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Solvency Risk and the Tax Sheltering Behavior of Property-Liability Insurers

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ABSTRACT

We analyze for property-liability insurers the choice between taxable and tax-exempt securities in an equilibrium model where solvency risk is explicitly considered. Contrary to the certainty model, we find that the insurer may not invest in tax-exempt bonds if the yield spread between taxable and tax-exempt bonds is large, even if the tax-exempt yield is larger than the after-tax yield of taxable bonds. If the yield spread is sufficiently small, the insurer may find it optimal to not invest in taxable bonds and thus forego offsetting underwriting losses with income from taxable bonds.

Introduction

Property-liability insurers are major participants in the market for tax-exempt (municipal) bonds. Since tax-exempt bonds almost always yield less than comparable taxable bonds in pretax terms, insurers are faced with the problem of optimally allocating investable funds between taxable and tax-exempt bonds. The allocation will depend on the tax-exempt yield relative to the taxable yield, a well known result in the literature. This article examines this relation between the relative yield of taxable and tax-exempt bonds and the use of tax-exempt bonds by property-liability insurers when solvency risk is explicitly taken into account.

The problem of optimally allocating the investable funds between taxable and tax-exempt securities has been analyzed in a certainty framework by Hendershott and Koch (1980) and Gleeson and Lenrow (1987). Later, Cummins and Grace (1994) extended the analysis to an uncertainty framework, taking premiums and the level of insurer assets as given; they also assume that insurance risks are fully diversifiable, so that expected profit maximization is

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appropriate. When insurance risks are not fully diversifiable, the allocation between taxable and tax-exempt securities will affect the competitive premium and/or the solvency risk. Hence, treating the premiums as given ignores the effect of the allocation (between taxable and tax-exempt securities) on the premium. This article uses a partial equilibrium framework; we assume that the securities and insurance markets are competitive, allowing the premiums to adjust.

In Hendershott and Koch's certainty model, the insurer invests in tax-exempt bonds whenever the tax-exempt yield exceeds the after-tax yield from taxable bonds and the underwriting losses are matched by income from taxable bonds. In our model, the insurer may optimally refrain from investing in tax-exempt bonds, if the spread between taxable and tax-exempt yields is sufficiently large, even if the tax-exempt yield is greater than the after-tax yield from taxable bonds. However, if the spread between taxable and tax-exempt yields is sufficiently small, the insurer may optimally refrain from investing in taxable bonds, even though the taxable yield is strictly greater than the tax-exempt yield. Cummins and Grace (1994) derive a similar result when insurers are subject to alternative minimum taxes. Our partial equilibrium model shows that the above result is true when the tax function is much simpler.

The Model

To study the insurer's investment choice, we work with a one-period model and make the following assumptions.

Assumption 1: Premiums are collected by the insurer at the beginning of the period and claims are paid at the end of the period. Assumption 1 is based on Smith's (1989) one-period model, which assumes that all underwriting expenses are incurred at the beginning of the period and claims are defined to include claims-adjustment expenses.

Assumption 2: The probability distribution of the claims at the end of the period is log normal. We will use the notation L_1 for the level of liabilities at the end of the period and the mean and standard deviation of log (L_1) will be represented by $\mu-\sigma^2/2$ and σ, respectively.[1] The uncertainty in claims, σ, is the only source of uncertainty that has not been eliminated in the portfolio of policies through diversification. The level of assets at the beginning and end of the period will be denoted A_0 and A_1, respectively. At the end of the period, if $L_1 > A_1$, the insurer is said to be in default.

Assumption 3: The insurer invests in riskless assets only. Restricting the investment to riskless assets simplifies the model and yet allows it to capture the essence of the tax sheltering problem of the insurers. Often insurers use risky assets, such as common stocks and long-term bonds, for tax sheltering. Assumption 3 simplifies the model and focuses on the tax sheltering decision

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[1] This assumption is equivalent to the diffusion process assumption for liabilities in Cummins (1991) with μ as the instantaneous rate of change and σ as the instantaneous standard deviation of changes in the level of liabilities over time.

of the insurer, avoiding the need to explore the insurer's choice of riskiness of the invested assets.

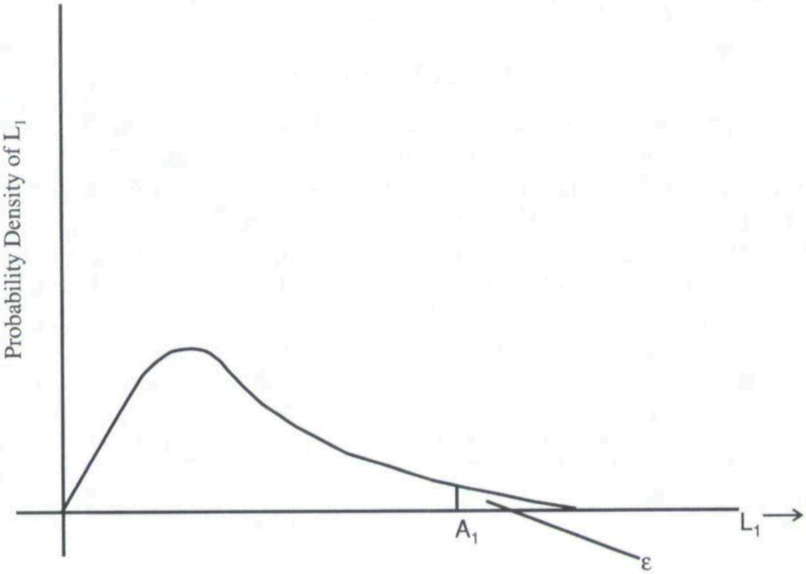
We will use θ ($0 \leq \theta \leq 1$) to denote the proportion of assets invested in tax-exempt bonds. The continuously compounded rate of return, α, on the assets will satisfy $e^{\alpha} = \theta e^{r_e} + (1-\theta)e^{r}$, where r_e and r are the continuously compounded rates of return on tax-exempt and taxable bonds, respectively.

Assumption 4: The insurer chooses the target default probability at the beginning of the period. The insurer then chooses the level of initial assets consistent with the target default probability. Since the probability distribution of L_1 is known (from Assumption 2), the level of assets needed at the end of the period, A_1, corresponding to any target default probability ε can be determined (see Figure 1). Further, since the assets are assumed to be riskless, the level of assets needed at the beginning of the period, A_0, corresponding to any ε can also be computed as

$$A_0 = A_1 e^{-\alpha} \tag{1}$$

once the rate of return on the assets α is known (that is, once θ is fixed).[2]

Figure 1
Level of Assets at the End of the Period
Determined by the Choice of Default Probability



[2] In practice, the level of initial assets required also will depend on the insurer's choice regarding the level of asset risk.

Assumption 5: The securities markets and the market for insurance policies are competitive. This assumption allows us to value equity claims using contingent claims methods. Since the initial assets are financed only from equity and premium payments, the competitive premium (net of underwriting expenses) can be computed as the difference between A_0 (known once θ is fixed) and the value of equity. In reality, the premium will also depend on other factors, such as the level of service provided by the insurer, which are not considered in this model. For simplicity, we also assume away any complications due to moral hazard or asymmetric information.

Assumption 6: The insurer's marginal tax rate is constant and the tax shield from net losses is unusable in future periods. The actual tax computation for property-liability insurers is more complex, especially in light of the partial taxation of tax-exempt interest since the 1986 Tax Reform Act (see Almagro and Ghezzi, 1988, for a detailed discussion). But the simplified model used here is sufficient to capture the essence of the tax sheltering activity using tax-exempt bonds. Personal taxes are ignored throughout the article.

The inflows to the insurer at the beginning of the period are premiums and shareholders' equity contribution. The outflows are underwriting expenses at the beginning of the period and policyholders' claims and tax claims at the end of the period. The residual is the value of equity at the end. Hence, using the notation π_p for premium net of underwriting expenses and E_0 and T_0 for the present value of equity and tax claims, respectively, we can write

$$A_0 = \pi_p + E_0, \text{ and} \tag{2}$$

$$A_0 = PV[Max(L_1, A_1)] + E_0 + T_0, \tag{3}$$

since the present value of equity claims will be equal to the equity contribution.[3] The notation PV stands for the present value operator.

According to Assumption 6, the taxable income for a given value of θ will be Max $[0, \pi_p + A_0 (1-\theta) (e^r-1) - L_1]$. The underwriting income is $\pi_p - L_1$, and the taxable part of the investment income is $A_0 (1-\theta) (e^r-1)$. Denoting the marginal tax rate as τ and using the Black-Scholes (1973) valuation method, the present value of tax claims can be evaluated as

$$\begin{aligned} T_0 &= \tau\, PV(Max\, [0, \pi_p + A_0 (1-\theta) (e^r-1) - L_1]) \\ &= \tau[\{\pi_p + A_0 (1-\theta) (e^r-1)\}e^{-r} N(-w_2) - PV(L_1)N(-w_1)], \end{aligned} \tag{4}$$

where $w_1 = \dfrac{\ln(PV(L_1)/[\pi_p+A_0(1-\theta)(e^r-1)])+(r+\sigma^2/2)}{\sigma}$, and

$w_2 = w_1 - \sigma$.

[3] Appropriate discounting in the computation of the present value of equity claims ensures that reasonable profits are available to equityholders. If the capital asset pricing model is applicable, the discount rate will depend on the beta of the claims.

At the end of the period after policyholders' claims are paid out, the residual represents the sum of equity and tax, which can be written, from equations (2) and (3), as

$$E_0 + T_0 = PV[\text{Max}\ (A_1\text{-}L_1, 0)]. \tag{5}$$

Again using the Black-Scholes valuation method,

$$E_0 + T_0 = A_1\ e^{-r}\ N(-d_2) - PV(L_1)N(-d_1), \tag{6}$$

where $d_1 = [\ln(PV(L_1)/A_1) + (r + \sigma^2\ \tau/2)]/\sigma$, and
$d_2 = d_1 - \sigma$.

Since $A_0 = \pi_p + E_0$ from equation (2), the premium (net of underwriting expenses) for a given value of θ can be computed using equations (4) and (6) as

$$\begin{aligned} \pi_p &= A_0 - [A_1\ e^{-r}\ N(-d_2) - PV(L_1)\ N(-d_1)] \\ &+ \tau[\{\pi_p + A_0\ (e^r\text{-}1)\}e^{-r}\ N(-w_2) - PV(L_1)N(-w_1)]. \end{aligned} \tag{7}$$

Insurer's Investment Decision

In a competitive market, the insurer will choose the proportion of assets invested in tax-exempt bonds θ to minimize premium (net of underwriting expenses) π_p for a given quality of the policies sold. Given the assumptions of the model, the quality is entirely determined by the target default probability.[4] Hence, for the minimization problem, default probability ε will be held constant. The optimal θ will be zero (no investment in tax-exempt bonds) in this minimization problem if $\left.\frac{\partial \pi_p}{\partial \theta}\right|_{\theta=0} \geq 0$. This condition holds when the rate of return on tax-exempt bonds r_e is sufficiently small (but greater than the after-tax yield from taxable bonds) (for proof, see the Appendix). Similarly, the optimal θ will be equal to unity (no investment in taxable bonds) if $\left.\frac{\partial \pi_p}{\partial \theta}\right|_{\theta=1} \leq 0$, which is true when r_e is sufficiently close to r (but less than r) (for proof, see the Appendix). We can now state the following result.

Proposition 1: If a positive probability of default exists, the insurer may optimally refrain from investing in tax-exempt bonds, even when the tax-exempt yield is greater than the after-tax yield from taxable bonds. However, the insurer may optimally refrain from investing in taxable bonds when the taxable yield is sufficiently close to (but larger than) the tax-exempt yield.

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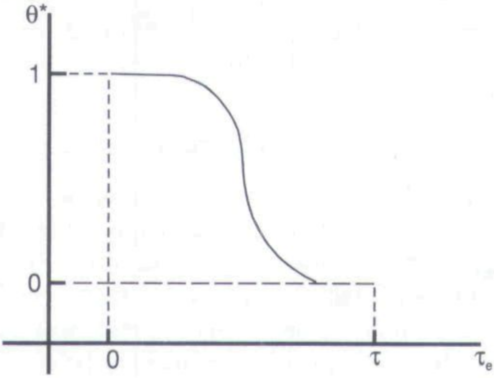
[4] When risky assets are included, the quality of the policies will depend on the distribution of A_1 as well. See PonArul and Viswanath (1992) for a detailed discussion of this point.

Let τ_e be the marginal tax rate implicit in the spread between the yields on taxable and tax-exempt bonds; that is, $(e^{r_e}-1)=(e^{r}-1)(1-\tau_e)$. Proposition 1 states that, as the tax-exempt yield approaches the after-tax yield from taxable bonds, τ_e approaches τ and the optimal θ becomes zero. Also, as the tax-exempt yield approaches the taxable yield, τ_e will approach zero and the optimal θ becomes one. The optimal investment in tax-exempt bonds as a function of the marginal tax rate implicit in the yield spread is shown in Figure 2).

Figure 2

Optimal Proportion of Assets Invested in Tax-Exempt Bonds (θ^*)
As a Function of the Tax Rate Implied
in the Spread Between Taxable and Tax-Exempt Yields (τ_e)



In the certainty models (such as Hendershott and Koch, 1980), the insurer will invest in tax-exempt bonds as long as τ_e is less than τ and all the underwriting losses are covered by income from taxable bonds. Cummins and Grace (1994) show that, in the presence of alternative minimum tax, if τ_e is less than but close to τ, the insurer will avoid tax-exempt bonds. This is similar to the first assertion in Proposition 1. However, Cummins and Grace take as given the initial assets and the premium, and in our model these two quantities are endogenously determined. Further, in our model the tax computation is simpler—we show that the alternative minimum tax is not necessary for a corner solution for θ to be optimal.

The intuition behind Proposition 1 is as follows: Consider an insurer fully invested in taxable bonds and let the implicit tax rate τ_e be large but less than the marginal tax rate τ. If one dollar is shifted from taxable to tax-exempt bonds, there will be a gain of $(e^r-1)(\tau - \tau_e)$ whenever the insurer pays taxes and a loss of $(e^r-1)\tau_e$ whenever it does not. If $(\tau-\tau_e)$ is sufficiently small, there will be a net expected loss, implying that it is optimal to stay fully invested in taxable bonds.

As the spread narrows, tax-exempt bonds become more attractive very quickly, and it can be optimal to invest completely in tax-exempt bonds even when the spread is positive. Consider an insurer fully invested in tax-exempt bonds and let the implicit tax rate τ_e be small but positive. If one dollar is shifted from tax-exempt to taxable bonds, there will be a gain of $(e^r-1)\tau_e$ whenever the insurer does not pay taxes and a loss of $(e^r-1)(\tau-\tau_e)$ whenever it does. If τ_e is sufficiently small, there will be a net expected loss, implying that it is optimal to stay fully invested in tax-exempt bonds.

The following numerical example illustrates Proposition 1. The default probability is set at 0.01 and the present value of L_1, $PV(L_1)$, is set to unity. The parameters μ and σ^2 are set at 0.08 and 0.0045, respectively (these values are borrowed from Cummins, 1991). The continuously compounded taxable yield is 0.08 and the tax exempt yield is allowed to vary from 0.048 to 0.08 (τ_e varies from 0.41 to 0.0). For each value of r_e, the first-order condition for the insurer's optimization problem is used to numerically solve for π_p and θ simultaneously. Table 1 presents the values of optimal θ as r_e varies from 0.048 to 0.08.

Table 1
Numerical Example Showing the Optimal Proportion of Assets Invested in Tax-Exempt Bonds θ^* As a Function of the Implicit Tax Rate
(Parameter Values: $r = 0.08$, $\mu = 0.08$,
$\sigma^2 = 0.0045$, $\varepsilon = 0.01$, $\tau = 0.40$)

r_e	τ_e	θ^*	θ^c
0.048	0.410	0.0000	0.0000
0.050	0.384	0.0000	0.2500
0.052	0.359	0.0000	0.2500
0.054	0.334	0.0000	0.2500
0.056	0.308	0.0000	0.2500
0.058	0.283	0.0000	0.2500
0.060	0.258	0.0340	0.2500
0.062	0.232	0.1660	0.2500
0.064	0.206	0.2863	0.2500
0.066	0.181	0.3996	0.2500
0.068	0.155	0.5099	0.2500
0.070	0.129	0.6207	0.2500
0.072	0.104	0.7383	0.2500
0.074	0.078	0.8690	0.2500
0.076	0.052	1.0000	0.2500
0.078	0.026	1.0000	0.2500
0.080	0.000	1.0000	1.0000

Note: r = continuously compounded taxable yield. μ = instantaneous rate of change in the level of liabilities. σ^2 = instantaneous variance of the changes in the level of liabilities. ε = default probability. r_e = continuously compounded rate of return on tax-exempt bonds. τ_e = implicit tax rate in the spread between the yields on taxable and tax-exempt bonds. θ^* = optimal proportion of assets invested in tax-exempt bonds. θ^c = optimal proportion of assets invested in tax-exempt bonds in a certainty model that assumes that underwriting losses will be 0.06 times the initial assets.

The last column of Table 1 has the values of optimal θ in a certainty model (θ^c) as in Hendershott and Koch (1980). For the certainty model it is assumed that the underwriting losses will be 0.06 times the initial assets. Hence, whenever $0 < \tau_e < \tau$, θ^c is 0.25 so that the income from taxable bonds will exactly offset the underwriting losses.

The results presented in Table 1 indicate that, when solvency risk is held constant, for high values of τ_e (low values of r_e), the optimal θ remains zero even when $\tau_e < \tau$. Then the optimal θ increases as τ_e decreases (r_e increases) until it reaches $\theta^* = 1$. This is in contrast to the certainty model where the optimal θ is constant and between 0 and 1 whenever $0 < \tau_e < \tau$.

Proposition 1 leads to certain empirically testable implications. As the difference between τ and τ_e decreases, the proportion of assets invested in tax-exempt bonds will decline. Proposition 1 implies that the relation between τ_e and θ^*, given τ, will be nonlinear. Furthermore, θ^* will be flat for very low and very high values of the spread.

Conclusion

We have examined the optimal tax sheltering problem faced by property-liability insurers in an equilibrium model where the effect of solvency risk on premiums is explicitly taken into account. We find that insurers may optimally choose to forego investing in tax-exempt securities if the yield differential between taxable and tax-exempt bonds is sufficiently large. Furthermore, if the yield differential is sufficiently small, the insurer may optimally choose not to invest in taxable bonds, and thus forego offsetting underwriting losses with income from taxable bonds.

Appendix

Proposition 1

If a positive probability of default exists, the insurer may optimally refrain from investing in tax-exempt bonds, even when the tax-exempt yield is greater than the after-tax yield from taxable bonds. However, the insurer may optimally refrain from investing in taxable bonds when the taxable yield is sufficiently close to (but larger than) the tax-exempt yield.

Proof

We first show that $\partial\pi_p/\partial\theta$ evaluated at $\theta = 0$ can be positive, even when $e^{r_e} > e^r - \tau(e^r - 1)$. From equation (7) we have

$$\pi_p = A_0 - [A_1 e^{-r} N(-d_2) - PV(L_1) N(-d_1)] \\ + \tau[\{\pi_p + A_0(1-\theta)(e^r - 1)\} e^{-r} N(-w_2) - PV(L_1)\, N(-w_1)]. \quad \text{(A1)}$$

Differentiating equation (A1) throughout by θ and noting that the term $[A_1 e^{-r} N(-d_2) - PV(L_1)(-d_1)]$ is the present value of $\mathrm{Max}(A_1 - L_1, 0)$ and hence does not depend on θ,

$$\frac{\partial\pi_p}{\partial\theta} = \frac{\partial A_0}{\partial\theta} + \tau e^{-r} N(-w_2)\left[\frac{\partial\pi_p}{\partial\theta} + (1-\theta)(e^r - 1)\frac{\partial A_0}{\partial\theta} - (e^r - 1)A_0\right]. \quad \text{(A2)}$$

Noting that $A_0 = A_1 e^{-\alpha}$ and using the definition of α,

$$\frac{\partial A_0}{\partial\theta} = A_1 e^{-2\alpha}(e^r - e^{r_e}). \quad \text{(A3)}$$

Using equation (A3) in (A2) and rearranging terms,

$$\frac{\partial\pi_p}{\partial\theta} = \frac{[A_1 e^{-2\alpha}(e^r - e^{r_e}) + \tau e^{-r} N(-w_2) A_1 [e^{-2\alpha}(1-\theta)(e^r - e^{r_e}) - (e^r - 1)e^{-\alpha}]]}{1 - \tau e^{-r} N(-w_2)}. \quad \text{(A4)}$$

The right-hand side of equation (A4) will be nonnegative at $\theta = 0$ if

$$(e^r - e^{r_e}) \geq \frac{\tau N(-w_2)(e^r - 1)}{1 + \tau e^{-r} N(-w_2)(e^r - 1)}. \quad \text{(A5)}$$

When $e^{r_e} = e^r - \tau(e^r - 1)$, equation (A5) is satisfied as a strict inequality. It follows by the continuity of all the functions in equation (A5) that it can hold as a strict inequality even for some values of r_e such that $e^{r_e} > e^r - \tau(e^r - 1)$.

Next we show that $\partial\pi_p/\partial\theta$ evaluated at $\theta = 1$ can be negative even when $r > r_e$. From equation (A4), $\partial\pi_p/\partial\theta \leq 0$ at $\theta = 1$ if

$$(r - r_e) \leq \ln[\tau\, e^{-r} N(-w_2)(e^r - 1) + 1]. \quad \text{(A6)}$$

The above inequality is a strict inequality for $r = r_e$. It follows from the continuity of all the functions in equation (A6) that it can hold as a strict inequality for some values of $r > r_e$.

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