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Severity Risk and the Adverse Selection of Frequency Risk

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ABSTRACT

This article shows how the introduction of severity risk into a simple model of insurance markets affects the optimal level of insurance. Also examined is how severity risk affects the equilibrium for an insurance market exhibiting adverse selection in the frequency risk. Individuals are assumed to possess identical loss severity distributions, but differ in their privately-known probabilities of having a loss. In particular, the effects of severity risk on the Nash equilibrium of Rothschild and Stiglitz (1976), on the anticipatory equilibrium of Wilson (1977), and on Miyazaki's (1977) extension of Wilson's equilibrium are analyzed. Severity risk is shown to affect the type of equilibrium contracts (pooling vs. separating), equilibrium levels of coverage, and overall societal welfare.

Introduction

The message of this article is a bit convoluted. Hopefully not our logic, but rather the setting. Insurance actuaries typically model frequency of losses and severity of losses separately. Aggregate loss distributions then are derived by compounding these two distributions. A general feeling in the insurance business is that individual characteristics affect frequency distributions much more than severity distributions, and that severity is mostly independent of the individual's type. Although such general statements are difficult to prove, the majority of real-world experience-rating formulas, which adjust insurance premiums to reflect an individual's history of claims, certainly support this hypothesis.

In attempting to understand basic insurance principles, we often simplify models so as not to obfuscate the basic point being made. Certainly, simple two-state models of insurance (with the severity fixed) have been of tremendous value in understanding many insurance concepts. For example, this sim-

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plification has allowed for analysis of insurance via simple (two-state) contingent-claims diagrams.

This article examines how the introduction of severity risk into a simple model of insurance markets affects several well-known results. In particular, we examine the qualitative effect of severity risk on the optimal level of insurance. We also examine how severity risk affects the equilibrium for an insurance market exhibiting adverse selection in the frequency risk. Individuals are assumed to possess identical loss severity distributions, but differ in their privately-known probabilities of having a loss. In particular, we examine the effects of severity risk on the Nash equilibrium of Rothschild and Stiglitz (1976), on the anticipatory equilibrium of Wilson (1977), and on Miyazaki's (1977) extension of Wilson's equilibrium to allow for subsidization across risk types. We show how severity risk affects the type of equilibrium contracts (pooling vs. separating), equilibrium levels of coverage, and overall societal welfare.

Although our story is one of severity risk, the same results would apply to insurance based on replacement cost in a model with only full losses. In this case, the added risk can be modeled as a price risk for replacement property.

Rational Insurance Purchasing

Consider a risk-averse individual with preferences represented by the twice continuously differentiable von Neumann-Morgenstern utility of wealth $u(y)$, where $u' > 0$ and $u'' < 0$. The individual has an initial wealth of $W > 0$, which is subject to a loss of expected size $L > 0$. The loss occurs with probability π, $0 < \pi < 1$, and we assume that there is no moral hazard. Given that a loss has occurred, the size of the loss is a realization of the random variable $L + \tilde{\varepsilon}$, where $E(\tilde{\varepsilon}) = 0$. The case where $\tilde{\varepsilon}$ is identically zero represents the model with no severity risk and is the classic case for examining adverse selection in insurance markets. In general, the support of $\tilde{\varepsilon}$ is bounded from below by $-L$; and we assume that the support of $\tilde{\varepsilon}$ is bounded from above by $W - L$, so as to avoid issues arising from possible limited liability.[1]

The individual is allowed to purchase insurance that pays the indemnity $I(\varepsilon) \geq 0$ following a loss of size $L + \varepsilon$, for a premium of $(1 + \lambda)\pi E[I(\tilde{\varepsilon})]$. Here, E denotes the expectation operator, and $\lambda \geq 0$ represents a proportional loading on the pure premium, $\pi E[I(\tilde{\varepsilon})]$. We also assume that the loss is perfectly observable and is an insurable loss, that insurance is of the common form of coinsurance, and that insurers have the capacity to pay their claims.[2]

[1] Technically, it is the distribution function over the real numbers that corresponds to $\tilde{\varepsilon}$ that has a support. It hopefully causes no confusion if we refer to the "support of $\tilde{\varepsilon}$."

[2] See Doherty and Schlesinger (1990) for a discussion of the case where insurers might default. See Fluet and Pannequin (1994) for a related examination of the relative efficiencies of deductibles versus coinsurance.

Consider the common coinsurance form of indemnity, and let $\alpha \in [0,1]$ denote the proportion of the loss paid by an insurer. The premium in this case is given by $P(\alpha) = \alpha(1 + \lambda)\pi L$. The individual's objective is, thus,

$$\underset{\alpha}{\text{maximize}}\ EU \equiv (1-\pi)u(W-P) + \pi Eu(W-P-(1-\alpha)(L+\tilde{\varepsilon})). \tag{1}$$

The first-order condition for equation (1) is

$$0 = \frac{dEu}{d\alpha} = -(1-\pi)\pi(1+\lambda)Lu'(y_1) + \pi L(1-(1+\lambda)\pi)Eu'(\tilde{y}_2) + \pi Cov(\tilde{\varepsilon},u'(\tilde{y}_2)), \tag{2}$$

where $y_1 \equiv W - P$, and
$\tilde{y}_2 \equiv W - P - (1-\alpha)(L+\tilde{\varepsilon})$.

The second-order condition is easily verified. It also is straightforward to verify that $\alpha^* = 1$ if $\lambda = 0$, and that $\alpha^* < 1$ if $\lambda > 0$, which are well-known results of Mossin (1968).

Let α_0^* and α_ε^* denote the optimal levels of coverage for the model without and with severity risk, respectively. If $\lambda > 0$, α_ε^* generally can be either higher or lower than α_0^*. Note that the addition of severity risk adds the third term to right-hand side of equation (2). This term is positive for $\alpha < 1$, reflecting the added benefit of insurance's protection against the risk $\tilde{\varepsilon}$. However, severity risk also affects marginal utility in the second term, since $\tilde{y}_2$ is itself a function of $\tilde{\varepsilon}$. This effect also will be positive whenever marginal utility is convex, which follows from Jensen's inequality. Such convexity is a sufficient condition for a precautionary savings motive, as shown by Kimball (1990), and follows, for example, under the stronger but quite common assumption of nonincreasing absolute risk aversion. Consequently, u' convex is sufficient to increase the demand for insurance in the presence of severity risk.[3] For the remainder of the article, we assume that u' is convex.

The above results are summarized as follows:

Proposition 1
(a) If $\lambda = 0$, $\alpha_0^* = \alpha_\varepsilon^* = 1$.
(b) If $\lambda > 0$ and u' is strictly convex, $\alpha_0^* < \alpha_\varepsilon^* < 1$.

Because adding severity risk induces a mean-preserving spread of the loss distribution, expected utility in the presence of noise will be lower for every level of insurance, with the exception of full coverage (see Rothschild and

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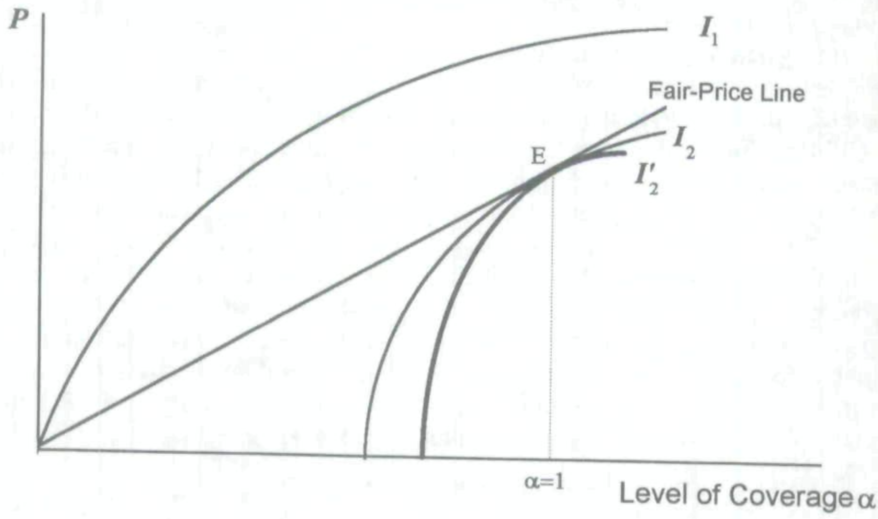
[3] If we add the constraint $\alpha \geq 0$, no insurance will be purchased for high enough λ. For expositional ease, we assume that λ is not so high as to preclude the purchase of insurance. Note that the condition "u' convex" is sufficient due to our assumption that background risk only attaches itself to the loss size L, and no background risk occurs when the loss is zero. This simple condition is not sufficient when loss size itself is random. For example, the analysis above may be compared to that of adding an independent background risk. If $\tilde{\varepsilon}$ is not multiplied by α above, the covariance term in equation (2) vanishes, but $\tilde{\varepsilon}$ also attaches itself to y_1 (see Eeckhoudt and Kimball, 1992).

Stiglitz, 1970). This addition of severity risk has the effect of making indifference curves "lower" in premium-coverage space as illustrated in Figure 1 (see Wilson, 1977).[4]

The indifference curve labeled I_1 in Figure 1 depicts all of the combinations of premiums and coinsurance coverage that yield expected utility identical to no coverage when no severity risk is present. Indifference curves are concave due to risk aversion. The fair-price line in Figure 1 represents the premium schedule $P = \alpha\pi L$. The figure illustrates the well-known result that the optimal level of coverage is full coverage ($\alpha^* = 1$) when the price is fair, and the level of expected utility attained is the level associated with the insurance contract E on the indifference curve labeled I_2.

Figure 1
Insurance Purchasing with Severity Risk



The above results are obtained in a straightforward manner by fixing marginal utility and calculating the marginal rate of substitution along an indifference curve in P - α space; namely,

$$\frac{dP}{d\alpha}\Big|_{Eu} = \frac{\pi L u'(y_2)}{(1-\pi)u'(y_1)+\pi u'(y_2)}, \tag{3}$$

where y_2 is nonstochastic, since no severity risk is present. It thus follows that $dP/d\alpha = \pi L$ (the slope of the fair price line) for any contract for which $\alpha = 1$. Contracts with $\alpha < 1$ have a steeper slope.

[4] By "lower" we mean a lower expected utility. This is a movement in a northwesterly direction, given the orientation in Figure 1.

Now we introduce severity risk $\tilde{\varepsilon}$. Following the purchase of coinsurance, the residual risk in the final wealth distribution in the loss state is $-(1-\alpha)\tilde{\varepsilon}$. Thus, if full coverage insurance is purchased ($\alpha = 1$), there is no residual risk and, hence, no change in expected utility. However, for $\alpha < 1$, residual risk leads to a lower expected utility for each insurance contract, and the indifference curve through E in the presence of severity risk lies everywhere below E, except at E itself (curve I_2' in Figure 1). Note that contracts along the locus I_2' in the presence of severity risk yield the same expected utility as contracts along the locus I_2 in the absence of severity risk.

To see that the family of indifference curves under severity risk are steeper for all $\alpha < 1$, as shown in Figure 1, calculate the marginal rate of substitution along an indifference curve in the presence of severity risk:

$$\frac{dP}{d\alpha}\Big|_{Eu} = \frac{\pi E[(L+\tilde{\varepsilon})u'(\tilde{y}_2)]}{(1-\pi)u'(y_1)+\pi Eu'(\tilde{y}_2)}. \tag{4}$$

Note that equation (3) is a special case of equation (4), with $\tilde{\varepsilon}$ identically zero. However, if $\alpha = 1$, then y_2 is nonrandom and the marginal rates of substitution in equations (3) and (4) coincide.

If $\alpha < 1$, then rewrite the numerator of equation (4) as $\pi E[(L+\tilde{\varepsilon})u'(\tilde{y}_2)] = \pi LEu'(\tilde{y}_2) + \pi \operatorname{cov}((L+\tilde{\varepsilon}), u'(\tilde{y}_2))$. Since u' is convex, $Eu'(\tilde{y}_2) > u'(y_2)$. Ignoring the covariance term for a moment, we see that $\pi LEu'(\tilde{y}_2)/[(1-\pi)u'(y_1) + \pi u'(y_2)]$ exceeds the value given in equation (3), since the proportional increase in the numerator is matched by an equal proportional increase in only one term of the denominator. Because the covariance term is also positive for $\alpha < 1$, it follows that equation (4) will be greater than equation (3) for any (P, α) pair such that $\alpha < 1$.

The above results imply that, for $\lambda = 0$, we obtain $\alpha_0^* = \alpha_1^* = 1$; for $\lambda > 0$, we obtain $\alpha_0^* < \alpha_1^* < 1$, confirming the results of Proposition 1.

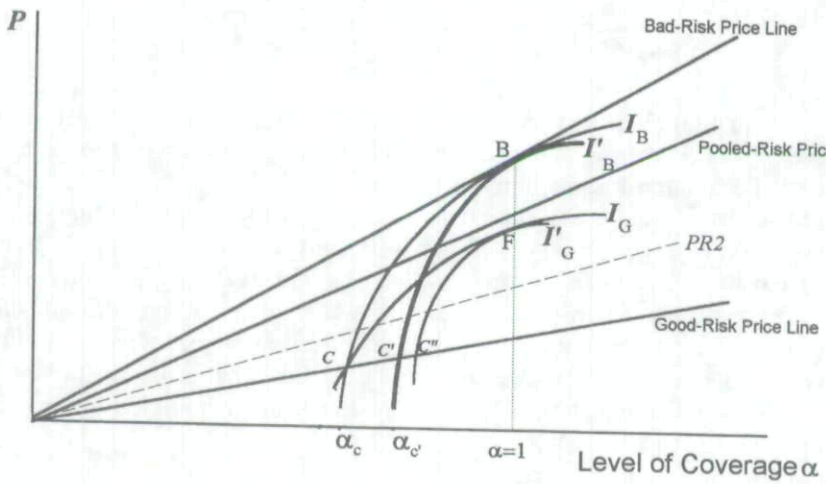
Adverse Selection in the Rothschild and Stiglitz Model

There exist two types of individuals who differ in their loss probabilities. The good risks have loss probability π_g and the bad risks π_b, where $\pi_g < \pi_b$. Insurers know both probabilities but cannot observe the probability type of any particular individual. Both types of individuals face the same severity risk, L + $\tilde{\varepsilon}$. If uncorrelated between individuals, severity risk is diversifiable by the insurer. Thus, the insurer can evoke the law of large numbers and treat all losses as worth L. However, consumer behavior is affected by the severity risk, and this behavior is taken into account by the insurer.

We consider first the Nash separating equilibrium of Rothschild and Stiglitz (1976) for a competitive insurance market. A Rothschild-Stiglitz equilibrium satisfies the properties that all contracts offered earn an expected profit of zero (i.e., long-run competition with a premium loading of $\lambda = 0$); and no contract exists that, if offered concurrently by a single insurer, would earn a profit (i.e., the zero-profit strategy is a Nash equilibrium). Rothschild and Stiglitz (1976) show that such an equilibrium, if it exists, can only be a separating equilibrium and never a pooling equilibrium.

This equilibrium is illustrated by the policy pair (B, C) in Figure 2, for the case without severity risk. In this equilibrium, the bad risks fully insure at a bad-risk fair price, at contract B. Insurance at a good-risk fair price is offered only in the limited quantity, α_c, and thus good risks self-select contract C. Bad risks are on indifference curve I_B, and good risks are on I_G. As shown in Figure 2, the separating Rothschild-Stiglitz equilibrium does exist since the pooled price line lies everywhere above the good-risk indifference curve, I_G.[5]

Figure 2
Separating Equilibria



With the introduction of severity risk, the bad risk indifference curve through B would shift to I_B'. Given our discussion of Figure 1, a new Rothschild-Stiglitz separating equilibrium would entail the pair of contracts (B, C′) in Figure 2, if the equilibrium exists. At such an equilibrium, less of a penalty is imposed on the good risks, in that they are allowed to purchase a higher level of coverage, α_c', at a good-risk fair price. However, the severity risk does more harm than good, as we show next.

[5] The pooled price line is the actuarially-fair price line for the mixed group of good and bad risks, assuming everyone buys the same contract. Existence of a Nash equilibrium requires a sufficient proportion of bad risks. If the pooled-risk price line intersected I_G, then it would be possible to offer a single contract that would be preferred by both types of consumers and would earn an expected profit. In such a case, no equilibrium would exist (see Rothschild and Stiglitz, 1976). Insurance purchases also are assumed to be perfectly observable, so that good-risk individuals cannot make multiple purchases to achieve more coverage (see Hellwig, 1988, for a discussion on relaxing this assumption).

Consider the Paretian change in welfare when severity risk is introduced. The bad risks retain full insurance contract B (which has no residual severity risk), and so their expected utility does not change. Consequently, contract C without severity risk and contract C′ with severity risk would yield the same expected utility to bad-risk individuals, since both C and C′ are equal in expected utility to contract B. Denoting the premium and coverage changes from contracts C to C′ as ΔP and Δα, respectively, implies that

$$(1-\pi_b)\,[u(W-P) - u(W-P-\Delta P)] = \pi_b(Eu[W-P-\Delta P-(1-\alpha-\Delta\alpha)(L+\tilde{\varepsilon})] - u[W-P-(1-\alpha)L]). \tag{5}$$

The left-hand side of equation (5) represents the loss in utility due to a higher premium when no loss occurs. The right-hand side is the expected utility gain from the higher net indemnity in case of a loss. Because individuals are identical except for their loss probabilities, the left-hand side of equation (5) is strictly greater than the right-hand side if π_b is replaced by π_g. This implies that expected utility for the good-risk individuals is higher with loss coverage at C and no severity risk, than at C′ with more coverage at a fair price but with severity risk added. Hence, the addition of severity risk to the loss size leaves the bad-risk individuals with no change in utility and lowers expected utility of the good risks. Consequently, societal welfare is reduced in the sense of Pareto efficiency.

The addition of severity risk also affects the existence of a separating equilibrium. When no severity risk is present, then good-risk individuals are indifferent to full coverage with contract F and the partial coverage contract C (see Figure 2). We also know that contract F will yield the same level of expected utility to good-risk individuals with or without severity risk. However, as we have just shown, contract C′ with severity risk is less preferred than contract C without severity risk. Thus, contract C′ is less preferred than contract F when severity risk is present. Consequently, contract C′ must lie to the left of the indifference curve through contract F in the presence of severity risk, labeled I_G' in Figure 2. Now, I_G' defines some contract C″ to the right of C′ on the good-risk fair price line, meaning that the good-risk indifference curve through C′ in the presence of severity risk (not shown in Figure 2) must lie everywhere above I_G'. In particular, it may lie partly above the pooled-risk price line. This would be the case, for example, if the location of the pooled-risk price line is changed to the PR2 price line in Figure 2. If so, a Rothschild-Stiglitz separating equilibrium will not exist. In such a case, where severity risk causes the insurance market to "disappear," obviously both types of individuals are strictly worse off.

The above results are summarized in the following proposition:

Proposition 2

In a Rothschild-Stiglitz adverse-selection model, the addition of severity risk will:

(a) decrease the signaling cost to the good-risk individuals in the sense of allowing them to obtain a higher level of insurance coverage, if an equilibrium exists.

(b) decrease societal welfare since good-risk individuals are strictly worse off, while bad-risk individuals are no better off.

(c) decrease the likelihood of an equilibrium. In particular, severity risk will raise the critical (i.e., the minimum) proportion of bad-risk individuals needed to ensure the existence of an equilibrium.

Adverse Selection in the Wilson Model

The Nash equilibrium concept used by Rothschild and Stiglitz (1976) is criticized by Wilson (1977), Miyazaki (1977), and others as being "too myopic" in the sense that each insurer assumes that the set of contracts offered by its rivals is independent of its own actions. Wilson (1977) assumes that insurers will take the reaction of rival firms into account and will not offer a contract if they cannot make a profit following the elimination of unprofitable, rival-firm contracts from the marketplace. In Wilson's model, an equilibrium always exists. Wilson shows in a two-state model that, for cases in which a Rothschild-Stiglitz separating equilibrium exists, a Wilson-type equilibrium is the identical pair of separating contracts. However, for cases in which the good-risk population is too large and no Rothschild-Stiglitz equilibrium exists, a pooling equilibrium exists in Wilson's model. In particular, all individuals buy the contract on the pooled price line that is most preferred by the good-risk individuals.[6]

For the case where severity risk eliminates the Rothschild-Stiglitz separating equilibrium, the changeover from a separating to a pooling equilibrium in Wilson's model has very clear welfare effects for each type of individual. In particular, the good-risk individuals are strictly worse off. This is clearly illustrated in Figure 2: The good risks originally are on indifference curve I_G, with contract C, and no severity risk. Since all contracts on the PR2 price line have a lower expected utility for good-risk individuals than contract C, the good risks are worse off. On the other hand, when there is severity risk, the good-risk individuals are better off at some pooled contracts than at C′; otherwise, a separating equilibrium would be optimal. Since the set of good-risk contracts preferred to C′ is a subset of those preferred by the bad-risk individuals, the bad-risk individuals must be strictly better off at the pooled equilibrium than at the separating contract B. This is obvious since contract B could always be offered alone by a firm, without any adverse-selection possibilities.

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[6] Any pooling contract with less coverage can be improved upon for both types of consumers. If a contract with more coverage is offered, only the bad risks would buy it, and it would lose money. A contract offered below the pooled-price line that would attract only the good-risk individuals in a Nash-equilibrium setting will not be offered in Wilson's model, since the existence of such a policy would force the original pooling policies to be withdrawn and the below-pooled-price-line policy would lose money.

Since the good risks are worse off, and bad risks are better off, following a severity-risk-induced change from a separating to a pooling equilibrium, no clear inferences can be made about societal welfare.

Consider next the possibility that the initial Wilson-type equilibrium with no severity risk is itself a pooling equilibrium. The analysis of the previous section precludes any possibility that the addition of severity risk will lead to a separating equilibrium. In other words, a pooling equilibrium with no severity risk always begets a pooling equilibrium when severity risk is added. By Proposition 1b, it follows that a pooling contract with a higher level of coverage will be optimal for the good-risk individuals in the presence of severity risk. Hence, the Wilson-type pooling equilibrium will entail a higher level of coverage.[7]

The good-risk individuals will be worse off at the new pooling equilibrium in the presence of severity risk, since, at every level of insurance coverage along the pooled-price line, the good-risk individuals have a lower expected utility with severity risk than without severity risk. The good risks optimally purchase a higher level of coverage in the presence of severity risk, which is beneficial to the bad risks.[8] However, this benefit of extra coverage at the (subsidized) pooled-risk price for the bad risks comes at the cost of added severity risk. Whether the level of extra coverge is enough to compensate the bad risks for the added severity risk is not determinate. The amount of extra coverage in a pooling equilibrium is determined by the marginal rate of substitution of the good-risk individuals and could be either more or less than the amount required to compensate the bad-risk individuals for the severity risk.

The consequences of severity risk under a Wilson-type equilibrium are summarized in Proposition 3 below.

Proposition 3

Under a Wilson-type equilibrium for an insurance market with adverse selection, the addition of severity risk will:

(a) result in either a separating or pooling equilibrium if the original no-severity-risk equilibrium was separating.

(b) decrease societal welfare if both the original and the severity-risk equilibria are separating equilibria, since the good risks are strictly worse off and the bad risks are exactly as well off.

(c) have an indeterminate effect on societal welfare if the equilibrium changes from separating contracts to a pooling contract, since the good risks are strictly worse off while the bad risks are strictly better off.

(d) result in a new pooling equilibrium with a higher level of coverage if the original no-severity-risk equilibrium was pooling.

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[7] The Wilson equilibrium occurs at the most preferred good-risk contract on the PR2 price line. Although this price line is actuarially fair for the mixed group of good and bad risks, it is viewed as if $\lambda > 0$ for all good-risk individuals. Hence, Proposition 1b may be applied.

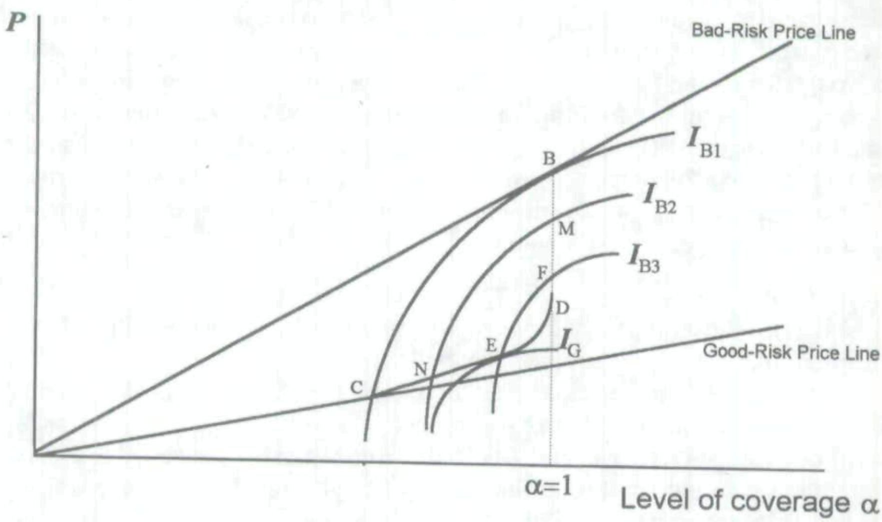
[8] The bad-risk individuals are better off as coverage increases along the pooled-price line until somewhere beyond full coverage.

(e) have an indeterminate or decreasing effect on societal welfare if the original equilibrium was pooling, since the good risks are strictly worse off while the bad risks can be either better off or worse off.

Adverse Selection in the Miyazaki Model

Miyazaki (1977) extends Wilson's equilibrium concept by allowing for subsidies between the good-risk and bad-risk individuals.[9] Miyazaki's equilibrium always exists and is a unique, separating equilibrium. An example of Miyazaki's equilibrium is illustrated in Figure 3. The locus of contracts on the curve passing through C, N, E, and D defines a set of good-risk contracts that could be offered by the insurer, together with full insurance to the bad-risk individuals, at a subsidized price. In combination, these contracts would earn a zero expected profit and would leave the bad-risk individuals indifferent between full insurance and the good-risk contract.

Figure 3
Miyazaki Equilibrium without Severity Risk



The potential Rothschild-Stiglitz equilibrium pair (B, C) satisfies these properties with a zero subsidy. The policy pair (M, N) represents another such set of contracts. For this pair, the bad-risk individuals purchase full coverage contract M, which loses money, but the good risks pay a premium loading (i.e., a subsidy) to obtain contract N, which is the contract along the bad-risk indifference curve through M that earns enough money from the good risks to net the insurer a zero overall expected profit. This pair of contracts induces

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[9] Miyazaki does not consider an insurance market per se, but his model is easily adaptable (see Spence, 1978).

consumers to self-select the appropriate contract. The bad risks are indifferent to M and N, while the good risks strictly prefer contract N. Another such contract pair is the pooling contract pair (D, D), which lies on the pooled-risk price line and fully insures everyone.

As drawn, none of these contract pairs would support an equilibrium. The Miyazaki equilibrium occurs at the contract pair (F, E). With contract E, the good-risk individuals achieve their highest expected utility (on indifference curve I_G) among their zero-profit alternatives. At contract F, the bad-risk individuals can reach indifference curve I_{B3}. Both good-risk and bad-risk individuals are better off in a Miyazaki equilibrium than in Rothschild-Stiglitz equilibrium.[10]

If we now introduce severity risk, indifference curves will become steeper, as discussed above. The zero-profit set of contracts for the good-risk individuals will now be those on the locus containing C′, N′, and D, as illustrated in Figure 4. Thus, for example, (M, N′) will replace (M, N) as a zero-profit pair. The pair (M, N) still earns zero expected profit but does not induce self-selection. Contracts N and N′ are the same distance above the good-risk fair price line, indicating the same subsidy, but N′ lies further to the right. When the bad-risk individuals receive contract M and severity risk is present, the good-risk individuals are offered a zero-profit choice along I_{B2}' rather than I_{B2}. Thus, the good risks can purchase a higher level of coverage which, in turn, implies that a lower subsidy per unit of coverage is needed to cover the bad-risk losses. The Miyazaki equilibrium in the presence of severity risk entails a good-risk contract somewhere along the locus of contracts containing C′, N′, and D. The following results are obtained (proofs are presented in the Appendix):

Proposition 4

Under a Miyazaki equilibrium for an insurance market with adverse selection, the addition of severity risk:

(a) decreases societal welfare, since the good-risk individuals are strictly worse off and the bad-risk individuals are no better off.

(b) decreases or leaves unchanged the subsidy paid by the good-risk individuals, both on a per-contract basis and on a per–unit-of-coverage basis, as compared to the original no-severity-risk equilibrium. The subsidy is unchanged if bad-risk welfare is unchanged and is strictly lower if bad-risk welfare is reduced.

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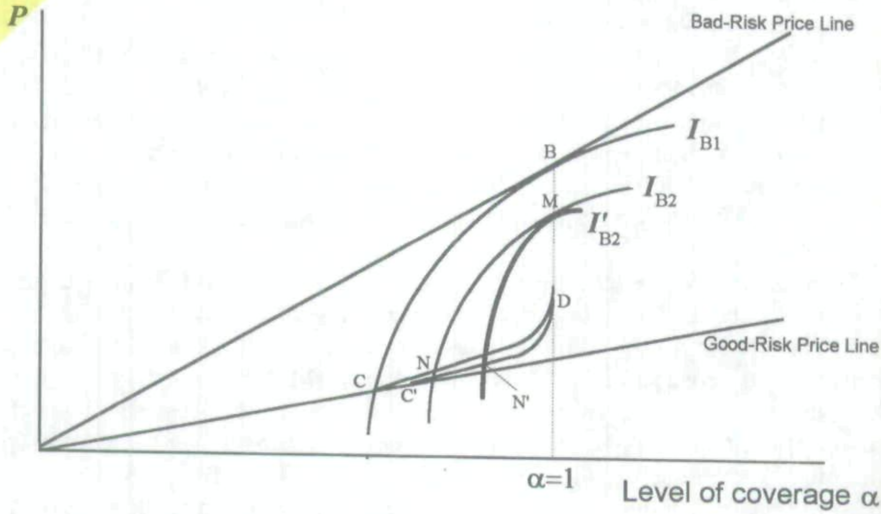
[10] The locus of zero-profit separating contracts for the good risks, CNED, need not be convex. If the most preferred good-risk contract is not unique, equilibrium will occur at the most preferred good-risk contract with the lowest subsidy (see Miyazaki, 1977, p. 411). The Rothschild-Stiglitz pair (B, C) potentially could be the contract on CNED most preferred by the good-risk individuals. In this case the individuals are obviously equally well off in both equilibrium settings (see Crocker and Snow, 1985).

Figure 4
Miyazaki Equilibrium with Severity Risk



Conclusion

This article analyzes the effects of severity risk on the rational purchase of proportional insurance, and on various equilibria in insurance markets characterized by adverse selection. Clear results are obtained in analyzing the effects of severity risk on the optimal level of insurance under the fairly weak assumption of prudence (i.e., convex marginal utility). In such instances, severity risk induces the purchase of more insurance.

For competitive insurance markets with adverse selection, severity risk is shown to have an impact on market equilibrium. In the Rothschild-Stiglitz (1976) equilibrium model, severity risk enables the good risks to purchase more coverage but decreases overall welfare and lessens the likelihood of an equilibrium's existence. For Wilson's (1977) model, in which an equilibrium always exists, the addition of severity risk leads to more coverage within the same type of equilibrium—that is, pooling or separating, or the addition of severity risk can lead to a change from a separating equilibrium to a pooling equilibrium (but never the reverse). In all of the possible cases, societal welfare is either indeterminate or lower following the addition of severity risk. For a Miyazaki (1977) equilibrium, in which the good risks subsidize the bad risks, the rate of the subsidy per unit of coverage was shown to be lower, but with an accompanying decrease in societal welfare.

Although our results are set in a comparison of severity risk versus no severity risk, they all apply to increases in severity risk, as long as the increase in risk is modeled as the addition of an independent white-noise term to the existing severity risk.[11]

Since severity risk is a very real part of actual insurance markets, understanding its implications within basic models of insurance-market behavior is important. As seen in this article, these implications are far from trivial. Hopefully, these results will prove useful in both positive and normative analyses of insurance markets.

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[11] This result follows easily from equation (2). This type of increase in risk is a subset of the broader class of "increases in risk" of Rothschild and Stiglitz (1970). The extension of our model does not hold for more general types of increases in risk. Since severity risk entails a component analagous to background risk (see footnote 3, above), this nonextension follows from Eeckhoudt, Gollier, and Schlesinger (1996). For a general discussion of this problem and a subset of risk increases for which extensions are possible, see Dionne, Eeckhoudt, and Gollier (1993).

Appendix

Before proving Proposition 4, we first establish the following lemmas.

Lemma 1

Let X represent an insurance contract without severity risk and Y a contract with severity risk such that both X and Y have the same premium. Then $X \sim_G Y$ if and only if $X \sim_B Y$, where $\sim_i$ represents indifference for type i individuals.

Proof

Since X and Y have the same premium, both types of individuals are indifferent to these contracts in the no-loss state. Therefore, $X \sim_i Y$ for i = B or G implies indifference in the loss state. Since both types have identical preferences, the lemma follows.■

Lemma 2

Let X be a contract without severity risk on the good-risk set of subsidizing contracts for the Miyazaki model; that is, X is a contract along the CD locus in Figure 4. Let Y be a contract with severity risk along the set of subsidizing contracts with severity risk (locus C′D in Figure 4). Then $X \sim_B Y$, if and only if X and Y generate identical expected profit for good-risk individuals.

Proof

Trivial since X and Y generate an identical bad-risk subsidy.■

Lemma 2 implies that bad risks are indifferent to contracts along CD (when no severity risk is present) and C′D (in the presence of severity risk), which lie on lines parallel to the good-risk fair price line in Figure 4.

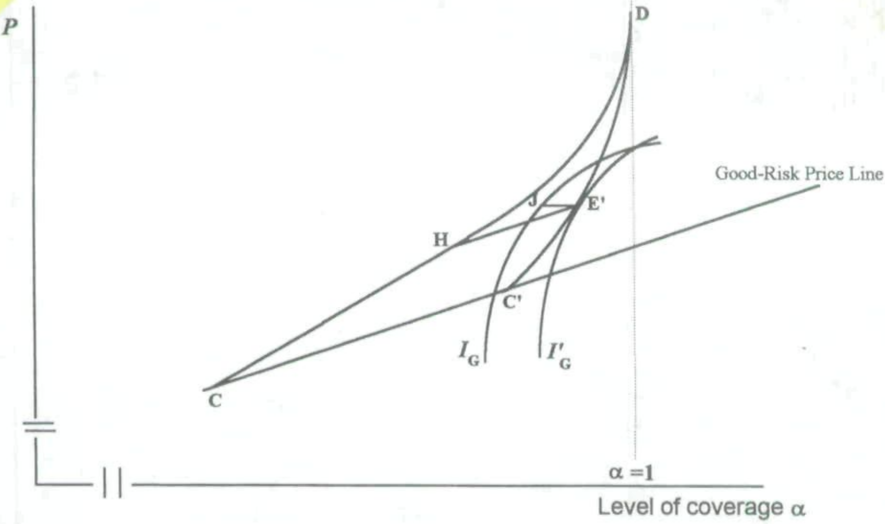
Proof of Proposition 4

(a) Let E denote the equilibrium good-risk contract without severity risk, along the CD locus in Figure 4. We first show that the good risks are worse off with severity risk. Suppose this were not true, so that the good risks are better off at E′, where E′ denotes the equilibrium good-risk contract in the presence of severity risk. Then $E' \geq_G E$, where $\geq$ denotes a weak preference. Define contract J without severity risk such that J has the same premium as E′ and $E' \sim_G J$. Also, define contract H without severity risk such that H is on the Miyazaki good-risk contract locus (CD in Figure 4) and $E' \sim_B H$. This situation is illustrated in Figure 5, which replicates part of Figure 4. Note that H lies "southwest" of E′ by Lemma 2.

Now $E' \sim_B J$ by Lemma 1. Therefore, for the two contracts without severity risk, H and J, we have $H \sim_B J$. However, it follows from equation (3) that the bad-risk indifference curve through J without severity risk is steeper than the good-risk curve I_G. Hence, J is strictly preferred to H by the bad risks—a contradiction! Therefore, the good risks are strictly worse off.

Next, we show that the bad risks are no better off in the presence of severity risk, thus completing the proof of (a). To this end, let E denote the equilibrium good-risk contract without severity risk, and let E′ denote the contract with severity risk on C′D such that $E' \sim_B E$. Now choose contracts K with no

Figure 5
Good Risks Worse Off with Severity Risk



severity risk on CD and K′ with severity risk on C′D such that K entails a premium increase over E and $K' \sim_B K$. Thus, segments KK′ and EE′ are parallel to the good-risk fair-price line by Lemma 2 (see Figure 6). We claim $K' <_G E'$; if this claim holds, the good-risk equilibrium contract with severity risk must lie southwest of E′ on the C′D locus, and, consequently, the bad risks cannot be better off in equilibrium.

To prove our claim, let Δ_1 denote the change in utility for good risks in the no-loss state for a switch from contract E to contract K in the absence of severity risk. Clearly, $\Delta_1 < 0$ due to the higher premium. Let Δ_2 denote the corresponding utility change in the loss state, $\Delta_2 < 0$. Similarly, let Δ_1' and Δ_2' denote utility changes for a switch from E′ to K′ with severity risk, where Δ_2' takes expectations over $\tilde{\varepsilon}$. Applying Lemma 2, we obtain

$$(1\text{-}\pi_b)(\Delta_1\text{-}\Delta_1') + \pi_b(\Delta_2\text{-}\Delta_2') = 0. \tag{A1}$$

Since the premium, P, is higher at E′ than at E, it follows from the concavity of utility that $(\Delta_1\text{-}\Delta_1') > 0$. Consequently,

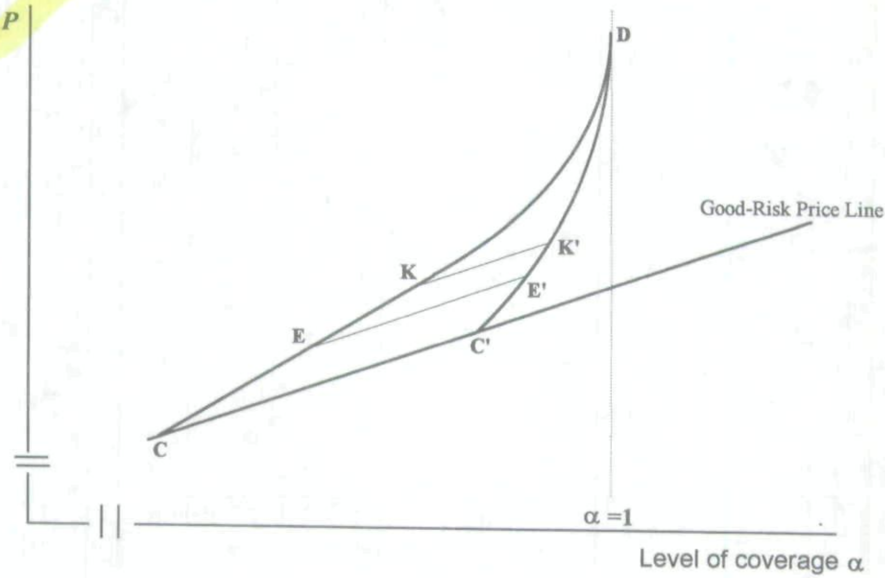
$$(1\text{-}\pi_g)(\Delta_1\text{-}\Delta_1') + \pi_g(\Delta_2\text{-}\Delta_2') > 0, \tag{A2}$$

or, equivalently,

$$(1\text{-}\pi_g)\Delta_1 + \pi_g\Delta_2 > (1\text{-}\pi_g)\Delta_1' + \pi_g\Delta_2'. \tag{A3}$$

The inequality in (A3) indicates that the change in expected utility for the good risks from E to K without severity risk exceeds the corresponding change from E′ to K′ with severity risk. But from the optimality of E for the good risks without severity risk, the left-hand side of inequality (A3) must be strictly

Figure 6
Bad Risks No Better Off with Severity Risk



negative. Consequently, (A3) implies that $K' <_G E'$. Our claim and, hence, part (a) of the proposition follow.

(b) Since the equilibrium good-risk contract with severity risk lies at or southwest of E′ in Figure 6, the per-contract subsidy paid by the good-risk individuals in equilibrium when severity risk is present will be identical or lower. Moreover, the slope from the origin to this equilibrium contract must be the same or lower than the slope from the origin to contract E. Hence, the subsidy per unit of coverage paid by the good risks under severity risk is the same when the bad-risk welfare is unchanged, and strictly lower when the bad risks are strictly worse off.■

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