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Flexible Spending Accounts as Insurance

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Abstract

We model flexible spending accounts (FSAs) as a special type of insurance policy. We prove the following results given losses drawn from a continuous distribution: (1) the optimal election amount, F^*, is increasing in the consumer's level of risk aversion; (2) F^* is increasing in the level of the maximum loss; If utility is decreasing in absolute risk aversion (DARA), then F^* is (3) decreasing in income and (4) increasing in the marginal tax rate.

Introduction

Flexible spending accounts (FSAs) for health care expenses offer an attractive way to avoid taxes. A recent survey of 681 major U.S. firms suggests that 79 percent of these employers offered such accounts in 1995 (up from 51 percent in 1990) and that a rapidly growing share of employees are taking advantage of these offerings (Hewitt Associates, 1996). The advantage to the employee can be significant, although a financial risk exists. The advantage is that qualified expenses escape both state and federal income taxation and all payroll taxes; the risk is that unused funds are forfeited to the employee's firm at the end of a calendar year. Employers also avoid payroll taxes on employee elections.

Though the actual reduction in tax revenues due to FSA usage is unknown, the potential effect on federal revenues is quite large. As an example, consider that estimated out-of-pocket medical expenses for 1997 were nearly \$190 billion (U.S. Bureau of the Census, 2000, Table 161). If only 15 percent of those expenses were paid for with FSA monies and the average individual faced a 15 percent federal income tax rate plus a 15 percent payroll levy (employer and employee portions), the reduction in federal tax revenue would be more than \$8.5 billion.

Recent work by Cardon and Showalter (2001) explores a model of FSA participation and usage. The most general model consists of two time periods: In period 1, a

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risk-averse individual chooses how much income to allocate to the FSA (the "election" amount). The amount in the FSA is tax-exempt and is used to pay for health expenditures up to the election amount. Health status in period 2 is subject to uncertainty, and thus the choice is made in an expected utility framework. In period 2, health status is revealed and the individual chooses between consumption and health care expenditures. Health expenditures beyond the election amount reduce consumption of the *numeráire good*. If health expenditures are less than the election amount, the difference is lost (the "use-it-or-lose-it" provision of FSAs). This model is difficult to analyze due to its generality and leads to ambiguous comparative statics results. In another, more tractable framework, instead of facing a continuum of choices for health expenditures, the individual faces a loss in the second period of a fixed amount with probability p. This model yields sharper comparative static results.

This article builds on this framework in two dimensions. First, we model losses as draws from a continuous distribution. Second, we show how distributional changes and differences in the level of risk aversion will affect the election amount. We discuss implications and possible extensions in the conclusion.

MODEL

An individual with initial pretax income $I > 0$ faces a loss $x \in [0, L]$ where the maximum loss $L < I$. The continuous, nondegenerate probability density function[1] for the loss is $g(x)$ with a distribution function represented by $G(x)$. The individual has preferences defined over consumption C, represented by a smooth, concave utility function $U(C)$ with $U' > 0$ and $U'' < 0$. The individual is eligible to participate in an FSA program that allows the loss (or a portion of the loss) to be paid out of pretax dollars. The FSA election amount F must be chosen prior to the realization of the loss. Losses in excess of F are not covered and unused funds are forfeited.[2]

One can consider FSAs in a traditional insurance framework. The employee purchases a policy that pays all expenses up to an upper limit F and that has premium $(1 - t)F$. Consumption C depends on the loss, the marginal tax rate t, and on the choice of F. If $x < F$, then consumption is $c_1 = (I - F)(1 - t)$. That is, when the loss is smaller than F, consumption is independent of the loss, since the unused amount of F cannot be used on general consumption. If $x > F$, then consumption is $c_2 = I(1 - t) + tF - x$. Expected utility is given by

$$E[U(C(F))] = \int_0^F U((I - F)(1 - t))g(x)dx + \int_F^L U(I(1 - t) + tF - x)g(x)dx. \quad (1)$$

[1] If the distribution is degenerate such that $x = L$ with probability 1, the election amount is L.

[2] By law, forfeited FSA monies revert to the employer. It is interesting to note that the full election amount must be available immediately, even if the employer allows deductions to be spread evenly across paychecks. This means that an employee could withdraw the full amount after the first paycheck, then quit, leaving the employer with a negative balance. In addition, FSAs typically are limited to $5,000 per year, although at least for the health care FSAs such a limit does not appear to be a legal requirement. We have not modeled these aspects of the FSA program, to keep the analysis focused on what we believe to be the most essential features of the program (a similar provision exists for child care expenditures).

The consumer optimizes expected utility by choosing an optimal election amount F.[3] The first-order condition for F is (using Leibniz's rule)

$$\frac{dE[U(C)]}{dF} = \int_0^F (t-1)\,U'(c_1)g(x)dx + \int_F^L t\,U'(c_2)g(x)dx = 0. \tag{2}$$

Let F^* be the solution to Equation (2). Note that the first integral above is negative and the second is positive. It is easily seen that $dE[U(C)]/dF$ is positive at $F = 0$ and negative at $F = L$. This implies that $F^* \in (0,L)$ when $t > 0$.

The intuition underlying the two integrals of Equation (2) is useful to explore. The first integral is expected marginal utility when the observed draw of loss x is less than the election amount F. When F exceeds x, some consumption is lost and the consumer would have preferred ex post to have chosen a smaller value of F. Thus this term is negative, and one can view this integral as a measure of the marginal cost of F.[4]

The second integral is the corresponding marginal benefit. This integral includes draws from the loss distribution that exceed the chosen F; by not choosing a higher F, the consumer has foregone the benefit of the tax-preferred status of FSAs with the associated higher consumption. To optimize, the consumer balances these tradeoffs such that the marginal cost equals the marginal benefit.

Continuing with the analysis, since c_1 is independent of x, rewrite Equation (2) as

$$\frac{dE[U(C)]}{dF} = (t-1)U'(c_1)\,G(F) + t\int_F^L U'(c_2)\,g(x)dx = 0. \tag{3}$$

Now expand U' about the point c_1 to obtain

$$U'(c_2) = U'(c_1) + U''(c_1)(F-x) + \frac{1}{2}U'''(c_1)(F-x)^2 + \frac{1}{6}U''''(\tilde{c})(F-x)^3,$$

for some number $\tilde{c}$ between c_1 and c_2. Note that $c_2 - c_1 = F - x < 0$ by definition of c_1 and c_2.

Inserting this into Equation (3) and rearranging gives

$$\begin{aligned}\frac{dE[U(C)]}{dF} &= U'(c_1)\left[t - G(F)\right] - t\,U''(c_1)(1-G(F))\,E[X-F|X>F] \\ &+ \frac{t}{2}U'''(c_1)(1-G(F))\left[Var(X|X>F) + \left[E[X-F|X>F]\right]^2\right] \\ &+ \frac{t}{6}\int_F^L U''''(\tilde{c})(F-x)^3 g(x)dx = 0.\end{aligned} \tag{4}$$

To illustrate the implications of this equation, suppose for the moment that the consumer is risk-neutral, which implies that the smooth, concave utility function $U(C)$

[3] Global concavity of expected utility in F follows from the concavity of U.

[4] It is more precise to consider this integral as the negative of the marginal cost, so cost would always be positive, and in equilibrium the positive marginal cost would equal the marginal benefit.

is linear and therefore the higher order derivatives are equal to zero.[5] In this special case, the optimal choice of F solves $t = G(F^*)$. This means that the consumer will set the optimal election amount F^* at the t^{th} percentile of the distribution of x.[6]

Next, consider the case if the consumer is risk-averse, but that U''' and all higher order derivatives are equal to zero. In this instance, the term added from the risk-neutral case is positive. This has the interesting implication that a risk-averse consumer will choose a higher F than the risk-neutral consumer; $t < G(F^*)$, so $F^* > t^{\text{th}}$ percentile. Also note that in this case, the first-order condition can be rearranged in terms of the measure of absolute risk aversion:

$$-\frac{U''(c_1)}{U'(c_1)} = \frac{-(t - G(F))}{t(1 - G(F))E[X - F|X > F]}.$$

This says that the standard measure of risk aversion is equal to the amount by which $G(F^*)$ exceeds the tax rate, scaled by the denominator. Finally, if $U''' > 0$, then F will be larger than would be chosen if the consumer were risk-averse but had $U''' = 0$. This is an example of prudence (Kimball, 1990).

These results have an interesting interpretation when one views an FSA as an insurance policy. Intuitively, for a constant price per unit of coverage, a risk-averse consumer will purchase more insurance the greater the level of risk aversion. With an FSA, the price per unit of coverage is $(1 - t)$ and the level of insurance is F, so the greater the level of risk aversion, the higher the F. Similarly, with prudence, the optimal level of coverage exceeds the level suggested by risk aversion alone.

We now prove four comparative static results: (1) F^* is increasing in risk aversion;[7] (2) F^* is increasing in the maximum loss L; given decreasing in absolute risk aversion (DARA) preferences, (3) F^* is decreasing in income and (4) increasing in t.

To prove the first, we use Pratt's (1964) result that if the utility function V is uniformly more risk-averse than U, then a concave function $h(U(C))$ exists such that $V(C) = h(U(C))$.

Proposition 1: *An increase in risk aversion increases the optimal FSA election amount.*

Proof of Proposition 1: Define $c_0 = (I - F)(1 - t)$ as the level of consumption where $x = F$ and note that $c_2 < c_0 = c_1$. Let F_u^* be the optimal choice and solution to Equation (2) and evaluate $dE[V(C(F))]/dF$ at that election amount:

$$\left.\frac{dE[V]}{dF}\right|_{F_u^*} = \left.\frac{dE[h(U)]}{dF}\right|_{F_u^*} = \int_0^F h'(U(c_1))U'(c_1)\,(t-1)\,g(x)dx + \int_F^L h'(U(c_2))U'(c_2)\,t\,g(x)dx \tag{5}$$

$$> h'(U(c_0))\left[\int_0^F U'(c_1)\,(t-1)\,g(x)dx + \int_F^L U'(c_2)\,t\,g(x)dx\right] = 0. \tag{6}$$

[5] Linearity in U requires the additional condition to the optimization problem that the density function is everywhere strictly positive ($g(x) > 0$) to assure a maximum.

[6] See Cuddington (1999) for this result derived in a different context.

[7] See Schlesinger (2000) for a survey of similar results on insurance.

The inequality follows from the concavity of h (so that $h'' < 0$). This means that $h'(U(c_0)) = h'(U(c_1))$ for $x < F$ and $h'(U(c_0)) < h'(U(c_2))$ for $x > F$. Thus the derivative is positive at F_u^*, implying $F_v^* > F_u^*$. Q.E.D.

The intuition for this result is that because V is a concave transformation of U, V puts more weight on high realizations of the loss, where the value of a larger F is positive.

The next result is that F^* is increasing in L, the maximum loss.

Proposition 2: *The optimal election amount F^* is increasing in L.*

Proof of Proposition 2: Let $g(x)$ be the nondegenerate density function with cumulative distribution function $G(x)$ and upper limit of losses L^1. It is useful to be explicit that $g(x) = 0$ and $G(x) = 1$ for $x > L^1$.

Let $h(x)$ be a density function with upper limit $L^2 > L^1$ and $h(x) > 0$ for $x \in (L^1,L^2)$. The cumulative distribution function is $H(x)$. Assume that $H(x) < G(x)$, $x \in (0,L^2)$.[8] Consumption as a function of x and the election amount, F, in the two periods is given by $c_1(F) = (I - F)(1 - t)$ and $c_2(x,F) = I(1 - t) + tF - x$.

Define $Z_1(F)$ and $Z_2(F)$ as follows:

$$Z_1(F) \equiv (t-1)U'(c_1(F))G(F) + \int_F^{L^2} tU'(c_2(x,F))g(x)dx \tag{7}$$

and

$$Z_2(F) \equiv (t-1)U'(c_1(F))H(F) + \int_F^{L^2} tU'(c_2(x,F))h(x)dx. \tag{8}$$

Note that $Z_1(F)$ is the first-order condition of the consumer's optimization problem when losses follow the distribution represented by G with an upper limit of losses L^1. This implies that $Z_1(F^G) = 0$ where F^G is the optimal election amount for distribution G. Similarly, $Z_2(F^H) = 0$ is the first-order condition when losses are represented by distribution H.

Define $Z(F) = Z_2(F) - Z_1(F)$. We will prove that $Z(F) > 0$ for all values of F, which will imply that $Z_2(F^G) > 0$. By the concavity of the optimization problem, this will imply that $F^H > F^G$.

Write $Z(F)$ as

$$Z(F) \equiv (t-1)U'(c(F))[H(F) - G(F)] + \int_F^{L^2} tU'(c_2(x,F))(h(x) - g(x))dx. \tag{9}$$

The first term is positive, given our assumptions. After integrating by parts, write the integral as

[8] This is similar to the concept of first-order stochastic dominance: A risk-averse individual would prefer to draw from distribution G instead of H.

$$tU'(c_2(L^2,F))(H(L^2) - G(L^2)) - [tU'(c_2(F,F))(H(F) - G(F))] + \int_F^{L^2} tU''(c_2(x,F))(H(x) - G(x))dx. \quad (10)$$

The first term equals zero, the second term is positive after accounting for the negative sign, and the third term is also positive. Therefore, $Z(F) > 0$ for all $F \in (0,L^2)$. But this implies that $Z(F^G) = Z_2(F^G) - Z_1(F^G) = Z_2(F^G) > 0$. Since Z_2 is decreasing in F due to the second-order conditions of the optimization problem, then $F^H > F^G$ to satisfy the first-order condition when losses follow H. Q.E.D.

Next we show that if preferences are DARA, then F^* is decreasing in I. To prove this, it suffices to show that $\partial^2 E[U(C)]/\partial I \partial F < 0$.

Proposition 3: *Assume that preferences are DARA. Define c_0 as in Proposition 1. Then F^* is decreasing in I.*

Proof of Proposition 3: Let $r(c)$ be the Arrow-Pratt measure of absolute risk aversion. Then the required derivative is

$$\begin{aligned}
\frac{\partial^2 E[U(C)]}{\partial I \partial F} &= \int_0^F -U''(c_1)(1-t)^2 g(x)\,dx + \int_F^L U''(c_2)\,t\,(1-t)\,g(x)\,dx \\
&= \int_0^F r(c_1)U'(c_1)\,(1-t)^2 g(x)\,dx - \int_F^L r(c_2)U'(c_2)\,t\,(1-t)\,g(x)\,dx \\
&= -(1-t)\left[\int_0^F r(c_1)U'(c_1)\,(t-1)\,g(x)\,dx + \int_F^L r(c_2)U'(c_2)\,t\,g(x)\,dx\right] \\
&< -(1-t)\left(r(c_0)\left[\int_0^F U'(c_1)\,(t-1)\,g(x)\,dx + \int_F^L U'(c_2)\,t\,g(x)\,dx\right]\right) = 0.
\end{aligned} \quad (11)$$

The inequality follows from the fact that $c_2 < c_0 = c_1$ implies $r(c_1) = r(c_0) < r(c_2)$ under DARA. Q.E.D.

We can also show that F^* is increasing in I under increasing absolute risk aversion (IARA) and constant under constant absolute risk aversion (CARA).

Finally we show that F^* is increasing in t under DARA.

Proposition 4: *Assume that preferences are DARA. Define c_0 as in Proposition 1. Then F^* is increasing in t.*

Proof of Proposition 4: As in Proposition 2, the result follows from showing that $\partial^2 E[U(C)]/\partial t \partial F > 0$. Differentiating Equation (2), we have

$$\begin{aligned}
\frac{\partial^2 E[U(C)]}{\partial t \partial F} &= \int_0^F \left[U''(c_1)(1-t)(I-F) + U'(c_1)\right] g(x)\,dx \\
&+ \int_F^L \left[-U''(c_2)\,t\,(I-F)\,g(x)\,dx + U'(c_2)\right]
\end{aligned}$$

$$
\begin{aligned}
&= \int_0^F \left[r(c_1)U'(c_1)(t-1)(I-F) + U'(c_1)\right] g(x)\,dx \\
&\quad + \int_F^L \left[r(c_2)U'(c_2)\,t\,(I-F)\,g(x)\,dx + U'(c_2)\right] \\
&= (I-F)\left[\int_0^F r(c_1)U'(c_1)(t-1)(I-F)\,g(x)\,dx + \int_F^L r(c_2)U'(c_2)\,t\;g(x)\,dx\right] \\
&\quad + \int_0^F U'(c_1)g(x)dx + \int_F^L U'(c_2)g(x)dx \\
&> (I-F)r(c_0)\left[\int_0^F U'(c_1)(t-1)(I-F)\,g(x)\,dx + \int_F^L U'(c_2)\,t\;g(x)\,dx\right] = 0.
\end{aligned} \tag{12}
$$

The inequality follows from the fact that $c_2 < c_0 = c_1$ implies $r(c_1) = r(c_0) < r(c_2)$ under DARA. Q.E.D.

As t rises, the "premium" falls, and the substitution effect suggests higher FSA contributions. Since the same increase in t lowers after tax wealth, the income effect will also be positive under DARA, reinforcing the substitution effect. Income effects will be zero and negative under CARA and IARA. Therefore, F^* is increasing in t even under CARA, and IARA is not sufficient for F^* to decrease in t.[9]

Discussion and Conclusion

In summary, we have proven the following results: For losses drawn from a continuous distribution, (1) the optimal election amount F^* is increasing in the consumer's level of risk aversion; (2) F^* is increasing in the level of the maximum loss L; With the additional assumption that utility is DARA, (3) F^* is decreasing in income and (4) increasing in the marginal tax rate.

The results of this analysis are perhaps surprising. For example, that FSA usage should decrease as income increases (Proposition 3) is contrary to the conventional wisdom embedded in most policy discussions concerning FSAs and their cousin, medical savings accounts (MSAs). The presumption has been that the wealthy will take fuller advantage of such accounts than the less affluent (e.g., Jefferson, 1999).

Holding the marginal tax rate constant, we have shown that this is not true. The intuition comes from an examination of the first-order conditions in Equation (2). An increase in income will decrease the marginal utilities $U'(c_1)$ and $U'(c_2)$. Thus, an increase in income lowers the marginal cost in the case that $x < F^*$ (the first integral becomes less negative); in isolation, this would tend to increase F^*. But the increase in

[9] Obviously, modeling the U.S. tax code as a proportional tax is a strong assumption. Capturing the full nonlinearity of the tax code is not possible in the simple framework we have used and would risk obscuring the relatively clear insights that the current model highlights. It is tedious, but not difficult, to show that were taxes modeled as a function of income, the income effect demonstrated in Proposition 3 would be offset by the tax effects from Proposition 4 and that thus the full effect of income is ambiguous. However, it is also interesting to note that in reality the marginal tax rate actually decreases over some ranges of income, so ranges of income would exist where the tax effect and the income effect would work in the same direction.

income also lowers the marginal benefit in the case that $x > F^*$ since the consumer will have more income in this case, and thus the marginal utility of one dollar of consumption is diminished. This would tend to decrease F^*. With the reasonable assumption of DARA utility, the latter effect dominates, and so F^* decreases in income. Of course, the effect of the marginal tax rate is in the opposite direction, but that implies a fundamental ambiguity in determining the relationship between income and FSA usage when there is a positive marginal income tax.

It is also surprising that the level of F^* increases with risk aversion. Typically, one thinks that risk-averse consumers will tend to avoid FSAs due to the use-it-or-lose-it provision (e.g., Cuddington, 1999). But this analysis suggests that the opposite ought to be true. Casual empiricism suggests that indeed use of FSAs does tend to be below that predicted by the fully rational model described above, which suggests that consumers might view the opposing risks of (1) losing the tax advantage monies from FSAs and (2) losing unspent FSA monies asymmetrically.

One unsurprising result is that F^* increases as the distribution becomes riskier (a higher upper bound for losses and a higher expected loss). But this result has a nice interpretation when one views FSAs as a special type of insurance policy: As the potential for losses increases, a risk-averse individual will increase the amount of coverage purchased. This is analogous to an increase in the stop-loss provision provided in standard insurance policies.

Two extensions of this analysis of FSAs seem particularly valuable. First, we have ignored the plausible assumption that health expenditures ought to enter the utility function.[10] Some health expenditures do have characteristics that suggest they ought to be treated like other consumption goods. For example, expenditures on orthodontics, vision correction surgery (e.g., LASIK), or teeth whitening would seem to fit in this category. However, many other types of health expenditures have characteristics that fit in a standard insurance framework, such as for accidents that lead to hospitalization, heart attacks, and contracting cancer. Such events result in a reasonably well-defined loss that can be compensated for in monetary terms to return to some defined level of utility. With this analysis, we have chosen the simpler path of treating health expenditures purely as the latter type; extending this to a more general framework would be a useful goal for future work. A second extension would be to use some modification of this framework to examine what would happen if FSAs were allowed to be rolled over from year to year, thus creating an MSA.

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[10] This amounts to the assumption that there is no moral hazard in an insurance context.

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