Bank Capitalization, Deposit Insurance, and Risk Categorization

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ABSTRACT

This article examines the complementary relationship between bank capitalization and deposit insurance as tools that reduce the exposure of risk-averse depositors to a bank's random portfolio returns. In a costly state verification framework where banks possess private information about solvency, we establish that debt contracts are optimal and that banks both capitalize and purchase insurance. Moreover, optimal insurance contracts charge premiums which are conditioned on bank risk as indicated by the level of capitalization. A full-insurance scheme without risk categorization results in an equilibrium where banks totally decapitalize.

Introduction

Recent proposals to reform bank deposit insurance have suggested a linking of premiums to observable variables that reflect underlying bank risk, yet the economics of such insurance are not well understood (see, for example, Kane, 1985, 1989, and U.S. Congress, 1990). Although substantial debate exists within the policy environment regarding the economic incentives engendered by the current program, no formal analysis has examined either the role of such insurance, as compared to self-insurance through bank capitalization, or the design of optimal insurance plans. This article provides a model of banks, capitalization, and deposit insurance in a costly state verification environment of the type considered by Williamson (1986) where banks possess private information about the realization of their random portfolio returns. We demonstrate that, in the absence of deposit insurance, banks capitalize to partially insure risk-averse depositors against bank insolvency, but that deposit

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insurance which spreads risk across banks permits efficiency gains. In addition, the optimal insurance plan entails partial insurance against bank failure and categorizes banks' risks by making the deposit insurance premiums depend upon their chosen levels of capitalization.

The primary antecedents to this work are Kareken and Wallace (1978), who examine the moral hazard in portfolio selection engendered by deposit insurance, the model of bank runs provided by Diamond and Dybvig (1983), and the optimality of risk categorization in (nondeposit) insurance contexts considered by Crocker and Snow (1986) and Bond and Crocker (1991). Kareken and Wallace present a partial equilibrium model examining a bank's choice of investment portfolio under a variety of regulatory and deposit schemes. Theirs is an environment of complete markets, where the form of debt contracting is imposed exogenously, and the primary concern is with the tendency of regulated banks to select overly risky portfolios when insurance premiums do not reflect underlying portfolio risks.[1] Diamond and Dybvig, on the other hand, examine the potential for banks to suffer runs when their investment portfolios exhibit short-run illiquidities.[2] They demonstrate the existence of multiple Nash equilibria, one of which is characterized as a bank run where all depositors attempt to withdraw their funds prematurely. Suspension of convertibility or deposit insurance is shown to eliminate such an equilibrium by guaranteeing the return of those depositors not participating in the run. But, although their model provides substantial insight into the equilibrium mechanics of bank runs, there is no guidance regarding the risk spreading role of deposit insurance or of the design of optimal insurance plans.

In contrast, this article considers an environment without bank runs or moral hazard and addresses instead the importance of insurance to risk-averse depositors who face the possibility of bank insolvency in the event of adverse portfolio returns. We draw on the costly state verification framework developed by Townsend (1979) and applied by Diamond (1984), Gale and Hellwig (1985), and Williamson (1986) in the context of financial markets to character-

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[1] The portfolio selection issue is also addressed by Sharpe (1978), who demonstrates that increases in portfolio risk make capital more valuable and FDIC deposit guarantees more costly, and by Buser, Chen, and Kane (1981), who argue that the moral hazard incentives are mitigated by regulatorily-imposed costs that increase with the level of bank risk. Also relevant, but in a somewhat different vein, is the recent work by Flannery (1991), who examines an environment of exogenous portfolio risk where the insurer estimates that risk with error and finds that, in contrast to the errorless case, the optimal pricing of insurance requires both risk-based capital requirements and insurance premiums.

[2] The runs are generated by impatient depositors withdrawing their funds prior to the maturation of the investment project, which generates a social cost because the associated investment returns are forfeit. Patient depositors may also participate in a run because the illiquid bank is unable to satisfy fully the premature claims by all of the depositors. If, however, the impatient depositors were able to sell their future claims to patient depositors (as in the case of, say, negotiable CDs or equity interest), then the illiquidity problems would be alleviated and runs eliminated (see Jacklin, 1987 for a nice discussion). Moreover, the guarantee provided by deposit insurance is largely redundant, as short-term illiquidities can be alleviated by the Federal Reserve in its role as lender of last resort (See U.S. Congress, 1990, p. 5).

ize optimal debt contracts between depositors and banks. Banks are assumed to have access to profitable investment opportunities of known, and exogenously determined, risk. In addition, we permit the banks to adopt an equity position in the investment portfolio through the provision of capital. We find that efficient contracts entail positive levels of capitalization, which eliminates the problem of credit rationing encountered by Williamson (1986), and provide an insurance benefit to depositors by reducing the probability of bankruptcy.[3] We also demonstrate that, in the absence of deposit insurance, the equilibrium level of capitalization increases as depositors become more risk averse.

Permitting access to an actuarially fair risk-spreading pool provides banks with an additional tool with which to protect depositors from fluctuations in portfolio returns. While capitalization results in self-protection by reducing the probability of insolvency, the benefit of deposit insurance is its ability to reduce the size of the loss suffered in the bankrupt state, so that these tools are complementary.[4] The optimal deposit insurance plan assigns premiums based on bank riskiness, as determined by capitalization levels, resulting in endogenous categorization similar to that examined by Bond and Crocker (1991). In addition, the equilibrium entails positive levels of capitalization, which reduces the probability of bankruptcy, as well as purchases of insurance to protect depositors against the potential for bank insolvency inherent in an environment of limited liability. Moreover, the optimal insurance plan furnishes less than full insurance, which provides depositors with an incentive to require that banks self-protect through capitalization.[5] On the other hand, the current system of full insurance without categorization is shown to provide no incentive to self-protect, generating increased bank risk through decapitalization.[6]

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[3] Indeed, in our model capitalization would be desirable even in the case of risk-neutral depositors because it alleviates the credit rationing examined by Williamson (1986). Interestingly, Gale and Hellwig find the level of equity participation to be irrelevant in their environment, a result that obtains because of risk-neutral depositors who obtain no benefit from the insurance provided by capitalization.

[4] This terminology is attributable to Ehrlich and Becker (1972).

[5] In contrast to the full insurance afforded depositors by current FDIC policy, coinsurance is quite common in other deposit insurance plans. In Britain, for example, 75 percent of deposits are covered up to a maximum of £20,000, while Italy insures 100 percent of the first L 200m and 75 percent of the next L 800m. In addition, Ireland, Argentina, and Chile provide for decreasing coverage as the size of the deposit increases. Germany provides an interesting twist; depositors are insured for up to 30 percent of the bank's equity capital.

[6] Our results on the design of optimal insurance plans apply to any actuarially fair program regardless of how it is administered. The question of whether such plans should be public or private turns on the issue of how best to insure against correlated risks, an issue not addressed here. Some have suggested (see, for example, U.S. Congress, 1990, p. 23) that correlated risks are inherently uninsurable, implying the need for governmentally-sponsored risk spreading programs. Alternatively, Doherty and Dionne (1993) argue that the mutuality inherent in government plans can be achieved by private means through the use of reinsurance. The results presented in this article, however, would apply in either case and provide guidance on the design of an actuarially fair deposit insurance program.

A Model of Bank Capitalization

This section provides a model of a competitive banking equilibrium in the absence of deposit insurance. Banks are assumed to be risk-neutral agents who raise investment capital and must choose between financing projects by supplying their own capital or through deposits from risk-averse depositors. The optimal contract between risk-averse borrowers and risk-neutral banks is shown to be a debt contract, and, in the competitive equilibrium, projects are financed using both deposits and bank capital, with the amount of capitalization being greater when depositors are more risk averse.

Our treatment of banks follows the seminal work of Diamond (1984), who viewed banks as financial intermediaries that perform the role of monitoring the investment projects of entrepreneurs for which depositors have provided funding. Diamond assumes that the outcomes of investment projects are privately observed by entrepreneurs, and can only be observed at a cost by lenders. If there are many lenders contracting directly with each entrepreneur, monitoring is duplicated because each depositor must incur the monitoring cost to observe the outcome of the project. Financial intermediaries, such as banks, arise in this context because they economize on monitoring costs and collect payments from the entrepreneurs on behalf of the lenders. However, this creates a monitoring problem for depositors at the bank, since they must still incur the positive monitoring cost to observe the realization of the bank's portfolio of investment projects. Diamond shows that, in addition to economizing on monitoring costs, intermediation provides an advantage as a consequence of diversification. As the number of projects financed by the bank increases without bound, the depositors' expected monitoring costs approach zero if project returns are independently distributed. In effect, depositors never have to monitor the bank because diversification guarantees that the obligation to depositors can always be met.

We follow the approach of Diamond by assuming banks to be risk-neutral agents that monitor investment projects for depositors, but we also assume that there are organizational costs limiting the size of the bank, which prevent banks from completely diversifying the risks of projects for depositors. To simplify, we fix the optimal number of investment projects for the bank to monitor and choose units such that the optimal number of projects requires one unit of capital to finance.

The limit on the size of banks which inhibits the diversification of portfolio risk is critical to the analysis, because the resulting uncertainty will motivate risk-averse depositors to require that banks capitalize and purchase insurance to provide protection from fluctuations in deposit returns. One potential source of this risk arises from restrictions on branch banking, which tend to prevent banks from diversifying as well as they might otherwise.[7] Another possibility involves banks in their role of searching for and monitoring the performance

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[7] We are indebted to a referee for this suggestion.

of profitable investment opportunities. To the extent that such activities rely on the acquisition of localized private information, banks may face increasing costs of portfolio diversification.[8] The recent experience with illiquidities in the banking and the savings and loan sectors suggests that, at least in the United States, uncertainty in portfolio returns is an important concern for risk-averse depositors.

Banks are assumed to have access to investment projects, each of which has a return denoted by the random variable x, which is described by the probability density function f(x) on the compact support $[\underline{x},\bar{x}]$ with an expected value of ρ. Bank owners are assumed to be risk neutral and to have capital of their own that can be invested in the bank's projects. Since the owners could always unilaterally invest their capital in the projects without depositors' participation, the opportunity cost for capital is given by the expected project return, ρ. Letting w(x) denote the payment by the bank to depositors when the return to the bank is x and K indicate the amount of owners' capital invested in the bank, the expected profits of the bank will be

$$\pi(w(x),K) = \int_{\underline{x}}^{\bar{x}} (x-w(x))f(x)dx - \rho K - \bar{C}, \tag{1}$$

where $\bar{C}$ denotes the resource cost of setting up the bank to attract deposits.[9] The bank owners' limited liability is reflected in the restriction that $w(x) \leq x$.

There is a continuum of identical, risk-averse depositors each with endowment d to be deposited at the bank. With an investment of K by bank owners, the bank must raise D = 1-K in deposits, which requires n = (1-K)/d depositors.[10] Depositors are assumed to be able to observe the realization of the bank's portfolio only by incurring a fixed and exogenously determined monitoring cost γ. The problem of finding the optimal contract between depositors and the bank is an example of contracting in the presence of costly state verification, which was first addressed by Townsend (1979) and applied in a financial context by Gale and Hellwig (1985). A contract between the bank and depositors consists of a payment schedule w(x) and a monitoring set M $\subset$

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[8] For example, a bank may have experience in screening real estate investment projects which results in a comparative advantage in identifying local, but correlated, investment opportunities. To diversify the portfolio would require the addition of more distant uncorrelated projects where the bank has less of a screening advantage.

[9] These costs include the provision of transaction services, such as check clearing or automatic teller machines, as well as the location of branch offices to obtain the deposits of small, geographically dispersed depositors. Thus, $\bar{C}$ is a transaction cost of using deposits to fund an investment project.

[10] We are implicitly assuming that bank owners can invest their capital and receive the opportunity cost ρ in multiple banks, so that the constraint on a representative bank's size (normalized to unity) does not constrain the owners' investment opportunities.

$[\underline{x},\overline{x}]$.[11] If the bank announces a state $x \varepsilon M$, then the depositors incur the monitoring cost γ (= $\underline{x}$) and receive payment $w(x) - \gamma$. If the bank announces a state in the complement of M, denoted M^C, then the depositors do not monitor and receive $w(x)$. In order for the contract to be incentive compatible, the payment $w(x)$ must be a constant, R, for all $x \varepsilon M^C$. If not, the bank would always announce the $x \varepsilon M^C$ that yielded the lowest payment.

Let $u(y)$ be the depositors' strictly increasing and concave utility function defined over individuals' wealth y, where $u'(0) = \infty$. The utility of a depositor from a deposit contract will be $u((w(x)-\gamma)d/(1-K))$ for $x \varepsilon M$ and $u(Rd/(1-K))$ for $x \varepsilon M^C$, where the bank's total payment w is divided equally among the n depositors participating in the project. Since d is constant throughout the analysis, the notation can be simplified by defining $U(y) \equiv u(yd)$. The expected utility of each depositor in the contract will be

$$V(w(x),M,R,K) = \int_M U((w(x) - \gamma)/(1-K))f(x)dx + \int_{M^c} U(R/(1-K))f(x)dx. \quad (2)$$

Under this specification, deposits are indivisible, so that consumers place all of their funds in a single bank. This aspect of the model prevents depositors from totally diversifying risk by holding arbitrarily small deposits at a large number of independent banks, resulting in an exposure to risk that drives the demand for capitalization and deposit insurance analyzed below. While the nondivisibility of deposits is critically important in this model, the assumption

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[11] We simplify by considering only deterministic monitoring strategies. This has the advantage of generating contracts similar to the contracts actually offered to depositors. Townsend (1979) notes that it may be possible to achieve superior outcomes with stochastic verification, since the possibility of monitoring with a positive probability may induce truthful revelation at a lower resource cost. This requires, however, that the informed agent bear a penalty cost as a consequence of being monitored, so that incentives for truthful revelation can be maintained by increasing the magnitude of the penalty while simultaneously reducing the probability of monitoring. To illustrate the conditions under which stochastic monitoring may be preferred, consider the case where agents have entered into a deterministic monitoring contract of the type examined in Lemma 1. In this situation, the banks have nothing to gain by misrepresenting portfolio returns in the monitoring region since, by assumption, such information will be revealed by the deterministic monitoring. Now, suppose that in the event monitoring occurs, and the bank is found to have misrepresented portfolio returns as $\hat{x}$ rather than their actual value x, the bank can be forced to pay the penalty M. We can show that random monitoring, where $B(\hat{x})$ denotes the probability of monitoring when the bank reveals $\hat{x}$, can maintain the incentives for truthful revelation. The bank would be worse off revealing $\hat{x}$ rather than x as long as $(1-B(\hat{x}))(x-\hat{x}) - B(\hat{x})M \leq 0$. In order for this to hold for every $x < R$, it is sufficient that $B(\hat{x}) \geq (R-\hat{x})/(R-\hat{x}+M)$. Thus, the random monitoring scheme $B(\hat{x})$, in conjunction with the penalty M, is sufficient to induce truthful revelation by banks, and at a lower cost than deterministic monitoring. On the other hand, if $M = 0$ (as we have assumed in this article), then for truthful revelation we must have $B(\hat{x}) = 1$, so that deterministic monitoring is required. Considering stochastic verification would substantially complicate the analysis but should yield the same basic role for capitalization discussed below.

Moreover, we are assuming that the depositors can credibly commit to monitor when banks announce a portfolio return in the monitoring set M. Without such a commitment, depositors could opt not to engage in costly state verification, leading to obvious incentive problems. The issue of credible commitment is addressed by Melumad and Mookherjee (1989), who argue that delegation of the auditing task to a third party is the most effective solution.

seems reasonable in light of the use of deposits for transactions purposes. Depositors face a trade-off between reduced risk from diversifying across banks, on the one hand, and the increased cost of financing transactions using funds drawn from a large number of minuscule accounts, on the other. To the extent that there exist increasing costs of using multiple accounts for transactions, depositors would choose less than total diversification, resulting in the uncertainty regarding deposit returns which provides the foundation for the following analysis.

The optimal deposit contract offered by banks will maximize expected profits of banks (1), subject to $V(w(x),M,R,K) \geq \overline{V}$, and the incentive compatibility constraints. The following result, which is proven in the Appendix, establishes that the optimal contract is a debt contract.

Lemma 1. The optimal contract between banks and depositors takes the form

$$w(x) = \begin{cases} R & \text{if} \quad x \in M^C \\ x & \text{if} \quad x \in M \end{cases}$$

where $M \equiv [\underline{x}, R)$.

This result is similar to that obtained by Townsend (1979), where the lender is risk neutral and the borrower is risk averse, and Williamson (1986), where both are risk neutral.

Competitive Equilibrium

Lemma 1 characterizes the optimal contract between banks and depositors for a given K and $\overline{V}$. The optimal level of K will be chosen to maximize profits of banks for a given $\overline{V}$, which yields a maximum expected profit for banks denoted $\pi(\overline{V})$. If there is a perfectly elastic supply of identical banks, a competitive equilibrium will occur at the expected utility level of depositors V^* where $\pi(V^*) = 0$. The solution to this problem will be illustrated using indifference curves and isoprofit contours in R-K space, as shown in Figure 1.

Given Lemma 1, the expected utility of depositors can be written as

$$V(R,K) = \int_{\underline{x}}^{R} U((x-\gamma)/(1-K))f(x)dx + U(R/(1-K))(1-F(R)). \tag{3}$$

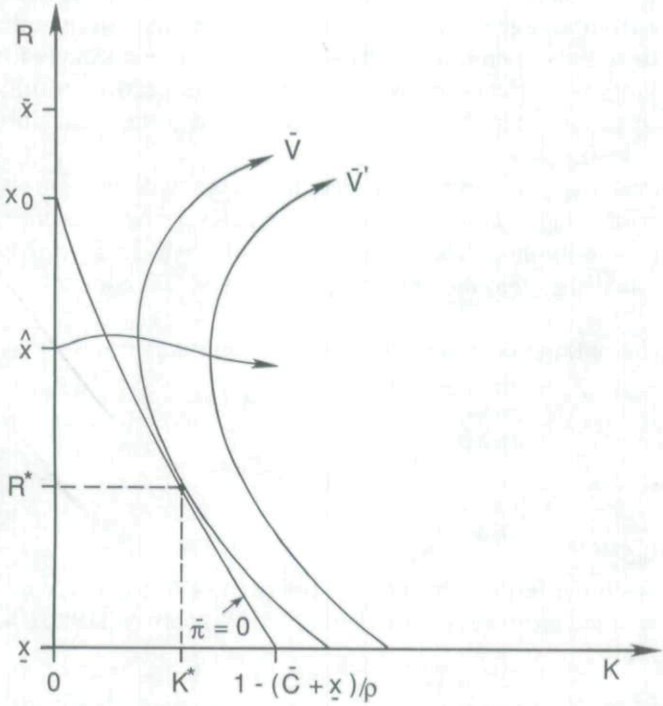
Holding R (the payout to all depositors) constant, an increase in K reduces the number of depositors and raises the expected return per depositor, as can be seen by

$$V_K(R,K) = \int_{\underline{x}}^{R} \frac{(x-\gamma)}{(1-K)^2} U'\left(\frac{(x-\gamma)}{1-K}\right) f(x)dx + (1-F(R)) \frac{R}{(1-K)^2} U'\left(\frac{R}{1-K}\right) > 0. \tag{4}$$

On the other hand, for given K, the effect of an increase in R is

$$V_R(R,K) = \frac{(1-F(R))}{1-K} U'\left(\frac{R}{1-K}\right) - f(R)\left[U\left(\frac{R}{1-K}\right) - U\left(\frac{R-\gamma}{1-K}\right)\right], \tag{5}$$

Figure 1



which is of ambiguous sign. There are two conflicting effects of an increase in R on the expected utility of depositors. An increase in R raises utility by raising the return in the solvent state, which is captured by the first term in equation (5). However, an increase in R makes bankruptcy (monitoring) more likely, which lowers the return to depositors as indicated by the second term in equation (5). As $R \rightarrow \overline{x}$, the first term approaches zero and the return will be decreasing in R, since, by the assumption of compact support, $f(\overline{x})$ must be nonzero. This yields positively-sloped indifference curves for R sufficiently large. Alternatively, as $R \rightarrow \underline{x}$, V_R becomes negative so that the indifference curves of depositors must be negatively sloped for R sufficiently small. Figure 1 illustrates the case in which the indifference curves are strictly convex to the origin.

As a consequence of Lemma 1, the expected profit for banks can be written as

$$\pi\,(R,K) \;=\; \int_R^{\overline{x}} (x - R)\, f(x)\, dx \;-\; \rho\, K \;-\; \overline{C}, \tag{6}$$

which is decreasing in K and nonincreasing in R. The isoprofit loci in Figure 1 must be negatively sloped, with slope

$$dR/dK \;=\; -\;\rho\,/\,(1 - F(R)). \tag{7}$$

The isoprofit loci are convex to the origin since the profit function is strictly quasi-concave in (R,K). A competitive equilibrium will occur at the point on the zero isoprofit locus yielding the highest expected utility to depositors.

The characteristics of an equilibrium are determined by the magnitude of the fixed cost of setting up the bank, $\overline{C}$. In the case where $\overline{C} = 0$, the competitive equilibrium occurs at the corner where $R = \underline{x}$ and $K = 1 - (\overline{C} + \underline{x})/\rho$, which yields a return of ρ on the deposits invested in the project. In this case, monitoring never occurs, and depositors receive the certain return ρ by using capital to leverage the gross return $\underline{x}$. When $\overline{C}$ is positive, however, the cost to depositors of increased capitalization is not only the opportunity cost of capital ρ, but also their share of the fixed costs $\overline{C}$. As capitalization increases, so that fewer depositors are required to finance the project, the remaining depositors are forced to support a larger portion of $\overline{C}$. For $\overline{C}$ sufficiently large, depositors prefer to spread the fixed costs by accepting lower levels of capitalization, resulting in an interior solution ($R > \underline{x}$) of the type depicted in Figure 1. Finally, for very large values of $\overline{C}$, a corner solution where $K = 0$ is a possibility.

An interesting point to note is the role of capitalization in establishing the existence of this equilibrium. If bank owners had no capital, then the zero profit locus would shrink to the single point x_o on the vertical axis. If the transactions cost $\overline{C}$ of using deposits to fund the project are sufficiently low, the point that maximizes expected utility of depositors along the vertical axis, $\hat{x}$, must be below the vertical intercept of the zero-profit locus,[12] which means that the banks would have to earn positive profits in equilibrium. This situation is analogous to the models of credit rationing analyzed by Stiglitz and Weiss (1981) and Williamson (1986), where an increase in the interest rate paid by borrowers does not make lending more attractive because it increases the probability of default. Capitalization eliminates this problem, since it allows banks to compete in another dimension to make lending more attractive to depositors. Even though increased capitalization reduces the return (R) on the debt contract, the reduction in deposits required to finance the project as a consequence of the capitalization results in an increase in the expected return $\left(\frac{R}{1-K}\right)$ to the depositors.

Optimal Capitalization

The insurance role of capitalization can be seen by considering the effect of the degree of risk aversion of depositors on their indifference curves. The slope of an indifference curve in Figure 1 is obtained by dividing equation (4) by equation (5), which can be written as

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[12] This result obtains because x_o converges to $\overline{x}$ as $\overline{C}$ approaches zero. On the other hand, for $\overline{C}$ sufficiently large, a no-capitalization corner solution may be attained at $R = x_o$, so that banks earn zero profit in the absence of capitalization.

$$\frac{dR}{dK} = -\left[\frac{(1-F(R))R + \int_{\underline{x}}^{R}(x-\gamma)\dfrac{U'\left(\dfrac{x-\gamma}{1-K}\right)}{U'\left(\dfrac{R}{1-K}\right)}f(x)dx}{(1-F(R)) - f(R)(1-K)\int_{R-\gamma}^{R}\left[\dfrac{U'\left(\dfrac{x}{1-K}\right)}{U'\left(\dfrac{R}{1-K}\right)}\right]dx}\right]\left(\frac{1}{1-K}\right). \tag{8}$$

This slope is the value of an increment of capital to depositors, measured in terms of the total payout to depositors. The numerator of this expression becomes larger as the ratio of the marginal utility of income in the bankrupt states to the marginal utility of income in solvent states increases. Similarly, the denominator is a decreasing function of the ratio of marginal utility of income evaluated at $x \in [R-\gamma,R]$ to the marginal utility of income at $x = R$. Therefore, the capital is more valuable to depositors the greater is the ratio $U'(y)/U'(z)$ for $y < z$.

The following result shows that this ratio increases as depositors become more risk averse, as measured by the Arrow-Pratt index of absolute risk aversion.

Lemma 2. Let U and V be concave utility functions, where $-U''(y)/U'(y) > -V''(y)/V'(y)$ for all y. Then for any $y < z$, $U'(y)/U'(z) > V'(y)/V'(z)$.

This lemma is proven by defining the function $H(y,z) = [U'(y)/U'(z)]/[V'(y)/V'(z)]$ and showing that $H(y,z)$ is a decreasing function of y when U is more risk averse than V. Since $H(z,z) = 1$, this shows that $H(y,z) > 1$ for every $y < z$, establishing the result of the lemma.

As a consequence, the more risk averse are depositors, the steeper will be the depositor's indifference curves in their negatively-sloped region. In the neighborhood of a competitive equilibrium, an increase in the steepness of the indifference curves must lead to an increase in the amount of capital in the equilibrium, so that capitalization provides a measure of protection against adverse portfolio returns for risk-averse depositors. We summarize the results of this section in the following theorem:

Theorem 1. A competitive equilibrium without deposit insurance will result in zero profits for banks and, if $\overline{C}$ is sufficiently small, positive capitalization for all $\gamma > 0$. Increases in the degree of risk aversion by depositors will increase the equilibrium level of capitalization by banks.

Deposit Insurance

We now consider the effects of introducing deposit insurance into the competitive equilibrium analyzed above, where a deposit insurance contract consists of a premium ϕ per unit of deposits and a payment schedule $I(x)$ for the insolvent states in which $x < R$.[13] At the beginning of the period, the insurer collects a premium ϕ from the bank. In return for this payment, the insurer pays an amount $I(x)$ to depositors in the event that insolvency occurs and the depositors, after incurring the monitoring cost γ, have ascertained the portfolio payoff to be x.[14] Since the bank must raise total capital of amount $1 + \phi D$ to cover the funding of projects and the insurance premium, the total deposits of the bank will be $D = (1\text{-}K)/(1\text{-}\phi)$ and the total premium paid by the bank to the insurer is $\phi(1\text{-}K)/(1\text{-}\phi)$. The break-even condition for the insurance contract is that the premium at least cover the expected payments made under the contract in the insolvent states,

$$\alpha\ \phi\,(1 - K)/(1 - \phi) - \int_{\underline{x}}^{R} I(x)f(x)dx \geq 0, \tag{9}$$

where α is the return earned on the premiums held in the insurer's investment portfolio. We assume that, in the case where the bank is insolvent and the depositor has invoked the monitoring technology, the insurer can freely observe the portfolio return x.

A question that arises is the relationship between the earnings potential of a bank's investment project, ρ, and the return available to the insurers on collected premiums, α. Our approach is to view banks and insurers as distinct institutions having different areas of comparative advantage. Banks consolidate funds of small depositors for investment purposes and search for profitable investment opportunities. On the other hand, the role of the insurer is to provide a risk-sharing pool to reduce depositors' exposure to the randomness of project returns. Insurers must incur the costs of collecting premiums and verifying the capitalization of the banks in cases where the premium depends on capitalization. Therefore, we assume that $\alpha < \rho$ reflects the comparative disadvantage of the insurer in finding investment opportunities and the cost of operating the insurance fund.

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[13] A question that might arise at this juncture is whether the sequential approach--showing a debt contract to be optimal under costly state verification, then deriving insurance results--is equivalent to solving both the insurance and monitoring problems simultaneously. The answer is yes, because the monitoring and insurance problems are separable. The debt contract arises between a depositor and a bank, while the insurance contract is between the same depositor and an independent insurer. The debt result holds for any risk-averse or risk-neutral depositor, and the effect of making insurance available is to make depositors more risk neutral. In particular, the fact that the insurance contracts involve full categorization means that the impact of changes in R and K on the probability of loss are captured in the premium the firm pays. No moral hazard problem is introduced between the bank and the insurance in this case. Thus, the introduction of insurance may affect the amount of self-protection depositors demand through capitalization and, therefore, the return they receive, but the debt structure of the deposit contract remains optimal.

[14] Note that we have assumed that, even with deposit insurance, depositors continue to do the verification. While this approach highlights the risk-reduction role of deposit insurance, it does ignore another potential advantage: that of economizing on monitoring costs by having only the insurer, as opposed to all the depositors, verify the return on the bank's portfolio in the bankruptcy state.

Note that in the absence of this assumption, there would be no role for banks. If $\alpha = \rho$, then depositors would be better off if banks simply invested all deposits in the insurance fund, since the insurance fund would pay the same rate as the bank and would have no risk of failure.

With deposits of amount $(1-K)/(1-\phi)$, the number of depositors required to finance the project will be $n = (1-K)/(1-\phi)d$. For $x < R$, the total return to depositors inclusive of the insurance payout will be $Z(x) \equiv x + I(x) - \gamma$, with an individual depositor receiving $Z(x)(1-\phi)d/(1-K)$. Before proceeding, we establish a preliminary result, formally derived in the Appendix, which will be useful in the analysis that follows.

Lemma 3. An optimal insurance contract generates a fixed payment Z to all depositors when $x \in [0,P]$, where $P \equiv \min(Z + \gamma, R)$.

This result is a direct consequence of risk-averse depositors and risk-neutral banks. Consider an insurance plan that results in a nonconstant return $Z(x)$ for $x \in [0,P]$, and let $\hat{Z} \equiv \int_0^P Z(x)f(x)dx/F(R)$ be the fixed-return payment to depositors financed out of the existing insurance premiums. The concavity of U and Jensen's inequality imply that $\int_{\underline{x}}^P \frac{U(Z(x))\, f(x)dx}{F(R)} < U(\hat{Z})$, so that any optimal contract must involve a constant payment to depositors in the region of positive insurance payouts.

The implication of Lemma 3 is illustrated in Figure 2.[15] An optimal insurance contract equalizes depositors' income at Z in the lowest portfolio return states, where $x \in [0, Z + \gamma]$. For the less severe bankruptcy states (i.e., $x \in [Z + \gamma, R]$) and the solvent states there is no insurance payout. The intuition is that the insurance plan pays the most in the worst states, where income is lowest, and, consequently, the marginal utility received from additional income is highest.

Letting $\theta \equiv (1-\phi)/(1-K)$, the expected utility of a representative depositor will be

$$V(\phi,Z,R,\theta) = U(\theta Z)F(P) + \int_P^R U(\theta x - \theta\gamma)f(x)\,dx + U(\theta R)(1-F(R)), \tag{10}$$

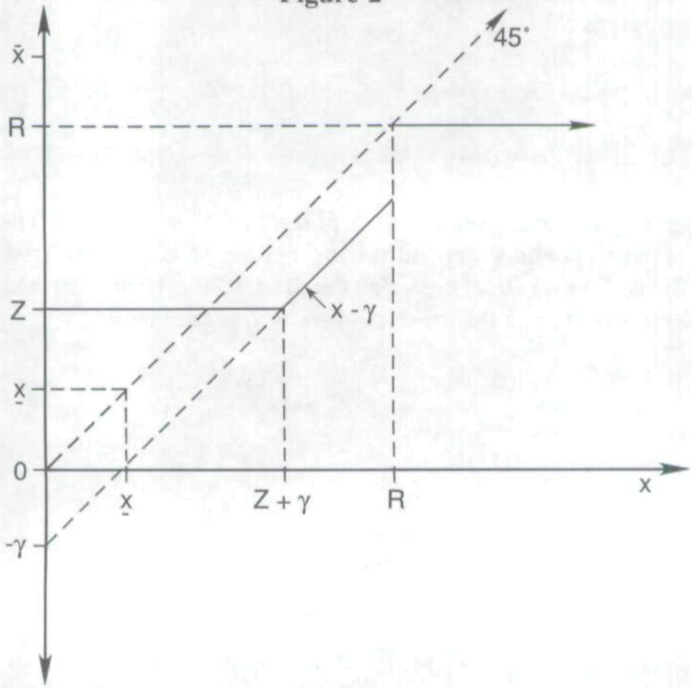
where, as before, $P \equiv \min(Z + \gamma, R)$. In addition, the insurance constraint is now

$$\frac{\alpha\,\phi}{\theta} - \int_0^P (Z - x + \gamma)f(x)\,dx \geq 0. \tag{11}$$

As in the previous section where there was no deposit insurance, the expected profits of banks will be equal to the expected returns from the project less payments to depositors and the costs of inputs provided by the owners of

[15] The case where $Z + \gamma < R$ is depicted. When $Z + \gamma > R$, depositors receive the constant return Z in all of the insolvent states.

Figure 2



the bank, which is given by equation (6), and is independent of the parameters of the insurance policy.

Optimal Insurance with Categorization

We now characterize the class of Pareto optimal allocations, which is obtained by choosing $\{\phi, Z, R, \theta\}$ to maximize the expected profits of banks, equation (6), given the resource constraint for insurers, inequality (9), and an expected utility constraint for depositors, $V \geq \overline{V}$. The set of Pareto optimal $\{V, \pi\}$ combinations is then obtained by varying $\overline{V}$.

This contracting problem can be placed in a market context by considering a bank as facing a given level of $\overline{V}$ that is provided to depositors by other firms in the market. The insurer can be treated as providing a premium schedule derived from inequality (11) which specifies the insurance premium as a function of the bank's choice of R, K, and coverage Z. This is an example of endogenous categorization by insurers, as in Bond and Crocker (1991), where insurance premiums are linked to observable actions that affect the probability of loss. This allows banks to compete both in the rate they pay and in the extent of insurance coverage that they provide to depositors, since both aspects of the deposit contract affect depositor utility. A competitive equilibrium with free entry of banks will occur at the value of $\overline{V}$ at which expected profits of the optimal contract are equal to zero.

The Lagrangean for the efficient contracting problem with deposit insurance can be written as

$$\max_{(\phi,Z,R,\theta)} L = \int_R^{\bar{x}} (x-R)f(x)dx - \rho K - \bar{C} + \mu[F(P)U(\theta Z) + \int_P^R U(\theta x - \theta\gamma)f(x)dx + (1-F(R))U(\theta R) - \bar{V}] + \lambda[\alpha\phi/\theta - \int_{\underline{x}}^{P} (Z+\gamma-x)f(x)dx], \tag{12}$$

where λ and μ are undetermined multipliers, and $K \equiv 1-(1-\phi)/\theta$. The nature of a solution will generally depend on the degree of depositors' risk aversion, which affects Z and, hence, the value taken by P. The first-order conditions for an interior maximum to this problem can be represented as

$$\frac{\partial L}{\partial \phi} = (\lambda\alpha - \rho)/\theta = 0 \tag{a}$$

$$\frac{\partial L}{\partial Z} = \mu\theta U'(\theta Z)F(P) - \lambda F(P) = 0 \tag{b}$$

$$\frac{\partial L}{\partial \theta} = \mu[ZU'(\theta Z)F(P) + \int_P^R U'(\theta x - \theta\gamma)(x-\gamma)f(x)dx + RU'(\theta R)(1-F(R))] - [\rho(1-\phi) + \lambda\alpha\phi]/\theta^2 = 0 \tag{c}$$

$$\frac{\partial L}{\partial R} = \mu[\theta U'(\theta R)(1-F(R)) - f(R)(U(\theta R) - U(\theta(P'-\gamma))] - \lambda(P'-R)f(R) - (1-F(R)) = 0, \tag{d}$$

where $P' \varepsilon \{Z+\gamma, R\}$ and $P' \neq P$.

Condition (a) establishes that $\rho = \lambda\alpha$, and condition (c) can be written to yield

$$\mu\theta^2[ZU'(\theta Z)F(P) + \int_P^R U'(\theta x - \theta\gamma)(x-\gamma)f(x)dx + RU'(\theta R)(1-F(R))] = \rho, \tag{13}$$

which, upon division by condition (b) yields

$$\theta ZF(P) + \left\{\int_P^R U'(\theta x - \theta\gamma)(\theta x - \theta\gamma)f(x)dx + (1-F(R))\theta RU'(\theta R)\right\}/U'(\theta Z) = \alpha. \tag{14}$$

The left-hand side of equation (14) is a continuous function of Z, which takes the value of 0 when Z = 0 and θR when Z = R. Consequently, an efficient insurance contract entails positive levels of insurance (Z > 0), but less than full coverage, as long as θR > α.

Using the zero-profit condition and the insurance constraint, we obtain $\theta R = \rho - (\rho - \alpha)\phi - \gamma F(R) - \bar{C}$ when deposits are fully insured. The return to depositors (θR) will be less than ρ due to the three resource costs: the fixed cost of setting up the bank ($\bar{C}$), the expected monitoring cost (γF(R)), and the cost of providing insurance ((ρ - α)ϕ). If θR ≤ 1, banks yield a payoff no larger than that which could be obtained by depositors on their own, and there is no reason for banks to exist. Thus, ρ > θR > 1 in an equilibrium with banks. Since we assume ρ > α, it will not be possible in general to rank θR and α. In the case of a pure risk-spreading pool where premiums are simply held by the insurers and earn a return of unity ($\alpha = 1$), we must have less than full insurance.

Theorem 2. The optimal insurance plan entails less than full insurance against bank insolvency if $\theta R > \alpha$. If $\alpha = 1$, depositors will not be fully insured.

This underinsurance result obtains because insurance may have an opportunity cost in the sense of foregone investment returns. This arises when deposits used to pay premiums (ϕD) earn an expected return (α) that is less than they would have had they been invested in the bank's portfolio (θR), resulting in a substitution away from insurance at the margin.

Optimal Capitalization with Insurance

Theorem 1 demonstrates that, in the absence of bank insurance, capital provides a measure of protection for risk-averse depositors facing the prospect of bank insolvency. In this section, we consider how the availability of actuarially fair deposit insurance affects banks' decisions to capitalize and, in particular, whether the presence of insurance eliminates the need for capitalization in the competitive equilibrium.

We may use the solution to the optimization problem (12) to establish the result. From condition (12b), it follows that $\lambda = \mu\theta U'(\theta Z)$ which, upon substitution into (12d), yields

$$\begin{aligned} &\mu\theta[U'(\theta R)(1-F(R)) - f(R)(U(\theta R) - U(\theta P') - \theta\gamma))/\theta] \\ &\quad - \mu\theta U'(\theta Z)(P'-R)f(R) = 1 - F(R). \end{aligned} \tag{15}$$

Dividing equation (13) by equation (15) gives

$$-\frac{\theta Z U'(\theta Z)F(P) + \int_P^R U'(\theta x - \theta\gamma)(\theta x - \theta\gamma)f(x)dx + \theta R U'(\theta R)(1-F(R))}{U'(\theta R)(1-F(R)) - f(R)(U(\theta R) - U(\theta P' - \theta\gamma))/\theta - U'(\theta Z)(P'-R)f(R)} = -\frac{\rho}{1-F(R)}, \tag{16}$$

where the left-hand side is the slope of a depositor's indifference curve (in R-K space) when actuarially fair insurance has been optimally provided, and the right-hand side is the slope of a bank's (zero) isoprofit contour. A sufficient condition for banks to capitalize in the presence of deposit insurance is that $\overline{C}$ be sufficiently small and that the slope of a depositor's indifference curve be positive when $R = \bar{x}$.

Two cases must be considered which correspond to different values of P. First, consider the case where $Z + \gamma > R$, so that $P = R$, and the slope of a depositor's indifference is given by

$$-\frac{\theta Z U'(\theta Z)F(R) + \theta R U'(\theta R)(1-F(R))}{U'(\theta R)(1-F(R)) - U'(\theta Z)(Z+\gamma-R)f(R) - f(R)(U(\theta R) - U(\theta Z))/\theta}. \tag{17}$$

The numerator is strictly positive and, as $R \to \bar{x}$, the denominator becomes negative, leading to positively sloped indifference curves. Next, consider the case where $Z + \gamma < R$, so that $P = Z + \gamma$, which yields the slope of an indifference curve to be

$$-\frac{\theta Z U'(\theta Z) F(Z+\gamma)+\int_{Z+\gamma}^{R} U'(\theta x-\theta\gamma)(\theta x-\theta\gamma) f(x) dx+\theta R U'(\theta R)(1-F(R))}{U'(\theta R)(1-F(R))-f(R)(U(\theta R)-U(\theta R-\theta\gamma))/\theta} \tag{18}$$

Once again, the numerator is positive and the denominator becomes negative as $R \rightarrow \overline{x}$.

A final point is that the numerators in expressions (18) and (17) represent the (positive) marginal utility depositors receive from increased capitalization. As a consequence, banks compete in the capitalization dimension, necessarily leading to zero profit in a competitive equilibrium. We may now summarize the results.

Theorem 3. A competitive equilibrium with categorized deposit insurance involves zero profit by banks and, for $\overline{C}$ sufficiently small, positive levels of capitalization.

Theorem 3 illustrates that deposit insurance and capitalization are not perfect substitutes because of the presence of monitoring costs. Deposit insurance is a more efficient form of insurance than owner's capital in the sense that it is not subject to the limited liability constraint, but capitalization is still desirable because it reduces the probability of monitoring. While it might seem plausible that an actuarially fair risk-spreading pool would provide an attractive substitute for insurance provided through costly capitalization, the effect of insurance on the equilibrium level of capitalization is, in general, ambiguous. This is due to the fact that capitalization is a form of self-protection which reduces the probability of insolvency. As is well known in other insurance contexts (Ehrlich and Becker, 1972; Dionne and Eeckhoudt, 1985), the incentive for individuals to invest in costly self-protection activities is not monotonically related to risk aversion, nor is market insurance generally a substitute for self-protection.

We have demonstrated that depositors benefit from banks' bundling of debt contrasts and deposit insurance, which raises the question of why such insurance is not associated with debt more generally. Our point is that traditional (nondeposit) debt is usually purchased by large investors having the ability to diversify their own risk, while the debt represented by a deposit contract is generally associated instead with small and illiquid investors with a limited ability to diversify the risks they face. We have modeled banks as institutions having access to projects that can provide a normal return to bank capital by using the leverage supplied by depositors' funds. In effect, we view banks to be institutions tailored to attract the funds of small, risk-averse depositors by offering an investment return bundled with an insurance package. Larger investors, on the other hand, have sufficient financial resources to self-insure and prefer nondeposit debt, which, unburdened by the need to finance insurance, offers higher returns.

Insurance Without Categorization

We now consider the case in which banks are not categorized on the basis of their capitalization and assume that all banks are offered an insurance policy that guarantees depositors a fixed fraction β of the solvent state payout R in the event of default, so that $I(x) \equiv \beta R - x + \gamma$ for $x \in [0,R]$. Banks are charged a premium ϕ per unit of deposits for this policy, but the premium is, for the purposes of this section, viewed by banks and depositors to be independent of capitalization (K) or payout (R) choices.

In this situation, an individual bank will choose R and K to maximize profits, given the insurance contract $\{\beta,\phi\}$ and the expected utility $\overline{V}$ available from competing banks,

$$\operatorname*{Max}_{\{R,K\}} \int_R^{\bar{x}} (x-R)f(x)dx - \rho K - \bar{C} + \mu[F(R)U(\beta R(1-\phi)/(1-K)) + (1-F(R))U(R(1-\phi)/(1-K)) - \bar{V}]. \tag{19}$$

For a given insurance policy, the competitive equilibrium occurs at the expected utility of depositors at which expected profits equal zero.

In order for this competitive equilibrium to be feasible, we also must impose the requirement that the insurance policy break even when evaluated at the competitive choices R^* and K^*. This requires the further condition

$$\phi(1-K^*)/(1-\phi) - \int_{\underline{x}}^{R^*} I(x)f(x)dx = 0. \tag{20}$$

Note that a solution $\{R,K,\phi,\beta\}$ to the problem described by expressions (19) and (20) for any given $\overline{V}$ must be a feasible solution to the problem with categorization, given by inequality (11), so that a solution to the latter must yield no lower profits to firms than a solution to the former.

The difference between the categorization and the no-categorization cases turns on the incentive of bank owners to invest their own capital in the bank. In particular, the question arises as to whether there will be any capital invested by bank owners in the (no-categorization) competitive equilibrium. The answer to this question can be seen most clearly by considering the indifference curves of depositors in (R,K) space, since a competitive equilibrium will occur at the highest depositor indifference along the zero-profit frontier for banks. Since depositors and banks view β and ϕ as fixed, the slope of an indifference curve is obtained by implicit differentiation of the expected utility constraint for depositors in expression (19), which yields

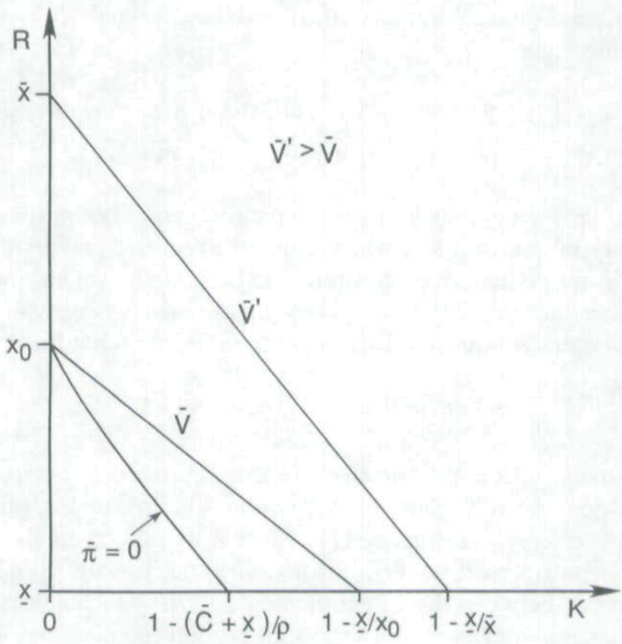
$$\frac{dR}{dK} = \frac{-R[y_B U'(y_B)F(R) + y_S U'(y_S)(1-F(R))]}{(1-K)[y_B U'(y_B)F(R) + y_S U'(y_S)(1-F(R)) - (U(y_S) - U(y_B))f(R)R]}, \tag{21}$$

where $y_B \equiv \beta R(1-\phi)/(1-K)$ is the bankruptcy payoff to depositors, and $y_S \equiv R(1-\phi)/(1-K)$ is the solvent state return to depositors.

If depositors are offered full insurance and suffer no loss in the event of bankruptcy, then $\beta = 1$, which implies $y_S = y_B$. In this case, equation (21) reduces to $\frac{dR}{dK} = -\frac{R}{1-K,}$ so that the depositors' indifference curves are linear.

An indifference curve with a vertical intercept at x has the horizontal intercept at $K = 1 - \underline{x}/x$, as depicted in Figure 3. The linearity of depositors' indifference curves, in conjunction with the strict convexity of the bank's zero isoprofit frontier, guarantees that the only competitive equilibrium entails $R = x_o$ and $K = 0$.[16] With full insurance and no risk categorization, there is no incentive for banks to provide capital since bankruptcy is not costly to depositors when there is full insurance.

Figure 3



Theorem 4. An insurance plan with full coverage and no categorization results in an equilibrium where banks totally decapitalize ($K = 0$).

When $\beta < 1$, there may still be an incentive for banks to provide capital because depositors suffer a loss in the event of bankruptcy. Less than full insurance provides an incentive for depositors to prefer safer banks (with greater K and lower R), because of the loss associated with bank insolvency. This result is analogous to that obtained by Shavell (1979) in a more traditional insurance context, where partial insurance is desirable in the presence of moral

[16] The zero isoprofit contour lies below the appropriate ($\bar{V}$) indifference curve in Figure 3 because its slope (by [7]) is monotonically decreasing in R. Two cases must be considered. First, for $x_0 > \rho$, it follows that $1 - \bar{x}/\underline{x} > 1 - \underline{x}/\rho$ and the result is immediate. Alternatively, for $\rho > x_0$, the slope of the isoprofit contour is always less than that of the indifference curve ($-x_0$), so that they never cross.

hazard because it provides an incentive for individuals to invest in costly self-protective activities.

A final point concerns the conditions under which this insurance scheme without categorization can be made equivalent to a categorization scheme by the application of minimum capital requirements on banks. The efficient contract can be duplicated by choosing a β consistent with the values of Z and R that solve the planner's problem (11) and a corresponding minimum capitalization requirement equal to the optimal K. As a result of Theorem 2, we must have $\beta < 1$. This equivalence between categorization and minimum capital requirements will cease to hold if there is more than one type of depositor. If there were heterogeneous depositors, a social optimum would require heterogeneous banks with each type of bank specialized to a particular depositor type. Different types of banks would generally offer different insurance policies and levels of capitalization, which could not be duplicated with a single minimum capital requirement.

Discussion

Our results indicate that efficiency dictates the conditioning of deposit insurance premiums on observable characteristics of the bank that are correlated with the underlying portfolio risk. We have, however, examined a highly stylized model of bank insurance, replete with simplifying assumptions, some of which deserve elaboration.

From the outset, we have assumed the distribution of portfolio returns (f) to be known, and identical, for all banks. Although convenient for the representative-bank modeling approach, this departure from reality obscures a basic difficulty in assigning risk-based premiums: that of assessing the underlying uncertainty of heterogeneous banks' portfolio returns. Moreover, we have assumed that the primary source of self-protection against risk, capitalization, is easily observable to both depositors and to insurers. In practice, ascertaining capitalization levels may entail monitoring and administrative expenses, and there are substantial incentives facing undercapitalized banks to mask their actual status.

In a related vein, there are sources of deposit risk other than capitalization, some of which may be observable and, therefore, could serve as additional categorization tools. One obvious source of risk would be moral hazard by the bank owners, who may invest in overly risky portfolios. This problem arises because the debt contract, which is necessary because of the costly state verification, generates convex payoffs to the banks, resulting in risk-loving behavior. Because of limited liability, banks are insured by depositors against the worst portfolio outcomes and prefer investment projects with higher variance in returns. The use of bank capital to partially fund investment projects may mitigate this problem somewhat by exposing banks to the downside risk, but the precise role of capitalization in a moral hazard contract is a topic for future research.

Conclusions

This article has presented a model of banks as risk-neutral financial intermediaries that fund investment projects generating uncertain returns by using their own capital in conjunction with resources raised from risk-averse depositors. Because of depositors' inability to freely observe project returns and the limited liability enjoyed by banks, the optimal compensation arrangement is shown to be a debt contract that exhibits the possibility of default. We find that bank capitalization, which reduces the probability of bankruptcy, and deposit insurance, which reduces the magnitude of the losses suffered under default, are complementary tools that can be used to protect depositors from the risk of bank insolvency. In particular, the optimal insurance plan entails positive, but less than full, coverage in the bankrupt state. This risk exposure provides depositors with the incentive to monitor bank capitalization, which is, in effect, a form of self-protection against insolvency.

In addition, the optimal insurance plan assesses premiums based upon bank riskiness as measured by the level of capitalization and the promised return to depositors. Thus, banks categorize themselves by their choice of observable characteristics which are related to their underlying potential to suffer insolvency. Moreover, we show that the current system of full insurance without such categorization results in moral hazard, where depositors no longer have the incentive to monitor bank riskiness, resulting in incentives for banks to totally decapitalize.

The incentive to reduce capitalization can be circumvented by the imposition of minimum capital requirements enforced by regulatory sanctions, as is presently the case with FDIC deposit insurance. Such an approach, however, requires knowledge by the regulators of the optimal capitalization levels, which may vary across banks depending upon their regional portfolio possibilities or depositors' risk preferences. The advantage of categorization, on the other hand, is to permit banks and depositors to select their optimal risk-return trade-offs based only upon the requirement that the premiums charged be actuarially fair. This reduces the regulatory burden because the need to determine optimal capitalization requirements is replaced by the considerably simpler task of computing actuarially fair premiums based upon observable bank characteristics.

APPENDIX

Proof of Lemma 1. In order for a deposit contract to be incentive compatible, the payment schedule must satisfy

$$w(x) = R \qquad \text{for } x \in M^C, \text{ and} \tag{A.1}$$
$$w(x) \leq R \qquad \text{for } x \in M. \tag{A.2}$$

In order to induce truthful revelation of x in the nonmonitoring region, equation (A.1) requires that the payment be the same in all states where the bank is not monitored. Inequality (A.2) is necessary for the bank to truthfully report a state in the monitoring region.

In addition to the incentive compatibility constraints, limited liability imposes the requirement that $w(x) \leq x$. Combining the limited liability requirement with inequality (A.2), we obtain that the maximum feasible payment in the monitoring region must be the smaller of x or R. We begin by showing that, under the optimal contract, the bank will always make this maximum feasible payment in the default region, so that

$$w(x) = \min(x,R) \qquad \text{for } x \in M. \tag{A.3}$$

To establish equation (A.3), assume the contrary, that there exists an interval $[a,b] \subset M$ such that $w(x) < \min(x,R)$. We will show that it is possible to improve upon this contract by raising the payment on [a,b] and reducing the payment in the nonmonitoring region.

Let h(x) be a function with the property that $h(x) \geq 0$ on [a,b] and $h(x) = 0$ elsewhere. Then there will exist an $\varepsilon > 0$ such that $w(x) + \varepsilon h(x)$ is feasible and satisfies inequality (A.2). Now increase the payment in the monitoring region by $h(x)d\varepsilon$ and reduce the payment R on M^C to keep the expected utility of depositors constant. (Since K remains fixed throughout the analysis, we can set K = 0 without loss of generality). Letting $P(R) \equiv \int_{M^c} f(x)dx$ be the probability that the bank pays R to depositors, we have, from differentiation of equation (2), that dV = 0 requires

$$\left(\int_a^b U'(w(x) - \gamma + \varepsilon h(x))h(x)f(x)dx\right)d\varepsilon + P(R)U'(R)dR = 0. \tag{A.4}$$

Differentiation of equation (1) yields the effect on bank profits to be

$$d\pi = -\left(\int_a^b h(x)f(x)dx\right)d\varepsilon - P(R)dR. \tag{A.5}$$

Since $U'(R) < U'(w(x) - \gamma + \varepsilon h(x))$ for $x \in [a,b]$, an increase in ε satisfying equation (A.4) must yield $d\pi > 0$. Therefore, no contract with $w(x) < \min(x,R)$ on the monitoring region can be optimal, which establishes equation (A.3).

In order to complete the proof of the lemma, it must be shown that the monitoring region is $M = [\underline{x},R)$. It is readily established that if $M \neq [\underline{x},R)$ the contract can be improved on. Note that by feasibility, $[\underline{x},R) \subset M$. Suppose that the monitoring region is $M = [\underline{x},R) \cup S$, where S is a nonempty subset of

$[R,\bar{x}]$. By equation (A.3), $w(x) = R$ for $x \varepsilon S$.

It follows that the contract with $M = [\underline{x},R) \cup S$ is equivalent to a contract with a monitoring region $M' = [\underline{x},T']$ where $F(T') - F(R) = \int_S f(x)\,dx$. It is then straightforward to show that it is possible to raise the profits of banks by a reduction in T' (which economizes on monitoring cost) and a simultaneous reduction in R that leaves the expected utility of depositors constant.

Proof of Lemma 3. This result may be demonstrated formally by taking K, R, and ϕ as given and selecting the insurance function I(x) to maximize depositors' expected utility

$$\int_{\underline{x}}^{R} U\left(\frac{(x-\gamma+I)(1-\phi)}{1-K}\right)f(x)\,dx + U\left(\frac{R}{1-K}(1-F(R))\right)$$

subject to the constraints $I(x) \geq 0$ and inequality (9). This is a bounded control problem (Kamien and Schwartz, 1981, p. 170) and is solved by letting I(x) be the control and defining the state variable s(x) where $\dot{s} \equiv I(x)f(x)$, $s(0) = 0$, and $s(R) = \phi(1-K)/(1-\phi)$. The Hamiltonian

$$H = U\left(\frac{(x-\gamma+I)(1-\phi)}{1-K}\right) f(x) + \lambda\dot{s} + \mu I(x)$$

yields the first order conditions $\dot{\lambda} = 0$, $\mu \geq 0$; $\mu\, I(x) = 0$; and

$$H_I = \frac{U'(\cdot)(1-\phi)f(x)}{1-K} + \lambda f(x) + \mu = 0.$$

When $\mu = 0$, so $I(x) > 0$, this implies that U' is a constant, so income is equalized across those states with a positive insurance payment. On the other hand, when $\mu > 0$ and $I(x) = 0$, we have $U'((x - \gamma)(1 - \phi) / (1 - K)) < -\lambda$, so that marginal utility is lower, and hence income higher, in the states with zero insurance payment. Thus, the payment region is of the form [0,P].

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