In the absence of mitigation, the insurer charges a premium of* $\pi_i^* = p + \alpha_i^*(\pi_i^*)/(-\alpha_i^{*'}(\pi_i^*))$ *and the utility-maximizing consumer purchases an amount of insurance,* α_i^*, *that satisfies* $\frac{\pi_i^*(1-p)}{p(1-\pi_i^*)} = \frac{U'(W-\pi_i^*\alpha_i^*V-(1-\alpha_i^*)V)}{U'(W-\pi_i^*\alpha_i^*V)}$.

In a perfectly competitive marketplace, insurers would charge the actuarially fair price of $\pi = p$. As is well-known, consumers would maximize utility by purchasing full insurance. The markup over the actuarially fair rate per dollar of coverage, p, is a function of both the amount of insurance purchased and the sensitivity of changes

[5] This assumption is for mathematical simplicity and does not materially affect the price charged by the insurance company.

[6] If individuals are heterogeneous, then under the assumption of perfect information insurers can use price discrimination across consumers to maximize profits.

in the amount purchased with respect to the rate. This result is well-known in classical economics: Since α is the consumer's demand function, the Lerner Index, $\frac{\pi-p}{\pi}$, which measures the degree of monopoly power, is inversely proportional to the demand elasticity, $-\alpha(\pi)/\pi\alpha'(\pi)$. The less sensitive consumers are to changes in prices, the higher the price charged by monopolists.

LOSS MITIGATION AVAILABLE

Now assume that consumers have the ability to reduce the size of loss through mitigation. Specifically, let $L(r)$ be the maximum possible loss if the consumer spends r ex-ante. If one does not undertake mitigation, then the size of loss will be V. An increase in mitigation decreases the size of loss such that $L(0) = V$ and $L'(r) < 0$. As mitigation expenditures increase, the maximum potential loss declines. Consumers have full knowledge about $L(r)$, and this function is deterministic in nature. Furthermore, assume that $L''(r) \geq 0$. The marginal effectiveness of mitigation decreases as the amount spent on mitigation increases.[7]

Assume that mitigation is observable but that the contract is based only on αV. That is, the insurance company still writes contracts on the amount of insurance purchased based on the value of the property at risk. In the absence of moral hazard or adverse selection, whether the insurer maximizes profits with respect to the value of the property, which is observable, or to the maximum probability of loss $L(r)$, is immaterial. For if the contact is written instead on the maximum size of loss, $L(r)$, in equilibrium, the fraction of the asset at risk covered, δ, satisfies $\delta^* L(r) = \alpha^* V$ and the amount spent on mitigation is not affected. (See Appendix A for further details.) If informational frictions such as moral hazard exist, then mitigation expenditures may conceivably signal the insured's quality. In this case, the insurer would prefer to write the contract based on the maximum size of loss, $L(r)$. Therefore, the insurer's problem is identical to the function given in Equation (1). The insurer chooses π to $\text{Max}_\pi\, N \cdot (\pi - p)\alpha V$. Individuals take π as given and maximize utility over both the amount spent on mitigation, r, and the amount of insurance purchased, αV, subject to the constraints that $\alpha \in [0,1], r \geq 0$:

$$\underset{\alpha,r}{\text{Max}}\, E[U] = (1-p)U(W - \alpha\pi V - r) + pU(W - \alpha\pi V + \alpha V - L(r) - r).$$

Lemma 2: *In the presence of insurance, the optimal amount spent on mitigation is* $r_m^* = \max\left(0, \left(L'\right)^{-1}\left[\frac{-1}{\pi_m^*}\right]\right)$.[8] *The utility-maximizing consumer insures a proportion,* α_m^*, *of the value of property, where* α_m^* *satisfies* $\frac{\pi_m^*(1-p)}{p(1-\pi_m^*)} = \frac{U'(W-\pi_m^*\alpha_m^* V - L(r_m^*) + \alpha_m^* V - r_m^*)}{U'(W-\pi_m^*\alpha_m^* V - r_m^*)}$. *The optimal rate satisfies* $\pi_m^* = p + \alpha_m^*(\pi_m^*)/(-\alpha_m^{*'}(\pi_m^*))$.

The continuous differentiability of $L(r)$ implies that it is possible for the optimal value of r in an unconstrained model to be less than zero. Define $\pi_m^{\min}$ such that $(L')^{-1}\left(\frac{-1}{\pi_m^{\min}}\right) = 0$. Then for all $\pi < \pi_m^{\min}$, the optimal response of the consumer is to set

[7] This does not preclude the possibility that to protect against earthquake losses consumers may have to spend a considerable amount initially to realize some rewards from mitigation. Obviously, this is not true for all losses. For example, purchasing home security systems and installing smoke detectors is relatively inexpensive.

[8] The function $(L')^{-1}(\cdot)$ is the inverse of the derivative of the mitigation function.

r_m^* to zero. Given the positive monotonic relationship between π and p, expenditures on mitigation will be rationally zero for events of low probability. Ehrlich and Becker (1972) assert that in the case of rare events, such as catastrophic earthquakes, the consumer might choose not to invest in any mitigation activity.

Since $\alpha'(r) < 0$ and $\frac{\partial r}{\partial \pi} < 0$, as prices decrease, consumers mitigate less and purchase more insurance. In a perfectly competitive marketplace, insurers would charge the actuarially fair premium of $\pi = p$. The optimal amount of mitigation satisfies $r^* = \max\left(0, (L')^{-1}\left[\frac{-1}{p}\right]\right)$, where $r^* \leq r_m^*$, the amount spent if the insurer were a monopolist. The consumer insures up to the value of the potential loss $L(r^*)$. Comparing the two scenarios, in the monopoly situation the insured purchases less insurance and spends more on mitigation than in the perfectly competitive market.

THE IMPACT OF DISASTER ASSISTANCE ON MITIGATION AND CONTRACT STRUCTURE

One common role that the U.S. government plays in the context of catastrophic events is provider of ex post assistance. For example, after a disaster is declared, disaster assistance is available in the form of low interest loans and cash grants. Individuals and businesses may be eligible for such assistance if they live, own a business, or work in a county declared a major disaster area, incur sufficient property damage or loss, and, depending on the type of assistance, do not have the insurance or other resources to meet their needs.

This section looks at the impact of such assistance if consumers and insurance companies alike rationally and accurately anticipate it. Specifically, assume that, after a loss has occurred, the government will pay for a fraction, $1 - D$, of the uninsured losses. That is, the government pays $(1 - D)(L(r) - \alpha V)$ of each loss. The insurance company's maximization problem does not change. Therefore, the insurer's problem is identical to the function given in Equation (1). The insurer chooses the rate π to $\underset{\pi}{\text{Max}}\, N \cdot (\pi - p)\alpha V$. Individuals take π as given and maximize utility over both the amount spent on mitigation, r, and the amount of insurance purchased, αV, subject to the constraints that $\alpha \in [0,1]$, $r \geq 0$. Their expected utility is altered because of the presence of rationally anticipated disaster assistance:

$$\underset{\alpha, r}{\text{Max}}\, E[U] = (1 - p)U(W - \alpha\pi V - r) + pU(W - \alpha\pi V + \alpha V - L(r) - r + (1 - D)(L(r) - \alpha V)). \tag{3}$$

Lemma 3: *If government assistance is available and rationally anticipated, the consumer purchases an amount of insurance, $\alpha_g^* V$, where α_g^* satisfies $\frac{(1-p)\pi_g^*}{p(D-\pi_g^*)} = \frac{U'(W-\alpha_g^*\pi_g^*V-r_g^*-D(L(r_g^*)-\alpha_g^*V))}{U'(W-\alpha_g^*\pi_g^*V-r_g^*)}$. The amount spent on mitigation solves $r_g^* = \max\left(0, (L')^{-1}\left[\frac{-1}{\pi_g^*}\right]\right)$. As in the previous two models, the rate charged by the insurance company satisfies $\pi_g^* = p + \alpha_g^*(\pi_g^*)/(-\alpha_g^{*'}(\pi_g^*))$, where π_g^* is the optimal premium rate given government assistance.*

As in the previous model, define $\pi_g^{\min}$ such that $(L')^{-1}\left(\frac{-1}{\pi_g^{\min}}\right) = 0$. Then for all $\pi < \pi_g^{\min}$, the optimal response of the consumer is to set r_g^* to zero. The presence of government assistance does not affect the amount spent on mitigation directly in that the individual will still mitigate until the marginal cost of mitigation equals the marginal cost of insurance. However, the presence of government assistance increases the elasticity of demand for insurance and therefore decreases the price charged by the insurer. Consumers, then, will be less likely to mitigate in the presence of government assistance.

The impact of a change in government assistance on the amount of insurance purchased is more difficult to calculate analytically. The numerical examples that follow suggest that as the amount of government assistance increases, the amount of insurance purchased by the individual will decrease. This is consistent with the results of Arvan and Nickerson (2000).

In a perfectly competitive marketplace, insurers would charge the actuarially fair premium of $\pi = p$. The optimal amount of mitigation satisfies $r^* = \max\left(0,(L')^{-1}\left[\frac{-1}{p}\right]\right)$. Since $p < \pi_g^*$, consumers mitigate less in a perfectly competitive marketplace. Unlike with the previous models, consumers will purchase less than full insurance. The consumer insures a proportion $\alpha^* < 1$ of the property, where α^* satisfies $\frac{(1-p)}{(D-p)} = \frac{U'(W-\alpha_g^* pV-r_g^*-D(L(r_g^*)-\alpha_g^* V))}{U'(W-\alpha_g^* pV-r_g^*)}$. Because of the government intervention, even actuarially fair insurance appears to be overpriced. The consumer responds by purchasing less than full insurance.

Government Intervention Through Subsidized Mitigation

The previous section demonstrates that the rational anticipation of government assistance reduces the incentive for consumers to undertake loss reduction activities. This is at odds with government goals of encouraging more loss mitigation. One manner in which governments or insurers can encourage mitigation is through subsidization.[9] This section looks at the impact of such subsidies if consumers and insurance companies alike rationally and accurately anticipate it. Specifically, assume that the subsidy program subsidizes s percent of mitigation expenditures. The consumer invests $(1-s)r$ in loss mitigation activities and faces a maximum possible loss of $L(r)$. The impact of this subsidy, from the consumer's perspective, is to make mitigation more cost-effective. As in the previous models, the insurer's maximization problem is unaffected. The insurer chooses the rate π to $\text{Max}_\pi\, N \cdot (\pi - p)\alpha V$. Individuals take π as given and maximize utility over both the amount spent on mitigation, $(1-s)r$, and the amount of insurance purchased, αV, subject to the constraints that $\alpha \in [0,1]$, $r \geq 0$. Their expected utility is altered due to this mitigation subsidy:

$$\underset{\alpha,r}{\text{Max}}\, E[U] = (1-p)U(W - \alpha\pi V - (1-s)r) + pU(W - \alpha\pi V + \alpha V - L(r) - (1-s)r).$$

Lemma 4: *If mitigation is subsidized, the optimal amount spent on mitigation is* $r_s^* = \max\left(0,(L')^{-1}\left[\frac{-(1-s)}{\pi_s^*}\right]\right)$. *The utility-maximizing consumer insures a proportion,* α_s^*, *of the value of property, where* α_s^* *satisfies* $\frac{\pi_s^*(1-p)}{p(1-\pi_s^*)} = \frac{U'(W-\pi_s^*\alpha_s^* V-L(r_s^*)+\alpha_s^* V-(1-s)r_s^*)}{U'(W-\pi_s^*\alpha_s^* V-(1-s)r_s^*)}$. *The optimal premium satisfies* $\pi_s^* = p + \alpha_s^*(\pi_s^*)/(-\alpha_s^{*\prime}(\pi_s^*))$, *where* π_s^* *is the optimal premium rate given government assistance.*

9 Although such programs do not exist as of yet for earthquake mitigation for individual property owners, many municipal utilities have introduced household subsidy programs for the purchase of water- and energy-efficient appliances.

As in the previous models, define $\pi_s^{\min}$ such that $(L')^{-1}(\frac{-(1-s)}{\pi_s^{\min}}) = 0$. Then for all $\pi < \pi_s^{\min}$, the optimal response of the consumer is to set r_s^* to zero. Even with subsidization, it is optimal to not mitigate against rare events.

The impact of a mitigation subsidy is ambiguous. In the absence of strategic pricing behavior, the immediate impact of the subsidy is to increase expenditures on mitigation and to decrease the amount of insurance purchased. However, as the elasticity of demand is increased, the monopolistic insurer responds by reducing the premium rate charged. The price decrease increases the amount of insurance purchased and decreases the amount spent on mitigation. The numerical examples that follow suggest that the former is dominant. As the amount of subsidy increases, the amount of insurance purchased by the individual decreases and the amount spent on mitigation activities increases. A natural consequence is that the insurer's profit earned per policy decreases.

In a perfectly competitive marketplace, insurers would charge the actuarially fair premium of $\pi = p$, and consumers would spend an amount $r^* = \max\left(0, (L')^{-1}\left[\frac{1-s}{p}\right]\right)$ on mitigation. As in the first two models, the consumer purchases full insurance up to the value of the property at risk, $L(r^*)$.

THE ASSUMPTION OF RISK NEUTRALITY

As discussed in the introduction, it is unclear whether insurance companies are risk-neutral or risk-averse with respect to selling earthquake insurance. The purpose of this section is to examine the impact of this assumption.

Suppose that the monopolistic insurer is now risk-averse in terms of offering earthquake insurance. Specifically, consider an insurer with initial capital, Ω, that issues policies to N identical consumers in an earthquake zone. Assume that the number of contracts is set exogenously.[10] Furthermore, assume that the insurer's utility function is given by $\Pi(\cdot)$, such that $\Pi'(\cdot) > 0$, and $\Pi''(\cdot) < 0$. Therefore, the monopolist chooses the premium rate, π, to

$$\underset{\pi}{\operatorname{Max}}\, p\,\Pi(\Omega - N\alpha(\pi)V(1-\pi)) + (1-p)\Pi(\Omega + N\pi\alpha(\pi)V). \tag{4}$$

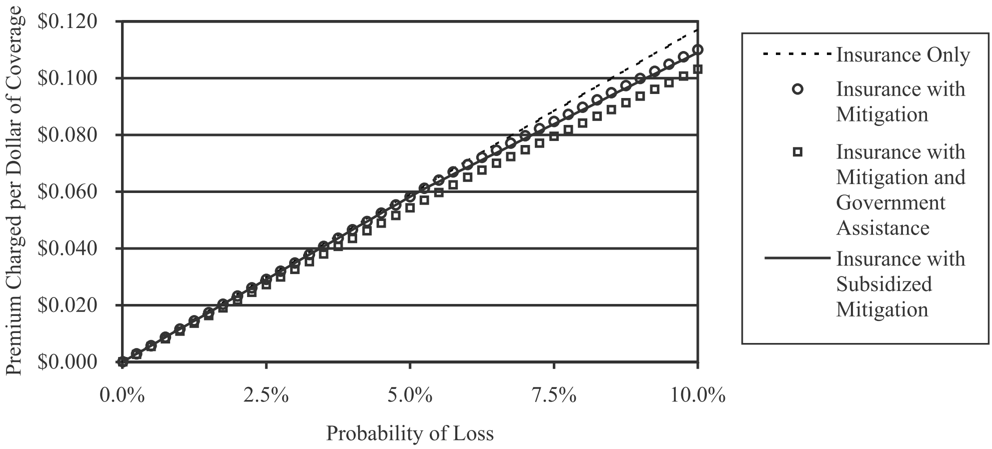
Lemma 5: *The utility-maximizing rate per dollar of coverage price charged by a risk-averse monopolistic insurer is given by* $\pi^* = p \cdot \frac{\Pi'(\Omega - N\alpha(\pi^*)V(1-\pi^*))}{p\Pi'(\Omega - N\alpha(\pi^*)V(1-\pi^*)) + (1-p)\Pi'(\Omega + N\pi^*\alpha(\pi^*)V)} + \alpha(\pi^*)/\alpha'(\pi^*)$. *The consumer's demand functions do not change, and as such, the response functions of the consumers under the various scenarios are given in Lemmas 1 through 4.*

The risk-averse insurer wishes to minimize the variance of profits in the loss and no-loss states. Thus, the risk-averse insurer will charge a higher premium than the risk-neutral insurer. Consumers respond by purchasing less insurance, which lowers the firm's profits in the no-loss state but also increases profits in the loss state. As N increases, the risk-averse insurer responds by charging higher premiums.

[10] Government restrictions on exit from the market for catastrophe coverage, as well as requirements for offering coverage, make this a reasonable assumption.

FIGURE 1

Premium Charged by Insurer With Respect to Probability of Loss



EXAMPLE

Since a comparison of the different scenarios is mathematically intractable, we use a numerical example to illustrate the results. We computed results for both the risk-neutral and the risk-averse cases, although we only show graphs for the risk-neutral scenario.[11] Overall, the risk-averse insurer charges a higher premium for coverage. Consumers react by purchasing less insurance and undertaking more mitigation.

Assume that the consumer's utility function is given by $U(W) = \log(W + \phi)$ for $\phi > 0$. As noted by Pratt (1964), this function exhibits decreasing risk aversion. From Mossin (1968), the premium that the consumer is willing to pay decreases as initial wealth (or the difference between W and V) increases, and the proportion of the loss insured will also decrease as initial wealth increases. Thus as V becomes larger, the consumer is willing to pay more for insurance and will insure a greater portion of the loss. For the example, assume that the consumer's total wealth, $W + \phi$, is $500,000.

Suppose also that $L(r) = Ve^{-\beta r}$, where β measures the effectiveness of mitigation. For this example, assume that the value of the property at risk, V, is $150,000. The effectiveness of mitigation, β, is chosen to be 0.00008, so that if a consumer spends $2,000 on mitigation, the maximum size of loss is $128,000. In these examples, the government subsidizes 10 percent of mitigation expenditures, $s = 0.1$, and pays for 10 percent of uncovered losses, $D = 0.9$.

Figure 1 shows the price charged with respect to the underlying probability of loss. As expected, prices increase as the probability of loss increases. The option to mitigate reduces the price charged by the insurance company. If mitigation is subsidized, the rate per dollar charged is slightly less than if mitigation were not subsidized. The price is lowered again if government assistance is rationally anticipated. The more elastic the demand for insurance (as in the presence of rationally anticipated government assistance and mitigation), the smaller the loading earned by the insurer.

[11] Graphs for the risk-averse insurer are available from the authors on request.

Figure 2
Consumer Expenditures on Mitigation With Respect to Probability of Loss

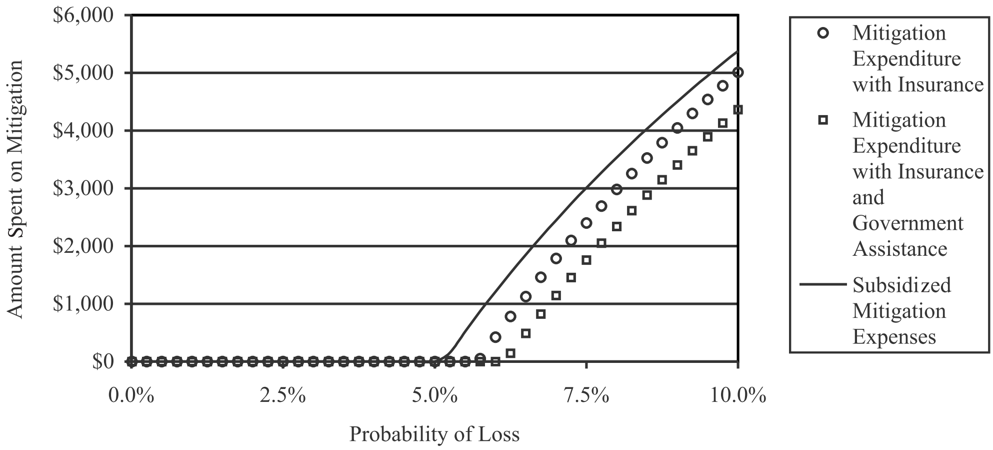


Figure 2 shows the relationship between the amount consumers spend on mitigation and the probability of loss. Figure 1 provides the corresponding premium schedule. For low probabilities of loss, consumers cannot be enticed to voluntarily undertake mitigation, even if mitigation is subsidized. Once the probability of loss passes a certain threshold, then the amount spent on mitigation increases as the probability of loss increases. Consumers spend more on mitigation, and are willing to mitigate at lower probabilities of loss, if ex post government assistance is not available. Consumers spend the most on mitigation if the government subsidizes mitigation.

The amount spent on mitigation with respect to the underlying probability of loss resembles the payout of a call option. This is not coincidental. Like insurance, mitigation is a contingent asset. Consumers invest up front in loss reduction activities and are only "in the money" (that is, they realize a reduction in the maximum possible loss) if a loss occurs.

Figure 3 shows the impact of the price of insurance on the amount of insurance purchased. This graph displays the proportion of the maximum possible loss, $L(r)$, that is insured. The proportion of the property insured is fairly constant if mitigation is not available. If mitigation is available, the proportion of the maximum possible loss insured declines as the probability of loss increases. The proportion insured is the same whether or not mitigation is subsidized. Consumers insure the smallest proportion of the loss if the government pays for some disaster assistance.

Table 1 provides a comparison of price levels and the amount of insurance purchased. As can be seen from this table, a minute change in the price of insurance produces large changes in the amount of insurance purchased. This extreme sensitivity has been noted by other researchers and indeed is a problem with the expected utility paradigm.[12]

[12] Grace et al. (2000) show that the demand for catastrophic insurance is very elastic with respect to price.

FIGURE 3
Proportion of Maximum Possible Loss Insured With Respect to Probability of Loss

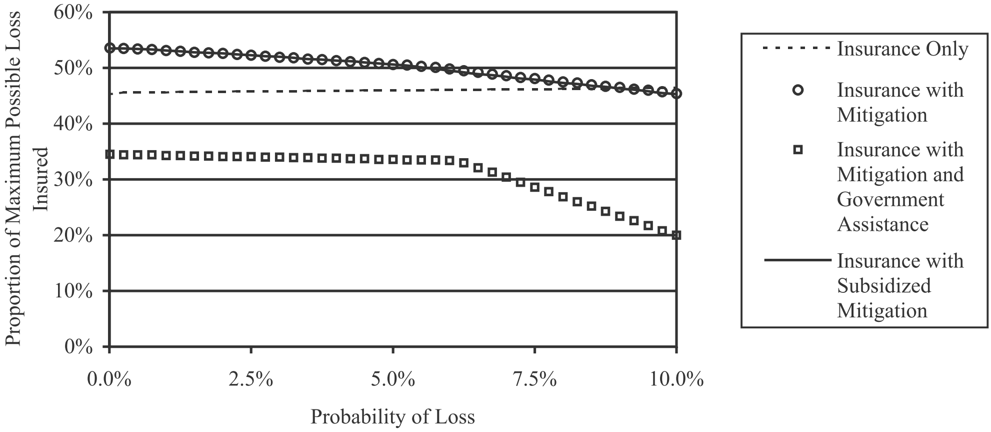


TABLE 1
Comparison of Premiums and Amount of Insurance Purchased When the Probability of Loss Is 2 Percent

Scenario	Insurer Preferences	Rate per Dollar of Coverage – π^*	Proportion of Property Insured – α^*
Insurance Available	Risk-Neutral	2.381¢	45.73%
	Risk-Averse	2.441¢	38.67%
Insurance and Mitigation Available	Risk-Neutral	2.326¢	52.55%
	Risk-Averse	2.392¢	44.39%
Insurance, Mitigation, and Government Assistance Available	Risk-Neutral	2.181¢	34.14%
	Risk-Averse	2.244¢	25.34%
Insurance With Subsidized Mitigation	Risk-Neutral	2.325¢	52.56%
	Risk-Averse	2.392¢	44.40%

*At 2 percent, mitigation is not optimal in any of the scenarios, so the proportion of the property insured equals the proportion of the property at risk.

Figure 4 shows the expected profits earned by the risk-neutral insurer under the various scenarios. According to the numerical results, the insurance company does not benefit from the presence of mitigation. Profits are maximized for the insurer if mitigation and government assistance are not available. Very little difference in expected profits earned exists if mitigation is available but not optimal and if mitigation were not offered. However, once mitigation becomes optimal, insurer profits decline rapidly.

Consumers purchase little coverage if disaster assistance is available and, as such, the profits earned by the insurance company are much less than if this was not available. If mitigation is subsidized, profits are greater than if ex post disaster assistance was rationally anticipated. Profits are less than if mitigation were not subsidized, once it is optimal to invest in loss reduction activities.

From a policy perspective, Figure 5 is perhaps the most informative. This graph displays the relationship between the probability of loss and the extent of the uncovered loss, $L(r) - \alpha V$. In all cases, once mitigation has been undertaken, the amount of uncovered loss falls. If mitigation is subsidized, the extent of the uncovered loss is smallest. The unprotected loss is greatest if the government provides ex post assistance, although the amount of uncovered loss falls rapidly for high probabilities of loss. In this scenario, the government is responsible for 10 percent of the loss. If mitigation was not available, consumers would only insure roughly 54 percent of the property value.

FIGURE 4
Expected Insurer Profits per Policy Sold With Respect to Probability

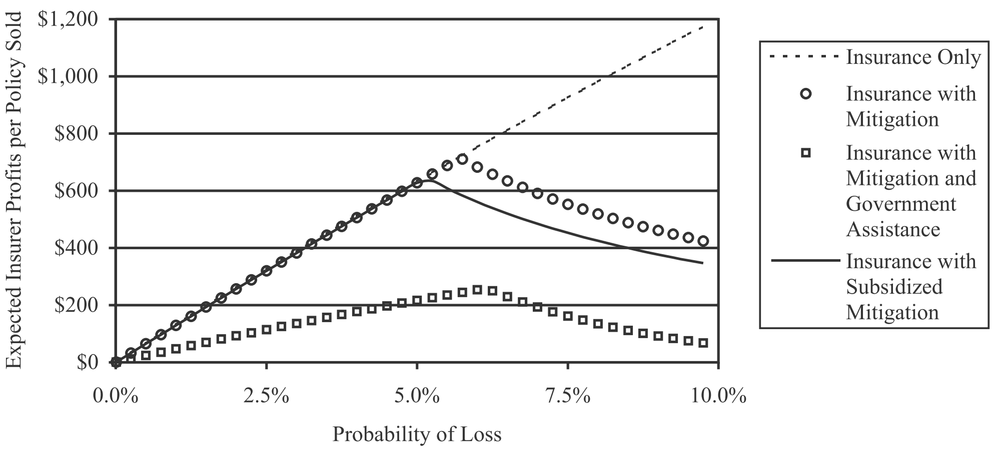
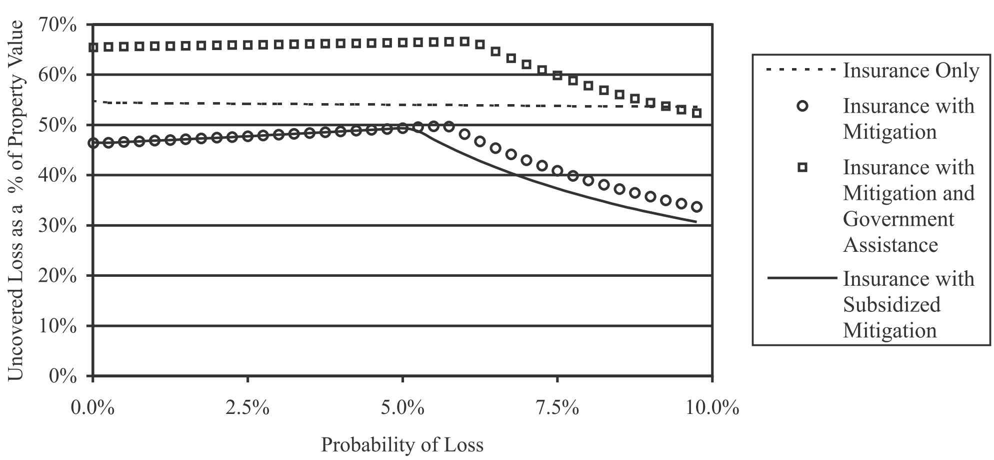


FIGURE 5
Extent of Uncovered Loss as a Percentage of Property Value With Respect to Probability



Conclusions and Future Work

The results in the previous section have important implications with regard to natural disaster protection legislation. Currently, the market for earthquake insurance is characterized by premia that do not provide discounts for investment in loss mitigation. Although insurers' underwriting criteria account for such factors as building construction, soil conditions, and seismic zones, insurers do not base their rates on individual mitigation efforts such as upgrading chimneys, securing water heaters, or anchoring walls to the foundation.[13] As noted previously, insurers do not have an incentive to introduce risk-based premia in order to encourage mitigation because it results in lower expected profits. However, following from the discussion in the introduction, it would be socially beneficial if individuals adopted cost-effective mitigation activities.

When mitigation is available, the decrease in premia charged is not a reward for undertaking mitigation but is instead the rational response of a monopolist to an increase in the elasticity of demand. The results displayed are sensitive to the marginal effectiveness of mitigation. If this effectiveness was to increase, then consumers would prefer to spend more on mitigation and less on insurance. If indeed regulators believe that mitigation above and beyond what is currently being undertaken is required, then one possible course of action is for the regulator to require the optimal level of mitigation to be employed by all building owners. Subsidies could be made available, such as existing programs that provide for rebates if water efficient appliances are purchased, to those who undertake mitigation. From the previous section, subsidies do a better job than ex post disaster assistance in encouraging mitigation and reducing the extent of the expected uncovered loss. However, this raises the issues of where the funds would come from and which agency would monitor the disbursement of such funds. Subsidized mitigation is used, to some extent, by the Federal Emergency Management Agency (FEMA). FEMA has a hazard mitigation grant program that provides grants to states and local governments to implement long-term hazard mitigation measures after a major disaster. Although individual homeowners and businesses may not apply directly to the program, a community may apply on behalf of homeowners and businesses.

An alternate solution would be for regulators to enforce risk-based premia that specifically reflect insureds' investment in mitigation. That is, the more the individual spends on mitigation, the lower the price per dollar of coverage. Although insurers would not choose such a premium structure (due to lower expected profits), if the federal government decided in favor of a federal reinsurance program, then the quid pro quo could be for insurers to encourage mitigation through risk-based premia. The behavior of consumers if risk-based premium structures exist needs to be examined. The cost-effectiveness of this regulatory scheme, compared to outright subsidization or ex post disaster assistance, also needs to be examined, but both of these issues are beyond the scope of this study.

Finally, note that the results in the preceding sections will be affected by moral hazard and adverse selection, if they exist. However, as noted by Jaffee and Russell (1997),

[13] This information was obtained through discussions with five major North American property-liability insurers.

since the probability of a catastrophic loss is not private information, there should be little adverse selection in the marketplace. Ex ante moral hazard does not exist, since the insured cannot affect the probability of a catastrophe. One can control any potential ex post moral hazard by careful contracting and vigilant investigation.

APPENDIX A

Proof of Lemma 1: In this model, the consumer chooses an amount of insurance, α_i^*, to maximize expected utility, and the insurer chooses the premium π_i^* to maximize its profits, based on consumer demand. From Equation (1), rewrite the monopolist's problem as $\text{Max}_\pi\, N \cdot (\pi - p)\alpha(\pi)V$. Therefore, the first derivative with respect to the premium charged solves

$$\pi_i^* = p + \alpha_i^*(\pi_i^*)/ - \alpha_i^{*'}(\pi_i^*).$$

And from the consumer's utility function, given in Equation (2), the first derivative with respect to the amount of insurance purchased is

$$\frac{\partial E(U)}{\partial \alpha} = -\pi V(1-p)U'(W-\pi\alpha V) + Vp(1-\pi)U'(W-\pi\alpha V - V + \alpha V). \quad \text{(A1)}$$

Setting Equation (A1) equal to zero yields $\frac{\pi_i^*(1-p)}{p(1-\pi_i^*)} = \frac{U'(W-\pi_i^*\alpha_i^* V - V + \alpha_i^* V)}{U'(W-\pi_i^*\alpha_i^* V)}$. Q.E.D.

Proof of Lemma 2: In this model, individuals take π as given and maximize utility over both the amount spent on mitigation, r, and the amount of insurance purchased, α, subject to the constraints that $\alpha \geq 0$, $r \geq 0$. From Equation (2), α satisfies

$$\frac{(1-p)\pi_m^*}{p(1-\pi_m^*)} = \frac{U'(W-\alpha_m^*\pi_m^* V + \alpha_m^* V - L(r_m^*) - r_m^*)}{U'(W-\alpha_m^*\pi_m^* V - r_m^*)}, \quad \text{(A2)}$$

and

$$\frac{\partial E[U]}{\partial r} = -(1-p)U'(W-\alpha\pi V - r) - p(L'(r)+1)U'(W-\alpha\pi V + \alpha V - L(r) - r). \quad \text{(A3)}$$

From the Kuhn-Tucker conditions, we know that $\frac{\partial E[U(r_m^*,\alpha_m^*)]}{\partial r_m^*} \leq 0$, $r_m^* \geq 0$ and $r_m^* \frac{\partial E[U(r_m^*,\alpha_m^*)]}{\partial r_m^*} = 0$. Therefore, from Equations (A2) and (A3), $-L'(r_m^*) = 1/\pi_m^*$.

As in Lemma 1, the solution to the monopolist's problem solves $\pi_m^* = p + \alpha_m^*(\pi_m^*)/(-\alpha_m^{*'}(\pi_m^*))$. Q.E.D.

Regarding $\delta^* L(r)$ and $\alpha^* V$, suppose that in the absence of moral hazard and asymmetric information, insurers write contracts based on $L(r)$, the maximum possible loss. The consumer's objective function is given by

$$\underset{\delta,r}{\text{Max}}\, E[U] = (1-p)U(W-\delta\pi L(r) - r) - pU(W - \delta\pi L(r) - (1-\delta)L(r) - r)$$

for $\delta \in [0,1]$, and $r \geq 0$. In equilibrium, the consumer purchases a proportion of coverage, δ^*, which satisfies

$$\frac{U'(W - \delta_m^* \pi_m^* L(r_m^*) - (1 - \delta_m^*)L(r_m^*) - r_m^*)}{U'(W - \delta_m^* \pi_m^* L(r_m^*) - r_m^*)} = \frac{\pi_m^*(1 - p)}{p(1 - \pi_m^*)} \tag{A4}$$

and spends r_m^* on mitigation, where r_m^* satisfies

$$\frac{U'(W - \delta_m^* \pi_m^* L(r_m^*) - (1 - \delta_m^*)L(r_m^*) - r_m^*)}{U'(W - \delta_m^* \pi_m^* L(r_m^*) - r_m^*)} = \frac{(1 - p)}{p} \cdot \frac{1 + \delta_m^* \pi_m^* L'(r_m^*)}{1 + (1 - \delta_m^*)L'(r_m^*) + \delta_m^* \pi_m^* L'(r_m^*)} \tag{A5}$$

The condition in Equation (A4) is the same as if the contract were written on V. The relationship in Equation (A5) equates the marginal deduction in the loss with the marginal cost of the loss reduction activities. Equating Equations (A4) and (A5) yields the optimal condition that $r_m^* = \max(0, (L')^{-1}[\frac{-1}{\pi_m^*}])$.

Proofs of Lemma 3 and Lemma 4 follow from above. Details are available from the authors.

Proof of Lemma 5: In this model, the insurance company chooses premium levels to maximize its expected utility. Taking the first derivative of Equation (4) with respect to π yields

$$\pi^* = p \cdot \frac{\Pi'(\Omega - N\alpha(\pi^*)V(1 - \pi^*))}{p\Pi'(\Omega - N\alpha(\pi^*)V(1 - \pi^*)) + (1 - p)\Pi'(\Omega + N\pi^*\alpha(\pi^*)V)} + \alpha(\pi^*)/(-\alpha'(\pi^*)).$$

Consumers then take premiums as given and maximize their utility. Q.E.D.

Appendix B

Table A1
Summary of Derived Results

Insurance Conditions	Mitigation Optimal	Amount Spent on Mitigation	Optimality Condition for Proportion of Property Insured
Insurance Available	N/A		$\frac{\pi_i^*\left(1-p\right)}{p\left(1-\pi_i^*\right)} = \frac{U'\left(W-\pi_i^*\alpha_i^*V-\left(1-\alpha_i^*\right)V\right)}{U'\left(W-\pi_i^*\alpha_i^*V\right)}$
Insurance Available and Mitigation	Yes	$\left(L'\right)^{-1}\left(\frac{-1}{\pi_m^*}\right)$	$\frac{\pi_m^*\left(1-p\right)}{p\left(1-\pi_m^*\right)} = \frac{U'\left(W-\pi_m^*\alpha_m^*V-L\left(r_m^*\right)+\alpha_m^*V-r_m^*\right)}{U'\left(W-\pi_m^*\alpha_m^*V-r_m^*\right)}$
	No	0	$\frac{\pi_m^*\left(1-p\right)}{p\left(1-\pi_m^*\right)} = \frac{U'\left(W-\pi_m^*\alpha_m^*V-\left(1-\alpha_m^*\right)V\right)}{U'\left(W-\pi_m^*\alpha_m^*V\right)}$
Government Assistance, Mitigation, and Insurance	Yes	$\left(L'\right)^{-1}\left(\frac{-1}{\pi_g^*}\right)$	$\frac{\left(1-p\right)\pi_g^*}{p\left(D-\pi_g^*\right)} = \frac{U'\left(W-\alpha_g^*\pi_g^*V-r_g^*-D\left(L\left(r_g^*\right)-\alpha_g^*V\right)\right)}{U'\left(W-\alpha_g^*\pi_g^*V-r_g^*\right)}$
	No	0	$\frac{\left(1-p\right)\pi_g^*}{p\left(D-\pi_g^*\right)} = \frac{U'\left(W-\alpha_g^*\pi_g^*V-DV\left(1-\alpha_g^*\right)\right)}{U'\left(W-\alpha_g^*\pi_g^*V\right)}$
Insurance With Subsidized Mitigation	Yes	$\left(L'\right)^{-1}\left(\frac{1-s}{\pi_s^*}\right)$	$\frac{\pi_s^*\left(1-p\right)}{p\left(1-\pi_s^*\right)} = \frac{U'\left(W-\pi_s^*\alpha_s^*V-L\left(r_s^*\right)+\alpha_s^*V-(1-s)\,r_s^*\right)}{U'\left(W-\pi_s^*\alpha_s^*V-(1-s)\,r_s^*\right)}$
	No	0	$\frac{\pi_s^*\left(1-p\right)}{p\left(1-\pi_s^*\right)} = \frac{U'\left(W-\pi_s^*\alpha_s^*V-\left(1-\alpha_s^*\right)V\right)}{U'\left(W-\pi_s^*\alpha_s^*V\right)}$

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