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Habit Formation and the Demand for Insurance

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ABSTRACT

This article models consumption and insurance decisions in a continuous-time, finite-horizon setting. We allow the consumer to acquire a "taste for the good life" by making current preferences for consumption dependent upon the individual's past consumption. The optimal consumption path is smoother and the optimal level of insurance greater in this setting than they are in an identical model without habit formation. Moreover, the optimal level of insurance increases over the planning horizon, approaching full coverage in the limit. These results help to explain the observed phenomenon of individuals' over-purchasing insurance, such as a propensity for low deductibles.

INTRODUCTION

Intertemporal models in finance and economics traditionally are based on time-separable preferences. Utility over time is a simple discounted aggregation (i.e., sum or integral) of utility at each point in time. A basic problem with this approach is that it does not allow for the disentangling of the elasticity of intertemporal substitution from the individual's degree of risk aversion (Hall, 1988; Zin, 1987). Several empirical puzzles arise as a result of this approach, such as the behavior of aggregate consumption, which appears to be much less volatile than predicted by theory.

Some empirical puzzles also have arisen in insurance markets. Some but not all of these puzzles might be explained by psychological effects of risk framing (Schoemaker, 1982). For instance, consumers seem to show a propensity for lower deductibles than standard theory would suggest (Pashigian, Schkade, and Menefee, 1966). Or, as noted by Borch (1983), "If a traveller insures his baggage at all, we will expect him to take insurance for its full value." Thus, some modification of existing models is needed if we desire to improve upon the theory in a positive (as

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opposed to a normative) sense.

Such an approach is taken by Briys and Loubergé (1985), who use a bounded rationality model. Another approach, not yet applied to insurance as far as we know, would be to use intertemporal preferences, such as those set forth by Kreps and Porteus (1978), Epstein and Zin (1989), and Duffie, Epstein, and Skiadas (1990), to model insurance-buying behavior.

The approach taken here is somewhat different and is based on the seminal works of Pollak (1970) and of Ryder and Heal (1973), which incorporate habit formation into a multiperiod von Neuman-Morgenstern expected utility model. The consumer is assumed to acquire a "taste for the good life," as past consumption negatively affects his or her current level of satisfaction. In particular, the more one has consumed in the past, the more current consumption is needed to bring the consumer up to any specified utility level. In other words, the consumer develops consumption habits over time that affect current levels of satisfaction. Abel (1990), Constantinides (1990), and Detemple and Zapatero (1991) discuss many of the merits of habit-formation models in more detail.

This article models consumption and insurance decisions in a continuous-time, finite-horizon setting. Our model is similar to those of Briys (1986) and Gollier (1994), except that we do not assume that preferences are fully time-separable. Instead, we take an approach similar to that of Sundaresan (1989) and Briys, Crouhy, and Schlesinger (1990), as summarized in Merton (1990, p. 208), with the major exception that wealth is not assumed to follow a diffusion process. In particular, we add a "habit formation" state variable to Briys's model, which is an exponentially weighted average of past consumption, and wealth follows a stochastic jump process to model insurable losses. It is shown that the optimal consumption path is smoother and the optimal level of insurance purchased is greater in markets exhibiting habit formation. Moreover, the optimal level of insurance increases over the planning horizon, approaching full coverage in the limit.

THE MODEL

Consider a risk-averse individual whose wealth at time t is denoted by W(t). The individual's wealth is assumed to grow at a risk-free rate, μ, except for the possibility of a total loss. The loss is modeled via a Poisson process q(t), which indicates the number of losses having occurred through time t. Thus, q(t) "jumps" by one if a loss occurs at time t. The process is fully specified by the parameter $\lambda > 0$, which indicates the probability of a loss over the next (infinitesimal) unit of time. Note that multiple losses are possible over the individual's planning horizon, which we consider to be the time interval [0,T].

More specifically, uninsured wealth without consumption is generated by the stochastic jump process:

$$dW(t) = \mu W(t)dt - W(t)dq(t), \tag{1}$$

and the endpoint constraint of initial wealth $W(0) = W_0 > 0$.

The individual decides upon a level of consumption, c(t), and the fraction of wealth to insure, ß(t), at each instant in time. Insurance prices are assumed to be linear in ß, with a constant loading factor $\theta > 0$. The insurance premium is, thus,

$$P[\beta(t),W(t)] = (1+\theta)\,\lambda\beta(t)W(t).^{1} \qquad (2)$$

The individual spends money on consumption and insurance premiums, and potentially loses money through one or more losses. At the same time, the individual receives interest income at a risk-free rate on savings and receives an insurance indemnity of $\beta(t)W(t)$ whenever a loss occurs at time t. Modifying the stochastic equation of motion (1) to account for consumption and insurance decisions,

$$dW(t) = \{\mu W(t) - P[\beta(t),W(t)] - c(t)\}\,dt + [\beta(t)-1]W(t)dq(t). \qquad (3)$$

Consumer preferences are modeled such that the marginal rate of substitution between consumption at time t and at time s depends upon the consumption path between these two times. In particular, the nonseparability of preferences over time is captured by a habit-formation variable (or "memory coefficient"), x(t), which is an exponentially weighted average of past consumption:

$$x(t) = a\int_0^t e^{a(s-t)}c(s)ds, \qquad (4)$$

where a is a positive scalar, $a < 1$. Noting that $x(0) = 0$, the habit-formation variable also can be specified completely by its equation of motion, obtained by differentiating in equation (4)

$$dx(t) = a[c(t)-x(t)]dt.^{2} \qquad (5)$$

The consumer's instantaneous utility is given by $U[c(t),x(t),t]$, where $U_c \equiv \partial U/\partial c > 0$, $U_{cc} \equiv \partial^2U/\partial c^2 < 0$, and $U_x \equiv \partial U/\partial x < 0$. The individual's objective is, then, to

$$\underset{c,\beta}{\text{maximize}}\, E_0 \int_0^T U[c(t), x(t), t]\,dt, \qquad (6)$$

where E_0 is the expectation operator based on time-zero information. For simplicity, it is assumed that the individual has no bequest or savings motive beyond time T and that the opportunity set facing the individual is fixed over time. The maximization in (6) is subject to the equations of motion given by (3) and (5).

The optimal paths of c and ß are restricted such that consumption at time t is always nonnegative, $c(t) > 0$, and results in a finite aggregate consumption over any finite time interval; the demand for insurance is nonnegative (note that $\theta > 0$ ensures that demand does not exceed full coverage); and the optimal strategy ensures that wealth remains strictly positive with probability one, at any time $t \in [0,T)$.

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[1] P is actually a periodic premium rate, and the premium is assumed to be paid continuously over time. Although modeled as coinsurance, the binary nature of the loss process allows one to interpret the insurance as a deductible policy with deductible level $D(t) \equiv [1-\beta(t)]W(t)$. It is impossible to distinguish between coinsurance and deductibles without multiple sizes of loss.

[2] In a discrete-time model, equation (5) is simply $x(t) = ac(t-1) + (1-a)x(t-1)$.

OPTIMAL CONSUMPTION AND INSURANCE

The above program is solved using dynamic-programming techniques. Let $J(\bullet)$ denote the indirect utility function

$$J[W(t), x(t), t] \equiv \max_{c,\beta} E_t \int_t^T U[c(s), x(s), s]\, ds,$$

where E_t denotes the expectation operator based on information available at time t. The state of the system at time t is fully described by the vector [W(t),x(t)].

The solution to program (6) must satisfy the Hamilton-Bellman-Jacobi equation of optimality:

$$\begin{aligned} -J_t(W,x,t) = \max_{c,\beta}\{&U(c,x,t) + J_W(W,x,t)[\mu W - \lambda(1+\theta)\beta W - c] \\ &+ J_x(W,x,t)[a(c-x)] + \lambda[J(\beta W,x,t) - J(W,x,t)]\}, \end{aligned} \tag{7}$$

where $J_t \equiv \partial J/\partial t$, and $J_x \equiv \partial J/\partial x$. Since $U_x < 0$, it follows that $J_x < 0$. Note that the time arguments for W, x, and c in equation (7) have been omitted for notational ease and are understood to be t unless otherwise noted. The assumption of no bequest or savings motive past time T imposes that

$$J[W(t),x(T),T] = 0. \tag{8}$$

Using the equation of optimality (7), the first-order conditions for the optimal consumption and insurance paths can be derived. These are

$$\partial U(c,x,t)/\partial c = J_w(W,x,t) - aJ_x(W,x,t) \qquad \forall t \tag{9}$$

and

$$(1+\theta)J_w(W,x,t) = J_w(\beta W,x,t) \qquad \forall t. \tag{10}$$

Note that when a = x = 0, there is no habit formation and the model is similar to those of Briys (1986) and Gollier (1994).

Condition (9) states that the marginal utility of spending an extra dollar on consumption today equals the marginal utility of saving an extra dollar (and then spending it optimally through time T) minus the utility lost by adding the extra consumption into the memory coefficient, x. Since $J_x < 0$ and U is concave in c, it follows for $a > 0$ that the optimal level of consumption will be lower, *ceteris paribus,* than a model with no habit formation.

Condition (10) states that the marginal utility cost of increasing the current insurance premium (where the utility cost stems from not having the premium money to spend through time T) equals the expected marginal utility gain from receiving a potentially higher insurance indemnity at time t.[3] As usual in insurance models, if the premium is actuarially fair, $\theta = 0$, full coverage is optimal; whereas for a positive premium loading for profit and expenses, $\theta > 0$, it follows that optimal coverage is partial insurance.

[3] Differentiating equation (7) with respect to ß, one sees that a factor of λW has been factored out of both sides of equation (10). The interpretation of equation (10) above takes this into account.

The Case of Isoelastic Utility

Unfortunately, the above first-order conditions typically do not lead to closed-form solutions and must be solved through numerical methods. In this section, a special case is considered in which a closed-form solution does exist. This allows for a better comparison with results obtained in models without habit formation. In particular, let the instantaneous utility function U(•) be a function of time only indirectly, through c and x, defined by $U(c,x) = \gamma^{-1}(c-x)^{\gamma}$, where $\gamma < 0$. Note that U exhibits constant relative risk aversion of $1 - \gamma$ with respect to $c - x$. It is also assumed that insurance premiums can be financed by interest income alone, that is, $\mu-\lambda(1+\theta) \equiv k > 0$.[4]

The above preferences are fairly common for models without habit formation; see, for example, Ingersoll (1987) and Merton (1990). They also have been used in continuous-time financial models with habit formation where the risky-asset value follows a geometric Brownian motion. Unfortunately, solutions to the current problem, where the stochastic part of wealth is governed by a jump process rather than a diffusion process, are more restrictive. Cases in which $k < 0$ and/or relative risk aversion is less than unity (i.e., $0 < \gamma < 1$) might not have closed-form solutions.

In the current setting, straightforward (but tedious) calculations lead to the following indirect utility function:

$$J(W,x,t) = \gamma^{-1} g^{\gamma-1}(k+a)^{-1}(kW-x)^{\gamma}, \tag{11}$$

where

$$g \equiv g(a,t) \equiv z\{(k+a)[1-\exp(z(t-T))]\}^{-1}, \tag{12}$$

and

$$z \equiv (\gamma-1)^{-1}\{\gamma[\mu-\lambda(1+\theta)^{\gamma/(\gamma-1)}] + \lambda[(1+\theta)^{\gamma/(\gamma-1)} - 1]\}. \tag{13}$$

Using the above indirect utility function and the first-order conditions (9) and (10), the optimal levels of consumption and insurance can be derived for each time $t \in [0,T]$:

$$c^* = g[kW-x] + x, \tag{14}$$

and

$$\beta^* = (1+\theta)^{1/(\gamma-1)} + [1-(1+\theta)^{1/(\gamma-1)}]\left(\frac{x}{kW}\right) = \left(1-\frac{x}{kW}\right)b + \left(\frac{x}{kW}\right), \tag{15}$$

where $b \equiv (1+\theta)^{1/(\gamma-1)}$.

Note that J is not well defined for $kW - x \leq 0$. However, as shown in the Appendix, $k > 0$ implies that $kW - x > 0$ $\forall t \in (0,T)$ and that $kW - x \to 0$ as $t \to T$.

Using the above information, it follows from equation (14) (since $\gamma < 0$ and $k > 0$ also imply that $g > 0$) that $c^*(t) > x(t)$ $\forall t < T$. Thus, the utility function U(c,x)

[4] This assumption is needed for the indirect utility function given below to be well defined. If it fails, a closed form solution for equations (8) and (9) above will not exist.

is also well defined. Taking limits as $t \to T$ in equation (14), one sees that $c^*(T) \to x(T)$.[5] Similarly, from equation (15) it follows that $\beta^* < 1$ for $t < T$ and $\beta^*(T) = 1$.

For the case with no habit formation ($a = x = 0$), note that $\beta^* = b$ is constant $\forall t$, which is a direct consequence of the constant relative risk aversion. This holds true with β^* given by equation (15) even if the relative loss size is random, rather than a full loss, as is shown by Gollier (1994). However, the level of insurance purchased at each point in time is higher with habit formation than without habit formation, $\beta^* > b$, with coverage increasing toward full coverage as the time horizon draws near. Indeed, from equation (15), the level of coverage at time t is seen to be a weighted average of the no-habit-formation optimal coverage level, $\beta^* = b$, and full coverage, $\beta^* = 1$. The reason for this ever-increasing coverage is that the individual must be certain of meeting his or her habit, that is, certain that $c > x$. Thus, the individual takes fewer chances with wealth and buys more insurance.

The relative sizes of kW versus x play an important role in the above analysis. The amount kW represents the highest level of incremental wealth that can be guaranteed over the next small time interval. This is the increment obtained when the individual purchases full insurance. The habit-formation variable x represents a minimal level of consumption that must be achieved over the next time interval. If x is much smaller than kW, it is relatively easy to guarantee that consumption level x will be achieved. However, as x approaches kW, any risk in the individual's wealth portfolio could potentially prevent the consumption level from reaching x. Both the absolute difference $x - kW$ and the relative level x/kW measure a type of "level of concern" for reaching minimum-consumption level x. Thus, for example, the weights on $\beta^* = b$ and $\beta^* = 1$ in equation (15) above are seen to comprise this "level of concern" factor. This becomes even more transparent, perhaps, by examining the retention rate implied by equation (15), namely

$$(1-\beta^*) = (1-b)\left(1-\frac{x}{kW}\right). \tag{16}$$

Some insight is gained by examining the relative risk aversion of the individual. From equation (11), we calculate this measure as follows:

$$R(x, W, t) \equiv \frac{-WJ_{WW}}{J_W} = \frac{1-\gamma}{1-\dfrac{x}{kW}}. \tag{17}$$

Note that with no habit formation ($x \equiv 0$), we obtain $R = 1 - \gamma$, which is the same as the relative risk aversion of the instantaneous utility function u with respect to c in this case. If we now allow for habit formation, then $0 < x/kW < 1$, and $x/kW \to 1$ as $t \to T$; so that R is increasing over time and $R \to \infty$ as $t \to T$. Thus, under habit formation, individuals become more risk averse as they age, even if their instantaneous utility function is invariant over time.

Since the insurance budget is always higher under habit formation, it follows, *ceteris paribus,* that consumption over the period generally will be lower. Indeed,

[5] Although $g \to \infty$ as $t \to T$, results for $kW - x$ show that $g[kW-x] \to 0$ (see the Appendix).

consumption starts off in early periods rather low, to compensate for the detrimental effects of habit formation. In other words, not only is the overall level of consumption lower, but its time path is altered to allow for relatively greater consumption levels in later time periods. Also, noting that c^* is effectively a weighted average of kW and x — that is, $c^* = g(kW) + (1-g)x$ — the time path of c^* is seen to be less volatile under habit formation than in models without such habit formation (see Ingersoll, 1987, and Briys, 1986). Thus, insurance allows the individual to obtain a "smoother" consumption path than would otherwise be possible, and habit formation induces the individual to make better use of this smoothing device.

CONCLUSION

This article examines the effects of habit formation on optimal consumption and insurance decisions. The level of insurance is seen to be greater than would exist without habit formation. This result may help to explain why many empirical and experimental studies show a propensity for individuals to "overinsure," as compared to previous theoretical predictions, which all are based on static models or on models with time-separable preferences. The individual behaves more prudently in order to ensure that an ever-increasing habitual level of consumption will be exceeded.

The model presented is admittedly oversimplified. There is no bequest motive, only a full loss can occur, and utility is undiscounted. Moreover, the use of the difference $c - x$ in the utility function is very strong. One could also use the relative sizes, c/x as the argument of the utility function, as in Abel (1990). Still, the simplicity of the model allows one to isolate the effects of habit formation. It is left to future research to determine how habit formation effects unfold in more complicated settings.

APPENDIX

It is shown here that $kW - x > 0$ for $t < T$, and $kW(T) - x(T) = 0$. Define $Y(t) \equiv kW(t) - x(t)$. It follows from the equations of motion, (3) and (5), that

$$dY(t) = m(t)Y(t)dt + nY(t-)dq(t), \tag{A1}$$

where $m(t) \equiv \mu - \lambda(1+\theta)^{\gamma/(\gamma-1)} - (k+a)g(t)$,
$n \equiv (1+\theta)^{\gamma/(\gamma-1)} - 1 > 0$, and
$Y(t-) \equiv \lim_{s \to t-} Y(s)$; that is, the limit from the left.

Now define

$$dA(t) = m(t)A(t)dt, \tag{A2}$$

and

$$dB(t) = nB(t-)dq(t). \tag{A3}$$

From these definitions, one can verify that

$$A(t) = A(0)\, e^{\int_0^t m(r)\,dr}, \tag{A4}$$

and

$$B(t) = B(0)[1+n]^{q(t)}. \tag{A5}$$

Thus,

$$\begin{aligned} d(A(t)B(t)) &= B(t)dA(t) + A(t-)dB(t) \\ &= B(t)A(t)m(t)dt + A(t-)B(t-)dq(t). \end{aligned}$$

Consequently, $Y(t) = A(t)B(t)$, which from equations (A4) and (A5) yields

$$Y(t) = Y(0)[1+n]^{q(t)}\, e^{\int_0^t m(r)\,dr}. \tag{A6}$$

From equation (A6), it follows that $\text{sgn}[Y(t)] = \text{sgn}[Y(0)]\ \forall t < T$ but $Y(0) = kW(0) > 0$, so that $Y(t) > 0\ \forall t < T$. Since $m(t) \to -\infty$ as $t \to T$, it also follows that $Y(T) = 0$.

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