© *The Journal of Risk and Insurance,* 1996, Vol. 63, No. 1, 95–110.

Extreme Values in Business Interruption Insurance

Daniel Zajdenweber

ABSTRACT

The size distribution of yearly claims in the French business interruption insurance branch is a Pareto law with an extremely long tail. The behavior of that law reflects the fact that the total value of yearly claims is dominated by a small number of huge claims. The estimated characteristic exponent of the tail is very close to one. The article analyzes the consequences of that particular value on the actuarial risk covered by the insurance and the reinsurance companies.

INTRODUCTION

Fire is the most frequent cause of business interruption. If a damaged firm is insured against business interruptions, insurers pay for the losses in sales, minus the costs spared because of the interruption of the production. Two facts motivate the analysis of business interruption.

First, on a microeconomic level, the value of a business interruption claim is seldom a simple proportion of the size of the physical damages due to fire. The value of business interruption damages is often insignificant compared to fire damages. This explains, at least partly, why so many firms are not insured against business interruption; in 1992, only 36 percent of the French firms insured against fire damages were also insured against business interruption (*Assemblée Plénière des Sociétés d'Assurances Dommages,* 1994). But sometimes the value of business interruption damages may be much greater than the value of the products, machines, furniture, or buildings burnt. For example, in 1990, the largest business interruption damages in France, which occurred in a chemical plant, were eleven times larger than the physical damages due to fire, and, in 1992, the value of France's second largest business interruption claim, actually paid to the ill-fated firm, amounted to three times

---

Daniel Zajdenweber is Professor of Economics at Parix-X-Nanterre University.

The author wishes to thank Sylvain Tribouillois and Fabrice Genest for providing data on business interruption claims in France, Mico Loretan and Carl Evertsz for the figures on the size-distribution of claims and the size distribution of French firms, and Lucien Birgé for helpful comments. The author is grateful to Benoît Mandelbrot for his invitation to develop the research in the Department of Mathematics at Yale University during 1989. The constructive suggestions of two anonymous referees and of Georges Dionne are gratefully acknowledged. Remaining errors are the author's responsibility. Financial support from the *Fédération Française des Sociétés d'Assurances* is also gratefully acknowledged.

the value of physical damages. Hence, business interruption is a unique branch of insurance that merits analysis for its own sake.

Second, on a macroeconomic level, the most striking feature of business interruption claims is the extreme variability of yearly claims. In France, for instance, the total amount of business interruption claims paid in 1988 by the insurance industry was nearly $200 million—more than twice as much as the $87 million paid in 1987. And the total amount of claims paid to the firms in 1988 was greater than the premiums collected ($193 million), a deficit situation that was repeated in 1990, 1991, and 1992 (APSAD, 1994). The variability can be easily explained by the occurrence of large claims, the amounts of which are greater than $16 million and even sometimes greater than $160 million in current value. Large business interruption claims are not numerous, but they make the tail of the yearly claim size distribution extremely long—so long that, as this article shows, if no upper bound is settled on the size of the largest claim, variance and even expectation may not exist.[1]

This statistical evidence has two important consequences on premium valuation. First, the expected value of business interruption claims in a country is highly sensitive to the value of the maximal claim that might happen. It increases with the size of that maximal claim, making the choice of its relevant value a critical decision for the insurers. Second, the effective size of that maximal claim is so large that a reinsurance treaty is always mandatory, in order to avoid insurer bankruptcies. Hence, the choice of the threshold value above which claims are reinsured also becomes a critical decision.

This article first develops a statistical analysis of the business interruption distribution by using data from the French insurance industry. Then, it is shown that the parameter estimates may affect the actuarial risk covered by the insurance industry. The first section describes the empirical evidence. The second section analyzes the theoretical law that can fit the data. The third section estimates the parameters of the claim size distributions, and the last section is devoted to the valuation of business interruption claims and to the valuation of reinsurance pure premiums.

EMPIRICAL EVIDENCE

The data used in this analysis originate from the statistics of the French insurance union, *Assemblée Plénière des Sociétés d'Assurances Dommages* (APSAD). It records all business interruption claims (and actual payments made) due to fire that are greater than $1,600. All the claims are located in France. The available data span the years 1975 through 1992. Yearly claims are ranked and valued in constant money value (reference year 1988). Except data on the abscissas of the figures and on Table 3, which are in French francs, all the claims are valued in U.S. dollars (using an exchange rate of six French francs for one U.S. dollar).

---

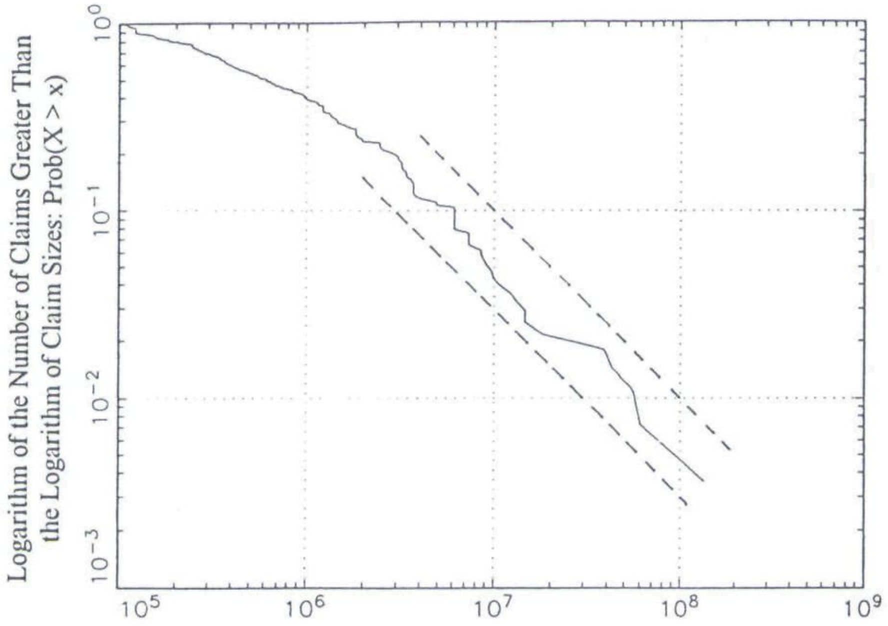
[1] Cummins et al. (1990) show that fire loss severity distributions also tend to be heavy-tailed.

Sixteen out of the 18 yearly size distributions (all but 1979 and 1981) look like the typical distributions shown in Figures 1 and 2. In these figures, the abscissa is the logarithm of the claim sizes. The ordinate is the logarithm of the number of claims that are greater than the size on the abscissa. It is thus the logarithm of the complement of the cumulative distribution function of the empirical distribution of yearly claims. The common slope of the parallel dotted lines is -1. The small steps on the curves are by-products of the tendency to report rounded values of the damages paid.[2]

Except for the years 1979 and 1981 (fortunately, no very large claims occurred these two years), the size distributions of yearly claims always display the same two features. Below a threshold of about $330,000 (Fr 2 million), the curves are concave.[3] But above that threshold, the tails fit a straight line with

Figure 1

Size Distribution of Losses in 1983

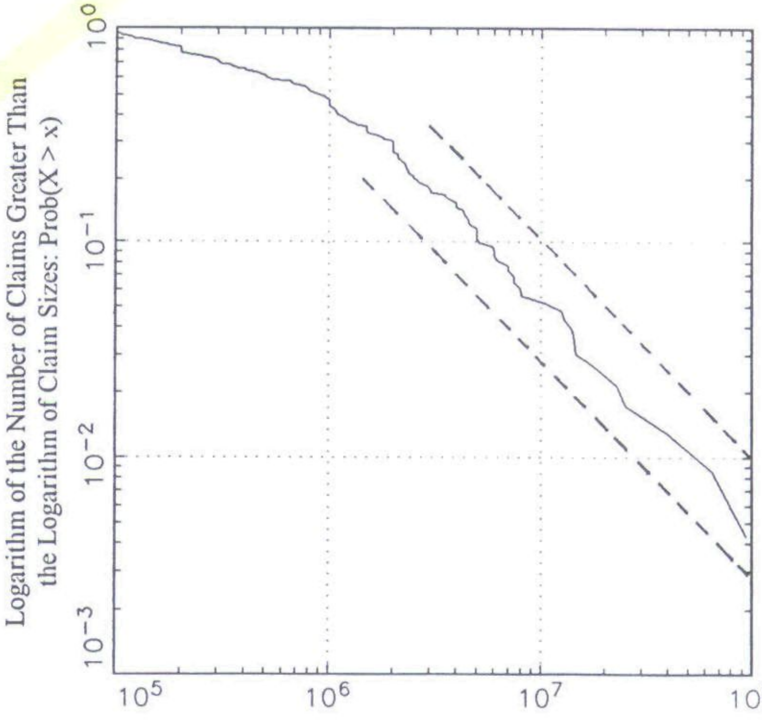


Logarithm of Claim Sizes, or Amount of Loss in French Francs

[2] Claims amounting to less than $16,000 are not shown in the figures. But the cumulative value of those small claims is always insignificant. For instance, in 1988, 1989, and 1990, their cumulative value amounted to a mere 0.3 percent of the total value of the yearly claims (in 1991, that proportion fell to 0.2 percent).

[3] In our sample, the value of this threshold is remarkably stable. But no technical or operational meaning was given by the APSAD that could explain that precise value ($330,000) and its stability. The most plausible origin of that stability might be purely empirical. As we shall see, the size distribution of the firms insured is also a Pareto distribution with a very long tail, and it is not changing through time, at least in our sample.

Figure 2
Size Distribution of Losses in 1986



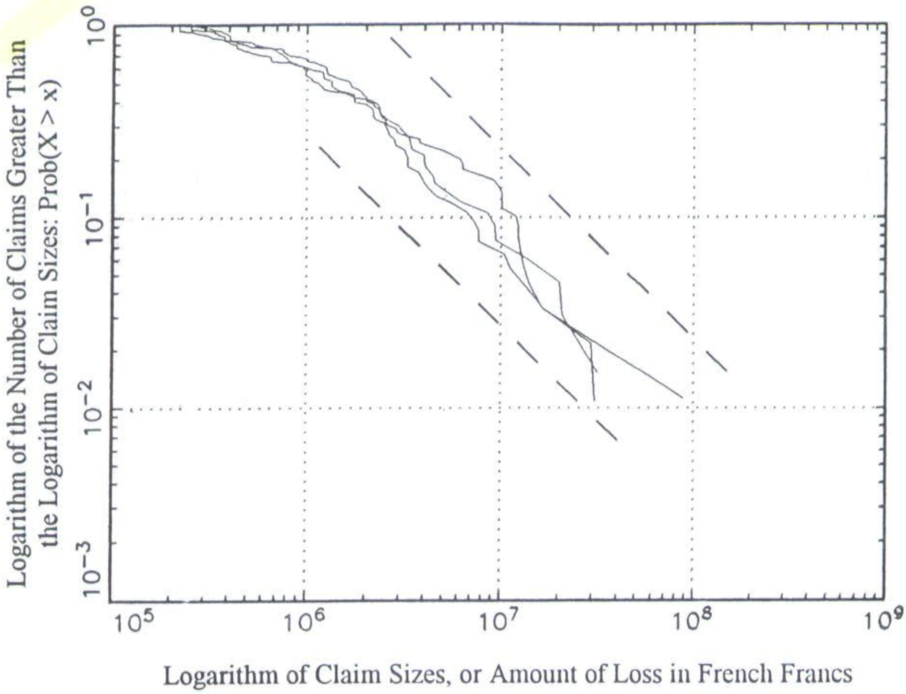
Logarithm of Claim Sizes, or Amount of Loss in French Francs

a slope close to -1. The behavior of the tail can be described, and the value of its slope can be approximately estimated by means of a graphical analysis. Figure 3 shows the superposition of three size distributions of yearly claims corresponding to 1975, 1976, and 1977. The common behavior is striking (the next section uses a more rigorous estimation technique). The straight line is also known in economic literature as the Pareto line. Each year, the cumulative value of the claims on the Pareto line amount to at least 80 percent of the total value of the claims. In 1988 and 1989 they shared 92 percent of that total value, and in 1992 that proportion was 94 percent. Thus, it is easy to understand why the Pareto line is the heart of the matter for insurers. Their managers speak about "lucky" years when no large claims occur, as in 1979 and in 1981. Nevertheless, those lucky years are exceptional and give only a temporary relief.

It is worth noting that the size distribution of business interruption claims in Western Germany in 1989 also fits a Pareto line with a slope close to -1. But the data recorded by the *Comité Européen des Assurances* in that country do not state claims less than \$3.3 million. Hence, the number of claims reported is small in absolute value (32 compared to 112 claims larger than \$330,000 re-

Figure 3

Size Distributions for Yearly Claims in 1975, 1976, and 1977



ported in France that year). Nevertheless, in Western Germany, the number of huge claims is relatively large, because the number of firms insured against business interruption in that country is greater than in France (where there were only 12 claims greater than \$3.3 million during the same year).

THEORETICAL PROBABILITY MODELS

Since the data in our sample only record the yearly values of the claims greater than a (relatively) high threshold, and since the size distribution is invariant, the theoretical model is found in the families of "max-stable" extreme distributions. One of the main theorems of the mathematical theory of extreme values states that there are only three types of max-stable probability distribution functions (Gumbel, 1958; Galambos, 1987).

The Pareto Distribution Function: $1-x^{-\alpha}$, $x > 1$, $\alpha > 0$

This distribution function has a "heavy" or "fat" upper tail. Its main mathematical feature is that it has an infinite variance when $\alpha \leq 2$ and an infinite expectation when $\alpha \leq 1$. Among the three max-stable probability distribution functions, only the Pareto distribution function has a tail that behaves like a straight line. The exponent α also gives the absolute value of the slope of the Pareto line on a double-log graph as those of Figures 1 and 2.

The Weibull Distribution Function: $1-(-x)^{-\alpha}, -1 < x < 0, \alpha < 0$

This distribution function has a short upper tail.[4] The Weibull law is found in the context of a maximum value that cannot be passed, for instance, in the case of a rectangular density. This is not the case in the data concerning business interruption: the maximum potential value of a claim in France is far greater than the record value already observed (as noted above). Another form of Weibull distribution function is found in the context of extreme values: $1-\exp(-x^{\alpha})$, $x > 0$. This function is unbounded and medium-tailed. But the tail has not the straight line behavior.

The Exponential Distribution Function: $1-e^{-x}, x > 0$

This distribution function has a medium upper tail. It is a particular case of the preceding Weibull distribution with $\alpha = 1$.

In practice, when dealing only with large values exceeding high thresholds ($330,000 in our sample), the outstanding result of the extreme values theory means that only one of those three distribution functions can be observed. However, since both the distribution function of the Weibull law and of the exponential law display a concave tail without a straight line when drawn on a double-log graph, only the Pareto law may be relevant to the data on the business interruption claims.

ESTIMATION OF THE SLOPES OF THE PARETO LINES

Since, for each year, we know the value of the empirical threshold ($330,000), the estimation of each yearly Pareto distribution is completely performed through the estimation of its characteristic exponent α.

Let N be the number of claims in the Pareto tail, x the size of a claim, x_0 the threshhold value below which the size distribution is no more a Pareto law (see footnote 2), and i the rank of a claim with the value x_i ($i = 1, 2, ..., N$). The maximum likelihood estimator of α (or Hill estimator, 1975) is

$$1/\hat{a} = (1/N)\sum \mathrm{Log}\, x_i - \mathrm{Log}\, x_0.$$

$$\text{Therefore, } E(1/\hat{\alpha}) = 1/\alpha, \text{and} \mathrm{Var}(1/\hat{\alpha}) = N\alpha^2.$$

Thus, for N sufficiently large, the confidence interval defined by means of the central limit theorem is given by

$$\sqrt{} \mathrm{Na}(1/\hat{a} - 1/a) \sim \Re(0,1),$$

[4] The Weibull distribution function can also be defined with a positive exponent. In that case, the distribution function becomes $1 - (-x)^{\alpha}$. The support of the distribution function lies within the positive or negative half-line when $\alpha > 0$ or $\alpha < 0$, respectively (Reiss, 1989).

which means that there is approximately a 95 percent probability that the true value of $1/\alpha$ lies within the interval

$$1/\hat{\alpha}(1 \pm 2/\sqrt{N}).$$

Table 1 presents all the estimations, except for the two "lucky" years 1979 and 1981 when no huge claims occurred, thus changing the Pareto line into a concave curve. The last column shows the values of the largest claim (Max Claim) each year. As noted in the empirical evidence, the threshold x_0 equals at least \$330,000 each year. Sometimes no claim has that precise value. In that case, the nearest value greater than \$330,000 is selected. The second column displays the number N_f of firms insured, and the third column displays the number N_c of large claims exceeding x_0.

The range of estimated values (0.8854–1.2510) is quite narrow. It is centered around an average value of 1.033, close to the hypothetical value of one, which we inferred from the graphical analysis of the data.[5] All the confidence intervals include that particular value.

There is no trend in the estimated values of the characteristic exponents. But, a trend obviously appears in the number of yearly claims greater than \$330,000, which is due to the increasing number of firms insured against business interruption. Nevertheless, since 1982, no trend has emerged in the ratio of the number of large claims to the number of firms insured. The ratio has fluctuated between 0.00455 (in 1982) and 0.00275 (in 1987), with an average of 0.00364. And no significant correlation is measured between the number of claims in the tail and the estimated characteristic exponents ($R = -0.05$).

In order to test the robustness of the estimator, the value of the threshold x_0 is changed. The years selected are 1988 through 1992, when the number of claims in the tail is large enough (larger than, or close to, 100 to avoid some spurious sampling fluctuations). A slightly higher threshold (\$384,000) is selected and a lower one (\$320,000) also, provided it is still on the Pareto line (not the case in 1988).[6] As shown in Table 2, the estimated characteristic exponents are not significantly altered, and they all fall in the confidence intervals estimated in Table 1.

---

[5] In Western Germany in 1989, $\hat{\alpha} = 1.0149$ with the confidence interval 0.75–1.57. The largest claim amounted to \$240 million. This huge claim is not far from the greatest historical business interruption claim in France of \$330 million. It is not included in our sample because it happened before 1970. It equalled nearly two years of business interruption premiums paid in France. The second largest historical business interruption claim happened in 1992, in an oil refinery. It represented half the total amount of premiums paid that year. The column Max Claim in Table 1 can be completed by the maximal yearly claims that happened between 1970 and 1974: 31,026,000 in 1970, 16,267,000 in 1971, 5,945,000 in 1972, 6,596,000 in 1973, and 3,667,000 in 1974.

[6] The Hill estimator is heavily biased when the lower values of x are not on a Pareto line. For instance, when x_0 is on the concave section of the positive tail of a Cauchy law (a stable distribution with two Pareto tails with $\alpha = 1$), the estimation gives a strong departure from the true value of one (Dekkers and deHaan, 1989; Haßmann, Reiss, and Thomas, 1993).

Table 1
Estimated Characteristic Exponents

Year	N_f	N_c	$\hat{\alpha}$	Confidence Interval	Max Claim
1975	n.a.	30	0.9396	0.78–1.48	6,789,000
1976	n.a.	42	0.9530	0.73–1.38	18,844,000
1977	n.a.	40	0.9475	0.72–1.39	6,611,000
1978	n.a.	46	1.0479	0.81–1.49	7,434,000
1979	n.a.	29	—	—	4,685,000
1980	n.a.	59	1.0320	0.82–1.39	11,689,000
1981	n.a.	48	—	—	4,315,000
1982	15,600	71	1.1117	0.90–1.46	18,667,000
1983	17,000	68	1.0646	0.86–1.40	24,116,000
1984	18,400	52	1.0219	0.80–1.41	27,191,000
1985	19,400	61	1.2510	0.99–1.68	6,584,000
1986	20,900	71	1.1723	0.95–1.54	16,776,000
1987	22,500	62	1.0487	0.83–1.40	5,549,000
1988	24,700	91	0.8854	0.73–1.12	24,547,000
1989	26,500	107	1.1653	0.98–1.44	13,183,000
1990	27,500	113	0.9032	0.76–1.11	52,542,000
1991	28,300	126	0.9833	0.83–1.20	15,378,000
1992	31,200	104	1.0061	0.84–1.25	147,832,000

Average value of $\hat{\alpha} = 1.033$ Median value of $\hat{\alpha} = 1.027$

Note: N_f = number of firms insured. N_c = number of large claims exceeding \$330,000. $\hat{\alpha}$ = estimated value of characteristic exponents.

These statistical results seem to confirm the particular value of "one" already presumed. The next section discusses the choice of the most representative value of α and its consequences on the valuation of premiums. But before leaving the statistical analysis, it is interesting to observe a puzzling coincidence between the size distribution of claims and the size distribution of firms that are insured against business interruption. The latter also has a Pareto distribution with a characteristic exponent $\alpha = 1$.

Table 2
Robustness of the Estimator

Year	$N_{(384,000)}$	$\hat{\alpha}_{(384,000)}$	$N_{(320,000)}$	$\hat{\alpha}_{(320,000)}$
1988	80	0.8747	—	—
1989	99	1.3169	116	1.2064
1990	107	1.0084	125	0.9557
1991	115	1.0164	146	1.0202
1992	94	0.9960	116	1.0189

Note: N_i = number of claims exceeding threshold value i. $\hat{\alpha}$ = estimated characteristic exponents.

Table 3 sums up the data on the size distribution of French firms in 1988, and Figure 4 shows the excellent adjustment between the size of the firms and the theoretical Pareto distribution with $\alpha = 1$.[7]

Table 3
1988 Size Distribution Data for French Firms

Size Classes (in millions of francs)	Average Sizes (in millions of francs)	Number of Firms Insured
Less than 4	n.a.	2,612
4-7	5.7	6,783
7-12	9.4	5,854
12-25	17.3	5,063
25-50	34.7	2,273
50-100	70.6	1,094
100-200	138.2	514
200-500	304.6	311
500-1,000	708.3	100
1,000-2,000	1,216.7	49
2,000-5,000	3,085.3	34
Over 5,000	10,561.0	16
		Total: 24,701

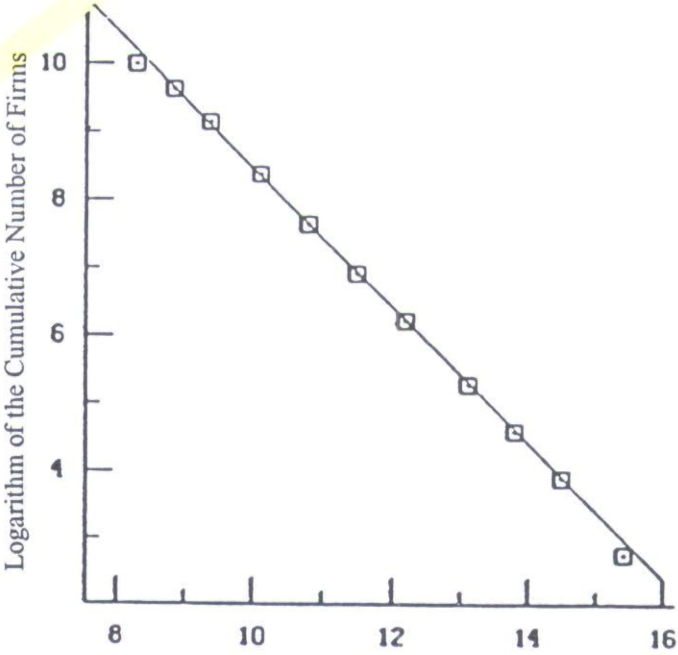
Since 1988, data from APSAD show that the size distributions of firms greater than Fr 50 miilion are not changing. Nevertheless, this coincidence does not give the origin of the particular value of one, because the stochastic relation between the size of a claim and the size of a firm is not linear. Most large firms gather many establishments which are more or less independent. Hence, a fire may stall the economic activity of a single establishment or the activities of many dependent establishments, but not all the economic activities of many independent establishments. Unfortunately, data stored by APSAD do not give details about establishment sizes. They only give firm sizes reported in each insurance policy. For instance, in 1992, the largest business interruption claim happened in an oil company with a global size of $3.5 billion. This is not the size of the exploded oil refinery itself, but the size of all the French oil refineries belonging to that oil company. Nevertheless, the stability of the size distribution of firms may also imply the stability of the size distribution of establishments that might have the same Pareto law. In that case, a plausible origin of the size distribution of claims would be found.[8]

---

[7] The low value of the characteristic exponent is quite surprising, because it implies an extremely unequal size distribution of firms. Most empirical distributions of firm sizes in a country are not so skewed. The most common values of α are found in the range of 1 to 2. Sometimes they are so close to 2 that it is nearly impossible to distinguish a Pareto distribution from the tail of a lognormal law (Gibrat, 1931; Steindl, 1965; Petruszewycz, 1972; Zajdenweber, 1976). A plausible explanation of the discrepancy between the two characteristic exponents is that large firms may be more often insured against business interruption than small firms. Hence, in the data recorded by APSAD, the relative number of large and giant firms has increased.

[8] The most satisfactory scientific inference would be the estimation of the true value of α

Figure 4
Cumulated Size Distribution of the French Firms



Expected Claim Valuation and Reinsurance Treaties

Business interruption insurance is a risky business, because insurers must face huge claims. And no insurer can cope alone with the size of the largest claims. Like their clients, they need to be insured against huge claims. They pool the risks among themselves by means of coinsurance arrangements, and they also transfer some claims to a second tier of the insurance market by means of reinsurance treaties.[9] In order to keep the analysis simple, we assume that all insurance companies in a country, like France, use the same kind of safety-first rationality: they transfer their large claims to an international pool of reinsurance companies.[10] We also assume that they all use the same threshold

---

by compounding three functions: establishment size distribution; the probability of having a claim during a year, conditioned by the establishment size; and the probability function setting the size of a claim, conditioned by the establishment size. Unfortunately, all the information necessary to make such an inference is not available. Until data with detailed information on establishments and on their mutual dependence are available, such estimations cannot be made. For an application of the compounding procedure in the case of house fire damages, see Mandelbrot (1964).

[9] In 1992, 94 percent of the number of large policies corresponding to firms greater than \$75 million were pooled (APSAD, 1994).

[10] This is the excess-of-loss treaty. All the claims greater than the threshold are paid by the reinsurance companies. Insurers pay only for the damages smaller than or equal to the threshold.

$m > x_0$, which means that all claims reinsured are on the Pareto tail. The problem is, thus, the valuation of the expected total value of the claims transferred. This task implies the choice of the statistical model for the size distribution of the yearly claims. Theory of extreme values and statistical evidence show that the Pareto law is the right theoretical model. Estimations presented in Table 1 indicate that the theoretical variance does not exist, because α (and the upper bound of each confidence interval) is much smaller than two. This statistical evidence is the consequence of the length of the tail of the claim size distributions.

But the same data indicate that even the expectation might not exist. Six out of 16 estimations are smaller than one, and one estimation almost equals one (like the average value of α). Only four estimations are greater than 1.1. The choice of the right statistical model becomes now the choice of the right value of the characteristic exponent α. But statistical data are not numerous enough and do not give sure criteria for choosing the precise value of α around one.[11] Hence, reinsurers need a rational basis for selecting a value corresponding to a law either with or without an expectation. Safety-first rationality being one rational basis for reinsurance, it will be also helpful in the choice of α. By choosing an α within the closed interval of estimated values 0.88–1.00, instead of the interval 1.03–1.25, should the true α be greater than one, then reinsurance companies will overestimate the probability of rare giant claims, an error that implies larger premiums paid by insurers to reinsurers. But by doing so, should α be equal to or smaller than one, they will avoid the dangerous error of underestimating the risk of ruin, that is, selecting $\alpha > 1$ instead of the true value ≤ 1—a major risk in the context of safety-first behavior. Since the empirical average α nearly equals one, and since the mathematical procedure is the same for all $\alpha \leq 1$, a reasonable compromise between statistical evidence and economic rationality is to choose the particular value of one. Hence, the Pareto law has an infinite expectation. Nevertheless, if we assume that there is an upper bound M for the maximal potential claim—a reasonable assumption since the maximal value of the sales per firm is finite—then expectation and variance exist, even with a true $\alpha \leq 1$.[12]

The bounded probability density of the claims transferred to the reinsurance pool is a rescaled Pareto law with $\alpha = 1$:

$$f(x) = (1-m/M)^{-1} m x^{-2} = M m x^{-2} (M-m)^{-1} \quad (m \leq x \leq M) \tag{1}$$

The mean and variance values are then equal to

Here, we assume that there is no ceiling on the payments from the reinsurers.

[11] In order to improve the precision of the estimator of the characteristic exponent of a Pareto distribution, Meyers (1994) has shown that Bayesian techniques may also be useful, especially when the number of observations in the tail is not large.

[12] We assume that M is sufficiently large, so that huge claims are always paid without ceiling. In practice, this means that M is greater than the largest claim in our sample. This assumption is also necessary to avoid statistical biases. With a relatively small M, the claim size distribution is truncated. The estimation of the parameter and even the choice of the max-stable law could be affected.

$$E(X) = \int_m^M x f(x)dx = Mm(\log M - \mathrm{Log}\, m)(M-m)^{-1} \qquad (m \le x \le M). \tag{2}$$

$$V(X) = E(X^2) - E^2(X) = \int_m^M x^2 f(x)dx - [Mm(\mathrm{Log}\, M - \mathrm{Log}\, m)(M-m)^{-1}]^2 \tag{3}$$

$$V(X) = Mm[1\text{-}Mm(\mathrm{Log}M\text{-}\mathrm{Log}m)^2(M\text{-}m)^{-2}] \tag{4}$$

$$E(X) \text{ and } V(X) \to \infty \text{ as } M \to \infty.$$

In the case of $\alpha < 1$, $E(X) = (1-\alpha)^{-1}M^{\alpha}m(M^{1-\alpha} - m^{1-\alpha})(M-m)^{-\alpha}$. (5)

The expected value of the yearly claims transferred to the reinsurance pool equals the expected value E(X) of the claims reinsured times the expected number of major claims exceeding the threshold m. That is, E(N).E(X) (where N = number of claims per year at least equal to m). This value corresponds to the yearly pure premium jointly paid by all the insurers in a country to the reinsurance pool.[13] Obviously, the pure premium increases with M. But the pure premium also increases when m decreases, because the expected number of claims greater than m increases as the threshold is reduced. Four examples using APSAD data already analyzed, to which we can join older data about the ten largest claims per year (collected since 1970), will illustrate that interesting feature.

Example 1

Let m = \$330,000 and M = \$330 million. Those values correspond to all claims in the Pareto tail, including an extremely large claim that happened once in the remote past. In France, some APSAD experts say that they are sure that the maximal potential business interruption claim is around \$330 million, nearly twice the size of the largest claim in our sample (based on private communication; no official figures on the maximal potential claim are published. That figure seems to be a compromise between historical evidence and judgment).

We have E(X) = \$2,281,841. The average number of claims in the Pareto tail since 1988 is 108; then, the expected value of the yearly claims equals 108.E(X) = \$246,438,837.

It is interesting to notice that the reinsurance pool insures all claims larger than \$330,000. Since 1988, the annual average value of these claims amounts to \$198 million. They all would have been paid, leaving an average annual surplus of \$48 million, that is \$240 million in five years. Should the maximal claim M happen, then reinsurance companies would be able to pay more than two-thirds of its amount, and the remaining third within two or three years.

[13] The pure premium does not include loading items such as overhead cost, profit margin, and precautionary reserves. Those reserves are determined by the variance of the claim size distribution. Hence, the cost of a reinsurance treaty is actually larger than the pure premium. The present research is centered on the valuation of pure premium, deferring the valuation of precautionary reserves to future research.

Example 2

Let m = \$14.5 million and M = \$330 million. In that case, only the largest claims are reinsured. We have E(X) = \$47,394,159.

Claims statistics, as it is expected in a context of rare events, show that the number of claims greater than \$14.5 million, per year, may be approximated by a Poisson distribution. From 1970 through 1992, 15 giant claims occurred and were distributed as follows: Twelve years without a giant claim, nine years with one claim, one year (1992) with two claims, and one year (1988) with four claims.

A Poisson law with expectation $\lambda = 0.65$ gives a fair approximation, that is 12 years without a large claim and 11 years with at least one large claim. The theoretical Poisson distribution is: no claim, 12 years; one claim, 7.8 years; two claims, 2.54 years; three claims, 0.55 year; and four claims, 0.09 year. Even if we limit the sample to the years 1975 through 1992, the Poisson approximation with $\lambda = 0.65$ remains quite good (nine years among 18 without a giant claim, seven years with one giant claim, etc.).[14] Hence, E(N) = 0.65. The expected value of the yearly claims, corresponding to the yearly pure premium to be paid to the reinsurance companies equals E(N)E(X) = \$30,806,203.

This premium amounts to 16.4 percent of the average yearly value of business interruption premiums paid to insurance companies between 1988 and 1992 (in constant money value).[15] Nevertheless, it is not large enough to recoup the losses due to the four giant claims in 1988 (\$72.5 million) or due to the two giant claims in 1992 (\$162.7 million). But it exceeds all the payments due to claims larger than \$14.5 million that happened only once in a year, building a surplus that can be used to pay the catastrophic claims in the future. Here we have the following example spanning 23 years: Total payments for the claims larger than \$14.5 million since 1970 = \$537 million. Total amount of theoretical premium since 1970 = \$708.5 million.

Since 1970, even with two giant claims with a size equal to the record claim of 1992, the reinsurance would have been in equilibrium. But with one catastrophic claim equaling the upper bound M = \$330 million, the excess-of-loss reinsurance treaty would have been in disequilibrium, with a not negligible deficit amounting to \$158.5 million. This deficit could have been cleared by means of a slight increase (20 percent) of the premium or by means of a lower m (see the discussion below), or without change in the premium, that is, by waiting at least 20 years until the surplus would have cleared the deficit.

---

[14] We assume that the Poisson parameter λ remains unchanged, even with an increasing number of firms insured. This hypothesis may be realistic because the number of extremely large firms corresponding to the giant claims has remained relatively stable through time. As noted by APSAD (1994), the increase in the number of firms insured is mainly due to medium and small firms, the potential claims of which are smaller than \$14.5 million. Interestingly, the number of giant claims within the five years 1970 through 1974 can also be approximated by the same Poisson distribution (three years without a claim, two years with one claim).

[15] This ratio may be compared to the average proportion of all the fire insurance premiums (including business interruption) transferred to the reinsurance companies between 1988 and 1994: 32.4 percent (APSAD, 1994).

Example 3

With m = \$145 million and M = \$330 million, we have a reinsurance policy designed for the catastrophic claims only. Both E(X) and Poisson parameter λ are changing. They become, respectively, E(X) = \$212,702,021 and λ = E(N) = 0.044. Hence, the expected value of the yearly claims equals \$9,358,889, a much smaller premium than in examples 1 and 2. Since 1970, only one catastrophic claim (\$147,885,241 in 1992) has happened. It represented more than two-thirds of the total reinsurance premiums paid between 1970 and 1992 and accumulated as surplus: 23 × 9,358,889 = \$215,254,446.

But should that catastrophic claim have reached the maximal potential value M, then all the accumulated surplus would have been eliminated, leaving a deficit of \$114,745,553. It would have been cleared either by waiting at least 12 years without a catastrophic claim, or by a substantial raise (53 percent) in the premium.

Example 4

Now M is changing, with m and λ remaining unchanged (m = \$14.5 million, λ = 0.65). Then the expected value of large claims (and consequently the reinsurance premium) grows with M.

Let M = \$1 billion, a catastrophic claim that has never happened in France. It amounts to five percent of the size of the largest insured French firm; there are 50 large insured firms with a size greater than \$330 million (or Fr 2 billion; see Table 3). Then E(X) = \$62,290,500. Hence, pure premium = \$40,488,830.

The new premium is 33 percent larger than the premium with M = \$330 million, and it generates a higher surplus: 40,488,830 × 23 - 537,000,000 = \$394,243,102. Nevertheless, in case of a super-giant claim equaling \$1 billion, the reinsurance companies would face a deficit of \$605,756,898. With the same Poisson probabilities for the giant claims, this deficit requires either at least 23 years to be cleared with the same premium, or a significant increase (64 percent) in the premium.

If we raise the threshold m up to \$145 million, in order to reinsure the catastrophic claims only, the pure premium corresponding to the expected value of yearly claims becomes \$14.4 million (rounded value). Since 1970, it would have accumulated a net surplus of \$183.5 million. Should a super-giant claim happen, with a size M, then the reinsurance companies would not have been able to pay, because the deficit (\$816.5 million) requires either about 57 years to be cleared, a very unrealistic delay, or an unacceptable raise in the premium (246 percent).

These examples illustrate the behavior of E(N)E(X): it increases with the upper bound M and decreases when the threshold m is raised. This means that if French insurers want to reinsure all their claims greater than m, a kind of mutualization is unavoidable: the larger the upper bound M is, the lower m must be. But M might be questionable, because huge claims with a size close to M are extremely rare. After all, in a Pareto law with $\alpha = 1$ and with m = \$330,000, the yearly probability of a claim larger than or equal to \$330 million is 1/1,000, and the yearly probability of a claim larger than or equal to \$1 billion is about 1/3,000. They may never happen in a country, even in the long run. Hence, an agreement between insurers and reinsurers on the actual value M of the maximal claim, that has not been observed in the past but could happen in the forthcoming year, is a requisite for all reinsurance treaties.

CONCLUSION

Unlike fires and natural hazards, business interruptions are not spectacular accidents. Damages appear in the sales department and in the balance sheets of the damaged firms; most of the time they remain unnoticed to people outside the ill-fated firm. Those damages, however, may be more severe than fire damages, and empirical claim size distributions fitting all the claims in a country like France prove that, every year, some business interruption claims may be catastrophic, even for the insurance industry. Some claims are so large that the theoretical tail distribution that fits the data, a max-stable Pareto law, loses its variance and its expectation. Empirical evidence shows that the estimated characteristic exponent α of the Pareto law is close to one, a value corresponding to infinite expectation. The main consequence of that statistical feature is that all pure premium calculations based on expectation require an upper bound, that is, a maximal value for the largest potential claim, should it be completely insured.

Moreover, large claims are so enormous that insurance companies must be reinsured. In that case, two critical values are requisite to all reinsurance treaties: an agreement between insurers and reinsurers about the value of the maximal potential claim M that might happen during the forthcoming year and a value of the threshold m, above which all claims are reinsured. Calculations prove that a sufficiently low value for m, that raises both the expected number of claims greater than m and the total amount of premiums, is required by the reinsurance companies to cope with large claims with a size close to the upper bound M. Of course, premium valuation is not restricted to pure premiums. The financial equilibrium of insurers and of reinsurers requires reserves in order to face yearly fluctuations, or mere bad luck. Further research, based on the variance of the bounded Pareto law and also based on ruin theory pertaining to long-tail distribution functions, will complete the valuation of business interruption insurance and reinsurance policies.

REFERENCES

Aiuppa, Thomas A., 1988, Evaluation of Pearson Curves as an Approximation to the Maximum Probable Aggregate Loss, *Journal of Risk and Insurance,* 55: 425–441.

Assemblée Plénière des Sociétés d'Assurances Dommages (APSAD), 1994, *Assurance Incendie des Risques d'Entreprises, Résultats 1992,* Mimeo (Paris: APSAD).

Comité Européen des Assurances, 1987, 1988, 1989, 1990, *Compte-Rendu de l'Assemblée Générale,* Mimeo (Paris: CEA).

Cummins, J. David et al., 1990, Application of the GB2 Family of Probability Distributions in Collective Risk Theory, *Insurance Mathematics and Economics,* 9: 257–272.

Dekkers, A. L. M. and Laurens de Haan, 1989, On the Estimation of the Extreme-Value Index and Large Quantile Estimation, *Annals of Statistics*, 17: 1795–1832.

Galambos, Janos, 1987, *The Asymptotic Theory of Extreme Order Statistics, Sec*ond Edition (Malabar, Fla.: Krieger).

Gibrat, Robert, 1931, *Les Inégalités Economiques* (Paris: Sirey).

Gumbel, Emil J., 1958, S*tatistics of Extremes* (New York: Columbia University Press).

Haßmann, S., R. D. Reiss, and M. Thomas, 1993, *XTREMES, Extreme Value Analysis and Robustness* (Siegen: Xtremes Group Siegen).

Hill, Bruce M., 1975, A Simple General Approach to Inference About the Tail of a Distribution, *Annals of Statistics*, 3: 1163–1174.

Mandelbrot, Benoît B., 1964, Random-Walks, Fire-Damage Amount and Other Paretian Risk Phenomena, *Operations Research*, 12(4): 582–585.

Meyers, Glenn, 1994, Quantifying the Uncertainty in Claim Severity Estimates for an Excess Layer When Using the Single Parameter Pareto, *Proceedings of the Casualty Actuarial Society,* to appear.

Petruszewiycz, M., 1972, Loi de Pareto ou Loi Lognormale: Un Choix Difficile, *Mathématiques et Sciences Humaines*, 39: 37–52.

Reiss, R.-D., 1989, *Approximate Distributions of Order Statistics (With Application to Nonparametric Statistics)* (New York: Springer).

Steindl, Josef, 1965, *Random Process and the Growth of Firms: A Study of the Pareto Law* (London: Griffin).

Zajdenweber, Daniel, 1976, *Hasard et Prévision* (Paris: Economica).