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Are Net Discount Rates Stationary?: The Implications for Present Value Calculations: Comment

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ABSTRACT

This comment reexamines the time series properties of the net discount ratio over the period 1964 through 1989. Using the standard augmented Dickey-Fuller tests, we fail to reject the null hypothesis of a unit root in the net discount ratio. Using a modification of this test that takes into account a possible one-time shift in the series mean, we reject the unit root hypothesis. Thus, we conclude that the net discount ratio is nonstationary, but the source of nonstationarity is a one-time shift that occurred between October 1979 and January 1980. The implications of this mean shift for forecasting the present value of future earnings are examined.

Introduction

In a recent article in this *Journal,* Haslag, Nieswiadomy, and Slottje (1991) present evidence that the nonlinear net discount ratio over the 1964 through 1989 period is a stationary series. This is an important finding, since it implies not only that a stable relationship exists between the growth rate of wages and interest rates, but also that valid inferences about the population moments of the distribution of the net discount ratio can be drawn from these data. As a

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result, historical averages of the net discount rate can be used confidently to estimate the present value of future earnings.[1]

Although Haslag, Nieswiadomy, and Slottje do examine the robustness of their results to changes in the interest rate, they do not examine robustness with respect to changes in the form of the unit root test. As pointed out by Campbell and Perron (1991), the outcomes of unit root tests are often quite sensitive to the specification of the tests. In this comment, we reexamine the time series properties of the nonlinear version of the net discount ratio and find that the results of Haslag, Nieswiadomy, and Slottje are critically dependent upon the specification of the unit root test they performed. In particular, we find that their tests fail to adequately eliminate the serial correlation from the residuals. Moreover, once we eliminate the serial correlation from these tests, we fail to reject the null hypothesis of a unit root for all of the net discount ratios tested.

A unit root process is essentially a time series process that experiences a random change in its mean every time period. The standard augmented Dickey-Fuller procedure tests whether a time series has a constant mean or a mean that changes randomly with each new observation. Under certain circumstances, this battery of tests will falsely suggest that a series is a unit root process. In particular, as demonstrated by Perron (1990), it is possible to misidentify a time series that has a one-time shift in its mean as a unit root process even though it is stationary around its two separate means. This distinction is important for forecasting purposes. In the case of a unit root process, all of the information that is relevant for forecasting is contained in the most recent observation. In the case of a series that is stationary around a one-time shift in its mean, all of the observations made since the mean shift contain information about the possible future path of the series.

Although our augmented Dickey-Fuller tests appear to indicate the presence of a unit root in the net discount ratio, the graphical evidence suggests that there was a substantial shift in the mean of the series during the period 1977 through 1981. As a result of this mean shift, these tests might incorrectly identify the net discount ratios to be unit root processes. To investigate this possibility, we employ the procedure recommended by Zivot and Andrews (1992), which tests the null hypothesis of a unit root against the alternative hypothesis that the series is stationary around a one-time shift in its mean.

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[1] Estimating present values using the net discount rate concept is accomplished by first estimating earnings using a base wage such as the actual wage of the worker or the wage of workers with similar characteristics (e.g., education, age, etc.). Next, the present value of future earnings is calculated by assuming that earnings remain at this level over work-life expectancy, and discounting using the net discount rate. If the net discount rate is stationary, this procedure should provide reliable forecasts for the typical worker or a worker whose wage pattern is expected to follow that of the economy-wide average. These assumptions, of course, may not apply to any particular worker. If it is known, for example, that a worker is likely to deviate from the average due to occupation, likely occupational change, or other reasons, then the simple net discount calculations must be adjusted to account for these factors.

Using this test, we reject the unit root hypothesis in favor of the alternative of a one-time mean shift for all of the net discount ratios tested.

Our results indicate that the nonlinear version of the net discount ratio is a nonstationary series over the entire 1964 through 1989 time period. However, the source of nonstationarity appears to be a one-time shift in the mean of the series. Once the shift in the mean is removed, the net discount ratio becomes stationary. The most important implication of this finding is that the optimal forecast for the net discount ratio is not its grand mean over the 1964 through 1989 time period, but rather the mean since its last shift.

Data, Methodology, and Results

Following Haslag, Nieswiadomy, and Slottje, we first construct four net discount ratios using monthly data on the growth rate of the real average hourly wage in the private nonagricultural sector and the ex post real return on four different Treasury security maturities.[2] Table 1 replicates the augmented

Table 1
Unit Root Test for Net Discount Ratio
1964:1 through 1989:4 (Four Lag Case)

$$(1)\ \Delta X_t = \alpha + \beta X_{t-1} + \sum_{i=1}^{4} \delta_i \Delta X_{t-i}$$

$$(2)\ \Delta X_t = \alpha + \beta X_{t-1} + \sum_{i=1}^{4} \delta_i \Delta X_{t-i} + \gamma t$$

Interest Rate	t-Statistic $\beta = 0$	F-Statistic $\beta = \gamma = 0$	Q-Statistic Model (1)	Q-Statistic Model (2)
Three-Month Rate	-4.02*	14.09*	24.8 (0.16)	21.5 (0.44)
Six-Month Rate	-4.98*	20.4*	28.0 (0.01)	23.4 (0.02)
One-Year Rate	-4.20*	14.02*	15.4 (0.22)	12.5 (0.40)
Three-Year Rate	-3.21*	8.28*	23.7 (0.02)	21.3 (0.05)

Note: The Q-statistics are evaluated at 12 lags of the estimated residuals. In parentheses below the Q-statistics are the probabilities of incorrectly rejecting the null hypothesis of no serial correlation in the estimated residuals. The 5 percent critical value for the t-ratio is -2.88. The 5 percent critical value for the F-ratio is 6.34.
* Significant at the 5 percent level.

[2] We used the same method described by Haslag, Nieswiadomy, and Slottje to construct the ex post real interest rate; that is, we subtracted the inflation rate that occurred over the term of the security from the nominal interest rate on the security.

Table 2
Unit Root Test for Net Discount Ratio
1964:1 through 1989:4 (Highest Significant Lag Case)

$$(1)\ \Delta X_t = \alpha + \beta X_{t-1} + \sum_{i=1}^{m} \delta_i \Delta X_{t-i}$$

$$(2)\ \Delta X_t = \alpha + \beta X_{t-1} + \sum_{i=1}^{m} \delta_i \Delta X_{t-i} + \gamma t$$

where m is chosen to be the highest significant lag.

Interest Rate	*t-Statistic* $\beta = 0$	*F-Statistic* $\beta = \gamma = 0$	*Q-Statistic Model (1)*	*Q-Statistic Model (2)*
Three-Month Rate	-1.47	2.67	1.95	1.94
	(17)	(17)	(0.99)	(0.99)
Six-Month Rate	-1.59	2.93	2.26	2.22
	(17)	(17)	(0.99)	(0.99)
One-Year Rate	-1.81	3.22	2.41	2.39
	(17)	(17)	(0.99)	(0.99)
Three-Year Rate	-2.29	4.83	2.33	2.49
	(17)	(17)	(0.99)	(0.99)

Note: The Q-statistics are evaluated at 12 lags of the estimated residuals. In parentheses below the Q-statistics are the probabilities of incorrectly rejecting the null hypothesis of no serial correlation in the estimated residuals. The numbers in parentheses below the t- and F-statistics are the numbers of lagged dependent variables used in the test (m). The 10 percent critical value for the t-ratio is -2.57. The 10 percent critical value for the F-ratio is 5.34.

Dickey-Fuller tests conducted by Haslag, Nieswiadomy, and Slottje utilizing four lags of the dependent variable. Our results, just as theirs do, reject the hypothesis of a unit root for all four measures of the net discount ratio.

As previously mentioned, however, the results of the augmented Dickey-Fuller test are sensitive to the number of lags included in the regression and give biased results in the presence of occasional shifts in the mean of the series. With respect to the number of lags, the test results can be biased by including either too few or too many lags. Campbell and Perron (1991) suggest that, to determine the appropriate number of lagged dependent variables, one should start with a large number of lags and continue to eliminate them one at a time until the last lag is significantly different from zero.

We employ this suggested technique and find the number of lags in all regressions to be 17. This far exceeds the four lags used by Haslag, Nieswiadomy, and Slottje. Table 2 shows the results of the augmented Dickey-Fuller tests when the lag length is chosen in this manner. Unlike the results in Table 1, we now fail to reject the unit root hypothesis in all of the net discount ratios tested.

Table 3
Unit Root Test for Net Discount Ratio 1964:1 through 1989:4
(Akaike Information Criteria (AIC) Lag Case)

$$(1)\ \Delta X_t = \alpha + \beta X_{t-1} + \sum_{i=1}^{m} \delta_i \Delta X_{t-i}$$

$$(2)\ \Delta X_t = \alpha + \beta X_{t-1} + \sum_{i=1}^{m} \delta_i \Delta X_{t-i} + \gamma t$$

where m is chosen such that the AIC is minimized.

Interest Rate	*t-Statistic* $\beta = 0$	*F-Statistic* $\beta = \gamma = 0$	*Q-Statistic Model (1)*	*Q-Statistic Model (2)*
Three-Month Rate	-1.60	3.98	0.45	4.19
	(10)	(8)	(0.99)	(0.98)
Six-Month Rate	-1.84	4.62	0.71	2.23
	(10)	(10)	(0.99)	(0.99)
One-Year Rate	-2.28	4.48	1.19	1.03
	(8)	(8)	(0.99)	(0.99)
Three-Year Rate	-1.82	2.65	5.7	6.24
	(8)	(8)	(0.93)	(0.90)

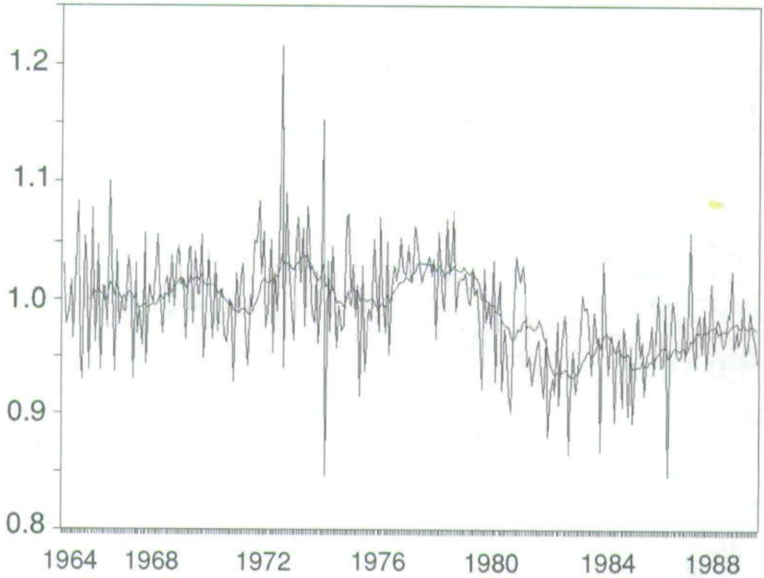
Note: The Q-statistics are evaluated at 12 lags of the estimated residuals. In parentheses below the Q-statistics are the probabilities of incorrectly rejecting the null hypothesis of no serial correlation in the estimated residuals. The numbers in parentheses below the t- and F-statistics are the numbers of lagged dependent variables used in the test (m). The 10 percent critical value for the t-ratio is -2.57. The 10 percent critical value for the F-ratio is 5.34.

To determine whether these results are robust with respect to lag length, we next employ the alternative Akaike lag length selection criteria. This procedure produces lags ranging from eight to ten. These also exceed the four lags used by Haslag, Nieswiadomy, and Slottje. The results of the augmented Dickey-Fuller tests using this criteria are shown in Table 3. Again, in all of the cases tested, we fail to reject the null hypothesis of a unit root.

Although the evidence in Tables 2 and 3 indicates that the net discount ratio is a unit root process, attention also must be given to alternative specifications of the deterministic components of the series. In particular, as described above, Perron (1990) has shown that the standard augmented Dickey-Fuller tests may falsely identify a series as a unit root process when in fact it is a stationary process around a one-time shift in its mean. Figure 1 plots the net discount ratio for a three-month Treasury bill over the 1964 through 1989 time period.[3] It appears that the net discount ratio fluctuated around two distinct means, with the shift occurring around 1980. If such a shift did occur, then it may be more reasonable to model the net discount ratio as stationary around a one-time shift in its mean, rather than as a pure unit root process.

[3] Graphs for other interest rate series follow a similar pattern and are available from the authors upon request.

Figure 1
Net Discount Ratio and Its 12-month Moving Average (Smooth Line), Calculated for a Three-Month Treasury Security



To test the hypothesis of a unit root conditional on a mean shift, we must first determine the date of the mean shift. Perron (1990) recommends choosing exogenous events such as oil price shocks or the Great Depression as possible dates of mean (or trend) shifts in the data. Alternatively, Zivot and Andrews (1992) suggest endogenously searching for the most likely shift in the mean of the series. They argue that the choice of exogenous events is unavoidably determined by prior inspection of the data, and therefore the test employed by Perron (1990) is biased in favor of rejecting the null hypothesis of a unit root. Zivot and Andrews (1992) therefore calculate a series of critical values which take into account the fact that the data are examined prior to choosing a mean shift date.

Because inspection of Figure 1 constitutes prior inspection of the data, we employ the endogenous search algorithm recommended by Zivot and Andrews (1992). To implement this test, we estimate a separate augmented Dickey-Fuller test contingent on a shift in the mean of the series at every possible date in the sample. The date associated with the minimum t-statistic for the null hypothesis of a unit root is then chosen as the most likely shift in the series. Thus, allowing for lags and differences, we estimate the augmented Dickey-Fuller test contingent on mean shifts starting in August 1967 and running through April 1989. The minimum t-statistics associated with the null hypothesis of a unit root are presented in Table 4. Note that the three-month, six-month, and one-year net discount ratios experienced a shift in their

Table 4
Unit Root Test for Net Discount Ratio
1964:1 through 1989:4
Conditional on Mean Shift

$$(1)\ \Delta X_t = \beta X_{t-1} + \sum_{i=1}^{m} \delta_i \Delta X_{t-i}$$

where X is the demeaned series and m is the highest significant lag.

Interest Rate	*t-Statistic* $\beta = 0$	*Q-Statistic*	*Date of Mean Shift*
Three-Month Rate	-4.61*	1.42	January 1980
	(15)	(0.99)	
Six-Month Rate	-4.59*	1.77	January 1980
	(15)	(0.99)	
One-Year Rate	-4.51*	1.59	January 1980
	(15)	(0.99)	
Three-Year Rate	-4.52*	1.56	October 1979
	(15)	(0.99)	

Note: The Q-statistics are evaluated at 12 lags of the estimated residuals. In parentheses below the Q-statistics are the probabilities of incorrectly rejecting the null hypothesis of no serial correlation in the estimated residuals. The numbers in parentheses below the t-statistics are the numbers of lagged dependent variables used in the test (m). The 1 percent critical value for the t-ratio is -4.45 and may be found in Zivot and Andrews (1992), Table 2, p. 156. The time of break relative to total sample, λ, = 0.60 for October 1979 and 0.61 for January 1980.
* Significant at the 1 percent level.

respective means in January 1980. The three-year net discount ratio experienced a shift in its mean in October 1979. The critical values for the null hypothesis differ from the standard Dickey-Fuller critical values in two respects. First, they depend upon the break fraction λ, which is 0.60 for the break in October 1979 and 0.61 for the break in January 1980.[4] Second, they are higher (in absolute value) because of the endogenous search for the mean shift.

The t-statistics reported in Table 4 indicate that the null hypothesis of a unit root is rejected at the 1 percent level for all of the net discount ratios. These results suggest that the net discount ratio is not a pure unit root process, but rather one that is stationary around a one-time shift in its mean.

The search procedure described above identified October 1979 and January 1980 as the most likely dates of shifts in the means of the net discount ratios. Yet it is clear from Figure 1 that over the period 1977 through 1981 the net

[4] The break fraction, λ, is the fraction of the full sample period prior to the break. The full sample period 1964:1 through 1989:4 consists of 304 monthly observations; however, after lagging and differencing, the sample is reduced to 285 observations. For the net discount ratio employing the three-month, six-month and one-year interest rates, the mean shift occurred at observation 179 (January 1980) or 0.61 of the entire sample. For the net discount ratio employing the three-year interest rate, the mean shift occurred at observation 175 (October 1979) or 0.60 of the entire sample.

discount ratio experienced a gradual change. Much of this change can be attributed to changes in the behavior of real interest rates over this period. Economists (Wilcox, 1983; Summers, 1983; Barro and Sali-i-Martin, 1990; and Antoncic, 1986) attribute this change to the confluence of several factors, including the reduction in world oil prices which raised the marginal product of capital, the removal of direct interest rate regulation by the Federal Reserve, improved sophistication of markets in anticipating inflation, and the Federal Reserve Bank's increased emphasis on controlling inflation.

Table 5 presents the subsample and full-sample means of the net discount ratio. Table 6 presents the subsample and full-sample means of the net discount rate. It is clear from these two tables that there was a shift in the mean of the net discount ratio and net discount rate during the 1964 through 1989 period. As a consequence of this shift, it is inappropriate to use the entire sample average to forecast the future mean of the series. Rather, the appropriate forecast of the mean of the net discount ratio (or rate) is the sample average since the shift. Using the sample average since the shift yields net discount rates two to three times greater than those found by Haslag, Nieswiadomy, and Slottje.

Conclusion

In sum, our results indicate that the nonlinear net discount ratio is best characterized as a stationary series around a one-time shift in its mean. This implies that the optimal forecast of the future net discount ratio (or rate) is its mean value since the last shift, not its grand mean over the 1964 through 1989 time period. Although this procedure produces the optimal forecast, a note of caution should be added. As Balke and Fomby (1991) point out, the mere fact that a series has experienced a mean shift in the past implies some probability that it will shift again in the future. From a forecasting perspective, the critical

Table 5
Net Discount Ratios
Pre- and Post-Mean Shift Sample Averages

Interest Rate	*Pre-Shift Average*	*Post-Shift Average*	*Full-Sample Average*
Three-Month Rate	1.01	0.96	0.99
	(0.04)	(0.04)	(0.05)
Six-Month Rate	1.01	0.96	0.99
	(0.05)	(0.04)	(0.05)
One-Year Rate	1.01	0.96	0.99
	(0.05)	(0.04)	(0.05)
Three-Year Rate	1.01	0.94	0.99
	(0.05)	(0.04)	(0.06)

Note: Standard deviations are in parentheses below the means. For the three-month, six-month, and one-year rates, the pre-shift sample period is 1964:1 through 1979:12 and the post-shift sample period is 1980:1 through 1989:4. For the three-year rate, the pre-shift sample period is 1964:1 through 1979:09 and the post-shift sample period is 1979:10 through 1989:4.

question is how *often* a shift might occur. A series with a frequently shifting mean is virtually as difficult to forecast as a pure unit root process. However, the factors discussed above that have been said to account for the shift in the mean of the net discount ratio seem to be either infrequent events or permanent changes in the economic environment. As a result, we might expect such shifts to be infrequent. Nonetheless, since future shifts are always possible, continuous monitoring of the series appears warranted.

Table 6
Net Discount Rates
Pre- and Post-Mean Shift Sample Averages

Interest Rate	*Pre-Shift Average*	*Post-Shift Average*	*Full-Sample Average*
Three-Month Rate	-0.008	0.04	0.01
	(0.05)	(0.05)	(0.05)
Six-Month Rate	-0.007	0.05	0.01
	(0.05)	(0.05)	(0.06)
One-Year Rate	-0.009	0.05	0.01
	(0.05)	(0.05)	(0.06)
Three-Year Rate	-0.008	0.07	0.02
	(0.05)	(0.05)	(0.06)

Note: Standard deviations are in parentheses below the means. For the three-month, six-month, and one-year rates, the pre-shift sample period is 1964:1 through 1979:12 and the post-shift sample period is 1980:1 through 1989:4. For the three-year rate, the pre-shift sample period is 1964:1 through 1979:09 and the post-shift sample period is 1979:10 through 1989:4.

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