

The Value of Information in Insurance Pricing

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ABSTRACT

This article develops a mathematical model of sequential insurance pricing. Prior estimates of expected losses are updated to reflect new information and the evaluation of risk by a sequence of decision-makers. Among the applications discussed are the estimation of adverse selection resulting from a price increase and the evaluation of "follow the leader" strategies in reinsurance underwriting.

Introduction

If an insurer charges too much for a risk, it is likely to lose that business, and if it charges too little, it is likely to lose money. Little research has been conducted, however, on the value of increasing the accuracy of pricing, on commonly used methods of increasing accuracy, or on the effects of raising or lowering prices. There have been many articles on improving accuracy through better classification and experience rating systems, and some insurers have used the systems set forth in these articles with great success.

Insurance company managers are interested in the speed with which pricing is done, but it seems that they often underestimate the value of accuracy as well as the effects of various methods that can be used to improve accuracy. Methods of quantifying both of these things are discussed in this article. I address questions of how much money should be spent on data, classification and experience rating systems, and underwriters' and actuaries' salaries. The effect of adverse selection on pricing adequacy and the effect on profitability of raising or lowering prices also are examined.

The Value of Accuracy

Considerations in Risk Selection

The expected profitability of a risk depends on the relation between the risk's expected losses and its experience-modified premium (if there is any experience modification). Another consideration in the selection process is

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whether the risk is better than average for its classification. Future experience rating credits and debits will not fully reflect experience unless the risk has 100 percent credibility. Therefore, in the long run, a risk with lower expected losses than the provision for losses in its *unmodified* premium will also tend to have expected losses that are less than the provision for losses in the *modified* premium. The reverse is true for risks with higher than average expected losses. This long term consideration involves an estimate of how long the risk will continue renewing its coverage with the company if it is selected.

Competitive strategy should be considered in deciding whether to insure a risk for a given premium or in deciding the level of premium to charge. The prices demanded by the competition, and the prices that buyers are willing to pay, clearly influence and affect the insurer. For example, at a low point in the insurance profitability cycle, an insurer may not even expect to make a profit. It may merely try not to lose too much money or too many customers who will be valuable in the profitable part of the cycle.

The Process of Risk Selection

During the process of risk selection, an insurer sometimes offers to insure a risk for some price. It then wins or does not win the contract depending upon what prices are quoted by other insurers. In other cases, the insurer receives an offer of a price from the buyer or the buyer's representative.

If the insurer *makes* an offer, it is possible to estimate in advance the average expected loss ratio for those cases in which the insurer will win the contract. The estimating method uses the insurer's estimate of the expected loss ratio, the accuracy of such estimates, and the probability, given each possible expected loss ratio of the risk, that the insurer will win the contract.

If the insurer *receives* an offer, the average expected loss ratio of those risks accepted depends on the probability distribution of possible expected loss ratios of the risk and on the accuracy and selectivity of the insurer.

In both cases, the situation can be analyzed through the mathematics of prior and posterior distributions. One must exercise judgment in estimating the probability distributions to be used in particular cases. No matter what other considerations are present when negotiating premiums or selecting risks, the expected losses are an extremely important consideration. Other considerations may affect the expected loss ratio that an insurer is willing to accept for a given risk at a given time.

Quantifying the Value of Information and Accuracy

Assume a prior probability distribution of expected losses for a risk, relative to certain information. Also assume that for a given estimator (which may be either a person or a method) and for any given value of the expected losses, there exists a probability distribution of possible estimates.

Given any probability distribution of expected losses prior to the time that an estimator makes an estimate, the fact that the estimator makes an estimate x determines a posterior distribution. This posterior distribution may, in turn,

be considered the prior distribution relative to a second estimator. The mean of the prior distribution is called the *prior estimate*, and the estimate made by the estimator is called the *estimate*. In many cases, the two estimates are based partly on the same information, and so it is likely that there will be a correlation between the probability distributions of the ratios of expected losses to the prior estimate and of expected losses to the estimate. Any discrepancy between the expected losses and the available information influences both estimates.

The above distributions of expected losses, and of estimates given certain expected losses, are assumed to be lognormal. It is also assumed that whenever the prior probability distribution of expected losses of a risk is lognormal with median m , and an estimate is made of the expected losses, the joint probability distribution of the logarithms of the ratios (expected losses)/(estimate) and (expected losses)/ m is bivariate normal. This assumption has the useful property that the posterior distribution, given any estimate x , is lognormal. The ratio (expected losses)/ m is used in a proof below.

The distribution of $\log(\text{expected losses}/\text{estimate})$ is assumed to have a median of zero and thus a mean of zero. It is not assumed that the distribution will actually have a mean of zero for any given estimation method. Zero is selected as the mean of the distribution of possible means, and the variance of the distribution of possible means increases the variance of the above distribution. If, for an estimation method, a nonzero number is believed to be the mean of the distribution of possible means, the method can be adjusted so that it is believed that the mean is zero.

The applications of the model in this article can be understood independently of the particular distributions and formulas used. The following propositions are useful in the subsequent analysis. See the Appendix for the proofs.

Proposition 1. Assume that the random variable x_1 , which equals $\log(\text{expected losses}/m)$, where m is the median of the prior distribution, and the random variable x_2 , which equals $\log(\text{expected losses}/\text{estimate})$, are bivariate normal. Assume that x_2 has a mean of zero, that the standard deviation of x_1 and x_2 are σ_1 and σ_2 , respectively, and that the correlation between x_1 and x_2 is ρ , where $\sigma_1 > 0$, $\sigma_2 > 0$, and $-1 < \rho < 1$. Then, given that the estimate of a risk's expected losses is x , the probability distribution of the risk's expected losses has a mean of $m(\exp(\mu + \sigma^2/2))$ and variance of

$$m^2(\exp(2\mu + \sigma^2))(\exp(\sigma^2) - 1), \quad (1)$$

where

$$\begin{aligned} \mu = & ((1 - \rho\sigma_2/\sigma_1)\sigma_1^2 \log(x/m))/ \\ & ((1 - \rho^2)\sigma_2^2 + (1 - \rho\sigma_2/\sigma_1)^2 \sigma_1^2), \text{ and} \end{aligned} \quad (2)$$

$$\sigma^2 = ((1 - \rho^2)\sigma_2^2 \sigma_1^2 / ((1 - \rho^2)\sigma_2^2 + (1 - \rho\sigma_2/\sigma_1)^2 \sigma_1^2)). \quad (3)$$

Proposition 2. Suppose that the conditions of Proposition 1 are satisfied. Then, given that the estimate of a risk's expected losses is less than or equal to a constant E , the mean of the probability distribution of the expected losses of the risk is

$$m \cdot \exp((\sigma^2 + a^2)/2) \phi[(1/v) \log(E/m) - a] / \phi[(1/v) \log(E/m)], \quad (4)$$

where $\phi(x)$ is the standard normal distribution function, and

$$\sigma^2 = (1 - \rho^2) \sigma_2^2 \sigma_1^2 / ((1 - \rho^2) \sigma_2^2 + (1 - \rho \sigma_2 / \sigma_1)^2 \sigma_1^2), \quad (5)$$

$$a = (1 - \rho \sigma_2 / \sigma_1)^2 \sigma_1^2 / ((1 - \rho^2) \sigma_2^2 + (1 - \rho \sigma_2 / \sigma_1)^2 \sigma_1^2)^{.5}, \quad (6)$$

$$v = ((1 - \rho \sigma_2 / \sigma_1)^2 \sigma_1^2 + (1 - \rho^2) \sigma_2^2)^{.5}. \quad (7)$$

The probability that the estimate of a risk's expected losses is equal to or less than E is

$$\phi((1/v) \log(E/m)). \quad (8)$$

Example 1—Fixed Rates, No Adverse Selection

Suppose that an insurer has decided, before evaluating a certain risk, that it will accept the risk's offer of premium P if its expected losses are estimated to be less than or equal to E . This decision could be based on many considerations, such as the volume of business the company desires, the long-term prospects of the risk, etc. Suppose that the prior probability distribution of expected losses for the risk, conditional on its risk class and experience rating, is lognormal with median m .

The accuracy of the method used to estimate the risk's expected losses affects the mean and variance of the probability distribution of average expected losses of the set of accepted risks. Given that the prior distribution and the estimation method satisfy the conditions of Proposition 1, assume that $E = 0.82m$ (i.e., $\log(E/m) = -0.2$). Assume that the probability of $\log(\text{expected losses}/m)$ being between -0.5 and 0.5 (i.e., $0.606m \leq \text{expected losses} \leq 1.649m$) is 0.683 (i.e., $\sigma_1 = 0.5$), and $\rho = 0.5$. Then the mean of the probability distribution of the expected losses of a risk is given by Proposition 2, and $\sigma^2 = (.1875\sigma_2^2)/(\sigma_2^2 - .5\sigma_2 + .25)$, $a = (.25(1 - \sigma_2^2))/(\sigma_2^2 - .5\sigma_2 + .25)^{.5}$, and $v = ((1 - \sigma_2^2)(.25) + .75\sigma_2^2)^{.5}$. The probability of the risk being accepted is also given by Proposition 2. Therefore, if $\sigma_2 = 0.333$, then the probability of the risk being accepted is 0.325 , and the mean of the probability distribution of the expected losses of the risks selected is $0.802m$. If $\sigma_2 = 0.667$, then the probability of the risk being accepted is 0.369 , and the mean of the probability distribution of the expected losses of the risks selected is $1.072m$. Although not proven here, an increase in accuracy decreases both the mean and the variance of the expected losses of the risks chosen.

Example 2—Variable Rates, Adverse Selection

The more accurate an insurer's estimates of expected losses, the less room is left for adverse selection. For example, a perfect estimate for every risk would make adverse selection impossible. Therefore, a lower loading for adverse selection is necessary when a company is making an offer to a risk it is better able to price. This is a reason for an underwriter to be wary of making offers to types of risks he or she is not expert in.

Suppose insurer A offers to insure a certain risk for a premium based on some profit margin and an estimate E of the risk's expected losses. Suppose insurer B is competing for the risk and that, relative to the knowledge of company A, the probability distribution of expected losses is lognormal, with median m . Let X_1 be the random variable $\log(\text{expected losses}/m)$, and let X_2 be the random variable $\log(\text{expected losses}/\text{estimate of company B})$. Suppose that X_1 and X_2 satisfy all of the conditions of Proposition 1 and that company B will make a better offer than company A only if B's estimate of the expected losses is equal to or less than E . According to Proposition 1, the probability p of this happening is $\phi((1/v) \log(E/m))$, and the mean μ_1 of the probability distribution of the expected losses of the risk, given that company B makes a better offer, is

$$m \cdot \exp((\sigma^2 + a^2)/2) \phi((1/v) \log(E/m) - a) / \phi((1/v) \log(E/m)). \quad (9)$$

Let μ_2 = the mean of the probability distribution of expected losses for the risk prior to company B deciding whether to make an offer. The expected value C of the losses, given that company B does not make a better offer, satisfies the equation $(1-p)C + p\mu_1 = \mu_2$. Therefore, $C = (\mu_2 - p\mu_1)/(1-p)$, and $\mu_1 < \mu_2$, $C > (\mu_2 - p\mu_2)/(1-p) = \mu_2$.

If the risk ends up on insurer A's books, the expected losses equal C , not μ_2 . The above equations make it possible to solve for C in terms of μ_2 . If insurer B is the only competing company, and if the probability that the risk will get a competing bid from company B is p' , then the expected losses of the risk, given that it becomes insured by company A, are $(1-p')\mu_2 + p'C$.

In a more realistic and complicated example, in which the risk gets competing bids from several insurers, this model could still be used. It would be necessary to go through the above reasoning for all competing insurers. The posterior distribution, given that insurer B does not make a lower bid than insurer A, would be the prior distribution relative to the estimate of insurer C.

The Effect of Rate Increases on Loss Ratios

When rates are increased, expected loss ratios may not be lowered as much as estimated, due to the fact that adverse selection tends to be increased. It is not necessarily true that an insurer will decrease its expected loss ratio at all by charging higher rates. By doing so, an insurer may allow the competition to take most of the better risks to the point that the insurer will have a higher expected loss ratio in spite of its higher rates. Adverse selection must be considered in estimating the effect of a rate increase on expected loss ratios.

On Improving Accuracy

Two Heads Are Better Than One

Proposition 1 is directly applicable to the question of how to weight two estimates of the expected losses of a risk. Suppose that the prior distribution referred to in Proposition 1 is the probability distribution of expected losses given the first estimate. Also, suppose that the estimate x referred to in Proposition 1 is the second of the two estimates. Then the probability distribution of the risk's expected losses, given the two estimates, has the mean and variance referred to in Proposition 1. That is, the mean is $m \cdot \exp(\mu + \sigma^2/2)$, and the variance is $m^2(\exp(2\mu + \sigma^2))(\exp(\sigma^2) - 1)$, where μ and σ are as defined in Proposition 1.

If the two original estimates are not identical, the estimate $m \cdot \exp(\mu + \sigma^2/2)$ is a more accurate estimator. If the estimators are people rather than methods, they can discuss their estimates with each other and they may discover mistakes, oversights, or poor judgments that were used. So the benefits of the "two heads" method are even greater than those of using $m \cdot \exp(\mu + \sigma^2/2)$.

The above method of using two estimates can be extended to n estimates by supposing that the prior distribution referred to in Proposition 1 is the probability distribution of expected losses given the first $n-1$ estimates.

The above analysis of whether two heads are better than one also applies to estimates of loss reserves. The lognormal also may be appropriate as a probability distribution of loss reserves.

Following the Lead

Forms of "following the lead" are sometimes used by both primary insurers and reinsurers. Various follow the lead strategies are used by primary insurers when they obtain information from competitors' rate manuals or from brokers about the premiums charged by their competitors.

A reinsurer that operates in the reinsurance broker market will sometimes be told by a broker, when given a reinsurance treaty to consider, that a certain reinsurer has "taken the lead" by agreeing to accept some share of the treaty at certain terms. The broker may also point out that certain other reinsurers have "followed the lead" by agreeing to take various shares. It may be that only the lead insurer has actually priced the treaty, and that the other insurers are simply following it.

Given that a certain reinsurer has been the only reinsurer to price a treaty, the fact that it found the treaty acceptable could be used by another reinsurer to produce a "prior probability distribution" of the expected losses of the risk. The mean of this distribution would be the premium minus the reinsurer's expenses and expected profit margin, and the variance would be based on the estimated accuracy of its pricing. However, if the rate was rejected by several other reinsurers before being accepted by the lead reinsurer, the estimated prior distribution would be much different. The significance of the fact that a certain reinsurer has taken the lead and that other reinsurers have followed depends on

estimates of: (1) How many reinsurers have accepted the rate, and how many have rejected it? (2) How many are merely “following the lead” without pricing the treaty themselves? (3) How accurate are the reinsurers which have rejected or accepted the rate, and what are their expected profit margins? (4) What are the correlations between the various pairs of reinsurers’ estimates? Due to the unknowns inherent in the follow the lead method, it should be used only as a supplement to other estimation techniques.

Stop and Go Pricing

Instead of deciding beforehand how much time to spend estimating whether a risk is acceptable at some fixed price, an insurer could use a stop and go strategy. A quick estimate can be made, and then, if the price is much too high or much too low, a decision is made. If the price is on the borderline, further time is spent attempting to estimate more accurately. (For example, a second opinion might be sought.)

Given the estimated probability distribution of expected losses for a risk, Propositions 1 and 2 can be applied to estimate the probability that the decision about the acceptability of the risk will be changed as a result of further analysis. It is also possible to estimate the expected losses of the risk given that the decision is changed. Therefore, the value of the second opinion can be estimated and compared to the expense.

Conclusion

The question of how to maximize profit through the level and accuracy of pricing can, to some extent, be answered mathematically. This analysis requires estimates relating to current market conditions, the available set of risks, and the accuracy of various methods of pricing. There are many important practical applications of the mathematical analysis.

Appendix

Lemma 1

See DeGroot (1986, p. 303). Suppose that the random variables X_1 and X_2 have a bivariate normal distribution, and that the correlation of X_1 and X_2 is ρ . Suppose also that $E(X_1) = \mu_1$, $E(X_2) = \mu_2$, $\text{var}(X_1) = \sigma_1^2$, $\text{var}(X_2) = \sigma_2^2$, where $\sigma_1 > 0$, $\sigma_2 > 0$, $-1 < \rho < 1$. Then the conditional distribution of X_2 , given that $X_1 = x_1$, is a normal distribution with mean of $\mu_2 + \rho\sigma_2((x_1 - \mu_1)/\sigma_1)$ and variance of $(1 - \rho^2)\sigma_2^2$.

Lemma 2

See DeGroot (1986, p. 324). Suppose that an element is chosen at random from a normal distribution for which the value of the mean θ is unknown ($-\infty < \theta < \infty$), and the value of the variance σ^2 is known ($\sigma^2 > 0$). Suppose also that the prior distribution of θ is a normal distribution with given values of the mean μ and the variance v^2 . Then the posterior distribution of θ , given that the element chosen equals x_1 , is a normal distribution with mean μ_1 and variance v_1^2 :

$$\mu_1 = (\sigma^2\mu + v^2x_1)/(\sigma^2 + v^2), \quad v_1^2 = (\sigma^2v^2)/(\sigma^2 + v^2).$$

Proof of Proposition 1

If $\log(\text{expected losses}/m) = x_1$, then, since X_1 and X_2 are bivariate normal and X_1 must have mean 0 since it has median 0 and is normal, the probability distribution of $\log(\text{expected losses}/\text{estimate})$ is normal with mean $(\rho\sigma_2x_1)/\sigma_1$ and variance $(1 - \rho^2)\sigma_2^2$ by Lemma 1. Therefore, the distribution of $\log(\text{estimate}/m)$, given that $\log(\text{expected losses}/m) = x_1$, is normal with mean $x_1(1 - (\rho\sigma_2)/\sigma_1)$ and variance $(1 - \rho^2)\sigma_2^2$. The set of all such distributions, as x_1 ranges between $-\infty$ and ∞ , is therefore a set of normal distributions, each of which has variance $(1 - \rho^2)\sigma_2^2$, and whose means are normally distributed with mean 0 and variance $[(\rho\sigma_2)/\sigma_1]^2\sigma_1^2$. If the estimate of expected losses is x , then $\log(\text{estimate}/m) = \log(x/m)$, and it follows from Lemma 2 that the posterior distribution of the means of the above set of normal distributions

$$\text{has mean } \frac{(1 - (\rho\sigma_2)/\sigma_1)^2\sigma_1^2}{(1 - \rho^2)\sigma_2^2 + (1 - (\rho\sigma_2)/\sigma_1)^2\sigma_1^2} \log(x/m) \text{ and variance } \frac{(1 - \rho^2)\sigma_2^2(1 - (\rho\sigma_2)/\sigma_1)^2\sigma_1^2}{(1 - \rho^2)\sigma_2^2 + (1 - (\rho\sigma_2)/\sigma_1)^2\sigma_1^2}.$$

As mentioned above, if $\log(\text{expected losses}/m) = x_1$, then the distribution of $\log(\text{estimate}/m)$ has mean $x_1(1 - (\rho\sigma_2)/\sigma_1)$. So the posterior distribution of

$\log(\text{expected losses}/m)$ has a mean μ , which is $1/(1 - (\rho\sigma_2)/\sigma_1)$ times the above mean, and a variance σ^2 , which is $1/(1 - (\rho\sigma_2)/\sigma_1)^2$ times the above variance; so

$$\mu = (1 - (\rho\sigma_2)/\sigma_1)\sigma_1^2 \log(x/m) / ((1 - \rho^2)\sigma_2^2 + (1 - (\rho\sigma_2)/\sigma_1)^2\sigma_1^2)$$

$$\sigma^2 = (1 - \rho^2)\sigma_2^2\sigma_1^2 / ((1 - \rho^2)\sigma_2^2 + (1 - (\rho\sigma_2)/\sigma_1)^2\sigma_1^2).$$

If the logs of a distribution are normally distributed with mean μ and variance σ^2 , the mean of the distribution is $\exp(\mu + \sigma^2/2)$, and the variance is $\exp(2\mu + \sigma^2)(\exp(\sigma^2) - 1)$. This gives the mean and variance of the distribution of $(\text{expected losses}/m)$, and Proposition 1 follows immediately. Q.E.D.

Proof of Proposition 2

It can be seen from the proof of Proposition 1 that the probability distribution of $\log(\text{estimate}/m)$ is normal with variance $v^2 = (1 - (\rho\sigma_2)/\sigma_1)^2\sigma_1^2 + (1 - \rho^2)\sigma_2^2$. (In the proof, given $\log(\text{expected losses}/m) = x_1$, the distribution of $\log(\text{estimate}/m)$ has mean $x_1(1 - (\rho\sigma_2)/\sigma_1)$ and variance $(1 - \rho^2)\sigma_2^2$, and the distribution of X_1 has variance σ_1^2 .) The probability p that the risk will be accepted is $\phi((1/v)\log(E/m))$ where $\phi(x)$ is the standard normal distribution function.

It can also be seen from the statement of the theorem that if $(1/v)\log(\text{estimate}/m)$ equals t , the probability distribution of $\log(\text{expected losses}/m)$ has mean (μ) and variance (σ^2) :

$$\mu = \frac{vt(1 - (\rho\sigma_2)/\sigma_1)^2\sigma_1^2}{(1 - \rho^2)\sigma_2^2 + (1 - (\rho\sigma_2)/\sigma_1)^2\sigma_1^2} = \frac{t(1 - (\rho\sigma_2)/\sigma_1)^2\sigma_1^2}{((1 - \rho^2)\sigma_2^2 + (1 - (\rho\sigma_2)/\sigma_1)^2\sigma_1^2)^{.5}}$$

and variance

$$\sigma^2 = \frac{(1 - \rho^2)\sigma_2^2\sigma_1^2}{(1 - \rho^2)\sigma_2^2 + (1 - (\rho\sigma_2)/\sigma_1)^2\sigma_1^2}.$$

The mean of the probability distribution of the expected losses of the risk, given that it is accepted (i.e., $\text{estimate} \leq E$), is, therefore,

$$\frac{\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\log(E/m)/v} m \cdot \exp((-t^2)/2) \exp(\mu + \sigma^2/2) dt}{\phi(\log(E/m)/v)}$$

There is a such that $\mu = at$, so for that a the above integral equals

$$\begin{aligned}
 & \frac{\int_{-\infty}^{\log(E/m)/v} m \cdot \exp((-t^2)/2) \exp(at + \frac{\sigma^2}{2}) dt}{\phi(\log(E/m)/v)} \\
 &= \frac{\int_{-\infty}^{\log(E/m)/v} m \cdot \exp((\sigma^2 + a^2)/2) \exp(-(t-a)^2/2) dt}{\phi(\log(E/m)/v)} \\
 &= \frac{m \cdot \exp((\sigma^2 + a^2)/2) \int_{-\infty}^{\log(E/m)/v-a} \exp(-t^2/2) dt}{\phi(\log(E/m)/v)} \\
 &= m \cdot \exp((\sigma^2 + a^2)/2) \frac{\phi(\log(E/m)/v - a)}{\phi(\log(E/m)/v)} \quad \text{Q.E.D.}
 \end{aligned}$$

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