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Managerial Compensation and Corporate Demand for Insurance

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ABSTRACT

This article examines the role of insurance in shaping efficient compensation contracts and thereby in determining the value of the firm, assuming rational expectations. With fixed compensation, insurance coverage sufficient to eliminate insolvency is optimal. With compensation based on the firm's liquidating value, full insurance is optimal because it allows for a higher reliance on performance compensation, which then induces higher managerial efforts in both business operation and risk management and therefore a higher value of the firm. With compensation based on stock price, partial coverage or no coverage is optimal due to the cost arising from the imperfect impounding of the effect of insurance in the stock price.

INTRODUCTION

The role of insurance in corporate risk management has been examined in a new light since Mayers and Smith (1982) put forth the argument that insurable risks can be eliminated at low costs by shareholders holding diversified portfolios. Thus, in the absence of transaction costs, bankruptcy costs, and agency problems, corporate purchase of insurance is at best a zero net present value proposition. Mayers and Smith (1990) and others suggest various hypotheses for why corporations may nevertheless purchase insurance.[1] One explanation is that some parties with which the firm contracts (e.g., managers) are risk averse and cannot diversify the risks

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[1] Shapiro and Titman (1985) briefly explain that corporations purchase insurance to take advantage of insurance companies' efficiency in claim processing, among other benefits. MacMinn (1987) and Mayers and Smith (1987) demonstrate that insurance contracts can be properly drawn to prevent or eliminate the asset substitution and underinvestment problems that often arise from the conflicts of interest between the shareholders and the manager. Both articles arrive at their results by assuming that insurance contracts can be tailored to suit the particular needs of the firm. MacMinn and Han (1990) show that limited liability causes the firm to invest less than that which maximizes the total value of the firm's stakeholders. They further demonstrate that such a problem can be eliminated by the firm's purchase of insurance, motivated by the manager's self-interest to protect his or her stake in the firm as a liability claim holder.

that arise from these contractual arrangements. In these circumstances, the corporation (shareholders) may find it beneficial to purchase insurance so that they can reduce the compensation for risk demanded by the other parties (managers).

This article studies the role of insurance as an efficient means to transfer nondiversifiable human capital risk under various managerial compensation schemes (or incentive contracts) and thereby examines its impact on the value of the firm. Managerial compensation matters because it determines both how the manager is paid and how much risk he or she is exposed to: the more the manager's compensation is tied to performance, the more risk he or she bears.

There is a body of literature (Holmstrom and Milgrom, 1987, 1991, for example) that examines the tension between reward and risk bearing in incentive contracts. This article focuses on how that tension affects corporate demand for insurance and how the demand for insurance in turn affects the manager's allocational efficiency among various duties.

As a risk averter, the manager demands a risk premium for the risk he or she is forced to bear. And that risk premium can be reduced if the firm shifts insurable risk to an insurer at an actuarially fair premium, which is costless to the firm from an *ex ante* point of view. The benefit goes beyond the reduction in the manager's risk premium if the manager's pay is tied to performance. Insurance reduces the firm's insurable risk, and thereby enables the insured firm to rely more on performance compensation without forcing the manager to bear excessive risk. The shareholders benefit from the resulting higher managerial efforts.

This study begins with the assumption of fixed compensation. In this case, the variability of the manager's compensation is due entirely to the firm's probability of insolvency. The optimal insurance coverage is that which eliminates the probability of insolvency, assuming that insolvency would be caused solely by the insurable loss. Any further coverage is beneficial to neither the manager nor the shareholders.

The analysis then proceeds with the assumptions that the manager's compensation is tied to the firm's liquidating value and that the manager's efforts can affect both the gross liquidating value and the insurable loss. The compensation can be based on the firm's net liquidating value (after the insurable loss), its gross liquidating value (before the insurable loss), or its gross liquidating value and insurable loss separately.

Under the first scheme, the manager is rewarded for efforts in both business operation and risk management and thus expends efforts in both areas. Insurance reduces the manager's risk exposure to the insurable loss and therefore enables the shareholders to rely more on performance compensation than on fixed compensation. The shareholders benefit from the resulting higher managerial effort in business operation and in risk management that leads to a lower expected insurable loss (and thus insurance premium). Consequently, full insurance coverage is optimal under this scheme. These results differ from Campbell and Kracaw's (1987) because of a crucial difference in assumption. This article assumes that the shareholders can observe and impound the manager's behavior, and Campbell and Kracaw assume otherwise.

Under the second scheme, the manager devotes all efforts to business operation with no effort expended in risk management, for it is not rewarded. Therefore, the shareholders gain nothing from and thus are indifferent to insurance.

The third scheme compensates the manager's effort in business operation and in risk management separately based on his or her effort in the two respective areas. As a result, this scheme induces the most efficient managerial effort allocation among the three considered here. It also entails that full insurance coverage is optimal for the same reason as that for the first scheme. The benefits of both effects make this scheme the dominant contract as measured by the value of the firm. This result shows that insurance plays an important role in shaping the efficient incentive contract and thus should be treated as an integral part of corporate financial decisions.

For a long-lived firm, the liquidation value is often unobservable. In its place, stock price is often used as the basis for performance compensation. Stock price is determined by investors based on signals about the firm's future payoffs. If the signals are noisy, then investors price the stock according to each signal's informativeness about the firm's future payoffs, which is inconsistent with the pricing system based on the manager's productivity, as shown by Paul (1992).

Because of imperfect signalings (due to noisy signals), only a portion of insurance coverage is impounded into the stock price. The unimpounded portion represents the cost of insurance. Consequently, insurance will be purchased only when the benefit outweighs the cost; even in such a case, only partial coverage will be purchased. Further, the manager's effort allocation is determined not by the manager's productivity in these two functions but by the risks involved in the two functions; and insurance offers no relief in this case.

The next section of this article describes the assumptions and the basic model used throughout the article. It is followed by three sections containing the analyses of the corporate demand for insurance and of the effect of that demand on the efficiency of the manager's effort allocation, assuming fixed compensation, liquidating-value-based compensation, and stock-based compensation, respectively.

THE BASIC MODEL

The firm is assumed to be all-equity, and the nonmanager shareholders are risk neutral. The manager, however, is risk averse because of the nondiversifiable human capital risk. The shareholders select insurance coverage and pecuniary compensation (for the manager) to maximize the value of the firm, that is, the present value of the firm's future payoffs. The manager's problem is to select pecuniary compensation to achieve a required level of utility in the case of fixed compensation and, for a given incentive contract, to select the effort level to achieve the required utility in the case of performance compensation. That is, the manager sees the firm's risk and insurance coverage and contracts the risk he or she is to bear with the shareholders by requiring pecuniary compensation that compensates for the risk bearing.

For simplicity, assume that the risk-free interest rate is zero and that the insurance premium is actuarially fair. With these two assumptions, the insurance premium is the expected value of the insurable risk. Also because of the assumptions, the premium can be conveniently treated as if it would always be paid with priority over other payments at the end of the period, even though the premium is conventionally paid at the beginning of the period. The notations used throughout the article are as follows:

π = the firm's gross liquidating value,
F = the manager's fixed compensation,
L = the insurable loss (denotes the expected loss in the sections analyzing incentive contracts),
p = the probability of no loss (used only in the next section),
γ = the Arrow-Pratt measure of absolute risk aversion,
δ = the percentage of loss covered by an insurance contract, and
W^0 = the manager's required utility.

The manager is assumed to have exponential utility, separable in wealth and in the cost of expending effort. The exponential utility assumption is made primarily because Holmstrom and Milgrom (1987) show that linear performance-based compensation schemes, popular in practice and assumed in the second half of the article, are optimal for individuals with constant risk aversion.[2]

With the exponential assumption, the manager's expected utility can be conveniently substituted by the certainty equivalent, denoted by W and written as follows:

$$W = E(\text{end-of-period wealth}) - \gamma/2 \, Var(\text{end-of-period wealth}) - C(a),$$

where $E(\cdot)$ = the expected value,
$C(a)$ = the cost of expending managerial effort a, and
γ = the Arrow-Pratt measure of absolute risk aversion.

So, the manager's objective can be specified as to select F, given δ, to achieve W^0 in the case of fixed compensation; and to select a for a given incentive contract in the case of performance compensation.

If V denotes the firm's value, then the shareholders' problem is to select a contract to maximize V subject to $W = W^0$. The manager's certainty equivalent must be binding, otherwise the manager would seek work elsewhere.

The wealth in the manager's certainty equivalent is limited to that generated from human capital by assuming that the marketable assets and nonmarketable assets are independent of one another.[3] Mayers (1973) shows that the optimal portfolio composition is dependent on the covariances between the marketable assets and the nonmarketable assets and that it reduces to the formula developed by Black (1972) if those covariances are zero. That is, if the marketable and nonmarketable assets are not correlated, then the manager makes investment decisions independent of the manager's nonmarketable assets.

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[2] Paul (1992) adopts the exponential utility assumption for the same reason in analyzing the effectiveness of stock-based compensation.

[3] Elimination of this independence assumption does not change the fundamental results of this study, but it changes the first-order conditions.

FIXED COMPENSATION

The analysis begins with fixed compensation. Holmstrom and Milgrom (1991) demonstrate that, in theory, there are many situations in which fixed compensation is optimal from the shareholders' perspective. In practice, fixed compensation is still the dominant form of compensation, despite the popularity of incentive compensation. Therefore, it is important to begin the analysis with the case of fixed compensation.

This section formally shows that insurance benefits the shareholders in the case of fixed compensation because of the firm's insolvency risk. To accentuate the importance of the firm's insolvency risk to corporate demand for insurance, this section assumes the firm's end-of-period payoff to be known with certainty. Such end-of-period payoff in a one-period model can be viewed as the firm's liquidating value. The only uncertainty comes from the firm's insurable risk: the magnitude of the risk (denoted by L) is known, but the occurrence (with a probability of $1 - p$) is uncertain. To eliminate the complication of introducing liability claimants into the analysis, the insurable loss is limited to property loss. Therefore, the firm's after-loss liquidating value is always non-negative.

The manager is compensated with a fixed salary and, as a result, will not expend more effort than is required to pass the firm's monitoring procedure, leading to a constant gross liquidating value. Assume that the liquidating value is always sufficient to cover the labor cost if no loss occurs; that is, $\pi > F^0$, where F^0 is the highest monetary compensation in the case where the manager receives nothing upon occurrence of insurable loss.[4] The actuarially fair premium, being the expected loss, is $(1 - p)\delta L$.

Table 1 summarizes the shareholder's and the manager's end-of-period payoffs under various conditions. If no loss happens, the manager is paid in full, F, and the shareholders receive the residual, $\pi - (1 - p)\,\delta\,L - F$. Otherwise, the manager's payoff depends on the extent of insurance coverage, δ. The case I condition defines the situation in which δ is chosen such that the gross liquidating value after the premium and the uninsured loss is no less than zero but smaller than F; in this case, the firm goes bankrupt, leaving the manager the residual and the shareholders nothing. The case II condition is one in which δ is sufficiently large, so that the manager can be paid in full after the premium and uninsured loss are covered.

Suppose δ is chosen such that case I applies. Then, the manager's uncertain wealth has an expected value of $pF + (1 - p)[\pi - (1 - p\delta)\,L]$ and a variance of $p(1 - p)\,[\pi - (1 - p\delta)\,L - F]^2$. If a sufficiently large δ is chosen—that is, if case II applies—then the manager's compensation is known with certainty and thus has a zero variance. The applicable representation of the manager's certainty equivalent depends on the shareholders' choice of δ. That is, the two conditions are mutually exclusive. The manager's problem is to select F such that the certainty equivalent, W, is equal to W^0, where W can be written as follows:

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[4] As will become clear below, F^0 is the F value that satisfies:

$$W^0 = pF + (1-p)\,(\pi-L) - \tfrac{1}{2}\gamma p\,(1-p)\,(\pi-L-F)^2 - C(a).$$

$$W = pF + (1-p)[\pi - (1-p\delta)L] - \frac{\gamma}{2} p(1-p)[\pi - (1-p\delta)L - F]^2 - C(a) \quad \text{Case I}$$
$$= F - C(a) \quad \text{Case II.} \qquad (1)$$

Table 1
Payoffs to the Shareholders and the Manager with Fixed Compensation

	To Manager	To Shareholders
No Loss: Probability p	F	$\pi - (1-p)\,\delta\,L - F$
Loss Occurs: Probability $(1-p)$		
Case I: $0 < \pi - (1-p)\,\delta\,L - (1-\delta)\,L < F$	$\pi - (1-p)\,\delta\,L - (1-\delta)\,L$	0
Case II: $\pi - (1-p)\,\delta\,L - (1-\delta)\,L \geq F$	F	$\pi - (1-p)\,\delta L - (1-\delta)\,L - F$

Recall that the effort level does not change with F. To maintain W at W^0, the manager either requires a higher F to compensate for the probability of not getting paid in full and for the undesirability of the uncertainty associated with the income, or the manager settles for a lower F as a tradeoff for the insurance coverage that reduces the uncertainty in income. The shareholders understand the manager's problem and choose δ to maximize the firm's expected value. Based on the payoffs to the shareholders presented in Table 1, the firm values under the two mutually exclusive conditions are

$$V = p[\pi - F - (1-p)\delta L] \quad \text{Case I}$$
$$= p(\pi - F) + (1-p)[\pi - (1-\delta)L - F] - (1-p)\delta L \quad \text{Case II.} \qquad (2)$$

Subsitituting equation (1) into (2) and differentiating V with respect to δ yields[5]

$$\frac{dV}{d\delta} = -\gamma\, p^2 (1-p) L[\pi - (1-p\delta)L - F] \quad \text{Case I}$$
$$= 0 \quad \text{Case II.} \qquad (3)$$

If δ is chosen so that the case I condition applies, then $dV/d\delta > 0$ because the terms in the bracket of equation (3) are negative by definition of the case I condition. As δ increases within the boundary of the case I condition, the firm's insolvency probability is not affected, but the residual that accrues to the manager in the event of insolvency increases. As a result, the manager requires a smaller risk premium as the firm purchases higher insurance coverage.

The condition $dV/d\delta = 0$ in case II indicates that, once δ is chosen so that the case II condition applies, no value can be extracted from further insurance cover-

[5] For δs satisfying the case I condition, the case I representation of equation (1) is substituted into that of equation (2). The same applies for δs satisfying the case II condition.

age. Let δ_1 be the value of δ such that $\pi - (1 - p)\delta L - (1 - \delta) L - F = 0$; that is, δ_1 is the smallest coverage that prevents the firm from insolvency. The above analysis indicates that the firm's optimal insurance coverage is anywhere in $[\delta_1, 1]$. By purchasing an insurance coverage that eliminates the firm's probability of insolvency, the shareholders save on the risk premium that the manager would have demanded otherwise. An immediate corollary of this result is that no insurance will be demanded by a firm with no insolvency probability.

The risk-shifting from the manager to the insurer, as described above, must be carried out through the firm because the manager lacks the insurable interest. The shareholders have the incentive to do this because the savings in labor costs accrue entirely to them. Proposition 1 summarizes the results.

Proposition 1

If the manager's pay is fixed, insurance benefits the shareholders because it saves them the manager's required risk premium at no ex ante *cost by eliminating the insolvency risk caused by the insurable risk. The optimal coverage is anywhere in* $[\delta_1, 1]$.

INCENTIVE CONTRACTS BASED ON LIQUIDATING VALUE

Fama (1980) argues that performance-based compensations are not necessary if labor markets are efficient. Holmstrom (1982) and Gibbons and Murphy (1992), however, show that labor market efficiency cannot entirely replace performance-based compensation.[6] Academic research largely views incentive contracts as an important form of compensation, and this view is further supported by the popularity of incentive contracts in the business world.

This and the following section examine the impact of insurance on the value of the firm and the allocational efficiency of the manager's effort in business operation and in risk management in the context of two types of incentive contracts: those based on liquidating value and those based on stock value.[7] A few assumptions are changed to facilitate the analysis in these two sections. First, assume that the firm's probability of insolvency is zero. This assumption allows for analyzing the main objectives here without unnecessary complications in the mathematical modeling of insolvency.

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[6] Holmstrom (1982) demonstrates that without incentive contracts managers tend to work too hard in early years when their performance bears implications for future compensation for a long time, and not hard enough in later years when their performance bears implications only for a limited amount of time. Gibbons and Murphy (1992) extend Holmstrom's model and show that incentive contracts alone are insufficient to motivate workers who have career concerns because career concerns provide implicit incentives that incentive contracts do not.

[7] In a single-period model, the firm's end-of-period value is the liquidating value. Therefore, the term "liquidating value" is used throughout this section even in the absence of insolvency. Liquidating value–based compensation in a single-period model is similar to profit-sharing in a multiple period model.

Second, uncertainty is introduced to the gross liquidating value by defining

$$X(a_1, \varepsilon) = \pi(a_1) + \varepsilon_1,$$

where a_1 is the manager's effort in business operation, and ε_1 is a random noise independent of π and has a distribution of $N(0, \sigma_1^2)$. The random noise can be thought of as the measurement error or as the randomness of the manager's productivity. The manager's effort in business operation is motivated by the portion of the liquidating value to be shared by the manager: the larger the portion, the more the managerial effort. If α denotes such a portion, then $\partial a_1/\partial\alpha > 0$. A higher managerial effort in turn produces a higher expected gross liquidating value. But the increase of the expected gross liquidating value declines with the effort level; that is, $D_1\pi(a_1) > 0$, and $D_1^2\,\pi(a_1) < 0$, where $D_1\pi$ and $D_1^2\,\pi$ denote the first- and second-order derivatives of π with respect to a_1, respectively.

Third, the insurable risk is assumed to have a normal distribution, rather than the binomial distribution assumed in the previous section. The random insurable risk, denoted by R, can be decomposed into two components: the expected value of R and a white noise with a distribution of $N(0, \sigma_2^2)$. The expected loss is a decreasing function of the manager's effort in risk management if the insurable risk is included in the incentive contract, because the manager will share the resulting reduction in the expected insurable loss. But the expected loss decreases in the manager's effort at decreasing rates. In sum, the insurable loss can be written as

$$R(a_2, \varepsilon_2) = L(a_2) + \varepsilon_2,$$

where L is the expected value of R, and a_2 is the manager's effort in risk management; the properties of L include $\partial a_2/\partial\alpha > 0$, $D_2L(a_2) < 0$, and $D_2^2\,L(a_2) > 0$, where D_2L and $D_2^2\,L$ denote the first-order and the second-order derivatives of L with respect to a_2, respectively. The cost of expending effort, $C(a_1, a_2)$, on the other hand, increases at increasing rates in effort; that is, $D_iC(a_1, a_2) > 0$ and $D_i^2\,C(a_1, a_2) > 0$ for $i = 1$ and 2.

There are potentially three compensation schemes based on liquidating value, if the manager's activities are grouped in two main categories: business operation and risk management. The first scheme is a contract that compensates the manager with a portion of the firm's net liquidating value, the gross liquidating value net of the insurable loss. This scheme is similar to the commonly practiced profit-sharing incentive program. Since the manager's portion of the firm's liquidating value is conventionally referred to as "weight," this type of contract is hereafter referred to as a one-weight contract including the insurable loss (or contract I).

The second scheme is a one-weight contract that compensates the manager with a portion of the firm's gross liquidating value. This contract (hereafter one-weight contract excluding the insurable loss or contract E) is examined to show how the exclusion of the insurable loss from an incentive contract affects the firm's insurance decision and the efficiency of the manager's effort allocation.

The third scheme is a two-weight contract (or contract T), in which the manager is rewarded for his or her effort in business operation with a portion of the gross liquidating value and is required to share a different portion of the uncovered insurable loss net of the insurance premium.[8] Because the manager is to share both the "profit" and loss, he or she will expend efforts in both functions in accordance with the "weights" in the contract. The anticipated managerial effort allocation in turn affects the shareholders' choice of weights and insurance coverage.

The optimal insurance coverage and managerial effort allocation are derived and analyzed for the three incentive contracts. The role of insurance is then examined by comparing the optimal value of the firm resulting from each of the three incentive contracts.

One-Weight Incentive Contract Including Insurable Risk

This contract compensates the manager with a fixed pay (F) and a portion (α) of the firm's end-of-period net liquidating value: $F + \alpha[X - \delta L - F - (1 - \delta) R]$. The manager's objective is to select a_1 and a_2 to maximize the following certainty equivalent, given α and δ:

$$W = F + \alpha (\pi - F - L) - \frac{\gamma}{2} \alpha^2 [\sigma_1^2 + (1 - \delta)^2 \sigma_2^2] - C(a_1, a_2). \tag{4}$$

The shareholders' objective is to select α and δ to maximize the firm's *ex ante* value (V_I), given as follows, subject to the manager's certainty equivalent:

$$\begin{aligned} V_I &= E\{(1 - \alpha) [X - \delta L - F - (1 - \delta) R]\} \\ &= (1 - \alpha) (\pi - F - L). \end{aligned} \tag{5}$$

As shown in the Appendix, the optimal solutions are

$$\alpha^I = \frac{D_1 \pi \dfrac{\partial a_1}{\partial \alpha} - D_2 L \dfrac{\partial a_2}{\partial \alpha}}{D_1 \pi \dfrac{\partial a_1}{\partial \alpha} - D_2 L \dfrac{\partial a_2}{\partial \alpha} + \gamma \sigma_1^2}, \tag{6}$$

and $\delta = 1$, where α^I denotes the optimal weight for contract I. That is, the optimal contract calls for full insurance coverage ($\delta = 1$) and a weight of α^I (smaller than 1) to reward the manager. The weight, α^I, induces the manager to expend a_1^I in business operation and a_2^I in risk management. Equation (6) also indicates that the larger the σ_1, the smaller the α^I. That is, the more uncertain the outcome of the manager's effort, the less the firm relies on performance compensation. This is true with all incentive contracts analyzed in this section.

Full insurance coverage is optimal for the following reasons. To the manager, it reduces his or her risk exposure and thus the risk premium in his or her

[8] As shown below, the optimal two-weight contract requires full insurance. As a result, the manager shares a portion of the reduction (due to his or her effort) in the expected insurable loss, rather than a portion of loss. The wording changes hereafter reflect this result.

certainty equivalent (shown in equation [4]). This enables the manager to accept a contract that offers a smaller fixed pay and a larger performance-based compensation (i.e., a smaller F and a larger α) without sacrificing his or her utility. To the shareholders, a larger α leads to higher managerial efforts in both business operation and risk management and thus a larger expected gross liquidating value and a smaller expected insurable loss. Together with a smaller F, full insurance coverage enhances the value of the firm and thus the shareholders' wealth, even though they are left with a smaller portion $(1 - \alpha)$ of the firm's net liquidating value. This point can be seen more clearly by examining equation (A9) in the Appendix, which shows that the shareholders benefit from a larger π and a smaller L led by the more managerial efforts induced by a larger α.

The above results differ from Campbell and Kracaw's (1987) because of a crucial difference in assumption.[9] This section assumes rational expectations in that the shareholders take into account the manager's anticipated behavior when selecting α and δ, as opposed to Campbell and Kracaw, who assume an embedded agency problem between the shareholders and the manager.

Incentive Contract Excluding Insurable Risk

This contract compensates the manager with a fixed pay and a portion of the firm's gross liquidating value. Suppose the manager's contract takes the form of $F + \alpha (X - F)$. Then the value of the firm is

$$\begin{aligned} V_E &= E[X - F - \alpha (X - F) - (1 - \delta) R - \delta L] \\ &= (1 - \alpha)(\pi - F) - L. \end{aligned}$$

By following the same procedure from equations (A2) through (A8) in the Appendix, it is shown that the optimal $\alpha(\alpha^E)$ is

$$\alpha^E = \frac{D_1 \pi \frac{\partial a_1}{\partial \alpha}}{D_1 \pi \frac{\partial a_1}{\partial \alpha} + \gamma \sigma_1^2}, \tag{7}$$

and that the optimal insurance coverage can be anywhere between no coverage and full coverage (i.e., insurance is irrelevant to the value of the firm). Since the weight, α^E, applies only to the gross liquidating value, it induces the manager to expend a_1^E in business operation but no effort in risk management. As a result of no managerial effort in risk management, the shareholders are indifferent about insurance, because, at an actuarially fair premium, the total effect of the insurable loss on the shareholders' wealth remains to be the expected insurable loss that re-

[9] Campbell and Kracaw show that the manager tends to under- or over-insure relative to what is desired by the shareholders.

flects no managerial effort in risk management (i.e., $L(a_2 = 0)$) as shown in equation (A9) in the Appendix.

Two-Weight Incentive Contract

The two-weight incentive contract rewards the manager's effort in business operation and in risk management based on his or her performance in the respective areas with different weights:

$$F + \alpha_1(X - F) - \alpha_2[\delta L + (1 - \delta) R],$$

where α_1 is the weight for the manager's performance in business operation and α_2 in risk management.

The manager's objective is to select a_1 and a_2 to maximize his or her utility or certainty equivalent:

$$W = F + \alpha_1 (\pi - F) - \alpha_2 L - \frac{\gamma}{2}[\alpha_1^2 \sigma_1^2 + \alpha_2^2 (1-\delta)^2 \sigma_2^2] - C(a_1, a_2).$$

The shareholders' objective is to select δ, α_1, and α_2 to maximize the value of the firm (V_T):

$$\begin{aligned} V_T &= E\{X - F - \alpha_1 (X - F) + \alpha_2 [\delta L + (1 - \delta) R] - \delta L - (1 - \delta) R\} \\ &= (1 - \alpha_1)(\pi - F) - (1 - \alpha_2) L. \end{aligned}$$

Following the same process from equations (A2) through (A8), the optimal solutions are

$$\alpha_1^T = \frac{D_1 \pi \dfrac{\partial a_1}{\partial \alpha_1}}{D_1 \pi \dfrac{\partial a_1}{\partial \alpha_1} + \gamma \sigma_1^2}, \quad \alpha_2^T = 1, \quad \text{and} \quad \sigma = 1.$$

The optimal contract includes purchasing full insurance coverage, awarding the manager all the reduction in the expected insurable loss and a portion of the firm's gross liquidating value. The reasons for full insurance coverage are the same as those for the one-weight contract including the insurable loss. It is worth noting, however, that, because the manager is awarded all the benefit of his or her effort in risk management, he or she would expend more effort than he or she would if offered the one-weight contract including the insurable loss.

Comparison of the Three Incentive Contracts

As shown above, the three incentive contracts induce different sets of managerial efforts and insurance coverage. Ultimately, the shareholders choose the one that maximizes the value of the firm (i.e., the efficient contract). Here we examine the role of insurance in shaping the efficient incentive contract. As shown in the Appendix,

$$V_T^* > V_I^* > V_E^*,$$

and

$$a_1^I > a_1^T = a_1^E \text{ and } a_2^T > a_2^I > a_2^E = 0, \text{ or } \alpha^I > \alpha_1^T = \alpha^E,$$

where * signifies the optimal value. The result indicates that the two-weight contract dominates the other two. $V_I^* > V_E^*$ because the manager expends more effort in both business operation and risk management given contract I. He or she does so because, under contract I, the manager is not exposed to the insurable loss and thus can accept a smaller fixed pay (F) and a larger portion (α) as compensation.

The reasons for $V_T^* > V_I^*$ are more complicated. The manager expends more effort in risk management but less effort in business operation under contract T than under contract I. The more effort in risk management reduces the expected insurable loss and thus increases the value of the firm, while the less effort in business operation decreases the value of the firm. The former is shown to outweigh the latter because contract I (with the same weight for both functions) forces the manager to expend more effort in business operation than the optimal level when that extra effort can be expended more productively in risk management. Thus, $V_T^* > V_I^*$.

Clearly, insurance cannot be dismissed simply by excluding the insurable loss from an incentive contract. On the contrary, insurance plays an important role in shaping the optimal two-weight incentive contract. In practice, the benefit of the two-weight contract could be partially offset by the costs incurred in differentiating and measuring the manager's efforts in the two functions. What is clear, though, is that insurance is an integral part of corporate financial decisions and should be treated as such.

Proposition 2

The optimal two-weight contract dominates the optimal one-weight contract including the insurable loss and the one excluding the insurable loss; that is, $V_T^* > V_I^* > V_E^*$. *Further, the managerial efforts in business operation and risk management are ranked as follows:* $a_1^I > a_1^T = a_1^E$, *and* $a_2^T > a_2^I > a_2^E = 0$.

INCENTIVE CONTRACTS BASED ON STOCK PRICE

Up to this point, the firm's liquidating value and insurable loss are assumed to be observable. For a long-lived firm, the actual liquidating value is not observable until the firm is liquidated. Both shareholders and managers may not outlive the firm and need a proxy for the liquidating value for various purposes. Stock price is one such proxy.

The motivation of stock-based compensation for the manager is to align the manager's interest with that of the shareholders, as measured by the firm's market value. As investors, the shareholders price the stock by weighing the signals according to their respective informativeness about the firm's future payoffs. As principals, they want an incentive scheme that rewards the manager's productivity. Paul (1992) points out that signals informative about the asset price may not be informative about the manager's effort level: "Intuitively, investors are interested in a signal to the extent that it resolves uncertainty about the firm's ultimate payoffs. The more uncertainty it resolves about firm payoffs, the more weight the signal receives in the stock price" (p. 483).

To avoid unnecessary complication, the analysis continues to be performed in a one-period setting, with the additional assumption that the manager is paid before the firm is liquidated. This is possible because Holmstrom and Milgrom (1987) show that a static model yields the same result as a dynamic model if the manager has constant risk aversion and the risky payoffs are jointly normally distributed. As a result, stock compensation can be modeled in a one-period setting by simply assuming the following instantaneous sequence of events: the signals are observed, the manager is paid, and the firm is liquidated.

Suppose there are two pieces of unobservable information about the firm: the liquidating value (X) and the insurable loss (R). Investors, unable to observe X and R, price the stock based on signals about them. The signals can be public information (such as financial reports) or private information. These signals give assessments of X and R with errors, portrayed as follows. If Z signals about X and Y about R, then define Z and Y as

$$Z = X + \theta_1$$

$$Y = R + \theta_2,$$

where θ_1 has a distribution of $N(0, \omega_1^2)$, and θ_2 has a distribution of $N(0, \omega_2^2)$. Also, assume that $Cov(\theta_1, \theta_2) = 0$. One may consider R as a property or liability loss, the magnitude of which is not known with certainty at the time compensation is determined (i.e., $R = L + \varepsilon_2$ as defined above). Y (signals) can then be thought of as the estimated loss reported in news and/or the firm's financial statements before a loss occurs or after a loss occurs but before the actual loss is known. Investors use these signals about the loss when setting the share price.

Suppose the manager's incentive contract takes a simple linear form: $W = F + \frac{\alpha}{1-\alpha} P$, where P is the market value of the firm and is equivalent to the expected value of the future payoffs given the signals received by investors.[10] Let $Q = X - (1 - \delta)R$. Then, the firm's value

$$P = E(Q \mid Z, Y) - \delta L - F - \frac{\alpha}{1-\alpha} P. \tag{8}$$

In the above equation, P is treated as a constant because P is known once Y and Z are known. The insurance premium δL and F are obviously known with certainty. Since Q, Y, and Z are jointly normally distributed,

$$P = c + b_1 Z - (1 - \delta)\, b_2 Y - \delta L - F - \frac{\alpha}{1-\alpha} P, \tag{9}$$

where

$$b_1 = \frac{\mathrm{Cov}(X, Z)}{\mathrm{Var}(Z)} = \frac{\sigma_1^2}{\sigma_1^2 + \omega_1^2},$$

$$b_2 = \frac{\mathrm{Cov}(R, Y)}{\mathrm{Var}(Y)} = \frac{\sigma_2^2}{\sigma_2^2 + \omega_2^2}, \quad \text{and}$$

$$c = (1 - b_1)\, \pi\, (a_1^e) - (1 - b_2)(1 - \delta^e)\, L\, (a_2^e)$$

with a_1^e and a_2^e being the manager's equilibrium efforts in business operation and in risk management, respectively, and δ^e being the firm's equilibrium insurance coverage. Investors cannot observe the manager's effort levels, but they understand that the manager would choose a_1^e and a_2^e, assuming rational expectations. Solving equation (9) for P yields

$$P = (1 - \alpha)\, [c + b_1 Z - (1 - \delta)\, b_2 Y - \delta L - F]. \tag{10}$$

P is clearly a function of the weights (b_1 and b_2) for the two signals and the chosen insurance coverage. However, the weights are not functions of the manager's marginal productivity and costs of expending effort but are functions of variances of the noise terms involved. Given the equilibrium managerial effort, P increases in b_1 and decreases in b_2. Since the manager is compensated with a fraction of P, he or she depends on b_1 and b_2 for wealth. The result of this dependency is a distorted allocation of effort between the two activities. To see, write b_1 and b_2 as follows:

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[10] For the purpose of comparison with the contracts analyzed above, the manager is awarded $\alpha/(1-\alpha)$ of the market value of the firm so that the manager gets α of the firm's expected payoffs, as shown in equation (10).

$$b_1 = \frac{1}{1+\omega_1^2/\sigma_1^2}, \qquad b_2 = \frac{1}{1+\omega_2^2/\sigma_2^2}.$$

Clearly, a larger σ_i leads to a larger b_i for i = 1 and 2. This is because the higher the uncertainty about the outcome of an activity, the more valuable a signal that resolves such uncertainty. Consequently, the manager will receive a large portion (b_1) of the increase in E(Z) that results from his or her effort in business operation if σ_1 is large; by the same reasoning, the manager will receive a large portion (b_2) of the reduction in the insurable loss that results from his or her effort in risk management if σ_2 is large.

The distortion of effort allocation can further be seen by assuming $\omega_1^2 = \omega_2^2$. Then, $b_2 > b_1$, if $\sigma_2 > \sigma_1$; that is, the manager will expend more effort in risk management than in business operation if the uncertainty of the insurable risk is higher than that of the gross liquidating value. This result contradicts that obtained in the previous section, where the larger the σ, the smaller the weight.

α^e and δ^e are determined in the same fashion as described previously: the shareholders choose α and δ to maximize E(P) at time zero, and the manager selects the optimal effort levels to keep $W = W^0$, given α and δ. As shown in the Appendix, the firm's optimal α is

$$\alpha^e = \frac{1}{1+H}, \tag{11}$$

where $H = \dfrac{\gamma\,[b_1\,\sigma_1^2 + b_2\,(1-\delta)^2\,\sigma_2^2]}{b_1\,D_1\,\pi\,\dfrac{\partial a_1}{\partial \alpha} - [b_2 + \delta\,(1-b_2)\,]\,D_2\,L\dfrac{\partial a_2}{\partial \alpha}}$. Given α^e, the manager's efforts in these two activities in equilibrium are determined by the first-order conditions (equations [A17] and [A18] in the Appendix). The two equations indicate that the ratio of the manager's marginal efforts in business operation and in risk management is $b_1/[b_2 + \delta\,(1 - b_2)]$, which is primarily determined by the weights assigned by investors based on the variances and δ, and not by the manager's marginal productivity in these two activities.

Also shown in the Appendix is that the firm's optimal insurance coverage is anything but full coverage: If $-(1 - b_2)\,L + \gamma\alpha^2 b_2 < 0$ (equation [A21]), then no coverage is optimal (i.e., $\delta^e = 0$); otherwise, partial coverage is optimal. This result is in sharp contrast with that obtained in the previous section. The reason can be seen from the partial derivative of E(P) with respect to δ: $-(1-b_2)\,L + \gamma\alpha^2 b_2\,(1-\delta)$, in which the first term is the net marginal cost of insurance, and the second term is the marginal benefit. The benefit of insurance coverage is the marginal reduction in the risk premium required by the manager. The net marginal cost warrants more explanation because it is composed of two terms: $-L$, the marginal cost of additional coverage; and b_2L, the benefit of additional coverage. With the insurance premium actuarially fair, the discrepancy between the cost and benefit comes from the uncertainty about the terminal outcome of the insurable loss after investors observe the signal. Thus, investors only impound a portion (b_2) of the effect of insurance coverage into the stock price—the unimpounded portion becomes the cost of insurance.

So far, the actual insurable loss is assumed not known with certainty at the time investors observe signal Y. Although this may well describe many circumstances in which financial statements or news releases reveal a firm's insurable loss without absolute accuracy, it is also likely that no information about a firm's insurable loss is available to investors prior to liquidation, or that the insurable loss is signaled with perfect accuracy. In the former case, the stock price will reflect the expected value of the insurable risk (because the expected loss is the investors' best guess in the absence of relevant signals) and will thus be independent of insurance coverage. As a result, no insurance will be purchased. In the latter case where the signal is perfect—that is, $b_2 = 1$—full insurance is optimal because there is no cost of insurance.

Proposition 3

With stock compensation, the manager is rewarded based on the risk levels of the business and of the insurable risk. The firm will not purchase insurance if the net marginal cost of insurance coverage (due to the imperfect signaling) outweighs the marginal reduction in risk premium required by the manager. The firm will purchase partial coverage otherwise.

CONCLUSION

This article examines corporate demand for insurance and its impact on the efficiency of managerial allocation of effort in business operation and risk management under different compensation schemes, assuming rational expectations. Insurance is shown to play an important role in shaping the efficient incentive contract and thus in determining the value of the firm.

When the manager rents his or her human capital, he or she is not only rewarded for productive work but is also forced to bear the risks associated with the firm. The part of the risk that is insurable can be readily transferred through insurance. Insurance benefits the shareholders by reducing the manager's required risk premium in the case of fixed compensation and, in the case of performance compensation, by enabling more reliance on performance compensation without forcing the manager to bear excessive risks. The optimal insurance coverage depends on the magnitude of the offsetting cost of insurance.

In the case of fixed compensation, the optimal insurance coverage is anywhere between the coverage that eliminates the insolvency probability and full coverage. If the firm's liquidating value is observable, the shareholders would choose a two-weight incentive contract that rewards the manager for his or her efforts in business operation and risk management separately in accordance with his or her performance in the respective areas. Such an incentive contract entails full insurance coverage because full coverage results in higher managerial efforts in both business operation and risk management and leads to a more efficient managerial allocation of effort in the two functions.

When the liquidating value is not observable, the stock price is often used in its place for contracting purposes. The stock price is determined by investors according to the effectiveness of the observed signals in resolving the uncertainty about the firm's liquidating value and insurable loss. In the absence of perfect signaling, the stock price only impounds a fraction of a dollar of expected benefit for a dollar of insurance premium. The fraction that is not impounded becomes the cost of insurance. Insurance will be purchased only when the benefit outweighs the cost. In this case, partial coverage is optimal. Stock-based contracts can also distort the manager's incentive by better rewarding efforts in activities with high risks, regardless of the manager's productivity in those areas. Insurance coverage offers no remedy for this distortion.

This study is performed under the assumptions that human capital and non-human capital are separable, and that the firm is all-equity. However, the manager's human capital as a proportion of the manager's total wealth may affect the manager's incentives and thus the firm's insurance decision. Likewise, the firm's leverage may make insurance look even better by signaling to the bondholders that the firm is serious about maximizing the total value for all stakeholders rather than simply maximizing the value of equity, as proclaimed by Mayers and Smith (1982) and Campbell and Kracaw (1990). The combination of these two concerns warrants further research.

APPENDIX

Derivation of Equation (6)

In the one-weight contract including the insurable loss (or contract I), the random payoff to the manager is $F + \alpha[X - \delta L - F - (1 - \delta)R]$, which has an expected value of $F + \alpha(\pi - F - L)$ and a variance of $\frac{\gamma}{2}\alpha^2[\sigma_1^2 + (1-\delta)^2\sigma_2^2]$. The manager's problem is to select a_1 and a_2 for a given α and δ to maximize his or her certainty equivalent, written as

$$W = F + \alpha(\pi - F - L) - \frac{\gamma}{2}\alpha^2[\sigma_1^2 + (1-\delta)^2\sigma_2^2] - C(a_1, a_2). \tag{A1}$$

The first-order conditions with respect to a_1 and a_2 are

$$\frac{\partial W}{\partial a_1} = \alpha D_1 \pi - D_1 C(a_1, a_2) = 0 \tag{A2}$$

and

$$\frac{\partial W}{\partial a_2} = -\alpha D_2 L - D_2 C(a_1, a_2) = 0, \tag{A3}$$

where D_1 and D_2 denote partial differentiations with respect to a_1 and a_2, respectively. The shareholders select α and δ to maximize the firm's *ex ante* value, V_I, under the constraint that $W = W^0$. The firm's *ex ante* value is

$$\begin{aligned} V_I &= E\{(1-\alpha)[X - \delta L - F - (1-\delta)R]\} \\ &= (1-\alpha)(\pi - F - L). \end{aligned} \tag{A4}$$

The manager's certainty equivalent must be binding, otherwise he or she will seek employment elsewhere. The fixed compensation, F, can be expressed in terms of W^0 by solving equation (A1) for F. By substituting the resulting expression of F into equation (A4), V_I reduces to

$$V_I = \pi - L - W^0 - C - \frac{\gamma}{2}\alpha^2[\sigma_1^2 + (1-\delta)^2\sigma_2^2]. \tag{A5}$$

The first-order condition of V_I with respect to α is

$$\frac{\partial V_I}{\partial \alpha} = D_1 \pi \frac{\partial a_1}{\partial \alpha} - D_2 L \frac{\partial a_2}{\partial \alpha} - D_1 C \frac{\partial a_1}{\partial \alpha} - D_2 C \frac{\partial a_2}{\partial \alpha} - \gamma\alpha[\sigma_1^2 + (1-\delta)^2\sigma_2^2]. \tag{A6}$$

Substituting the manager's first-order conditions, equations (A2) and (A3), into equation (A6) gives

$$(1-\alpha)\,D_1\pi\frac{\partial a_1}{\partial \alpha}-(1-\alpha)\,D_2L\frac{\partial a_2}{\partial \alpha}-\gamma\,\alpha[\sigma_1^2+(1-\delta)^2\sigma_2^2]=0. \tag{A7}$$

The first-order condition of V_I with respect to δ is

$$\gamma\alpha^2(1-\delta)=0. \tag{A8}$$

It follows that the optimal solution is $\delta^*=1$. Substituting $\delta=1$ into equation (A7) and solving for α yields

$$\alpha^I=\frac{D_1\pi\frac{\partial a_1}{\partial \alpha}-D_2L\frac{\partial a_2}{\partial \alpha}}{D_1\pi\frac{\partial a_1}{\partial \alpha}-D_2L\frac{\partial a_2}{\partial \alpha}+\gamma\sigma_1^2}, \tag{6}$$

where α^I denotes the optimal weight for the one-weight contract including the insurable loss.

Proof of Proposition 2

Hereafter, the value of the firm and manager's certainty equivalent under each of the three incentive contracts based on liquidating values are listed to facilitate the comparison:

One-weight contract including insurable loss (or based on net liquidating value):

Manager's Utility: $W=F+\alpha(\pi-F-L)-\frac{\gamma}{2}\alpha^2[\sigma_1^2+(1-\delta)^2\sigma_2^2]-C(a_1,a_2)$

Firm Value: $V_I=(1-\alpha)(\pi-F-L).$

One-weight contract excluding insurable loss (or based on gross liquidating value):

Manager's Utility: $W=F+\alpha(\pi-F)-\frac{\gamma}{2}\alpha^2\sigma_1^2-C(a_1,a_2)$

Firm Value: $V_E=(1-\alpha)(\pi-F)-L.$

Two-weight contract:

Manager's Utility: $W=F+\alpha_1(\pi-F)-\alpha_2L-\frac{\gamma}{2}[\alpha_1^2\sigma_1^2+\alpha_2^2(1-\delta)^2\sigma_2^2]-C(a_1,a_2)$

Firm Value: $V_T=(1-\alpha_1)(\pi-F)-(1-\alpha_2)L$

For comparison, the manager's fixed compensation must be endogenized in the value of the firm because, although the fixed payments in the three contracts are denoted with the same notation F, they have different values. In addition, the value of the firm must also reflect the optimal insurance coverage. Simple algebraic operations show that the value of the firm under both one-weight contracts can be written in a general form:

$$V_j = \pi - L - \frac{\gamma}{2}\alpha^2 \sigma_1^2 - C(a_1, a_2) - W^0, \tag{A9}$$

where j = I or E. The value of the firm under the two-weight contract can be expressed as follows:

$$V_T = \pi - L - \frac{\gamma}{2}\alpha^2 \sigma_1^2 - C(a_1, a_2) - W^0. \tag{A10}$$

The optimal firm values (signified with *) are compared below.

Comparison of V_E^* and V_I^*. The optimal V_E can be expressed in its complete form as $V_E^*(a_1 = a_1^E, a_2 = 0, \delta)$ to indicate that the value reflects the manager's optimal efforts and insurance coverage: a_1^E in business operation, no effort in risk management, and any insurance coverage of the firm's choice. By the same token, the optimal V_I can be written as $V_I^*(a_1 = a_1^I, a_2 = a_2^I, \delta = 1)$.

As the optimal value, $V_I^*(a_1 = a_1^I, a_2 = a_2^I, \delta = 1)$ is greater than $V_I(a_1 = a_1^E, a_2 = a_2^E, \delta = 1)$ or any other V_I reflecting suboptimal managerial efforts. If it can be established that $V_I(a_1 = a_1^E, a_2 = a_2^E, \delta = 1)$ is greater than V_E^*, then it follows that $V_I^* > V_E^*$. By equation (A9),

$$V_I(a_1 = a_1^E, a_2 = a_2^E, \delta = 1) - V_E^* = -L(a_2^E) - C(a_1^E, a_2^E) + L(0) + C(a_1^E, 0). \tag{A11}$$

To establish the sign of the above equation, recall the first-order condition with respect to a_2 excluding the insurable loss (the same as equation [A3]). By equation (A3), $-\alpha^E D_2L - D_2C = 0$. Then, for any $a_2 < a_2^E$, $-\alpha D_2L - D_2C > 0$. Since $-D_2L$ is positive, $-D_2L > -\alpha D_2L$. It follows that

$$-D_2L - D_2C > 0 \quad \text{for all } a_2 < a_2^E.$$

Integrating positive numbers ($-D_2L - D_2C$) over a range of positive numbers (from 0 to a_2^E) yields a positive number; that is,

$$\int_0^{a_2^E} (-D_2 L(a_2) - D_2C(a_1^E, a_2))\, da_2 > 0.$$

By simple integration,

$$\int_0^{a_2^E}(-D_2L(a_2)-D_2C(a_1^E,a_2))da_2=-L(a_2^E)-C(a_1^E,a_2^E)+L(0)+C(a_1^E,0)>0, \quad \text{(A12)}$$

which indicates that equation (A11) is positive. By the previous arguments, it also shows that $V_I^* > V_E^*$.

Comparison of V_T^* and V_I^*. V_T^* is the value of the firm with the manager's optimal efforts; that is, $V_T^* = V_T^*(a_1 = a_1^T, a_2 = a_2^T, \delta = 1)$. Since $a_1^T \neq a_1^I$ as shown below, $V_T^*(a_1 = a_1^T, a_2 = a_2^T, \delta = 1) > V_T(a_1 = a_1^I, a_2 = a_2^T, \delta = 1)$. V_T^* is shown to be greater than V_I^* by way of demonstrating that $V_T(a_1 = a_1^I, a_2 = a_2^T, \delta = 1) > V_I^*$ via the same process used to show equation (A11). The results of the two comparisons above establish that

$$V_T^* > V_I^* > V_E^*.$$

Comparison of a_1^T, a_1^I, and a_1^E. The first-order condition with respect to α for the one-weight contract excluding the insurable loss and with respect to α_1 for the two-weight contract, derived via the same process from equations (A2) through (A7), are identical:

$$(1-\alpha_j)\, D_1\, \pi \frac{\partial a_1}{\partial \alpha_j} - \gamma\, \alpha_j\, \sigma_1^2 = 0, \quad \text{(A13)}$$

where j = 1 for the two-weight contract, and j is blank for the one-weight contract excluding the insurable loss. This implies that the optimal weights and therefore the optimal managerial efforts in business operation under the two contracts are the same. That is, $\alpha_1^T = \alpha^E$ and $a_1^T = a_1^E$.

The first-order condition with respect to α for the one-weight contract including the insurable loss, according to equation (A7) at $\delta = 1$, is

$$(1-\alpha)\, D_1\, \pi \frac{\partial a_1}{\partial \alpha} - (1-\alpha)\, D_2\, L \frac{\partial a_2}{\partial \alpha} - \gamma\, \alpha\, \sigma_1^2 = 0. \quad \text{(A14)}$$

Substituting equation (A13) into equation (A14) yields

$$-(1-\alpha)\, D_2 L \frac{\partial a_2}{\partial \alpha} > 0,$$

which indicates that at $\alpha = \alpha_1^T$, the manager's marginal benefit still outweighs the marginal cost. Thus, $\alpha^I > \alpha_1^T$ and $a_1^I > a_1^T$ $(= a_1^E)$.

Derivation of Equation (11)

The manager is compensated with $F + \frac{\alpha}{1-\alpha} P$, and thus the manager's certainty equivalent is

$$W = E\left(F + \frac{\alpha}{1-\alpha} P\right) - \frac{\gamma}{2}\left(\frac{\alpha}{1-\alpha}\right)^2 \mathrm{Var}(P) - C. \tag{A15}$$

By equation (10), W can be transformed into

$$W = F + \alpha[c + b_1\pi - (1-\delta)b_2 L - \delta L - F] - \frac{\gamma}{2}\alpha^2[b_1\sigma_1^2 + b_2(1-\delta)^2\sigma_2^2] - C. \tag{A16}$$

The manager's first-order conditions are

$$\frac{\partial W}{\partial a_1} = \alpha\, b_1\, D_1\, \pi - D_1\, C = 0 \tag{A17}$$

and

$$\frac{\partial W}{\partial a_2} = -\alpha[b_2 + \delta\,(1 - b_2)]\, D_2 L - D_2 C = 0. \tag{A18}$$

The shareholders' objective is to select α and δ to maximize the firm's *ex ante* market value, that is, E(P). Solving equation (A16) for F and substituting the resulting expression into E(P) gives

$$E(P) = [c + b_1\pi - (1-\delta)b_2 L - \delta L] - W^0 - \frac{\gamma}{2}\alpha^2[b_1\sigma_1^2 + b_2(1-\delta)^2\sigma_2^2] - C. \tag{A19}$$

The first-order condition with respect to α yields the following optimal solution of α, α^e:

$$\alpha^e = \frac{1}{1+H}, \tag{11}$$

where $H = \dfrac{\gamma[b_1\sigma_1^2 + b_2(1-\delta)^2\sigma_2^2]}{b_1 D_1 \pi \dfrac{\partial a_1}{\partial \alpha} - [b_2 + \delta\,(1-b_2)] D_2 L \dfrac{\partial a_2}{\partial \alpha}}$.

The first-order condition with respect to δ is

$$\frac{\partial\, E(P)}{\partial\, \delta} = -(1-b_2)\, L + \gamma\, \alpha^2\, b_2\, (1-\delta) = 0. \tag{A20}$$

It is not clear that the solution to equation (A20) falls in [0, 1]. To examine, evaluate the partial derivative at $\delta = 0$:

$$\frac{\partial\, E(P)}{\partial\, \delta}\Big|_{\delta=0} = -(1-b_2)\, L + \gamma\, \alpha^2\, b_2. \tag{A21}$$

With $b_2 < 1$, the sign of equation (A21) is indeterminate. If equation (A21) < 0, then the optimal insurance coverage is zero. Otherwise, the optimal coverage is somewhere in (0, 1]. To see whether $\delta = 1$ is optimal, evaluate equation (A20) at $\delta = 1$:

$$\frac{\partial E(P)}{\partial\, \delta}\Big|_{\delta=1} = -(1-b_2)\, L < 0. \tag{A22}$$

The above analysis indicates that, if insurance coverage is demanded, the optimal solution is somewhere in (0, 1). The closed-form solution can be obtained easily but is omitted here because it does not provide any further insight other than the solution itself.

It is worth noting that in deriving all of the first-order conditions, the c in the E(P) expression is treated as a constant although it contains the equilibrium values of α and δ. The reason is that, when the manager makes decisions about a_1 and a_2, he or she understands that c is determined by investors' belief about the manager's effort levels. Therefore, the manager's choice of a_1 and a_2 only affects $E(Z|a_1)$ and $E(Y|a_2)$. In an equilibrium of rational expectations, the manager's actual action and the investors' belief will converge. The same explanation applies to the derivation of the shareholders' first-order conditions.

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