© The Journal of Risk and Insurance, 1992 Vol LIX, No. 4, 655-659

The Purchase of Insurance by a Risk-Neutral Firm for a Risk-Averse Agent: An Extension

L. Dwayne Barney, Jr.

ABSTRACT

In a previous issue of this journal, Parry and Parry (1991) analyze the purchase of director and officer liability insurance by a firm for a prospective board member. This article determines the sign of certain comparative statics results that Parry and Parry reported as indeterminate. Simple expressions for relevant derivatives are obtained using the envelope theorem.

The Parry and Parry Model

Parry and Parry (1991) develop a model to examine the purchase of director and officer insurance by a firm for a prospective board member. Within their framework, the objective of the firm is to maximize the value of the firm net of the compensation package offered to the prospect. The firm is constrained to offer a compensation package resulting in a level of expected utility which is at least as great as the prospect's utility in the absence of accepting the board position.

Using the authors' notation, the formal model is stated as follows:

$$\max_{R,F} N(1, R, F) \equiv M(1) - rR - F \tag{1}$$

subject to

$$PU(W_0 + F - L + R) + (1 - P)U(W_0 + F) = U(W_0), \tag{2}$$

where N(1, R, F) = the value of the firm net of the compensation package,
- M(1) = the gross value of the firm if the prospect accepts the board position,
- r = the per unit director and officer insurance premium,
- R = the amount the insurance policy reimburses the board member if the member loses a lawsuit,

L. Dwayne Barney, Jr., is Associate Professor of Finance at Boise State University. Comments from an associate editor and participants at the Boise State University Finance Seminar Series are greatly appreciated.

F = the fee paid for serving on the board,
P = the probability the prospect will lose a suit of L dollars,
W_0 = the prospect's initial wealth, and
$U(W)$ = an increasing and concave utility function.

The utility function specified by Parry and Parry does not depend upon the prospect's work effort. As seen in equation (2), a consequence of ignoring labor effort is that the fee paid for serving on the board (F) will be zero if there is a zero probability of losing a lawsuit.

The Lagrangian (Z) for this problem is

$$Z = M(1) - rR - F + \mu[PU(W_1) + (1-P)U(W_2) - U(W_0)], \tag{3}$$

where μ is the Lagrange multiplier,

$$W_1 = W_0 + F - L + R, \text{ and } W_2 = W_0 + F.$$

The first-order necessary conditions (see Parry and Parry, p. 35) are

$$PU(W_1) + (1-P)U(W_2)-U(W_0) = 0, \tag{4}$$

$$-1 + \mu[PU'(W_1) + (1-P)U'(W_2)] = 0, \tag{5}$$

$$-r + \mu PU'(W_1) = 0. \tag{6}$$

The solution to equations (4) through (6) depends upon the relationship between P and r, which is assumed by Parry and Parry to be $P \leq r$. For the case of $P = r$, the policy is actuarially fair, and the solution is full indemnity ($W_1 = W_2$). Otherwise, $P < r$ and, at the optimum, $W_1 < W_2$.

In conducting comparative statics analysis, Parry and Parry derive numerous intuitively appealing results concerning the impact of changes in r, P, W_0, and L on the optimal compensation package (R^*, F^*). They also examine the cost of the optimal compensation package $CP^* \equiv rR^* + F^*$. In examining the impact of a change in r on CP^*, Parry and Parry (p. 43) obtain the following expression:

$$\frac{dCP(F^*,R^*;r)}{dr} = (1/H)(A+B)\{(1-r)A - rB\} + R^*, \tag{7}$$

where H is the positive determinant of the bordered Hessian,

$$A \equiv PU'(W_1) \text{ and } B = (1 - P)U'(W_2).$$

Although this expression is correct, it can be considerably simplified by utilizing the first-order necessary conditions. Note that equations (5) and (6) can be rewritten as follows:

$$\frac{1}{\mu} = A + B \tag{5a}$$

Multiplying both sides of equation (5a) by r gives

$$\frac{r}{\mu} = A. \tag{6a}$$

Multiplying both sides of (5a) by r gives

$$\frac{r}{\mu} = r(A+B). \tag{8}$$

Then, substituting equation (6a) into the left side of equation (8) yields

$$A = r(A + B),$$

which implies

$$A(1 - r) - rB = 0. \tag{9}$$

Referring to equation (7) and utilizing equation (9) implies

$$\frac{dCP(F^*,R^*;r)}{dr} = R^* > 0. \tag{10}$$

Parry and Parry state that the "effects of a change in r or P are indeterminate, but if the insurance is actuarially fair (from the prospect's perspective), then an increase in r or L increases the cost to the firm of the prospect's compensation package" (p. 38). Equation (10) shows that the result is in fact determinate, regardless of whether the insurance is actuarially fair.

The Envelope Theorem

The envelope theorem provides a much simpler method for obtaining the effects of changes in parameters on the cost of the optimal compensation package.[1] To apply the theorem, note that the indirect objective function is related to the cost of the optimal compensation package by the equation

$$N^*(1, R^*, F^*) = M(1) - CP^*(F^*, R^*; r). \tag{11}$$

Thus, for any parameter x_i,

$$\frac{dN^*}{dx_i} = -\frac{dCP^*}{dx_i}$$

By the envelope theorem, the rate of change in N^* with respect to a parameter can be found by differentiating the Lagrangian (3) with respect to the parameter, holding all choice variables constant at their optimal levels. Thus,

[1] Silberberg (1978) provides a thorough presentation of the envelope theorem and its consequences.

$$\frac{dN^*}{dr} = -R^* \Rightarrow \frac{dCP^*}{dr} = R^* > 0,$$

$$\frac{dN^*}{dP} = \mu\,[U(W_1) - U(W_2)] \Rightarrow \frac{dCP*}{dP} = \mu\,[U(W_2) - U(W_1)] \geq,$$

$$\frac{dN^*}{dL} = -\mu PU'(W_1) = -r \Rightarrow \frac{dCP^*}{dL} = r > 0,$$

$$\frac{dN^*}{dW_o} = \mu[PU'(W_1) + (1-P)u'(W_2) - U'(W_o)]$$

$$\Rightarrow \frac{dCP^*}{dW_o} = -\mu[PU'(W_1) + (1-P)U'(W_2) - U'(W_o)].^2$$

Modifying their model, Parry and Parry specify the loss (L) to be a function of the per-unit insurance premium r and environmental factors e, where $\frac{\partial L}{\partial r} \equiv L_r \geq O$ and $\frac{\partial L}{\partial e} \equiv L_e \geq O$. Once again, the envelope theorem can be used to determine the effects of parameter changes on the cost of the optimal compensation package. Application of the envelope theorem yields the following results:

$$\frac{dN^*}{dr} = -R^* - -\mu PU'(W_1)L_r = -R^* - rL_r \Rightarrow$$

$$\frac{dCP^*}{dr} = R^* + r\ L_r > O,$$

$$\frac{dN^*}{de} = -\mu PU'(W_1)L_e = -rL_e \Rightarrow \frac{dCP^*}{de} = rL_e > O.$$

An alternative case is considered, where the probability of loss is a function of r and e, with $\frac{\partial P}{\partial r} \equiv P_r \geq O$ and $\frac{\partial P}{\partial e} \equiv P_e \geq O$. In this case, the relevant envelope results are

$$\frac{dN^*}{dr} = -R^* + \mu[U(W_1) - U(W_2)]P_r \Rightarrow \frac{dCP*}{dr} = +R^* + \mu[U(W_2) - U(W_1)]P_r > O.$$

$$\frac{dN^*}{de} = \mu[U(W_1) - U(W_2)]P_e \Rightarrow \frac{dCP^*}{de} = \mu[U(W_2) - U(W_1)]P_e \geq O.$$

---

[2] The solution to this model provides a Pareto optimal contract between the risk-neutral firm and the risk-averse prospect. Comparative statics results like those obtained here also could be derived from the model of Pareto optimal insurance attributable to Raviv (1979), where Raviv's risk-neutral insurer takes the place of this model's risk-neutral firm. The author thanks an associate editor of this journal for pointing out the similarities between the two formulations.

Parry and Parry developed an interesting model of director and officer liability insurance and conducted appropriate comparative statics analysis. This article simplifies and extends some of their comparative statics results through the use of the envelope theorem. Applying the theorem facilitates the determination of the sign of derivatives which Parry and Parry initially reported as being indeterminate, attesting to the envelope theorem's power in analysis.

References

Parry, Mark E. and Arthur E. Parry, 1991, The Purchase of Insurance by a Risk-Neutral Firm for a Risk-Averse Agent, *Journal of Risk and Insurance,* 58: 30-46.

Raviv, Artur, 1979, The Design of an Optimal Insurance Policy, *American Economic Review,* 69: 84-96.

Silberberg, Eugene, 1978. *The Structure of Economics: A Mathematical Analysis* (New York: McGraw-Hill).