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The Inefficiency of Private Constant Annuities

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ABSTRACT

This article investigates why individuals save to finance consumption during retirement, and we focus this investigation on the inefficiency of constant annuities. To thoroughly examine this problem, we first derive the demand function for the annuities in the case where the capital market is imperfect and the annuities are constrained to be constant throughout the retirement period. With these constraints, we then show that the individual holds assets not only in the form of actuarial notes but also in the form of monetary wealth.

Introduction

People prepare for retirement by using a scheme such as an annuity system to reduce the inefficiency caused by the uncertainty of not knowing how long they will live. In this sense, the annuity system is a kind of insurance that reduces the essential cost of preparing for retirement. Annuity systems are classified mainly into public and private systems. Public annuity systems are organized through the governmental process, and the demand for public pensions is not controllable by the individual. In contrast, the demand for private annuities is determined by individual choice. This article focuses on the private annuity system. Prior research on this topic includes Yaari (1965), Taubman (1985), Diamond and Mirrlees (1985), and Green (1985).

Should individuals demand more of private annuities in an economy where pay-as-you-go public annuities are provided? Several reasons for an inegalitarian economy exist. Because individuals are not homogeneous and job earnings vary among individuals, the optimal level of mandatory public pension that maximizes social welfare can be positive. This socially optimal public pension level, however, does not necessarily equal the optimal annuities level

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demanded by each individual. The gap between these two optimal levels creates the demand for private annuities.

In a theoretical model based on a representative individual, it is hard to explain why an individual might demand both public and private annuities. Individual "myopia" cannot fully explain this situation, because the myopic individual may not demand private annuities at all. The mandatory pay-as-you-go public pension vanishes at the optimum if the individual is not myopic and the sum of population growth rate and productivity growth rate is smaller than the interest rate. We assume a representative individual, and we consider a situation where the individual demands only private annuities. In particular, we address here a private annuity system organized by an insurance company; the corporate pension system is not considered.

Our discussion of the insurance aspect of the annuity system owes much to the pioneering work of Yaari (1965), who demonstrated that the optimal behavior of a non-altruistic individual is to hold all assets in the form of actuarial notes when the probability of death is positive. Empirical studies, such as that of Yagi and Tachibanaki (1989), however, show that almost every individual, young and old alike, holds assets partly in the form of monetary wealth. It is hard to find empirical evidence that all of this monetary wealth is held from the bequest motive.

Thus, it is necessary to investigate the reasons why an individual feels it necessary to own part of his or her assets in monetary wealth to finance living costs during retirement. If the reason stems only from the irrationality of the individual, then the issue is of less interest. If, however, the positive ratio of monetary wealth can be partly explained by the optimal behavior of the individual, then investigating the demand for private annuities produces a view different from Yaari's.

Friedman and Warshawsky (1988, 1990), asking why some individuals do not demand annuities at all, attributed monetary holdings to expectations of a low level of return on private annuities. In contrast, here we consider the reasons why individuals might hold monetary savings in an economy where private annuities that earn higher rates of return are available. We attribute the cause to constraints on private annuity systems. In particular, we consider a situation in which the capital market is imperfect and annuity payments are constrained to be constant during retirement. We discuss the reasons why the real annuity market departs from the ideal market that Yaari described.

This article provides the details of an investigation into why an individual saves to finance consumption during retirement and investigates the inefficiency of constant annuities. To thoroughly examine this problem, we first derive the demand function for annuities in the case where the capital market is imperfect and annuities are constrained to be constant throughout the retirement period. With these constraints, individuals hold assets in the form of both actuarial notes and monetary wealth. The ratio between annuities and monetary wealth is determined by several parameters, such as the time preference rate and death probabilities. We analyze how the demand for annuities is affected by changes in these parameters. We then explore how constraints on the annuity markets distort the consumption stream. The consumption stream without any constraints, which Yaari derived, is considered to be the first-best optimal consumption stream in an economy where insurance is available. The consumption stream with constraints diverges from the first-best consumption stream, and this divergence can be seen as a distortion created by the constraints. In other words, the constraints imposed by the insurer and the imperfect capital market create a loss in efficiency. We next investigate how the degree of efficiency loss (welfare loss) is affected by changes in the parameter values, which represent characteristics of the individual and the annuity market. We subsequently assess these problems by developing a three-period model.

The Demand for Private Pensions

Reasons for Constant Annuities

Why does an insurer want to sell an actuarial note that guarantees to pay constant annuity payments throughout the retirement period? An interview with a Japanese insurer revealed that there is no variable annuity in Japan that permits an individual to determine the stream of annuity payments. Yaari considered an annuity system in which an insurer provides a retired individual with annuity payments so that the optimal consumption level in each period is fully financed by the annuities.

One reason why an insurer might prefer to sell annuities with constant payments stems from the difficulties in calculating an actuarially fair level of annuity. When a variable annuity is available, the optimal stream of annuity payments differs between an individual who has a long life and one who has a short life. In this case, the difficulty of calculating the actuarially fair level of annuity increases the cost of providing an annuity. According to the interview, reducing expenses for inspections is one of the reasons for providing constant annuities.

When the capital market is perfect and an individual can borrow money using future annuity payments as collateral, the annuity system that provides constant annuity payments does not differ from an ideal annuity system. In this case, each individual can reallocate a stream of annuity payments so that the optimal consumption stream in the retirement period can be fully financed by the annuity payments. It is impossible, however, to expect any financial institution to lend money based on future annuity payments as collateral, because the individual cannot receive annuities upon death.

Model

We formulate a three-period model for examining the effect of constant annuity payments on the demand for annuities, assuming an imperfect capital market. In formulating this model, we consider a representative individual whose lifetime comprises three periods. The first is the working period, during which the individual has earnings of w. The second and third periods are retirement periods that differ in their death probabilities. The proportion of individuals who die at the beginning of the second period is p_1, and the proportion of individuals who die at the beginning of the third period is given by p_2. We consider the case where the subjective death probability and the average death probability are equal.

It is well known that an individual who has a bequest motive holds monetary assets as well as annuities. Because our purpose is to analyze the demand for both annuities and monetary savings of an individual who has no bequest motive, we assume that utility of the individual comes only from consumption, and we do not consider utility from the bequest motive. The utility function U is given as

$$U(c_1,c_2,c_3) = (1-p_1)(1-p_2)U(c_1,c_2,c_3) + (1-p_1)p_2V(c_1,c_2) + p_1H(c_1), \quad (1)$$

where c_i = consumption during the ith period,
U = the utility function when the individual lives all three periods,
V = the utility function when the individual lives two periods, and
H = the utility function when the individual dies at the beginning of the second period.

To express the time preference explicitly, we specify the utility functions as

$$U(c_1,c_2,c_3) = u(c_1) + \frac{u(c_2)}{1+\rho} + \frac{u(c_3),}{(1+\rho)^2} \quad (2)$$

$$V(c_1,c_2) = u(c_1) + \frac{u(c_2),}{1+\rho} \quad (3)$$

and

$$H(c_1) = u(c_1), \quad (4)$$

where ρ = a time preference rate, and
u is assumed to be a continuously differentiable concave function.

The budget constraint during the first period is described as

$$w = c_1 + z + s_1, \quad (5)$$

where z = the amount of an actuarially fair annuity note, and
s_1 = savings in the form of monetary assets during the first period.

The individual faces the following budget constraint during the second period:

$$(1 + r)s_1 + a = c_2 + s_2, \quad (6)$$

where r = the interest rate,
a = an annuity payment, and
s_2 = savings in the form of monetary assets during the second period.

In this model, it is assumed that the market interest rate r is given exogenously. The budget constraint during the third period is given by

$$(1 + r)s_2 + a = c_3. \tag{7}$$

The actuarially fair annuity payments a are determined so that the following relation holds:

$$z = \frac{1-p_1}{1+r}\,a + \frac{(1-p_1)(1-p_2)}{(1+r)^2}\,a. \tag{8}$$

Rewriting equation (8), we get

$$a = \frac{(1+r)^2}{(1-p_1)(1+r) + (1-p_1)(1-p_2)}\,z. \tag{9}$$

The appendix, which derives the individual's optimization problem, shows that both s_1 and s_2 are positive simultaneously only when the death probability is zero. Thus, it is possible to divide the cases according to whether s_1 is positive or s_2 is positive. When $r = \rho$ and the first-best consumption during the second and third period is the same, it is possible to finance consumption during these periods by annuities only. In this case, both s_1 and s_2 are zero, and neither the constant annuity constraint nor the nonnegative savings constraint is binding. Since the first-best consumption stream is feasible in this case, it is not considered here.

The case when s_1 is positive (Case 1) arises when the interest rate is smaller than the time discount rate, which implies that the first-best optimal consumption is larger during the second period than during the third period. Case 2, when s_2 is positive, arises when the interest rate is larger than the time discount rate, implying that the first-best optimal consumption is larger during the second period than during the third period.

The appendix also illustrates the case when the nonnegative constraint on saving is binding. This case arises when the difference between the interest rate and the time discount rate is small. The discussion becomes unnecessarily complicated in the case when the nonnegative constraint on savings is binding, so we consider only the case when the nonnegative constraint on consumption and savings is not binding. This approach allows greater understanding of the economic implications.

Case 1: $r < \rho$. Since the death probabilities are positive during the two retirement periods, the actuarial note's rate of return exceeds the interest rate earned on monetary savings. In this situation, the individual has a high time preference rate and optimizes by consuming more in the second period than in the third period. Assuming constant annuities and an imperfect capital market—which implies nonnegative savings—the individual might hold monetary assets during the first period to fulfill the consumption gap between the second and third periods. By contrast, there is no reason for the individual to hold monetary assets during the second period. It is more profitable to finance third-period consumption by annuity payments as long as the rate of return on the actuarial note is larger than the interest rate. For this reason, s_2 can be set to zero.

At the beginning of the first period, the individual determines the optimal consumption plan and the optimal amount to invest in the actuarial note under the constraints of constant annuity payments and an imperfect capital market so as to maximize the utility function of equation (1). Combining the budget constraints from equations (5), (6), (7), and (9), this optimization problem is summarized as

$$\text{Max} \quad U(c_1,c_2,c_3) = (1-p_1)(1-p_2)U(c_1,c_2,c_3) + (1-p_1)p_2V(c_1,c_2) + p_1H(c_1)$$

$$\text{s.t.} \quad w = c_1 + \frac{c_2}{1+r} + \frac{(1-p_1)(1+r) + (1-p_1)(1-p_2) - (1+r)}{(1+r)^2}\, c_3 \tag{10}$$

$$c_1,c_2,c_3,s_1 \geq 0.$$

From the Lagrangean,

$$\begin{aligned} L &= U(c_1,c_2,c_3) \\ &+ \mu[w - c_1 - \frac{c_2}{1+r} - \frac{(1-p_1)(1+r)+(1-p_1)(1-p_2)-(1+r)}{(1+r)^2}\, c_3 \\ &+ q_1(c_2-c_3) + q_2c_1 + q_3c_2 + q_4c_3, \end{aligned} \tag{11}$$

the optimality conditions are derived as

$$\frac{\partial L}{\partial c_1} = (1-p_1)(1-p_2)U_1 + (1-p_1)p_2V_1 + p_1H' - \mu + q_2 = 0, \tag{12}$$

$$\frac{\partial L}{\partial c_2} = (1-p_1)(1-p_2)U_2 + (1-p_1)p_2V_2 - \frac{\mu}{1+r} + q_1 + q_3 = 0, \tag{13}$$

$$\frac{\partial L}{\partial c_3} = (1-p_1)(1-p_2)U_3 - \mu\frac{(1-p_1)(1+r)+(1-p_1)(1-p_2)-(1+r)}{(1+r)^2} - q_1 + q_4 = 0, \tag{14}$$

$$\frac{\partial L}{\partial \mu} = w - c_1 - \frac{c_2}{1+r} - \frac{(1-p_1)(1+r)+(1-p_1)(1-p_2)-(1+r)}{(1+r)^2}c_3 = 0, \tag{15}$$

$$q_1(c_2-c_3) = 0, \tag{16}$$

and

$$q_{i-1} + c_i = 0 \qquad\qquad i = 1,2,3. \tag{17}$$

Assuming that nonnegative constraint is not binding on c_1, c_2, c_3, and s_1 and using equations (2), (3), and (4), the optimality conditions (12), (13), and (14) can be rewritten as

$$\frac{\partial L}{\partial c_1} = u' - \mu = 0, \tag{18}$$

$$\frac{\partial L}{\partial c_2} = \frac{1-p_1}{1+\rho}u' - \frac{\mu}{1+r} = 0, \tag{19}$$

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and

$$\frac{\partial L}{\partial c_3}=\frac{(1-p_1)(1-p_2)u'}{(1+\rho)^2}-\mu\frac{(1-p_1)(1+r)+(1-p_1)(1-p_2)-(1+r)}{(1+r)^2}=0. \tag{20}$$

Case 2: r > ρ. In this case, the individual has a small time preference rate and optimizes by consuming more in the third period than in the second. Assuming constant annuities and an imperfect capital market, the individual might hold monetary assets during the second period to fulfill the consumption gap between the second and third periods. Second-period consumption can be financed only by annuities. For this reason, s_1 can be set to be zero.

The optimization problem of Case 2 is summarized as

$$\begin{aligned} Max \quad U(c_1,c_2,c_3) &= (1-p_1)(1-p_2)U(c_1,c_2,c_3)+(1-p_1)p_2V(c_1,c_2)+p_1H(c_1) \\ s.t. \quad w &= c_1+\sigma c_2+\frac{\sigma}{1+r}c_3 \\ & c_1,c_2,c_3,s_2 \geq 0, \end{aligned} \tag{21}$$

where

$$\sigma=\frac{(1-p_1)(1+r)+(1-p_1)(1-p_2).}{(2+r)(1+r)} \tag{22}$$

From the Lagrangean,

$$\begin{aligned} F &= U(c_1,c_2,c_3) \\ &+\mu[w-c_1-\sigma c_2-\frac{\sigma}{1+r}c_3] \\ &+q_1(c_3-c_2)+q_2c_1+q_3c_2+q_4c_3, \end{aligned} \tag{23}$$

The optimality conditions are derived as

$$\frac{\partial F}{\partial c_1}=(1-p_1)(1-p_2)U_1+(1-p_1)p_2V_1+p_1H'-\mu+q_2=0, \tag{24}$$

$$\frac{\partial F}{\partial c_2}=(1-p_1)(1-p_2)U_2+(1-p_1)p_2V_2-\sigma\mu-q_1+q_3=0, \tag{25}$$

$$\frac{\partial F}{\partial c_3}=(1-p_1)(1-p_2)U_3-\frac{\sigma}{1+r}\mu+q_1+q_4=0, \tag{26}$$

$$\frac{\partial F}{\partial \mu}=w-c_1-\sigma c_2-\frac{\sigma}{1+r}c_3=0, \tag{27}$$

$$q_1(c_3-c_2)=0, \tag{28}$$

and

$$q_{i+1}c_i=0 \qquad i=1,2,3. \tag{29}$$

Assuming that the nonnegativity constraint is not binding on c_1, c_2, c_3, and s_2 and using equations (2), (3), and (4), the optimality conditions (24), (25), and (26) can be rewritten as

$$\frac{\partial F}{\partial c_1} = u'(c_1) - \mu = 0, \tag{30}$$

$$\frac{\partial F}{\partial c_2} = \frac{1-p_1}{1+\rho} u'(c_2) - \sigma\mu = 0, \tag{31}$$

and

$$\frac{\partial F}{\partial c_3} = \frac{(1-p_1)(1-p_2)u'(c_3)}{(1+\rho)^2} - \frac{\sigma}{1+r}\mu = 0. \tag{32}$$

Demand for Constant Annuities and Individual Characteristics

What is the relationship between the demand for annuities and the characteristics of the individual? To answer this question, we analyze how changes in the time preference rate and death probabilities affect the demand for annuities. In contrast to the case where a variable annuity is available, the result of comparative statics analysis is not so obvious when the capital market is incomplete and annuity payments are constant over time. One purpose of the comparative statics analysis is to make clear how the constraints affect an individual's demand for annuities.

As in the previous section, the comparative statics analyses are examined for both Case 1 and Case 2. The following analysis considers only the case when the nonnegativity constraint is not binding for all variables.

Case 1: $r < \rho$. First, we analyze the effect of a change in the time preference rate on the demand for annuities. From the comparative statics analysis we derive the result

$$\frac{da}{d\rho} = \frac{1}{|A|}(\psi_{11}\psi_{42}(\psi_{3\rho}\psi_{24} - \psi_{2\rho}\psi_{34}) + \psi_{22}\psi_{3\rho}), \tag{33}$$

where

$$\psi_{11} = u''(c_1) < 0, \tag{34}$$

$$\psi_{22} = \frac{(1-p_1)}{1+\rho} u''(c_2) < 0, \tag{35}$$

$$\psi_{24} = -\frac{1}{1+r} < 0, \tag{36}$$

$$\psi_{34} = -\frac{(1-p_1)(1+r) + (1-p_1)(1-p_2) - (1+r)}{(1+r)^2}, \tag{37}$$

$$\psi_{42} = -\frac{1}{1+r} < 0, \tag{38}$$

$$\psi_{2\rho} = -\frac{(1-p_1)}{(1+\rho)^2} u'(c_2) < 0, \tag{39}$$

$$\psi_{3\rho} = -2\frac{(1-p_1)(1-p_2)}{(1+\rho)^3} u'(c_3) < 0, \tag{40}$$

and the determinant $|A|$ of the Hessian matrix can be shown to be negative. Thus, whether $da/d\rho$ is positive or negative depends on the sign of $\psi_{3\rho}\psi_{24} - \psi_{2\rho}\psi_{34}$. This term can be rewritten as

$$\psi_{3\rho}\psi_{24} - \psi_{2\rho}\psi_{34} = \frac{(1-p_1)}{(1+r)(1+\rho)^2}\left(\frac{2(1-p_2)u'(c_3)}{(1+\rho)} - u'(c_2)K\right), \tag{41}$$

where

$$K = \frac{(1-p_1)(1+r) + (1-p_1)(1-p_2) - (1+r)}{1+r}.$$

Thus, the demand for the pension decreases as the degree of time discount rate increases if

$$\frac{2(1-p_2)u'(c_3)}{(1+p)} > u'(c_2)K. \tag{42}$$

The economic meaning of K is given by the budget constraint, equation (10). As shown in the budget constraint, K/(1+r) is the price of consumption during the third period. When the price of consumption is negative, it is optimal for the consumer to borrow money to spend in the third period. This violates the nonnegativity constraint on consumption and savings. Since we only consider the case when the problem has an interior solution, $K > 0$ must be assumed. The maximum of K is derived as follows. K is rewritten as

$$K = -p_1 + \frac{(1-p_1)(1-p_2)}{1+r}. \tag{43}$$

The maximum value is attained when $p_1 = 0$ and $p_2 = 0$. Since $r > 0$, the maximum value of K is less than 1.

We examine the result of the comparative statics analysis. Since the degree of time discount rate is correlated to the degree
of myopia, this comparative statics analysis also examines the effect of the degree of myopia on the demand for the annuities. Condition (42) can be interpreted by using optimality conditions (19) and (20). Using these two conditions we can derive

$$\frac{1-p_2}{1+\rho} u'(c_3) = u'(c_2)K. \tag{44}$$

Thus, condition (42) must hold as long as an interior solution exists.

An increase in myopia decreases the demand for annuities, because increased myopia increases consumption during the first period and decreases consumption during the third period. The economic implication, however, is not so straightforward. An increase in myopia expands the gap between second- and third-period consumption. An increase in the consumption gap increases monetary savings and generates inefficiency caused by different rates of return available on annuities and monetary savings. In addition, the consumption gap generates inefficiency by distorting consumption away from first-best consumption. These two effects decrease the demand for annuities because they increase the inefficiency of constant annuities. The increased myopia, however, undervalues the inefficiency of saving for third-period consumption. This makes more complex the effect of changes in myopia on the demand for annuities. This complex effect occurs only in the case where the annuity payments are constant during the two retirement periods. The welfare change caused by the changes in myopia is discussed below.

Second, we analyze the effect of changes in the death probability at the beginning of the second period. From comparative statics analysis we derive the result

$$\frac{da}{dp_1}=\frac{1}{|A|}\Big\{\psi_{11}(\psi_{22}\psi_{34}\psi_{4p_1}+\psi_{42}(\psi_{24}\psi_{3p_1}-\psi_{34}\psi_{2p_1})) + \psi_{22}\psi_{3p_1}\Big\}\ , \tag{45}$$

where $\psi_{1p_1}=0,$ (46)

$$\psi_{2p_1}=-\frac{u'(c_2)}{1+\rho}<0, \tag{47}$$

$$\psi_{3p_1}=-\frac{(1-p_2)u'(c_3)}{(1+\rho)^2}+\frac{(1+r)+(1-p_2)}{(1+r)^2}\mu \tag{48}$$

$$\psi_{4p_1}=\frac{(1+r)+(1-p_2)}{(1+r)^2}c_3>0. \tag{49}$$

Thus, an increase in the death probability at the beginning of the second period increases the demand for annuities if K > 0 and $\psi_{3p_1}>0$. As discussed above, K is assumed to be positive. Using equations (19) and (20), ψ_{3p_1} can be rewritten as

$$\begin{aligned}\psi_{3p_1}&=-\frac{(1-p_2)u'(c_3)}{(1+\rho)^2}+\frac{[(1+r)+(1-p_2)]}{(1+r)^2}\frac{(1+r)(1-p_1)}{1+\rho}u'(c_2)\\&=-\frac{(1-p_2)u'(c_3)}{1+\rho)^2}+\frac{(1+r)(1-p_1)+(1-p_1)(1-p_2)}{(1+\rho)(1+r)}u'(c_2)\\&=-\frac{(1-p_2)u'(c_3)}{(1+\rho)^2}+\frac{K}{1+\rho}u'(c_2)+\frac{1+r}{(1+\rho)(1+r)}u(c_2)\\&=\frac{1+r}{(1+\rho)(1+r)}u(c_2)>0\ .\end{aligned} \tag{50}$$

The results of comparative statics analysis show that the demand for annuities increases when the death probability at the beginning of the second period increases. An increase in the death probability increases the rate of return on annuities, and this stimulates demand.

Third, we analyze the effect of change in the death probability at the beginning of the third period. From the comparative statics analysis we derive the result

$$\frac{da}{dp_2}=\frac{1}{|A|}\left\{\psi_{11}(\psi_{22}\psi_{34}\psi_{4p_2}+\psi_{42}\psi_{24}\psi_{3p_2})+\psi_{22}\psi_{3p_2}\right\}, \tag{51}$$

where $\psi_{1p_2}=0,$ (52)

$$\psi_{2p_2}=0, \tag{53}$$

$$\psi_{3p_2}=-\frac{(1-p_1)u'(c_3)}{(1+\rho)^2}+\frac{(1-p_1)}{(1+r)^2}\mu \tag{54}$$

$$\psi_{4p_2}=\frac{(1-p_1)}{(1+r)^2}c_3>0. \tag{55}$$

The comparative statics analysis shows that an increase in the death probability at the beginning of the third period increases the demand for annuities if K > 0 and $\psi_{3p_2} > 0$. Using equations (19) and (20), ψ_{3p_2} can be rewritten as

$$\begin{aligned}\psi_{3p_2}&=-\frac{(1-p_1)u'(c_3)}{(1+\rho)^2}+\frac{1-p_1}{1+\rho}\left(K+\frac{1+r-(1-p_1)(1-p_2)}{1+r}\right)u'(c_2)\\&=\frac{1-p_1}{(1+\rho)^2}[(1-p_2)-1]u'(c_3)+\frac{1-p_1}{(1+\rho)(1+r)}[1+r-(1-p_1)(1-p_2)]u'(c_2)\\&=\frac{1-p_1}{(1+\rho)^2}\left(-p_2+\frac{(1-p_2)[(1+r)-(1-p_1)(1-p_2)]}{K}\right)u'(c_3).\end{aligned} \tag{56}$$

Thus, ψ_{3p_2} is positive when p_2 is small, and it is negative when p_2 is large. The effects of an increase in the death probability at the beginning of the third period on the demand for annuities are twofold. One is the positive effect via increases in the rate of return, and the other is the negative effect via the decrease in expected utility for a given amount of consumption. The result of comparative statics analysis shows that the positive effect dominates the negative effect when the death probability is relatively small.

Case 2: $r > \rho$. In this case, the individual does not save in the first period. Thus, the demand for an actuarial note is
given by w-c_1. To examine the effects of changes in the parameter values on the demand for annuities, we analyze the effects on first-period consumption because an increase (decrease) in consumption during the first period automatically implies a decrease (increase) in demand for the actuarial note, respectively.

First, we analyze the effect of change in the time preference rate on first-period consumption. From the comparative statics analysis we derive the result

$$\frac{dc_1}{d\rho}=\frac{1}{|A|}(\phi_{3\rho}\phi_{22}\phi_{34}-\phi_{2\rho}\sigma\phi_{33})>0, \tag{57}$$

where

$$\phi_{22}=\frac{1-p_1}{1+\rho}u''(c_2)<0, \tag{58}$$

$$\phi_{33}=\frac{(1-p_1)(1-p_2)}{(1+\rho)^2}u''(c_3)<0, \tag{59}$$

$$\phi_{34}=-\frac{\sigma}{1+r}<0, \tag{60}$$

$$\phi_{2\rho}=-\frac{1-p_1}{(1+\rho)^2}u'(c_2)<0, \tag{61}$$

$$\phi_{3\rho}=-2\frac{(1-p_1)(1-p_2)}{(1+\rho)^3}u'(c_3)<0. \tag{62}$$

Thus, an increase in the degree of myopia decreases the demand for an actuarial note.

Second, the effect of change in the death probability in the second period can be derived as

$$\frac{dc_1}{dp_1}=\frac{1}{|A|}(\phi_{3pl}\phi_{22}\Phi_{34}-\phi_{2pl}\sigma\phi_{33}+\phi_{4pl}(-\phi_{22})\phi_{33}), \tag{63}$$

where

$$\phi_{2p_1}=-\frac{1}{1+\rho}u'(c_2)-\sigma_{p_1}\mu, \tag{64}$$

$$\phi_{3p_1}=-\frac{1-p_2}{(1+\rho)^2}u'(c_3)-\frac{\sigma_{p_1}}{1+r}\mu, \tag{65}$$

$$\phi_{4p_1}=-\sigma_{p_1}c_2-\frac{\sigma_{p_1}}{1+r}c_3>0, \tag{66}$$

$$\sigma_{p_1}=-\frac{(1+r)+(1-p_2)}{(2+r)(1+r)}<0. \tag{67}$$

Thus, first-period consumption increases when $\phi_{2p_1} \leq 0$ and $\phi_{3p_1} \leq 0$. Using equations (31) and (32), we can show that $\phi_{2p_1} = 0$ and $\phi_{3p_1} = 0$, which implies that an increase in the death probability in the second period increases consumption during the first period and decreases the demand for an actuarial note.

Finally, the effect of changes in the death probability in the third period on first-period consumption can be derived as

$$\frac{dc_1}{dp_2} = \frac{1}{|A|}(\phi_{3p_2}\phi_{22}\phi_{34} - \phi_{2p_2}\sigma\phi_{33} + \phi_{4p_2}(-\phi_{22})\phi_{33}), \tag{68}$$

where $\phi_{2p_2} = -\sigma_{p_2}\mu > 0,$ (69)

$$\phi_{3p_2} = -\frac{1-p_1}{(1+\rho)^2}u'(c_3) - \frac{\sigma_{p_2}}{1+r}\mu, \tag{70}$$

$$\phi_{4p_1} = -\sigma_{p_2}c_2 - \frac{\sigma_{p_2}}{1+r}c_3 > 0, \tag{71}$$

$$\sigma_{p_2} = -\frac{(1-p_1)}{(2+r)(1+r)} < 0. \tag{72}$$

Using equations (31) and (32), ϕ_{2p_2} and ϕ_{3p_2} may be rewritten as

$$\phi_{2p_2} = -\frac{1-p_1}{(1+\rho)((1+r)+(1-p_2))}u'(c_2) \tag{73}$$

$$\phi_{3p_2} = -\frac{(1-p_1)(1+r)}{(1+\rho)^2((1+r)+(1-p_2))}u'(c_3) \tag{74}$$

Substituting equations (73) and (74) into the first and the second term of the right-hand side of equation (68), we derive

$$\begin{aligned}\phi_{3p_2}\phi_{22}\phi_{34} - \phi_{2p_2}\sigma\phi_{33} = {} & \frac{\sigma(1-p_1)^2}{(1+\rho)^3[(1+r)+(1-p_2)]}[u'(c_3)u''(c_2) \\ & + (1-p_2)u'(c_2)u''(c_3)] < 0.\end{aligned} \tag{75}$$

Thus, an increase in the death probability during the second period increases first-period consumption and decreases the demand for an actuarial note.

The comparative statics analysis revealed the factors which affect change in both the consumption stream and in the demand for annuities or for an actuarial note. The analysis is, however, limited to examining the welfare changes caused by variations in the parameter values. The next section defines the efficiency loss caused by constant annuities and simulates the effects of changes in the parameter values on changes in the efficiency loss and the demand for an actuarial note. The numerical analysis examines the differences between the case with constraints and the case without constraints.

The Inefficiency of Constant Annuities

In this section we analyze how the inefficiency of constant annuities varies according to changes in individual myopia and death probabilities. This analysis clarifies the situation where the cost of a constant annuity is large. We compare the consumption stream and the utility attained in the case where there is no constraint (the first-best case) and the case with constraints (the second-best case).

The analytical investigation demonstrated how demand for annuities changes according to changes in the parameter values, such as the individual time discount rate and death probabilities. The analytical investigation, however, is limited to examining how the differences in the demand for the annuities, consumption stream, and utility between the first-best case and the second-best case change according to changes in the parameter values. We, therefore, consider this problem by using a numerical example.

Demand for Private Annuities Without Constraint

Without the constraints stated above, the first-period budget constraint, equation (5), becomes

$$w = c_1 + z \tag{76}$$

because there is no reason for an individual to save during the first period to finance consumption during the second and third periods. The second-period budget constraint, equation (6), becomes

$$a_2 = c_2, \tag{77}$$

where a_2 is the value of the second-period annuity. The budget constraint during the third period is given by

$$a_3 = c_3, \tag{78}$$

where a_3 is the value of the third-period annuity. The amount of an actuarial note, z, which pays benefits a_2 and a_3, is determined so that the following relation holds:

$$z = \frac{1-p_1}{1+r}a_2 + \frac{(1-p_1)(1-p_2)}{(1+r)^2}a_3 \tag{79}$$

Using equations (76), (77), (78), and (79), the individual's budget constraint can be rewritten as

$$w = c_1 + \frac{1-p_1}{1+r}a_2 + \frac{(1-p_1)(1-p_2)}{(1+r)^2}a_3. \tag{80}$$

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The optimal consumption stream in the first-best case is determined by maximizing expected utility, equation (1), subject to the budget constraint, equation (80).

Comparison of Individual Behavior

Here we compare the consumption stream, demand for annuities, and utility between the case without constraints and the case with constraints. We denote the variables in the first-best case by the superscript N and denote the variables in the second-best case by the superscript R. For example, the first-best consumption during the ith period is denoted by c^N_i, and the second-best consumption is denoted by c^R_i.

To implement the numerical calculation, we specify the utility function as

$$U(c_1,c_2,c_3) = \log\ c_1 + \frac{1}{1+\rho}\log\ c_2 + \frac{1}{(1+\rho)^2}\log\ c_3, \tag{81}$$

$$V(c_1,c_2) = \log\ c_1 + \frac{1}{1+\rho}\log\ c_2, \tag{82}$$

$$H(c_1) = \log\ c_1. \tag{83}$$

The first-best consumption streams c^N_i are calculated as

$$c_1^N = w/\left(1 + \frac{1-p_1}{1+\rho} + \frac{(1-p_1)(1-p_2)}{(1+\rho)^2}\right), \tag{84}$$

$$c_2^N = \frac{1+r}{1+\rho}c_1^N, \tag{85}$$

$$c_3^N = \frac{(1+r)^2}{(1+\rho)^2}c_1^N. \tag{86}$$

The annuity amounts purchased by the representative individual, z^R and z^N, are given by equations (8) and (79), respectively.

The second-best consumption stream c^R_i is calculated for Case 1 and Case 2. From the optimality conditions (12), (13), (14), and (15), the consumption stream in Case 1 is calculated as

$$c_1^R = w/\left(1 + \frac{1-p_1}{1+\rho} + \frac{(1-p_1)(1-p_2)}{(1+\rho)^2}\right) \tag{87}$$

$$c_2^R = \frac{(1+r)(1-p_1)}{1+\rho}c_1^R \tag{88}$$

$$c_3^R = \frac{(1+r)^2}{(1+\rho)^2}\frac{(1-p_1)(1-p_2)}{(1-p_1)(1+r)+(1-p_1)(1-p_2)-(1+r)}c_1^R. \tag{89}$$

From equations (84) and (87), first-period consumption is the same for both cases. From equations (85) and (88), second-period consumption is larger in the first-best case. The ratio of changes in the second-period consumption gap between the two cases is calculated as

$$\frac{c_2^N - c_2^R}{c_2^N} = p_1. \tag{90}$$

Thus, the gap ratio increases as the death probability at the beginning of the first period increases.

From equations (86) and (89), third-period consumption is smaller in the first-best case. The ratio of changes in the third-period consumption gap between the two cases is calculated as

$$\frac{c_3^N - c_3^R}{c_3^N} = 1 - \frac{(1-p_1)(1-p_2)}{(1-p_1)(1+r)+(1-p_1)(1-p_2)-(1+r)}. \tag{91}$$

In this example, the change in the gap ratio is independent of the degree of myopia.

In the same manner, we calculate the consumption stream in Case 2 as follows:

$$c_1^R = w/\left(1 + \frac{1-p_1}{1+\rho} + \frac{(1-p_1)(1-p_2)}{(1+\rho)^2}\right). \tag{92}$$

$$c_2^R = \frac{1-p_1}{\sigma(1+\rho)} c_1^R, \tag{93}$$

$$c_3^R = \frac{(1+r)(1-p_1)(1-p_2)}{\sigma(1+\rho)^2} c_1^R. \tag{94}$$

The results of the numerical calculations are illustrated in Tables 1 through 6. To implement the calculations, we assume that the individual purchases the actuarial note at age 45, the working period is from age 45 to 59, the first retirement period is from 60 to 74, and the second retirement period is from 75 to 89. When annual interest rate r_{1year} is 5 percent, $(1 + r_{1year})^{15} = 2.08$. We use this value in the calculations. The death probabilities used in the numerical calculation are set very close to the real death probabilities in Japan. According to estimates by the Ministry of Welfare in Japan, the death probability at age 60 on the condition that the individual is alive at 45 is 0.054, and the death probability at age 75 on the condition that the individual is alive at 60 is 0.411. Finally, the wage income during the working period is set to be 1,000.

Tables 1 and 2 show how individual consumption changes as the death probabilities and the time discount rate change. The numerical results are consistent with the analytical results derived above. Numerical calculations also make it possible to illustrate the consumption differences in each period between the first-best case and the second-best case. As shown in Table 1, the constant annuity constraint decreases second-period consumption and increases third-period consumption in Case 1. On the other hand, Table 2 shows that the constraint decreases third-period consumption and increases second-period consumption in Case 2. In both Case 1 and Case 2, the constraint decreases the consumption gap between the second and third periods.

Table 1
Consumption Stream When r < ρ

p_1	p_2	ρ	c_1^N	c_2^N	c_3^N	c_1^R	c_2^R	c_3^R
0.050	0.410	3.0	725.19	502.54	348.24	725.19	477.41	427.53
0.050	0.410	3.1	732.72	491.37	329.52	732.72	466.80	404.55
0.050	0.410	3.2	739.85	480.65	312.26	739.85	456.62	383.36
0.050	0.415	3.0	725.47	502.73	348.38	725.47	477.59	428.53
0.050	0.415	3.1	732.98	491.55	329.64	732.98	466.97	405.49
0.050	0.415	3.2	740.11	480.82	312.37	740.11	456.78	384.24
0.050	0.420	3.0	725.74	502.92	348.51	725.74	477.77	429.55
0.050	0.420	3.1	733.25	491.73	329.76	733.25	467.14	406.44
0.050	0.420	3.2	740.36	480.98	312.48	740.36	456.94	385.13
0.055	0.410	3.0	726.24	503.26	348.75	726.24	475.58	438.72
0.055	0.410	3.1	733.75	492.07	329.99	733.75	465.00	415.12
0.055	0.410	3.2	740.87	481.31	312.69	740.87	454.84	393.36
0.055	0.415	3.0	726.51	503.46	348.88	726.51	475.77	439.86
0.055	0.415	3.1	734.01	492.24	330.11	734.01	465.17	416.19
0.055	0.415	3.2	741.12	481.48	312.80	741.12	455.00	394.37
0.055	0.420	3.0	726.79	503.65	349.01	726.79	475.95	441.02
0.055	0.420	3.1	734.28	492.42	330.23	734.28	465.34	417.28
0.055	0.420	3.2	741.37	481.64	312.90	741.37	455.15	395.39
0.060	0.410	3.0	727.29	503.99	349.25	727.29	473.75	450.60
0.060	0.410	3.1	734.78	492.76	330.45	734.78	463.19	426.35
0.060	0.410	3.2	741.88	481.97	313.12	741.88	453.06	403.98
0.060	0.415	3.0	727.57	504.19	349.39	727.57	473.93	451.89
0.060	0.415	3.1	735.05	492.94	330.57	735.05	463.36	427.56
0.060	0.415	3.2	742.14	482.14	313.23	742.14	453.21	405.12
0.060	0.420	3.0	727.84	504.38	349.52	727.84	474.11	453.21
0.060	0.420	3.1	735.31	493.11	330.69	735.31	463.53	428.80
0.060	0.420	3.2	742.39	482.30	313.33	742.39	453.36	406.29

Note: r = the interest rate; ρ = time preference rate; p_1 = proportion of individuals who die at the beginning of the second period; p_2 = proportion of individuals who die at the beginning of the third period; c_1^N, c_2^N, and c_3^N = the first-best consumption during the first, second, and third periods, respectively; c_1^R, c_2^R, and c_3^R = the second-best consumption during the first, second, and third periods, respectively.

Estimates of Welfare Loss

Other interesting features are shown in Tables 3 and 4, which demonstrate how the welfare loss changes as the parameters change. The welfare loss is defined as the utility attained in the first-best case minus the utility attained in the second-best case. Column 10 in Tables 3 and 4 indicates that the ratio of welfare loss caused by constant annuities increases as the degree of myopia

Table 2
Consumption Stream When r > ρ

p_1	p_2	ρ	c_1^N	c_2^N	c_3^N	c_1^R	c_2^R	c_3^R
0.050	0.410	1.0	398.32	828.09	1721.54	398.32	955.30	1171.74
0.050	0.410	1.1	429.76	812.22	1535.05	429.76	936.99	1044.81
0.050	0.410	1.2	458.52	794.36	1376.19	458.52	916.39	936.68
0.050	0.415	1.0	399.08	829.66	1724.81	399.08	958.91	1166.20
0.050	0.415	1.1	430.48	813.59	1537.64	430.48	940.34	1039.65
0.050	0.415	1.2	459.22	795.57	1378.28	459.22	919.50	931.90
0.050	0.420	1.0	399.84	831.23	1728.08	399.84	962.53	1160.61
0.050	0.420	1.1	431.21	814.97	1540.25	431.21	943.70	1034.45
0.050	0.420	1.2	459.91	796.77	1380.37	459.91	922.63	927.07
0.055	0.410	1.0	399.59	830.72	1727.01	399.59	958.33	1175.46
0.055	0.410	1.1	431.05	814.67	1539.67	431.05	939.82	1047.95
0.055	0.410	1.2	459.83	796.63	1380.12	459.83	919.01	939.36
0.055	0.415	1.0	400.34	832.29	1730.28	400.34	961.95	1169.90
0.055	0.415	1.1	431.78	816.04	1542.27	431.78	943.17	1042.78
0.055	0.415	1.2	460.53	797.84	1382.21	460.53	922.13	934.56
0.055	0.420	1.0	401.10	833.87	1733.56	401.10	965.59	1164.28
0.055	0.420	1.1	432.51	817.42	1544.87	432.51	946.54	1037.56
0.055	0.420	1.2	461.22	799.04	1384.30	461.22	925.26	929.72
0.060	0.410	1.0	400.86	833.37	1732.51	400.86	961.39	1179.21
0.060	0.410	1.1	432.35	817.13	1544.32	432.35	942.65	1051.12
0.060	0.410	1.2	461.15	798.92	1384.08	461.15	921.65	942.05
0.060	0.415	1.0	401.62	834.94	1735.78	401.62	965.01	1173.62
0.060	0.415	1.1	433.08	818.50	1546.92	433.08	946.01	1045.92
0.060	0.415	1.2	461.84	800.12	1386.17	461.84	924.77	937.23
0.060	0.420	1.0	402.38	836.52	1739.07	402.38	968.65	1167.98
0.060	0.420	1.1	433.81	819.88	1549.52	433.81	949.39	1040.68
0.060	0.420	1.2	462.54	801.33	1388.26	462.54	927.91	932.38

Note: r = the interest rate; ρ = time preference rate; p_1 = proportion of individuals who die at the beginning of the second period; p_2 = proportion of individuals who die at the beginning of the third period; c_1^N, c_2^N, and c_3^N = the first-best consumption during the first, second, and third periods, respectively; c_1^R, c_2^R, and c_3^R = the second-best consumption during the first, second, and third periods, respectively.

increases in Case 1, while it decreases in Case 2. This difference arises because the effect of changes in the time discount rate on the consumption gap is different during the second and third periods. In Case 1, the first-best consumption during the third period is smaller than it is during the second period. An increase in the time discount rate expands the consumption gap. In Case 2, the first-best consumption is larger during the third period than it is during the second period. Thus, an increase in the time discount rate decreases the consumption gap. The welfare loss caused by the constancy of the annuity increases as the first-best consumption gap increases.

To convert the welfare loss into monetary terms, we calculated the compensating variation, defined as

$$E(q^R,U^N) - E(q^N,U^N), \qquad (95)$$

Table 3 entry

Welfare Loss of Uniform Pension Benefit When $r < \rho$

(1) p_1	(2) p_2	(3) ρ	(4) $c_1^N - c_1^R$	(5) $c_2^N - c_2^R$	(6) $c_3^N - c_3^R$	(7) $\frac{c_2^N - c_2^R}{c_2^N}$	(8) $\frac{c_3^N - c_3^R}{c_3^N}$	(9) $WEL^N - WEL^R$	(10) $\frac{WEL^N - WEL^R}{WEL^N}$	(11) *Compensating Variation*
0.050	0.410	3.0	0.0	25.13	-79.29	0.050	-0.228	0.00347	0.00039	2.518
0.050	0.410	3.1	0.0	24.57	-75.03	0.050	-0.228	0.00376	0.00043	2.755
0.050	0.410	3.2	0.0	24.03	-71.10	0.050	-0.228	0.00400	0.00046	2.964
0.050	0.415	3.0	0.0	25.14	-80.15	0.050	-0.230	0.00346	0.00039	2.511
0.050	0.415	3.1	0.0	24.58	-75.84	0.050	-0.230	0.00374	0.00042	2.748
0.050	0.415	3.2	0.0	24.04	-71.87	0.050	-0.230	0.00399	0.00046	2.957
0.050	0.420	3.0	0.0	25.15	-81.03	0.050	-0.233	0.00344	0.00039	2.503
0.050	0.420	3.1	0.0	24.59	-76.68	0.050	-0.233	0.00373	0.00042	2.741
0.050	0.420	3.2	0.0	24.05	-72.66	0.050	-0.233	0.00398	0.00045	2.950
0.055	0.410	3.0	0.0	27.68	-89.97	0.055	-0.258	0.00360	0.00040	2.619
0.055	0.410	3.1	0.0	27.06	-85.13	0.055	-0.258	0.00393	0.00045	2.887
0.055	0.410	3.2	0.0	26.47	-80.67	0.055	-0.258	0.00421	0.00048	3.124
0.055	0.415	3.0	0.0	27.69	-90.98	0.055	-0.261	0.00359	0.00040	2.609
0.055	0.415	3.1	0.0	27.07	-86.08	0.055	-0.261	0.00392	0.00044	2.878
0.055	0.415	3.2	0.0	26.48	-81.57	0.055	-0.261	0.00420	0.00048	3.115
0.055	0.420	3.0	0.0	27.70	-92.00	0.055	-0.264	0.00357	0.00040	2.599
0.055	0.420	3.1	0.0	27.08	-87.05	0.055	-0.264	0.00390	0.00044	2.868
0.055	0.420	3.2	0.0	26.49	-82.48	0.055	-0.264	0.00418	0.00048	3.106
0.060	0.410	3.0	0.0	30.24	-101.35	0.060	-0.290	0.00369	0.00041	2.686
0.060	0.410	3.1	0.0	29.57	-95.89	0.060	-0.290	0.00406	0.00046	2.987
0.060	0.410	3.2	0.0	28.92	-90.86	0.060	-0.290	0.00438	0.00050	3.253
0.060	0.415	3.0	0.0	30.25	-102.51	0.060	-0.293	0.00367	0.00041	2.673
0.060	0.415	3.1	0.0	29.58	-96.99	0.060	-0.293	0.00404	0.00046	2.975
0.060	0.415	3.2	0.0	28.93	-91.90	0.060	-0.293	0.00436	0.00050	3.241
0.060	0.420	3.0	0.0	30.26	-103.69	0.060	-0.297	0.00365	0.00041	2.660
0.060	0.420	3.1	0.0	29.59	-98.11	0.060	-0.297	0.00402	0.00046	2.963
0.060	0.420	3.2	0.0	28.94	-92.96	0.060	-0.297	0.00434	0.00050	3.230

Note: r = interest rate; ρ = time preference rate; p_1 = proportion of individuals who die at the beginning of the second period; p_2 = proportion of individuals who die at the beginning of the third period; c_1^N, c_2^N, and c_3^N = the first-best consumption during the first, second, and third periods, respectively; c_1^R, c_2^R, and c_3^R = the second-best consumption during the first, second, and third periods, respectively. WEL^N = the welfare in the first-best case; and WEL^R = the welfare in the second-best case.

Table 4 - entry

Welfare Loss of Uniform Pension Benefit When $r > \rho$

(1) p_1	(2) p_2	(3) ρ	(4) $c_1^N - c_1^R$	(5) $c_2^N - c_2^R$	(6) $c_3^N - c_3^R$	(7) $\frac{c_2^N - c_2^R}{c_2^N}$	(8) $\frac{c_3^N - c_3^R}{c_3^N}$	(9) $WEL^N - WEL^R$	(10) $\frac{WEL^N - WEL^R}{WEL^N}$	(11) Compensating Variation
0.050	0.410	1.0	0.0	-127.21	549.80	-0.154	0.319	0.080	0.00483	32.330
0.050	0.410	1.1	0.0	-124.77	490.24	-0.154	0.319	0.055	0.00359	23.829
0.050	0.410	1.2	0.0	-122.03	439.51	-0.154	0.319	0.037	0.00257	16.932
0.050	0.415	1.0	0.0	-129.25	558.61	-0.156	0.324	0.080	0.00484	32.425
0.050	0.415	1.1	0.0	-126.75	497.99	-0.156	0.324	0.055	0.00359	23.834
0.050	0.415	1.2	0.0	-123.94	446.38	-0.156	0.324	0.036	0.00256	16.868
0.050	0.420	1.0	0.0	-131.30	567.48	-0.158	0.328	0.080	0.00485	32.509
0.050	0.420	1.1	0.0	-128.73	505.79	-0.158	0.328	0.055	0.00359	23.828
0.050	0.420	1.2	0.0	-125.86	453.29	-0.158	0.328	0.036	0.00255	16.794
0.055	0.410	1.0	0.0	-127.62	551.55	-0.154	0.319	0.079	0.00482	32.261
0.055	0.410	1.1	0.0	-125.15	491.72	-0.154	0.319	0.055	0.00358	23.775
0.055	0.410	1.2	0.0	-122.38	440.76	-0.154	0.319	0.036	0.00257	16.890
0.055	0.415	1.0	0.0	-129.66	560.38	-0.156	0.324	0.080	0.00483	32.356
0.055	0.415	1.1	0.0	-127.13	499.49	-0.156	0.324	0.054	0.00359	23.779
0.055	0.415	1.2	0.0	-124.29	447.65	-0.156	0.324	0.036	0.00256	16.826
0.055	0.420	1.0	0.0	-131.72	569.27	-0.158	0.328	0.080	0.00484	32.439
0.055	0.420	1.1	0.0	-129.12	507.31	-0.158	0.328	0.054	0.00358	23.773
0.055	0.420	1.2	0.0	-126.22	454.58	-0.158	0.328	0.036	0.00255	16.753
0.060	0.410	1.0	0.0	-128.02	553.31	-0.154	0.319	0.079	0.00480	32.192
0.060	0.410	1.1	0.0	-125.53	493.20	-0.154	0.319	0.054	0.00358	23.720
0.060	0.410	1.2	0.0	-122.73	442.03	-0.154	0.319	0.036	0.00256	16.849
0.060	0.415	1.0	0.0	-130.07	562.16	-0.156	0.324	0.079	0.00482	32.286
0.060	0.415	1.1	0.0	-127.51	501.00	-0.156	0.324	0.054	0.00358	23.724
0.060	0.415	1.2	0.0	-124.65	448.93	-0.156	0.324	0.036	0.00255	16.785
0.060	0.420	1.0	0.0	-132.14	571.08	-0.158	0.328	0.079	0.00483	32.369
0.060	0.420	1.1	0.0	-129.51	508.84	-0.158	0.328	0.054	0.00357	23.718
0.060	0.420	1.2	0.0	-126.58	455.88	-0.158	0.328	0.036	0.00254	16.712

Note: r = interest rate; ρ = time preference rate; p_1 = proportion of individuals who die at the beginning of the second period; p_2 = proportion of individuals who die at the beginning of the third period; c^N_1, c^N_2, and c^N_3 = the first-best consumption during the first, second, and third periods, respectively; c^R_1, c^R_2, and c^R_3 = the second-best consumption during the first, second, and third periods, respectively. WEL^N = the welfare in the first-best case; and WEL^R = the welfare in the second-best case.

2D

where q^R = the price vector which appeared in the budget constraint of equations (10) or (21),

q^N = the price vector in the first-best case budget constraint of equation (80), and

U^N = the utility level achieved when the price vector is q^N and earnings income, w, is 1,000.

The price vector q is composed of the price of consumption during the first, second, and third periods. For example, the price of c^R_1 is 1, the price of c^R_2 is 1/(1+r), and the price of c^R_3 is K/(1+r) in Case 1. Thus, $E(q^R, U^N)$ is the expenditure required to attain the utility in the first-best case, when the price vector is q^R. Since the compensated income increases with the divergence from the utility level attained in the first-best case, the value of compensating variation increases as welfare loss increases.

The compensating variation has several advantages as a measure of welfare loss. First, it is easier to grasp the degree of the welfare change because it is measured in monetary terms. Second, when we compare the welfare change among different people, simple comparison of welfare level does not make sense and compensating variation or equivalent variation should be used (see Deaton and Muellbauer, 1980, pp. 184-189).

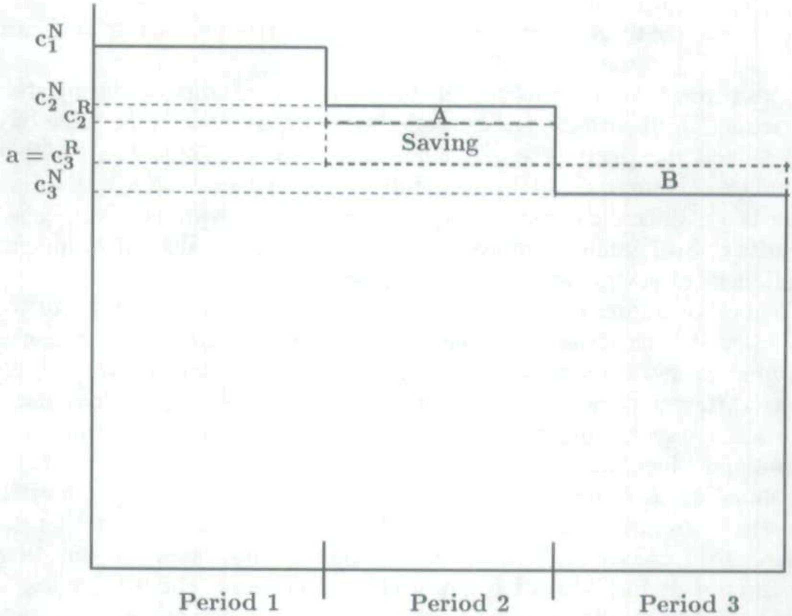
Column 11 in Tables 3 and 4 show that, as the degree of myopia increases, the welfare loss increases in Case 1 and decreases in Case 2. The fact that the welfare loss caused by the constant annuity increases as the first-best consumption gap increases has a profound implication. The welfare loss arises from the differences in the rate of return on annuities and monetary savings and from distorted consumption. Thus, the welfare loss partly reflects the degree of distortion caused by the constant annuity. When the time discount rate changes, however, the discounted value of the distortion also changes. This weakens the effect in Case 1 but strengthens the effect in Case 2.

Figures 1 and 2 illustrate the consumption stream with and without constraints for different discount rates. To simplify the discussion, we consider only Case 1. The consumption stream without constraints is represented by the dotted line. When the discount rate is larger than the interest rate, optimal consumption without constraints during the second period may be smaller than it is during the first period, and consumption may be smaller during the third period than it is during the second period. The gap in consumption increases as the discount rate increases as long as other conditions are equal. Figure 1 illustrates the case of a small discount rate, and Figure 2 illustrates the case of a large discount rate.

When annuity payments are constant throughout retirement and the capital market is incomplete, the demand for annuities that fully finance second-period consumption is too large to finance third-period consumption. On the other hand, the demand for annuities that finance third-period consumption is not enough to finance second-period consumption. Therefore, the demand for annuities is set to a level between the first-best consumption during the second period and that during the third period.

When annuity payments are constant, an individual must finance second-period consumption partly by savings. These savings are not utilized if the individual dies at the beginning of the second period. This is the essential

Figure 1
Divergence from the First-Best Consumption Stream in the Case of Weak Myopia



Note: c^N_1, c^N_2, and c^N_3 = the first-best consumption during the first, second, and third periods, respectively; c^R_2 and c^R_3 = the second-best consumption during the second and third periods, respectively.

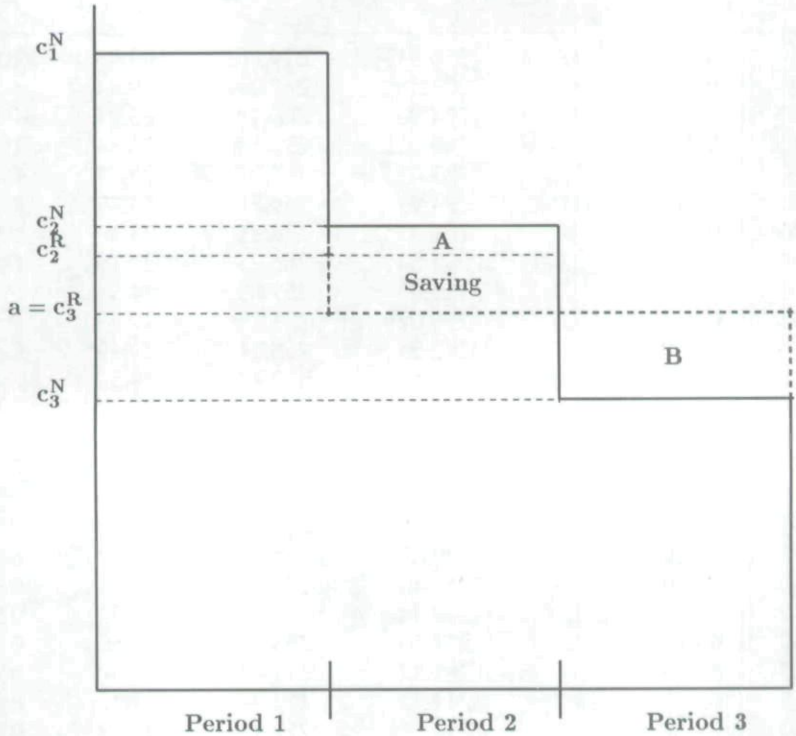
reason why constant annuity payments create inefficiency. The amount of savings necessary to finance second-period consumption increases as the consumption gap increases. This explains the results of the numerical calculation.

The effects of changes in the death probabilities are more complicated. As shown in Table 3, the welfare loss in Case 1 increases as p_1 increases but decreases as p_2 increases. In Case 2, the welfare loss decreases as p_1 increases but increases as p_2 increases, as shown in Table 4. Thus, changes in p_1 have different effects than changes in p_2.

The changes in the degree of myopia affect the consumption stream only through changes in the discounted marginal utility. Changes in death probabilities affect the consumption stream not only through the changes in the expected marginal utility but also through changes in the lifetime income, because an increase in death probabilities improves the rate of return on annuities. Thus, the welfare loss caused by the constant annuity increases as the relative advantage of annuities over savings increases.

The changes in welfare loss that correspond to the changes in p_1 and p_2 come from a change in the consumption gap between the second and third periods and from a change in the rate of return on the annuities. In Case 1, the weight of second-period consumption is relatively larger than the weight of third-period consumption. In contrast, for Case 2, while the effect of changes

Figure 2
Divergence from the First-Best Consumption Stream
in the Case of Strong Myopia



Note: c^N_1, c^N_2, and c^N_3 = the first-best consumption during the first, second, and third periods, respectively; c^R_2 and c^R_3 = the second-best consumption during the second and third periods, respectively.

in p_1 on the rate of return prevails equally in both the second and third periods, the effect on the expected utility differs between the periods. When the weight on the third period is small, the increase in the death probability decreases the welfare loss, because the positive effect of the increase in the rate of return dominates.

The level of welfare loss is larger in Case 2 than in Case 1 because of differences in the demand for annuities. Since the individual has a higher demand for annuities in Case 2, the inefficiency caused from the constant annuity is larger.

Constant Annuities and the Missing Demand for Actuarial Notes

How does an insurer lose potential demand for actuarial notes because of constant annuities? Table 5 shows that the missing demand for actuarial notes in Case 1 increases as the degree of myopia increases and decreases as the death probabilities increase. The last column of this table shows how the proportion of annuities in the total assets changes as the parameters change.

Table 5
Missing Demand for Private Pension When $r < \rho$

p_1	p_2	ρ	z^N	z^R	z^N-z^R	$\frac{z^R}{z^R+s_1}$
0.050	0.410	3.0	250.81	274.80	23.99	0.9127
0.050	0.410	3.1	237.33	267.27	29.94	0.8880
0.050	0.410	3.2	224.90	260.14	35.24	0.8645
0.050	0.415	3.0	250.93	274.53	23.59	0.9140
0.050	0.415	3.1	237.43	267.01	29.57	0.8892
0.050	0.415	3.2	224.99	259.88	34.89	0.8657
0.050	0.420	3.0	251.05	274.25	23.19	0.9154
0.050	0.420	3.1	237.54	266.74	29.19	0.8905
0.050	0.420	3.2	225.09	259.63	34.53	0.8670
0.055	0.410	3.0	256.02	273.75	17.73	0.9352
0.055	0.410	3.1	242.25	266.24	23.99	0.9099
0.055	0.410	3.2	229.55	259.12	29.57	0.8859
0.055	0.415	3.0	256.20	273.48	17.27	0.9368
0.055	0.415	3.1	242.42	265.98	23.56	0.9114
0.055	0.415	3.2	229.71	258.87	29.16	0.8873
0.055	0.420	3.0	256.40	273.20	16.80	0.9385
0.055	0.420	3.1	242.59	265.71	23.11	0.9130
0.055	0.420	3.2	229.87	258.62	28.74	0.8888
0.060	0.410	3.0	261.56	272.70	11.13	0.9592
0.060	0.410	3.1	247.48	265.21	17.72	0.9332
0.060	0.410	3.2	234.50	258.11	23.60	0.9085
0.060	0.415	3.0	261.82	272.42	10.60	0.9611
0.060	0.415	3.1	247.72	264.94	17.22	0.9350
0.060	0.415	3.2	234.72	257.85	23.13	0.9103
0.060	0.420	3.0	262.09	272.15	10.05	0.9631
0.060	0.420	3.1	247.97	264.68	16.70	0.9369
0.060	0.420	3.2	234.96	257.60	22.64	0.9121

Note: r = interest rate; ρ = time preference rate; p_1 = proportion of individuals who die at the beginning of the second period; p_2 = proportion of individuals who die at the beginning of the third period; z^N = the amount of an actuarially fair annuity note in the first-best case; z^R = the amount of an actuarially fair annuity note in the second-best case; s_1 = savings in monetary assets during the first period.

The proportion of annuities decreases as the degree of myopia increases and increases as the death probabilities increase. In Case 2 (see Table 6), the missing demand is zero for all parameter values. The ratio of monetary savings to annuities decreases as the degree of myopia increases, remains constant with respect to p_1, and decreases as p_2 increases. The changes in death probability alter the relative advantage of annuities to monetary savings. When the degree of myopia changes and the death probability is constant, the changes in the proportion of the annuities are caused by changes in the inefficiency of constant annuities.

Because the rate of return increases as the death probability increases, the amount of an actuarial note (premium) that provides one unit of annuity decreases as the death probability increases. Thus, the missing demand for annuities decreases and the ratio of annuities in total assets increases as the death probabilities increase.

Table 6
Missing Demand for Private Pension When r > ρ

p_1	p_2	ρ	z^N	z^R	z^N-z^R	$\frac{s^R}{a^R}$
0.050	0.410	1.0	601.67	601.67	0.000	0.0685
0.050	0.410	1.1	570.23	570.23	0.000	0.0360
0.050	0.410	1.2	541.47	541.47	0.000	0.0071
0.050	0.415	1.0	600.91	600.91	0.000	0.0656
0.050	0.415	1.1	569.51	569.51	0.000	0.0332
0.050	0.415	1.2	540.77	540.77	0.000	0.0044
0.050	0.420	1.0	600.16	600.16	0.000	0.0626
0.050	0.420	1.1	568.78	568.78	0.000	0.0303
0.050	0.420	1.2	540.08	540.08	0.000	0.0016
0.055	0.410	1.0	600.40	600.40	0.000	0.0685
0.055	0.410	1.1	568.94	568.94	0.000	0.0360
0.055	0.410	1.2	540.16	540.16	0.000	0.0071
0.055	0.415	1.0	599.65	599.65	0.000	0.0656
0.055	0.415	1.1	568.21	568.21	0.000	0.0332
0.055	0.415	1.2	539.46	539.46	0.000	0.0044
0.055	0.420	1.0	598.89	598.89	0.000	0.0626
0.055	0.420	1.1	567.48	567.48	0.000	0.0303
0.055	0.420	1.2	538.77	538.77	0.000	0.0016
0.060	0.410	1.0	599.13	599.13	0.000	0.0685
0.060	0.410	1.1	567.64	567.64	0.000	0.0360
0.060	0.410	1.2	538.84	538.84	0.000	0.0071
0.060	0.415	1.0	598.37	598.37	0.000	0.0656
0.060	0.415	1.1	566.91	566.91	0.000	0.0332
0.060	0.415	1.2	538.15	538.15	0.000	0.0044
0.060	0.420	1.0	597.61	597.61	0.000	0.0626
0.060	0.420	1.1	566.18	566.18	0.000	0.0303
0.060	0.420	1.2	537.45	537.45	0.000	0.0016

Note: r = interest rate; ρ = time preference rate; p_1 = proportion of individuals who die at the beginning of the second period; p_2 = proportion of individuals who die at the beginning of the third period; z^N = the amount of an actuarially fair annuity note in the first-best case; z^R = the amount of an actuarially fair annuity note in the second-best case; s^R_2 = savings in monetary assets during the second period in the second-best case; a^R = annuity payment in the second-best case.

Concluding Comments

Although inflation issues are relevant to private annuities, we do not address them here. When we incorporate inflation into our model, the degree of inefficiency of the constant annuity changes. In Case 1, the inefficiency caused from the constant annuity may decrease, while it may increase in Case 2. The exact conclusion, however, should be derived after examining the effect of inflation on the consumption stream. The issue of inflation is a topic for future examination.

Appendix

The optimization problem of the individual in Section 3 is derived more rigorously as follows.

$$\begin{aligned} Max \quad U(c_1,c_2,c_3) = (1-p_1)(1-p_2)U(c_1,c_2,c_3) + (1-p_1)p_2V(c_1,c_2) + p_1H(c_1) \\ s.t. \quad w = c_1 + z + s_1, \\ (1+r)s_1 + a = c_2 + s_2, \\ (1+r)s_2 + a = c_3, \\ a = \xi z, \\ c_1,c_2,c_3,s_1,s_2,z \geq 0, \end{aligned}$$

where

$$\xi = \frac{(1+r)^2}{(1-p_1)(1+r)+(1-p_1)(1-p_2)}. \tag{102}$$

From the Lagrangean,

$$\begin{aligned} L = U + \mu_1(w-c_1-z-s_1) \\ +\mu_2((1+r)s_1+a-c_2-s_2) + \mu_3((1+r)s_2+a-c_3) + \mu_4(a-\xi z) \\ +q_1c_1 + q_2c_2 + q_3c_3 + q_4z + q_5s_1 + q_6s_2, \end{aligned} \tag{103}$$

the optimality conditions are derived as

$$\# \leftarrow \frac{\partial L}{\partial c_1} = U_1 - \mu_1 + q_1 = 0, \tag{104}$$

$$\frac{\partial L}{\partial c_2} = U_2 - \mu_2 + q_2 = 0, \tag{105}$$

$$\frac{\partial L}{\partial c_3} = U_3 - \mu_3 + q_3 = 0, \tag{106}$$

$$\frac{\partial L}{\partial z} = -\mu_1 - \xi\mu_4 + q_4 = 0, \tag{107}$$

$$\frac{\partial L}{\partial a} = \mu_2 + \mu_3 + \mu_4 = 0, \tag{108}$$

$$\frac{\partial L}{\partial s_1} = -\mu_1 + (1+r)\mu_2 + q_5 = 0, \tag{109}$$

$$\frac{\partial L}{\partial s_2} = -\mu_2 + (1+r)\mu_3 + q_6 = 0, \tag{110}$$

$$q_ic_i = 0 \qquad i = 1,2,3, \tag{111}$$

$$q_4z = 0 \tag{112}$$

$$q_{i+4}s_i = 0 \qquad i = 1,2, \tag{113}$$

$$\mu_i \geq 0 \qquad i = 1,2,3, \tag{114}$$

and

$$q_i \geq 0 \qquad i = 1,\ldots,6. \tag{115}$$

In the following discussion, we presume that c_1, c_2, c_3, z, and a are all nonnegative. From the optimality conditions above, we show that s_1 and s_2 cannot be positive simultaneously. Suppose that both s_1 and s_2 are positive. From complimentary slackness conditions (111), (112), and (113), $q_i = 0$ must hold for i = 1,..., 6. From equation (107), we derive

$$\mu_4 = \frac{-\mu_1}{\xi}. \tag{116}$$

Substituting equation (116) into equation (108), we get

$$\mu_3 = \frac{\mu_1}{\xi} - \mu_2. \tag{117}$$

Substituting this into equation (110), we arrive at

$$\mu_2 = \frac{1+r}{2+r}\frac{\mu_1}{\xi}. \tag{118}$$

After substituting equation (118) into equation (109) and using equation (102), we finally derive

$$\left(\frac{(1-p_1)(1+r)+(1-p_1)(1-p_2)}{2+r} - 1\right)\mu_1 = 0. \tag{119}$$

Recalling that $0 \leq p_1 \leq 1$ and $0 \leq p_2 \leq 1$, equation (119) holds only when $p_1 = p_2 = 0$. Thus, $s_1 > 0$, $s_2 > 0$, and $z > 0$ are consistent only when $p_1 = p_2 = 0$. When the individual demands the annuities and death probabilities are positive, the individual never saves in both the first and second periods.

When $s_2 = 0$, we get from equation (110)

$$\mu_2 = (1+r)\mu_3 + q_6. \tag{120}$$

From equations (105) and (106),

$$\frac{U_2}{U_3} = \frac{(1+r)\mu_3}{\mu_3} + \frac{q_6}{\mu_3} \geq 1+r. \tag{121}$$

In the Cobb-Douglas case defined by equations (81), (82), and (83),

$$\frac{U_2}{U_3} = \frac{1+\rho}{1-p_2}\frac{c_3}{c_2}. \tag{122}$$

Substituting equation (122) into equation (121), we get

$$\frac{c_3}{c_2} \geq \frac{(1-p_2)(1+r)}{1+\rho}. \tag{123}$$

Thus, $s_2 = 0$ is consistent with optimality conditions when ρ is large enough compared with r.

When $(1-p_2)(1+r) > 1 + \rho$, c_3 must be larger than c_2. In this case, the constraint of nonnegative savings in the first period becomes effective by using $s_2 = 0$, from equations (6) and (7). In this case, q_5 becomes positive from equation (113). Using equations (109), (104), and (105), we arrive at

$$\frac{U_1}{U_2} = \frac{(1+r)\mu_2}{\mu_2} + \frac{q_5}{\mu_2} \geq 1 + r. \tag{124}$$

In the Cobb-Douglas case, this reduces to

$$\frac{c_2}{c_1} \geq \frac{(1-p_1)(1+r)}{1+\rho}. \tag{125}$$

In the same manner, we can consider the case when $s_1 = 0$ and $s_2 > 0$. From equation (113), $q_5 > 0$ and $q_6 = 0$. Thus, the inequality of equation (124) is derived. Using equations (110), (105), and (106), we derive

$$\frac{U_2}{U_3} = \frac{(1+r)\mu_3}{\mu_3} = 1 + r. \tag{126}$$

The results derived above suggest that the nonnegative constraint on either s_1 or s_2 may be binding when the difference between the interest rate and the time discount rate is small. This case unnecessarily complicates the discussion. Thus, we consider only the case when the nonnegative constraint is binding neither on s_1 nor s_2.

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