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Forbearance and Pricing Deposit Insurance in a Multiperiod Framework

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ABSTRACT

A multiperiod deposit insurance pricing model is developed in this article, which utilizes an asset value reset rule comparable to the typical practice of insolvency resolution by insuring agencies. The fairly-priced premium rate of our model can substantially differ from Merton's (1977). After incorporating capital forbearance and moral hazard into the model, our results show that the fairly-priced premium rate is not neutral to forbearance policy even in the absence of moral hazard. Our model formalizes the process of how excessive risk-taking under capital forbearance could lead to instability in the deposit insurance system.

Introduction

Since Merton (1977) first suggested an analogy between deposit insurance and a put option to value deposit insurance contracts, there has been a tradition of modeling deposit insurance as a one-period European put option (Merton, 1978; McCulloch, 1985; Marcus and Shaked, 1984; Ronn and Verma, 1986; Pennacchi, 1987; and Duan, Moreau, and Sealy, 1992). In this literature, the put option formula is derived under the assumption that, at the time of audit, either deterministic or stochastic, the put is exercised if the insured institution is found to be insolvent. The deposit insuring agent renegotiates the terms for the next period if the insured institution is solvent. Allen and Saunders (1993) depart from this tradition and argue that deposit insurance is best described as

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a callable put in the sense that deposit insurance is a perpetual put option with the insuring agent holding the right to terminate the put prematurely.[1]

In this article, we propose an alternative way of interpreting deposit insurance in a multiperiod framework. The defaulting banks in our model are assumed to have their assets reset to the level of the outstanding deposits plus accrued interests when an insolvency resolution takes place. According to the deposit insurance contract, the amount required to reset assets is indeed the legal liability of the insuring agent. The historical experience of deposit insurance in the United States supports this set-up. The majority of defaulting depository institutions were resolved through either purchase-and-assumption or the government-assisted merger method. Bartholomew (1991) reported data for 1,730 thrifts that were resolved during the period from 1980 through 1990. Of these 1,730 thrifts, 1,478 institutions (or 85.4 percent) were resolved through this form of reorganization. According to Table 125 of the Federal Deposit Insurance Corporation's (FDIC) 1990 Annual Report, 1,813 banks were closed during the period from 1945 through 1990. Among them, 1,261 banks (or 69.6 percent) were resolved through this form of reorganization.

Since the majority of defaulting banks after reorganization continue to operate with deposit insurance, these banks can thus be regarded as receiving an at-the-money put option at the point of insolvency resolution. From this perspective, deposit insurance can be viewed as a stream of one-period Merton-type put options with occasional asset value resets. The banks are assumed to pay out cash dividends whenever the asset value exceeds the level required by a threshold debt-to-asset ratio. This threshold debt-to-asset ratio can be regarded as the maximum level of paid-in capital above which the bank equity holders would consider excessive and start to distribute cash dividends. In other words, this threshold level is dictated by the dividend policy of the bank. The deposits in our model are assumed to bear interests with the interest payments being added back to the deposit base. The deposit base is therefore growing at a rate equal to the interest rate.

The premium rate levied by the insuring agent is assumed to be constant over a particular coverage horizon. The fixed premium rate coverage horizon can be one year or any number of years. In reality, charging a fixed premium rate over a period of several years is the standard practice of most deposit insuring agencies. A fairly-priced deposit insurance premium rate can be determined by setting equal the present value of premium proceeds and that of puts until the terminal point of the coverage horizon. The stream of one-period Merton-type put options can be priced by the risk-neutral valuation technique. Although a closed-form solution cannot be derived, the present value can be computed using a Monte Carlo simulation method.

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[1] In Allen and Saunders's (1993) model, capital forbearance or transaction costs prevent the insuring agent from immediately terminating the put when the insured bank becomes insolvent. Without capital forbearance and transaction costs, a callable put will be exercised promptly and the issuer will face no claims. In other words, it degenerates into a case of no insurance.

Our multiperiod deposit insurance model can be compared with that of Merton (1977) through the use of the fairly-priced deposit insurance premium rate. A deposit insurance premium rate determined by Merton's model cannot be fairly priced according to the multiperiod model. In other words, a deposit insurance subsidy exists even if Merton's model is used to set the government's levy. The magnitude of this subsidy, when Merton's premium rate is used, depends on the level of the market interest rate, the leverage ratio, and the risk of the bank's asset portfolio. We find the difference can be substantial. Compared to our multiperiod model, Merton's premium rates are, as expected, found to vary more in relation to different leverage ratios and portfolio risk levels.

The FDIC and the now-defunct Federal Savings and Loan Deposit Insurance Corporation permitted the troubled depository institutions to remain open, believing that these institutions were suffering temporary financial setbacks and would later return to sound financial conditions. According to Bartholomew (1991), the thrift regulator took an average of 38 months to close and resolve the failed savings and loan institutions from 1980 through 1990. This practice of capital forbearance was criticized by Kane (1987) and Kaufman (1987) among others. They argued that capital forbearance encourages "zombie" institutions, in the words of Kane (1987), to engage in excessive risk-taking. In a multiperiod setting, capital forbearance effectively postpones recognizing and realizing losses. Postponing a timely closure simply substitutes an immediate cash settlement with future liabilities. Without excessive risk-taking by the troubled institutions, it is not clear whether capital forbearance is a bad practice.[2]

We incorporate capital forbearance and moral hazard into the multiperiod deposit insurance pricing model to examine their impact on the fairly-priced deposit insurance premium rate. Although it is beyond the scope of this article to address the risk-incentive problem, our model does suggest that the fairly-priced premium rate is not neutral to forbearance policy even in the absence of moral hazard. With excessive risk-taking, the fairly-priced premium rate rises with a higher degree of capital forbearance.

A Multiperiod Model for Deposit Insurance Pricing

Following Merton (1977), the bank asset value dynamic is assumed to be a lognormal process. Because we want to incorporate insolvency resolution into the multiperiod model, it is necessary to allow some discrete adjustments to the asset value at the points of audit. In other words, the lognormal process

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[2] Acharya and Dreyfus (1989) argue that early closure instead of granting capital forbearance is optimal if the insurer dynamically adjusts the insurance premium to the actuarially fair level. In contrast, Dreyfus, Saunders, and Allen (1994) argued that, based on liquidation costs, it may be optimal under certain conditions to grant capital forbearance. Using a mechanism design approach, Nagarajan and Sealey (1995) demonstrated that capital forbearance is useful as part of an incentive compatible policy to prevent the insured banks from risk-taking.

assumption only applies to the periods between any two consecutive auditing times. We denote the sequence of auditing points by t_i, i = 1,.... To simplify the exposition, we define the annualized continuously compounded asset return over the interval from t_{i-1} to t_i by R_{t_i}. It follows from the lognormal process assumption that

$$R_{t_i} \sim N[\mu_{t_{i-1}} (t_i-t_{i-1}), \sigma^2_{t_{i-1}} (t_i-t_{i-1})], \tag{1}$$

where $\mu_{t_{i-1}}$ and $\sigma_{t_{i-1}}$ are annualized mean return and standard deviation of the bank asset return known at time t_{i-1}. The variables $\mu_{t_{i-1}}$ and $\sigma_{t_{i-1}}$ are assumed to be measurable with respect to the information set generated by the continuously compounded returns up to and including time t_{i-1}. In other words, they can be stochastic by being functions of past returns.

Let A_t be the bank asset value at time t. The initial face value of total interest-bearing bank deposits is denoted by D. The earned interest is assumed to be plowed back into the deposit base. Since the deposits are insured, the interest rate applicable must be the risk-free rate of return, r. This gives rise to an increasing deposit base with the rate of increase equal to the risk-free rate of return. We assume that the bank asset value is subject to reset at the time of audit. First, because the insuring agent adopts purchase-and-assumption or government-assisted merger as a way to conduct insolvency resolution, the insuring agent essentially provides a lump sum transfer to the acquirer of the failing bank. This lump sum transfer is in an amount sufficient to cover the face value of deposits and accrued interests. According to the deposit insurance contract, this amount is indeed the liability of the insuring agent at the time of insolvency. The second source of asset value reset comes from distribution of cash dividends. We introduce a threshold level of debt-to-asset ratio, q. This debt-to-asset ratio can be regarded as setting the maximum level of bank capital above which equity holders would consider excessive and start to withdraw capital by paying cash dividends. If the bank pays no dividends, the threshold debt-to-asset level, q, becomes zero.

In our multiperiod deposit insurance pricing model, capital forbearance is incorporated in a spirit similar to Ronn and Verma's (1986). A depository institution faces an insolvency resolution only when its asset value falls below ρDe^{rt_i}, where ρ is less than or equal to one.[3] Although capital forbearance alters the condition for triggering an asset value reset, the reset will, if taking place, fully restore the asset value to the bank's outstanding deposit liabilities,

[3] If the government-set capital standard is strictly enforced, banks will be required to have a capital infusion or face closure before they become insolvent. Failure to close a bank or forcing a capital infusion when the capital standard is violated can be considered as granting capital forbearance. By this interpretation, any closure rule based on zero net worth contains forbearance.

De^{rt_i}. The amount needed to restore the asset value is the liability of the insuring agent. Let $t_0 = 0$ and define $A^*_{t_i} = A_{t_{i-1}} e^{Rt_i(t_i - t_{i-1})}$. For any t_i, i = 1,..., we have

$$A_{t_i} = \begin{cases} \dfrac{De^{rt_i}}{q}, & \text{if } A^*_{t_i} \geq \dfrac{De^{rt_i}}{q} \\ A^*_{t_i}, & \text{if } \dfrac{De^{rt_i}}{q} > A^*_{t_i} \geq \rho De^{rt_i} \\ De^{rt_i}, & \text{if otherwise.} \end{cases} \tag{2}$$

The asset value reset rule without capital forbearance is determined by setting $\rho = 1$. Facing the insuring agent is a stream of put option-like liabilities. Each of these put options at time $t_i < T$ gives rise to a cash payment, denoted by C_{t_i}, in an amount equal to

$$C_{t_i} = \begin{cases} 0, & \text{if } A^*_{t_i} \geq \rho De^{rt_i} \\ De^{rt_i} - A^*_{t_i}, & \text{if otherwise.} \end{cases} \tag{3a}$$

If $\rho = 1$, the liability facing the insuring agent reduces to the familiar put option payoff; that is, $C_{t_i} = \text{Max}(De^{rt_i} - A^*_{t_i}, 0)$. For a coverage horizon $T = t_n$, regardless of whether ρ equals one, the cash payment must reflect the termination of coverage at time T. That is,

$$C_T = \text{Max}(De^{rT} - A^*_T, 0). \tag{3b}$$

The time-t_{i-1} value of the payment at time $t_i < T$ per dollar of deposits can be priced, similar to that of Merton (1977), to yield

$$I_{t_{i-1}}(\rho) = N[\sigma_{t_{i-1}}\sqrt{t_i - t_{i-1}} - d_{t_{i-1}}(\rho)] - \frac{A_{t_{i-1}}}{De^{rt_{i-1}}} N[-d_{t_{i-1}}(\rho)], \tag{4a}$$

where N(·) denotes the cumulative standard normal distribution function; and

$$d_{t_{i-1}}(\rho) = \frac{\ln \dfrac{A_{t_{i-1}}}{\rho De^{rt_{i-1}}} + \dfrac{\sigma^2_{t_{i-1}}}{2}(t_i - t_{i-1})}{\sigma_{t_{i-1}}\sqrt{t_i - t_{i-1}}}.$$

For the cash payment at the terminal time $T = t_n$, its value at the preceding time point, t_{n-1}, can simply be computed by letting $\rho = 1$. Specifically,

$$I_{t_{n-1}} = N(\sigma_{t_{n-1}}\sqrt{t_n - t_{n-1}} - d_{t_{n-1}}(1)) - \frac{A_{t_{n-1}}}{De^{rt_{n-1}}} N(-d_{t_{n-1}}(1)). \tag{4b}$$

The formula in (4b) is the same as Merton's (1977), whereas the formula in (4a) is also that of Merton if $\rho = 1$.

The time-t_0 value of an individual put option at time t_i is the present value of the product of I_{t_i} and De^{rt_i}. The present value operator can be derived by the use of the risk-neutral valuation technique. Following Cox and Ross (1976) and Harrison and Kreps (1979), the continuously compounded return on the bank's assets, under the risk-neutralized pricing measure, must distribute normally as follows:

$$R_{t_i} \sim N[(r - \frac{\sigma^2_{t_{i-1}}}{2})(t_i - t_{i-1}), \sigma^2_{t_{i-1}}(t_i - t_{i-1})]. \tag{5}$$

A contingent payoff is priced by taking the expectation with respect to the distribution in relation (5) and then discounting at the risk-free rate of return. Suppose that the insuring agent targets a fixed premium rate coverage horizon T where $T = t_n$. It becomes more relevant to ask about the total value of the multiperiod coverage. The total intrinsic value of a sequence of deposit insurance guarantees up to time T (or the size of the guaranty funds required for providing this deposit insurance), denoted by F_T, can thus be calculated by

$$\begin{aligned} F_T &= \sum_{i=0}^{n-1} e^{-rt_i} E^*_{t_0}[I_{t_i}] De^{rt_i} \\ &= D \sum_{i=0}^{n-1} E^*_{t_0}[I_{t_i}], \end{aligned} \tag{6}$$

where $E^*_{t_0}[\cdot]$ denotes the expectation at time t_0 with respect to the distribution characterized in relation (5). A closed-form solution for F_T does not exist because of the complex nature of asset value resets. The value can nevertheless be evaluated by a Monte Carlo simulation method. The asset value at various points of audit can be simulated by starting from an initial value for A_0, generating random returns by condition (5) and applying the asset value reset rule in equation (2). A Monte Carlo method adds a useful flexibility to the multiperiod deposit insurance pricing model. It is relatively straightforward to examine the impact of a change in the portfolio risk level accompanied by an insolvency resolution. The change in portfolio risk is often contingent on the asset value, and thus makes portfolio risk stochastic. With the Monte Carlo method, our model can still be used to compute the deposit insurance value.

Fairly-Priced Deposit Insurance

The typical deposit insurance program sets a flat-rate premium not only constant across all insured depository institutions, but also fixed over an extended period of time.[4] We assume a constant hypothetical premium rate, say

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[4] Prior to 1993, the FDIC and other insuring agencies around the world used a flat-rate premium system. Although most insuring agencies continue to use a flat-rate system, the FDIC began in January 1993 to classify the insured institutions into nine categories according to their capital

δ, over the period from time 0 to T. We further assume that the payment and auditing times coincide. With these assumptions, the payment to the insuring agent at time t_i is δDe^{rt_i}, and the present value of all payments for the coverage up to time $T = t_n$ becomes δnD. The amount of deposit insurance subsidy is the difference between the intrinsic value of the coverage and the present value of the premium payments. Let δ^*_T denote the premium rate that makes the subsidy equal to zero. In other words, δ^*_T is the fairly-priced deposit insurance premium rate for the fixed premium rate coverage horizon $T = t_n$. The fairly-priced premium rate is determined by

$$\begin{aligned} \delta^*_T &= \frac{F_T}{nD} \\ &= \frac{1}{n}\sum_{i=0}^{n-1} E^*_{t_0}[I_{t_i}]. \end{aligned} \tag{7}$$

For banks with different risk characteristics, the above formula will yield different fairly-priced premium rates. So the model provides a way to compute the risk-based premium rate for various fixed-rate coverage horizons. The fairly-priced premium rate is always a constant between zero and one. This is because I_{t_i}, the value of deposit insurance per dollar of deposits, must be bounded below by zero and above by one. If the coverage horizon is extended to infinity, it is clear that the premium rate, δ^*_T will converge to some constant in the interval [0,1]. If one uses Merton's (1977) model, deposit insurance is fairly priced only if it is set equal to I_{t_0}. Denote this premium rate by δ_M. The asset value reset rule will have no effect on δ_M because of Merton's liquidation assumption. Clearly, the fairly-priced premium rate in equation (7) disagrees with δ_M unless the fixed premium rate coverage horizon equals one period.

The fairly-priced premium rates corresponding to different coverage horizons along with δ_M are presented in Table 1. These values are computed by setting $\rho = 1$, that is, no capital forbearance. The continuously compounded risk-free rate is set at 6 percent.

It is well known that the deposit insurance premium rate computed according to Merton's formula is independent of the risk-free rate of return. This property can also be seen by letting $t_{i-1} = 0$ in equation (4a). For a coverage horizon longer than one period, the fairly-priced premium rate is not independent of the risk-free rate. These premium rates are, however, not found to be very sensitive to the risk-free interest rate. The fairly-priced premium rates are reported in Table 1 with the columns being different levels of portfolio risk. In this table, the portfolio risk remains fixed throughout any particular coverage horizon. In other words, there is no change in portfolio risk policy when a bank is reorganized. The fixed premium rate coverage horizons in Table 1 are

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ratios and supervisory reports. As of November 1, 1993, the premium rate set by the FDIC ranges from 23 to 31 basis points for these nine categories of insured institutions.

Table 1
Fairly-Priced Deposit Insurance Premium Rates

	Annual Standard Deviation of Asset Return				
	0.03	*0.05*	*0.08*	*0.1*	*0.2*
Debt-to-Asset Ratio = 0.85					
δ_M	1.7E-6	0.0832	6.76	23.73	254.27
$\delta^*_{T=5}$	51.41	91.25	166.24	223.65	571.95
$\delta^*_{T=10}$	83.76	136.71	214.65	274.47	625.14
$\delta^*_{T=20}$	101.60	160.36	240.26	299.49	652.85
Debt-to-Asset Ratio = 0.9					
δ_M	0.0176	3.34	37.01	79.15	398.79
$\delta^*_{T=5}$	61.23	110.32	189.57	251.21	611.36
$\delta^*_{T=10}$	89.19	146.42	227.61	288.09	644.52
$\delta^*_{T=20}$	104.21	164.95	245.52	305.71	660.89
Debt-to-Asset Ratio = 0.95					
δ_M	5.50	40.67	129.35	198.74	581.00
$\delta^*_{T=5}$	90.76	144.97	229.45	291.85	655.87
$\delta^*_{T=10}$	104.65	164.36	246.38	308.23	667.43
$\delta^*_{T=20}$	111.77	173.84	255.99	315.94	673.19
Debt-to-Asset Ratio = 1					
δ_M	119.68	199.45	319.07	398.78	796.56
$\delta^*_{T=5}$	119.31	189.35	280.86	345.84	706.24
$\delta^*_{T=10}$	119.12	186.87	273.36	334.89	691.50
$\delta^*_{T=20}$	119.10	185.43	268.37	329.61	684.97

Note: Merton's premium rate δ_M and the fairly-priced premium rate δ^*_T are stated in basis points. The fixed-rate coverage horizon T is 5, 10, or 20 years. The risk-free rate equals 6 percent. The threshold debt-to-asset ratio q is 0.92. All values are computed using 10,000 simulation runs.

5, 10, and 20 years, respectively. These premium rates are computed for different initial levels of leverage (deposit-to-asset ratio) ranging from 0.85 to 1.00. We do not consider the case where the deposit-to-asset ratio is higher than one. Limiting the analysis to these cases amounts to considering only the solvent banks.

In the case of an insolvent bank, one can interpret it as having an asset value reset before being considered. It is not surprising to see that the fairly-priced premium rate increases with portfolio risk and the leverage ratio. For example, $\delta^*_{T=5}$, with portfolio risk at 8 percent, increases from 189.57 to 229.45 basis points when the debt-to-asset ratio rises from 0.9 to 0.95. Similarly, the fairly-priced premium rate for T = 5 and debt-to-asset ratio = 0.9 increases from 189.57 to 251.21 basis points when portfolio risk is increased from 8 to 10 percent. The fairly-priced premium rate increases with the coverage horizon for the leverage ratios: 0.85, 0.9, and 0.95. In the case of a leverage ratio equal to one, the relationship is reversed. Interestingly, the increasing or decreasing pattern is preserved when portfolio risk is altered.

The data presented in Table 1 suggest that the premium rate, if set according to Merton's (1977) formula, can be over- or underpriced. This is intuitive because the fairly-priced premium rate may be viewed as an average of several one-period premium rates. Because debt-to-asset ratio = 1 is the highest leverage ratio permitted by the asset value reset rule, Merton's premium rate—that is, the one for the first period—must be the highest, with probability one, among all one-period premium rates. This in turn implies that the fairly-priced premium rate must be lower than Merton's when the leverage ratio is sufficiently high. The reverse will clearly hold true when the leverage ratio is sufficiently low. Compared to the fairly-priced premium rates for different coverage horizons, Merton's premium rates vary more for any given leverage ratio. Merton's premium rates are also found to be more sensitive to different levels of asset volatility. Both phenomena can again be attributed to the averaging effect of the fairly-priced premium rate.

When the fairly-priced premium rate is compared to the actual premium rate, one can conclude that the premium rate set by the FDIC is likely to be on the lower side. Over a reasonable range of values for the model parameters, all fairly-priced premium rates, reported in Table 1, are higher than 31 basis points, which is the highest possible premium rate for any insured institution according to current FDIC practice.

Capital Forbearance and Moral Hazard

Modeling Forbearance

Capital forbearance is the discretionary enforcement of capital standards by the regulators. In Ronn and Verma's (1986) model and many others, forbearance arises from the insuring agent's intentional delay in forcing a bank closure. The source of capital forbearance in the Ronn and Verma model is that bank closure occurs only when its asset value slides below a fixed percentage of the insured liabilities at the time of audit. In the absence of capital forbearance, deposit insurance is still valuable to insured banks and a subsidy can continue to exist. This is because deposit insurance is a European-style put option so that a bank, regardless of its financial condition, cannot be closed prior to the time of audit. Allen and Saunders (1993) view deposit insurance as a callable put in the sense that deposit insurance is a perpetual put option with the insuring agent holding the right to terminate the put prematurely. In their model, the government insuring agent grants the insured banks forbearance by deviating from the exercise policy that a private insurer would pursue. Forbearance is the reason that deposit insurance exists in their model. If the insuring agent obeys the private market principle, all troubled banks would be closed promptly and the insuring agent would never face any claims.

The model developed above can now be used to examine the effects of capital forbearance policy. The immediate impact of granting capital forbearance is best demonstrated by the following derivative property:

$$I_{t_{i-1}}^{\prime}(\rho) = \frac{A_{t_{i-1}}}{De^{rt_{i-1}}} e^{-0.5d_{t_{i-1}}^{2}(\rho)} (1 - \frac{1}{\rho}) d_{t_{i-1}}^{\prime}(\rho) > 0. \tag{8}$$

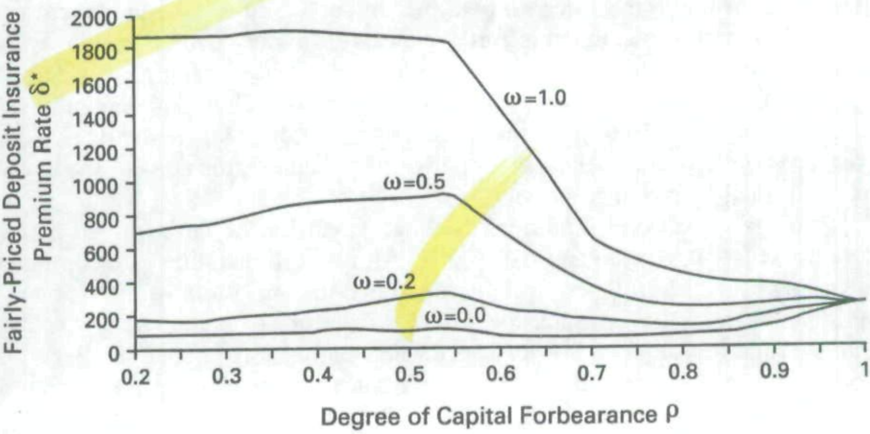
This derivative result implies that, holding other variables fixed, the time-t_{i-1} cost of time-t_i cash payment resulting from the deposit insurance guarantee is lower if capital forbearance is in place. In the multiperiod framework, capital forbearance is simply a policy of delaying the time to confront insolvency resolution. It need not, however, imply a lower aggregate value for a stream of deposit insurance guarantees. The reason for having a lower value for the next immediate cash payment is that the insuring agent postpones the insolvency resolution and converts its current cash liabilities to future cash payments. In other words, the delay amounts to allowing the failing institution not to be marked-to-market promptly. The insuring agent thus avoids an immediate cash payment although the future liabilities increase as a result of continuing to provide insurance coverage to the troubled institution at the same premium rate.

In the numerical analysis, this intertemporal exchange is found to be not completely off-setting. In Figure 1, the coverage horizon is fixed at ten years. The curve labeled by $\omega = 0$ depicts the relationship between ρ and δ^*. The fairly-priced premium rate first declines and then rises as forbearance increases (i.e., ρ decreases). Deposit insurance is therefore not neutral to forbearance policy. This phenomenon is due to the natural lower bound of the asset value at zero. The marginal reduction in the value of the immediate payment due to a delay in the cash settlement is high when capital forbearance is small, that is, ρ is near one. Since the asset value cannot fall below zero, a deposit insurance liability cannot increase much when the asset value is already low. No capital forbearance, on the other hand, implies not only an immediate cash infusion by the insuring agent but also issuing an at-the-money put to the insured bank. The combined value of two liabilities is higher than that of letting the deposit insurance put stay in-the-money under forbearance. A high level of capital forbearance essentially allows bank value to stay close to its natural lower bound. The marginal reduction due to any further increase in forbearance is therefore expected to be small. Since capital forbearance causes an increase in the value of future payments, the marginal increase will catch up with the marginal reduction, which causes the fairly-priced premium rate to rise again.

Moral Hazard and Forbearance

The aforementioned conclusion is of course reached in the absence of moral hazard. Kane (1987) and Kaufman (1987) argue that capital forbearance induces a failing bank to adopt risk-shifting portfolio strategies, a situation known in the literature as moral hazard. To shed light on the relationship between capital forbearance and moral hazard, we formalize the risk-taking behavior in a simple and stylized fashion. It frequently has been argued that risk-taking behavior can be caused by the flat-rate deposit insurance system. This risk-taking behavior is not induced by capital forbearance; rather it is a

Figure 1
Fairly-Priced Deposit Insurance Premium Rates
in Relation to Capital Forbearance



Note: A higher value of degree of capital forbearance ρ corresponds to a lower degree of capital forbearance. ω = the risk-taking intensity. The fairly-priced premium rates are stated in basis points. The fixed-rate coverage horizon T equals 10 years. The risk-free rate is 6 percent. The threshold debt-to-asset ratio q equals 0.92. The portfolio risk (annualized standard deviation of the bank asset return) is 8 percent, whereas the initial leverage ratio is 0.95. All values are computed using 10,000 simulation runs. The variable ρ changes in increments of 0.05.

result of the premium rate being independent of risk characteristics. In this article, our attention is restricted to capital forbearance-induced moral hazard.

We assume that risk-taking behavior is governed by the outcomes of the bank's asset value, which is classified into three categories. When value is greater than deposit commitments—that is, the bank is solvent by the private market criterion—portfolio risk remains intact. When asset value slides below deposit commitments but stays above the threshold point at which the insuring agent steps in, moral hazard occurs and the bank is assumed to increase its portfolio risk by 100ω percent. This risk-taking behavior is induced by the forbearance policy of the insuring agent not to close the failing bank promptly. The third category is when the failing bank's asset value slides further below the threshold point at which the insuring agent can no longer forbear. When this situation occurs, insolvency resolution takes place and the insuring agent forces the asset portfolio risk of the failing bank back to the normal level. Formally, the portfolio risk σ_{t_i} is determined by the following rule:

$$\sigma_{t_i} = \begin{cases} \sigma_{t_{i-1}}, & \text{if } A_{t_i}^* \geq De^{rt_i} \\ (1+\omega)\sigma_{t_{i-1}}, & \text{if } De^{rt_i} > A_{t_i}^* \geq \rho De^{rt_i} \\ \sigma_0, & \text{if otherwise.} \end{cases} \tag{9}$$

The risk-taking behavior prescribed in equation (9) does not model the endogenous response of the insured bank to forbearance policy. In other words, the risk-taking intensity in equation (9) is exogenously given. If the choice of risk-taking intensity is made endogenous, the bank would prefer to engage in risk-taking to the highest degree possible. In such a case, an equilibrium can exist only if some counteracting force, either a regulatory penalty or managerial agency costs, is present to prevent the bank from engaging in excessive risk-taking. Although the issue related to endogenous choice of risk-taking is an important one, it is beyond the scope of this article. Our model does, however, provide a useful analytical framework for future extensions to the analysis of optimal forbearance policy.

Let $\delta^*_T(\rho,\omega)$ be defined, similar to equation (7), as the fairly-priced premium rate with forbearance and risk-taking. All relevant quantities are computed according to the portfolio risk rule outlined in equation (9). Portfolio risk stays the same as the initial constant for different values of ω as long as $\rho = 1$. This is to say that moral hazard occurs only when capital forbearance is granted to banks. The fairly-priced premium rate provides a simple way to study moral hazard under forbearance policy.

Figure 1 portrays the functional relationship between the fairly-priced premium rate and capital forbearance with a coverage horizon of ten years. Different values for ω correspond to different intensities of risk-taking by the troubled bank. The case that $\omega = 1$ represents the troubled institution doubling its portfolio risk, whereas $\omega = 0$ indicates no risk-taking. The shape of the curve corresponding to $\omega = 0$ was discussed above. The remaining curves indeed exhibit an expected property. That is, a higher intensity of risk-taking implies a higher fairly-priced premium rate for any level of capital forbearance. The risk-taking intensity can be so high (see the curve for $\omega = 1$) that it overpowers the initial delaying effect created by capital forbearance. As argued by many researchers, capital forbearance is potentially dangerous when accompanied by risk-taking. Our model provides a useful way for analyzing its implications.

Table 2 reports the results with $\rho = 0.97$. This is a value used in Ronn and Verma (1986) and many subsequent studies. Three different values for the risk-taking intensity—1.0, 0.5, and 0—are considered. All relevant parameters, except those explicitly stated, share the same values as those in Table 1. The results suggest that risk-taking behavior does have a significant impact on the fairly-priced premium rate. In the case of debt-to-asset ratio = 1, for example, a high level of risk-taking such as $\omega = 1$ could reverse the declining pattern, reported in Table 1, of the fairly-priced premium rate in relation to the coverage horizon. The figures in this table confirm that more risk-taking, made possible by capital forbearance, increases the exposure of the deposit insuring agent.

To compare with the callable-put deposit insurance pricing model of Allen and Saunders (1993), we report in Table 2 the premium rates calculated by their model. As discussed above, their model under no capital forbearance is equivalent to the case of no deposit insurance. Allen and Saunders's model does, however, offer an interesting comparison when capital forbearance is

Table 2
Fairly-Priced Deposit Insurance Premium Rates with Capital Forbearance

	Annual Standard Deviation of Asset Return				
	0.03	*0.05*	*0.08*	*0.1*	*0.2*
			Debt-to-Asset Ratio = 0.9		
$\omega = 1.0$					
$\delta^*_{T=5}$	65.48	123.00	214.41	282.92	660.01
$\delta^*_{T=10}$	112.74	179.12	271.42	339.47	708.69
$\delta^*_{T=20}$	146.05	210.69	303.13	368.78	732.61
$\omega = 0.5$					
$\delta^*_{T=5}$	56.25	109.17	195.56	261.37	632.46
$\delta^*_{T=10}$	89.86	151.80	241.63	306.74	674.66
$\delta^*_{T=20}$	121.81	177.05	267.74	332.40	696.31
$\omega = 0.0$					
$\delta^*_{T=5}$	45.69	95.72	178.02	241.24	604.41
$\delta^*_{T=10}$	58.07	124.14	210.95	273.81	636.42
$\delta^*_{T=20}$	64.47	138.35	228.26	290.89	652.60
δ_{AS}	0.00	0.44	23.50	58.79	199.60
			Debt-to-Asset Ratio = 1		
$\omega = 1.0$					
$\delta^*_{T=5}$	139.77	217.16	317.32	385.68	753.98
$\delta^*_{T=10}$	158.00	230.25	324.63	390.32	754.18
$\delta^*_{T=20}$	169.20	238.85	330.95	396.87	755.84
$\omega = 0.5$					
$\delta^*_{T=5}$	113.54	191.58	292.24	359.76	726.27
$\delta^*_{T=10}$	124.23	196.32	290.95	357.79	721.46
$\delta^*_{T=20}$	129.84	199.94	292.21	356.38	718.74
$\omega = 0.0$					
$\delta^*_{T=5}$	80.88	163.59	264.88	332.06	698.44
$\delta^*_{T=10}$	76.03	158.27	254.73	319.99	683.71
$\delta^*_{T=20}$	73.58	155.54	249.94	313.24	677.13
δ_{AS}	5.17	69.53	169.47	208.15	273.80

Note: The degree of capital forbearance ρ is set equal to 0.97. The risk-taking intensity ω is 1, 0.5, or 0. The fairly-priced premium rate δ^*_T is stated in basis points. The fixed-rate coverage horizon T is 5, 10, or 20 years. The risk-free rate equals 6 percent. The threshold debt-to-asset ratio q is 0.92. All values, except the premium rates based on Allen and Saunders (1993), denoted by δ_{AS}, are computed using 10,000 simulation runs.

present. Since their model does not contain the risk-taking component, we can only compare their premium rates, denoted by δ_{AS}, with our fairly-priced premium rates under the case that $\omega = 0$. The figures in Table 2 clearly suggest that the callable-put approach yields substantially lower deposit insurance premium rates. This difference between two modeling approaches is, however, not surprising. In the context of Allen and Saunders, the deposit insurance contract is immediately called off when bank asset value falls below a preset level. This premature exercise by the insuring agent reduces the value of the deposit insurance coverage. Allen and Saunders's model implicitly assumes liquidation when a closure occurs. If the troubled institution is reorganized and continues its deposit-taking activities as assumed in our model, the callable-put approach is expected to underestimate the value of deposit insurance coverage.

Model Limitations and Empirical Analysis

Our multiperiod deposit insurance pricing model is subject to several limitations. The asset return is assumed to be lognormally distributed between two consecutive auditing times. There is increasing evidence (see Bollerslev, Chou, and Kroner, 1992) that financial asset returns can be better described by the ARCH model of Engle (1982) or its variants. An option pricing model in the ARCH framework proposed by Duan (1995) discusses the potential of this new pricing model to provide better price estimates for traded options. Failure to take into account heteroskedasticity may also cause our model to exhibit systematic biases in determining the fairly-priced deposit premium rate. This issue is clearly worthy of further investigation.

The second limitation of our model stems from the assumption that the fixed coverage rate horizon is known, and the forbearance policy remains unchanged throughout the coverage horizon. In practice, the insuring agent must respond to the political and economic forces propelled by major events, which include legislation such as the Financial Institutions Reform, Recovery and Enforcement Act of 1989 and the FDIC Improvement Act of 1991. In order to reflect this reality, it may be desirable to incorporate some form of state-contingent coverage horizon and/or capital forbearance policy into the pricing model. The third limitation rests with the assumption that the deposit base is deterministic and grows at the risk-free rate. Allowing the deposit base to be a stochastic process is another way of improving our multiperiod deposit insurance pricing model.

The pricing model proposed in this article can be empirically implemented, which is important for at least two reasons. First, it helps determine whether the limitations discussed above materially affect the pricing results. Second, a pricing model is useful only if it actually can be implemented. The difficulty of using option-based models for deposit insurance pricing has always been the inability on the part of analysts to observe the market value of bank assets and the volatility parameter of asset returns. This difficulty can nevertheless be overcome with the recent methodological advancement in Duan (1994). Since bank equity value, denoted by E_t, can be viewed as the value of a call option on the bank's assets, the observed time series of market equity values can be treated as a transformed data set with the transformation containing the unknown model parameters and the variables from the bank's balance sheet. In the context of our model, this data transformation can be written as a function in the form of $E_t = f(V_t,D_t,r_t;\sigma,\rho,\omega,q)$, where σ, ρ, ω, and q are the parameters of the model; and V_t, D_t, and r_t are the unobserved bank asset value, the book value of debt, and the one-period interest rate at time t, respectively.

This pricing formula for bank equity can be computed by aggregating the present values for a sequence of cash dividends and a call option-like payoff at the terminal point of the fixed-rate coverage horizon. Duan's maximum likelihood transformed data estimation method can thus be applied to obtain the estimates for the unknown model parameters. These parameter estimates can then be used to compute the corresponding time series of implied bank asset values. With these estimates, the fairly-priced deposit insurance premium rate at any point in time can be calculated using the formula in this article. Since the estimation method is based on maximum likelihood and the deposit insurance pricing model is a continuous function, the fairly-priced premium rate must also be a maximum likelihood estimate. The standard inference procedure can thus be applied to testing the hypothesis of underpricing by the insuring agent. It can also be used to determine the confidence interval for any variable of interest.

Conclusion

A multiperiod deposit insurance pricing model is developed in this article. The model utilizes an asset value reset rule comparable with the typical practice of insolvency resolution by insuring agencies. In many cases, the fairly-priced premium rate is found to be substantially different from that of Merton (1977). In contrast to the one-period Merton-type models, several interesting aspects of deposit insurance can be considered. We examine the effect of varying the fixed premium rate coverage horizon and find that it affects the fairly-priced deposit insurance premium rate. This multiperiod model also makes it possible to formally examine forbearance and the accompanying risk-taking behavior. Our results show that the fairly-priced premium rate is not neutral to capital forbearance policy. The risk-taking intensity determines how the fairly-priced premium rate responds to forbearance policy. Our model formalizes the process of how excessive risk-taking under capital forbearance leads to instability in the deposit insurance system.

Casting the issue of deposit insurance pricing in a multiperiod context is clearly a sensible way of modeling deposit insurance. The resulting model provides a way for assessing the costs of the coverage granted by the insuring agency. The model also offers insights as to how forbearance will affect the costs of deposit insurance. The model developed in this article provides an analytical tool useful in the ongoing policy debate on capital forbearance.

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