Mathematical and Statistical Models

Compound Model for Two Dependent Kinds of Claim, by Christian Partrat (Université Claude Bernard, Villeurbanne Cedex, France)

In this paper we deal with the aggregate amount of claims for a compound class of policies submitted to claims of two kinds whose yearly frequencies are a priori dependent.

This dependence is modelled by some bivariate discrete distributions: Poisson and mixed Poisson (notably Poisson-Gamma and Poisson-Inverse Gaussian). These distributions are checked with some recent data concerning natural events insurance in the United States and third-party liability automobile insurance in France. *Insurance: Mathematics & Economics*, December 1994, 15(2/3): 219-231. (Reprinted with permission of North-Holland Publishing Company.)

A Family of Discrete Distributions, by R. S. Ambagaspitiya (University of Calgary)

Ambagaspitiya and Balakrishnan (1994a) used the identity

$$P_n = \frac{a}{a+b}\left(b+\frac{a}{n}\right)P_{n-1}(a+b,b), n=1,2...,$$

with $p_0(a,b) = \exp(-a)$ to obtain a recursive formula for computing the distribution function of the compound generalized Poisson distribution. In this paper, we consider the discrete distribution family with the property

$$P_n(a,b) = \left(h_1(a,b)+\frac{h_2(a,b)}{n}\right)P_{n-1}(a+b,b), n=1,2,...,$$

which is a generalization of the first identity. We prove that weighted generalized Poisson distributions and weighted generalized negative binomial distributions with weights of the form w(a + bn; b) are two subclasses in the family. We provide a recursive formula for computation of respective compound distributions. Also we discuss the stability of the recursion as well as handling overflow/underflow problems. *Insurance: Mathematics and Economics,* May 1995, 16(2): 107-127. (Reprinted with permission of North-Holland Publishing Company.)

Some Alternatives for the Individual Model, by R. Kaas (University of Amsterdam) and H. U. Gerber (Ecole des HEC, Lausanne, Switzerland)

Three approximations for the total claims of a risk portfolio (individual model) are compared: the traditional collective (compound Poisson) model, a compound binomial model, and a compound pseudo-binomial model (nearly homogeneous portfolio.) Of these, only the collective model is a prudent choice in terms of stop-loss order. The binomial model has a larger variance, but the nearly homogeneous approximation might have a lower one. The binomial model also leads to an exponentially larger distribution. *Insurance: Math-*

ematics & Economics, December 1994, 15(2/3): 127-132. (Reprinted with permission of North-Holland Publishing Company.)

Predictive Stop-Loss Premiums and Student's t-Distribution, by Werner Hürlimann (Allgemeine Mathematik, Winterthur-Leben, Winterthur, Switzerland)

A Bayesian stop-loss prediction model is constructed from a mixture of a normal with a gamma conjugate prior. Its predictive density is a location-scale transformation of Student's t-distribution. When no data are available for statistical prediction, the marginal distribution coincides with Bowers' distribution, introduced in Hürlimann (1993b), which generates the best stop-loss upper bounds by given mean, variance and range $(-\infty, \infty)$ of a distribution. The present prediction model is justified using a conditional measure of safeness. Furthermore, it is shown how estimates of the approximation error made when replacing the predictive exact stop-loss premiums by a simple normal approximation can be obtained using computer simulations. *Insurance: Mathematics and Economics,* May 1995, 16(2): 151-159. (Reprinted with permission of North-Holland Publishing Company.)

Actuarial Equivalence, by Henk Wolthuis (University of Amsterdam)

In life insurance mathematics the actuarial equivalence principle is the basic concept to define premiums and premium reserves. The underlying valuation method of this principle is used here on the one hand to generalize equivalence relations of financial mathematics for the continuous-time Markov model for life contingencies and on the other to derive other actuarial equivalence relations for this model. For instance future or past payments due in states of the Markov process are replaced by equivalent insurance benefits payable at the moment of transition from one state to another state. *Insurance: Mathematics & Economics,* December 1994, 15(2/3): 163-179. (Reprinted with permission of North-Holland Publishing Company.)

Utility Theory

Efficiency and Equality in a Simple Model of Efficient Unemployment Insurance, by Andrew Atkeson (University of Pennsylvania) and Robert E. Lucas, Jr. (University of Chicago)

This paper describes the efficient allocation of consumption and work effort in an economy in which workers face idiosyncratic employment risk and considerations of moral hazard prevent full insurance. We impose a lower bound on the expected discounted utility that can be assigned to any agent from any date onward and show, with this feature added, that the efficient unemployment insurance scheme induces an invariant cross-sectional distribution of individual entitlements to utility. The paper thus provides a simple prototype model suited to the study of the *normative* question: what is the trade-off between equality and efficiency in resource allocation? *Journal of Economic*