

Insurance Futures and Hedging Insurance Price Risk

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ABSTRACT

This article discusses the Chicago Board of Trade's proposal to establish a market for futures contracts (and options on futures) written on insurance business. A European call option on an insurance future is equivalent to a stop-loss reinsurance contract on the portfolio of insurance policies underlying the futures contract. Put options correspond to the discounted expected value of retained losses. After a credible experience record is developed, an insurer can determine the correlation of its own portfolio of policies with the market portfolio and use the futures options market as a partial substitute for traditional stop-loss reinsurance. The advantages of such a market are that it provides a tool for hedging business risk, it allows an entity to participate in the market portfolio's profitability without being a licensed insurer, it may have lower transactions costs than traditional reinsurance, and it provides a mechanism for price discovery. But, despite the advantages of an insurance futures market, there are serious barriers to its success, which are discussed briefly.

Introduction

In May 1990 the Chicago Board of Trade (CBOT) announced a novel insurance futures contract designed to allow the insurance industry to hedge business results. We develop a valuation model for the proposed futures contracts. The model incorporates some features used in popular models for financial futures and options markets, such as the well-known Black-Scholes (1973) option pricing model and the Black (1976) futures option pricing model.

There are three insurance futures contracts under consideration by the CBOT, each pertaining to a different line of insurance: health, homeowners,¹

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¹ The homeowners futures contract is based mostly, but not exclusively, on property damage. The futures price index is based on losses paid within six months, which excludes most liability and medical payments losses.

and catastrophic property damage.² Options on these futures are also under consideration. In addition, futures markets for marine insurance are being developed in London. Because of possible competition, only general information is being provided to the public until the proposals are approved by the Commodities Futures Trading Commission.

Preliminary announcements by the CBOT indicate that an insurance futures contract would be based on a pool of qualifying policies representing a national cross-section, and a new policy pool would be compiled each January.³ Demographic information and other aggregate policy characteristics will be published upon pool formation. Qualifying policies must originate in the month of pool formation and must have fixed premiums over the first year. Qualifying insurers will be anonymous, and no insurer may comprise more than 15 percent of the pool. The CBOT will designate a pool manager (e.g. a statistical agent such as the Insurance Services Office) to establish the pool and monitor aggregate premium and claims activity.

We describe the value upon which the futures contract is based in more detail below, but for our introductory discussion it is sufficient to define it as $100,000 \cdot LR$, where LR is the loss ratio (paid claims divided by earned premiums).⁴ A futures trader who believes current insurance futures prices are low would buy insurance futures. No cash outlay is required, beyond establishing a margin account. The trader merely notifies a broker and pays a small transaction fee. As the futures price increases, the buyer's margin account is credited with the increase. If the futures price decreases, the trader's margin account is drawn down correspondingly. Margin accounts are adjusted for gains and losses on a daily basis. A trader who expects a price decline would sell insurance futures.

Although the contract is ultimately settled on the basis of actual claims and premiums, the day-to-day settlement prices are determined by market consensus—the same way the more familiar commodities markets work. But, unlike the more familiar commodities futures, insurance futures obligations cannot be settled by delivery of a commodity. Insurance futures are similar to the very successful futures written on stock price indices. In the case of a futures contract written on a stock price index, the futures obligations are settled in cash on the basis of the stock price index value at the time the futures contract matures. The value of $100,000 \cdot LR$ plays the same role for insurance futures that the stock price index plays for stock price futures. Thus,

² The catastrophe futures contract is based on an aggregate weighted average of property losses across the following lines: private passenger automobile physical damage, commercial automobile physical damage, fire, allied, homeowners multiple peril, commercial multiple peril, farmowners multiple peril, earthquake, and inland marine.

³ Details of the futures contract may vary slightly by line of insurance. For example, there may be an additional health insurance pool formed each July, and there may be additional catastrophe contracts based on regional pools.

⁴ The settlement value is $50,000 \cdot LR$ for the proposed homeowners futures contract and $25,000 \cdot LR$ for the proposed catastrophe futures contract.

information about claims payments will be very important to insurance futures traders.

Each month, until the last contract expires, insurers participating in the pool will report their premium and claims activity to the statistical agent, and this information, in aggregate, will be made public. Other events will also have an impact on futures prices. For example, a hurricane might have an impact on homeowners insurance futures prices. We can expect insurance futures prices to vary from day to day just as the more familiar futures prices of commodities and financial instruments do, even though actual data will be reported only once a month. The mark-to-the-market mechanism will be used to pass these market value changes to buyers and sellers daily based on each day's closing price.

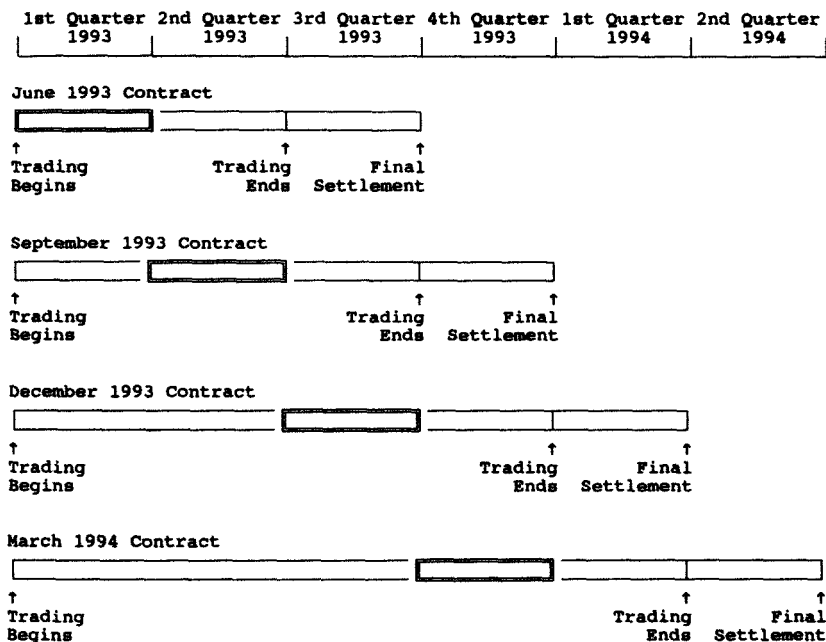
On the contract's maturity date, the ratio of paid claims to earned premiums per \$100,000 of premiums will be computed for the designated time period. On the maturity date, the settlement price is equal to this number. Although the computed ratio of claims to premiums may be greater than one, this is unlikely because the premiums are loaded for expenses, profits, and contingencies.

There will be four contracts available for each pool, pertaining to losses in each of four quarters over a one-year period. The loss period for each contract consists of one quarter. The reporting period for each contract includes the loss period and the quarter following the loss period. A contract is based on premiums earned and claims paid in the loss period plus claims reported and paid in the second quarter of the reporting period that are associated with events occurring in the loss period. Although all contingent events affecting the value of the final settlement index will have occurred by the end of the reporting period, the final settlement index will not be computed until approximately three months later to allow time for the collection of data from participating insurers. At that time, the contract will mature and trading will cease. The contract month is defined as the last month of the reporting period. For example, the contract for the first quarter will expire at the end of June, and final settlement for the June contract will occur at the end of September. Trading in all four contracts will begin when the pool is formed. Figure 1 illustrates the timing associated with a hypothetical pool formed in January 1993.

The computed loss ratio does not take into account operating expenses or earnings on invested assets. Because premiums are relatively certain, the volatility of the futures price will result from the uncertainty in claims.

A distinguishing feature of these futures contracts is that the underlying instrument is a portfolio of insurance contracts, for which there is no spot market. Thus, the price information that is needed to set the daily closing futures prices is obtained in other ways. First, some market participants are insurers who know their own results and can therefore better estimate the market portfolio value. Second, from time to time during the life of the futures contract the pool manager announces the loss results for policies in

Figure 1
Futures Contracts on Hypothetical
Pool Formed in January 1993



the market portfolio. Other, less direct information that the market might take into account in trading includes news of natural disasters, epidemics, etc.

Unlike stock index futures, we see a potential for fraud and abuse of insurance futures through manipulation of the loss ratio index. For example, a participating insurer (or one of its employees) could delay the reporting of losses, thereby biasing the information that is reported by the pool manager. Withholding of claims information may affect futures prices directly, because any losses not reported by the end of the settlement period would not be included in the final settlement index, nor would they be included in the index for the subsequent quarter. (Because each contract is based on losses that occur during a designated quarter, the index for the subsequent quarter would not be affected by losses that occurred prior to its designated quarter.) Such inside information could be an enormous advantage.

Although some insurers have been using financial futures successfully for years to hedge interest rate risk, a survey by Hoyt (1989) indicates that the use of financial futures in life insurer operations is surprisingly infrequent. The users tend to be companies with interest-sensitive products, such as guaranteed investment contracts. As life insurers become more competitive with other financial institutions, it seems reasonable that many of them will develop the expertise required to use financial futures, and hedging of interest rate risk is likely to become common practice.

Hedging of underwriting risk in property-liability insurance is a more difficult problem. An insurer can hedge underwriting uncertainty of its own portfolio directly through various types of stop-loss reinsurance agreements. Insurance futures contracts allow hedging of underwriting risk, but the hedge is not perfect because the insurance futures are based on the market's rather than the insurer's insurance portfolio. This article shows how the insurance futures market relates to traditional reinsurance hedging and investigates the possible use of the proposed contracts. We also address practical issues such as the impact of hedging techniques on financial statements, the possible problems with regulators, and the impact on insurer solvency.

Futures Contracts

A futures contract binds two parties to a transaction in the future, at a contractually specified price. When the contract is written there is no exchange of cash. Rather, the futures price is simply recorded as a basis for gains and losses. The buyer of a contract has a long position, while the seller has a short position. A position may be held until maturity, or it can be closed out before maturity by taking an offsetting position.

To close out a position before maturity, a long trader sells a contract, and a short trader buys a contract. A trader's net gain or loss from a contract is equal to the difference between the closing and opening prices of the underlying object. For example, if the closing price is higher, the long trader experiences a gain and the short trader experiences a loss.

Some questions about insurance futures must be resolved before they are accepted by traders and insurance regulators. Because there is no spot market, the potential for information asymmetries exists. Traders will be unwilling to enter the market if they believe others have superior information regarding the value of the instruments. Even limiting insurers to 15 percent of the policy pool does not protect against collusion by insurers who provide information for the index. Making insider trading activities public would violate the anonymity of pool participants. However, traders must be assured of the absolute integrity of the index or these contracts will fail. Howard (1990) points out that some insurance regulators are skeptical of the new securities, and there is some question about how the use of insurance futures and options should be treated on insurers' financial statements. Even though an insurer owning a futures option has a potentially valuable exercise right (and no liability), regulators may treat such contracts as nonadmitted assets. The CBOT is pushing for recognition of insurance futures, and the health and homeowners insurance contracts have reportedly been approved as qualified investments by the Illinois Insurance Department (Cox, 1992).

Options on Futures Contracts

An option on a futures contract, sometimes called a futures option, gives the option owner the right to open a futures position at the contract's striking price. Marking to the market begins when the option is exercised, so the gain on an

exercised option is recognized immediately. In general, an American option can be exercised any time on or before the option's expiration date, while a European option can only be exercised on the expiration date and not before.

A Hedging Example

Our example concerns a health insurer expecting a loss ratio of 0.65 on a block of group health insurance in force. That is, the insurer expects to pay claims of \$65,000 per \$100,000 of premiums. The insurer is naturally long in this block of business, or, equivalently, it has a naturally short position with respect to its claims level because it will be worse off if its claims are greater than expected. The idea underlying a hedge is to take a position in the futures market that is opposite your natural position. Because the insurer is short with respect to its claims level, it takes a long position in the futures market by purchasing group health futures that mature in one month. Suppose that the current futures price per contract is \$62,000; that is, the current market consensus is a loss ratio of 0.62 on the market portfolio of insurance policies. Suppose that the insurer opens long futures contracts, one for each \$100,000 of premiums it has on its own portfolio. One possible outcome when the contract matures (or is closed out by an offsetting position) is that the insurer's portfolio and the market portfolio both experience more claims than anticipated. Suppose the insurer experiences a loss ratio of 0.70, realizing an underwriting loss of \$5,000 per \$100,000 of premiums relative to its expected loss ratio of 0.65. If the futures price is \$66,000 at the end of the month, the insurer can close out its contract, realizing a gain of \$4,000 per contract. The results per \$100,000 of premiums are summarized below:

Insurer portfolio loss:	$\$70,000 - \$65,000 = \$5,000$
Long futures position gain:	$\$66,000 - \$62,000 = \$4,000$
Net result for hedged portfolio:	$\$4,000 - \$5,000 = -\$1,000.$

As the example suggests, insurers typically will not be able to perfectly hedge their underwriting results through the use of futures contracts. One reason is that the insurer's portfolio of policies may not be perfectly correlated with the market portfolio. Another reason is that futures contracts are sold only in increments of \$100,000, while the block of business might have actual premiums of some odd amount, such as \$830,000. In this case, the insurer would buy eight contracts, and only \$800,000 in premiums would be hedged.

If the insurer's portfolio and the market portfolio experience lower claims than anticipated, the hedge would cause the insurer to miss out on some underwriting gains. For example, if both portfolios experience a decrease in the loss ratio (of the same magnitude as above) instead of an increase, the results would be as follows:

Insurer portfolio gain:	$\$65,000 - \$60,000 = \$5,000$
Long futures position loss:	$\$62,000 - \$58,000 = \$4,000$
Net result for hedged portfolio:	$\$5,000 - \$4,000 = \$1,000.$

Futures hedges reduce the potential for upside risk as well as downside risk, because they effectively allow the insurer to fix its loss ratio at the current

value, at least to the extent that its portfolio is correlated with the market portfolio.

It is important to note that the futures contract is based solely on claims from events during the loss period that are reported before the end of the reporting period of the contract. The hedger must take into account any additional runoff of incurred claims.

An Example Using Options on Futures

The reason for using futures contracts to hedge underwriting risk is to reduce the volatility of an insurer's cash flows. The smoothing of cash flows will reduce the insurer's risk, but it will also reduce the potential for gains in the event of favorable underwriting experience. By using options on futures, an insurer can protect against losses while retaining most of its upside potential. This is analogous to the use of stock options to create portfolio insurance on investment portfolios.

Suppose that, instead of buying a futures contract, the insurer purchases a call option on a futures contract with a striking price equal to the current futures price of \$62,000. Because the insurer is hedging results over the next 30 days, it will choose an option that expires after 30 days. Now, if the insurer suffers an unexpected underwriting loss of \$5,000 and the futures price increases to \$66,000 at the end of the month as in the previous example, the insurer could exercise the call or sell it in the options market. The exercise value of the call is $\$66,000 - \$62,000 = \$4,000$; this is a lower bound on the option price. Thus the insurer could realize an immediate gain of at least \$4,000 by selling the option. This gain will partially offset the \$5,000 underwriting loss. Thus, if the futures price increases, the put option provides the same hedge as the long futures position described above. Although the option did not expose the insurer to additional risk or margin calls, the insurer had to pay for the call option. The net gain or loss to the insurer must also include the cost of the option.

If the futures price decreases and the insurer experiences less claims than anticipated, then the call option will be worthless and the insurer would let it expire without any further obligation. The insurer would realize an underwriting gain of \$5,000 relative to its expectations, but its net gain would be reduced by the cost of the call option. The effectiveness of this strategy depends on the extent to which the company's own loss ratio and the futures price are correlated. In the extreme case where they are perfectly correlated, the insurer's total loss potential (relative to its original expected loss ratio) is limited to the cost of the call option.

Futures Options as Reinsurance

Buying a call option on a futures contract is a way to purchase a stop-loss reinsurance on an aggregate portfolio of policies. In the previous example, the net retention by the ceding insurer is \$62,000 per contract (\$62,000 per \$100,000 of premiums on the policies underlying the futures index, not on the insurer's own policies). Market portfolio claims in excess of \$62,000 per

\$100,000 will drive the futures price above the exercise price of \$62,000, resulting in a payoff to the ceding insurer by the reinsurer (the seller of the call option) in an amount equal to the excess of claims over \$62,000. This is precisely the payoff that would be specified by a stop-loss reinsurance contract. The cost of the call option is the reinsurance premium.

Interpreting insurance futures and futures options as reinsurance may create regulatory problems. If regulators view the contracts as reinsurance, they might try to require the CBOT to register as an insurer. Howard (1990) addresses this issue, noting that the CBOT is making sure that it communicates to regulators that it does not intend to conduct insurance business.

One can imagine conditions under which the CBOT would indeed have obligations equivalent to those of a reinsurer. For example, suppose that the index is moving steadily against a futures seller (i.e., the index is increasing). The CBOT limits its liability by requiring the seller to mark to the market. If at the end of a trading day the seller cannot mark to the market, the CBOT is obliged to make good on the contract to the buyer so it may have a loss (limited to a day's movement of the index). The CBOT closes out the seller and matches the buyer with a new seller. In large markets with many parties having natural interests in buying and selling, such as bond futures, there may be little difficulty in finding a new seller. However, if the price is expected to keep increasing, so that very few traders are willing to sell, then the CBOT may have more than a single day of price movement to cover. If this should occur, it will find itself in the insurance business.

The new insurance futures will compete directly with the reinsurance industry. However, the usefulness of reinsurance will not disappear. Reinsurance provides a number of functions that are not provided by insurance futures. The potential benefits of reinsurance include a banking function that helps to finance a ceding insurer's growth (see Reinartz, 1969). Reinsurers do this by assuming a portion of the unearned premium reserves that the ceding insurer is required by state regulators to maintain and by paying a reinsurance commission to the ceding insurer. This can help the ceding insurer to achieve a favorable ratio of surplus to unearned premium reserves, which regulators use as a measure of solvency. Reinsurance also allows insurers to quickly get in or out of specific geographic markets. Insurance futures representing a nationally diversified cross-section of policies would not enable insurers to target specific geographic regions. Another advantage of reinsurance over insurance futures is that reinsurance contracts cover the ceding insurer's own policies rather than an index of policies that are less than perfectly correlated with its own. For this reason, reinsurance allows insurers to accept larger policies.

The advantages of insurance futures over reinsurance are that insurance futures would be more liquid, and trading insurance futures would enable insurers to avoid certain disclosure requirements that are required in formal reinsurance agreements. The process of contract negotiation would also be avoided because the insurance futures would be standardized contracts. Insurance futures would entail transactions costs, but these might be small relative to the costs of reinsurance contracting.

Other Uses for Insurance Futures

Traders in futures markets are often placed in two categories: hedgers and speculators. Hedgers are those who have a naturally long or naturally short position in the underlying asset. Each hedge requires a speculator to take the other side of the futures transaction or another hedger who has a naturally opposite position. Obviously, insurers have naturally short positions in underwriting losses and therefore would tend to purchase insurance futures or call options to hedge their positions. It is unclear where the demand for a short futures position might originate or who—other than speculators—would be willing to sell call options.

An insurer may believe it can do a better job of underwriting than the market, and this insurer can use insurance futures to speculate. (A numerical example below shows how an insurer may be led to speculate.) We believe that insurance futures would provide economic advantages beyond providing a tool for hedging and speculating. Potential users of insurance futures include not only direct insurers but also reinsurers and self insurers. Purchasing a put option allows one to participate in the market portfolio's profitability without having to be a licensed insurer. A reinsurer might find that lower transactions costs make selling insurance futures a cheaper way to write more reinsurance business. However, futures sellers share in the profitability of other insurers only to the extent that the profitability is not recognized by the market. This is an important distinction between writing insurance policies and selling insurance futures. When writing a policy, the insurer earns the profit. When selling insurance futures, the trader profits only if the futures price declines.

Trading in futures can reduce the volatility of insurer cash flows and decrease the probability of insolvency. But futures have other important economic benefits (Carlton, 1984), including the liquidity of standardized contracts. The existence of an organized market also provides a mechanism for price discovery. Insurance regulators could use futures prices as a measure of the adequacy of existing rates. Moreover, an efficient market would give insurers a consensus expectation of underwriting risks in particular lines of insurance. Thus, futures may improve the efficiency in insurance markets.

A Model of Insurance Futures Markets

The assumptions underlying the Black-Scholes option pricing model are quite restrictive but nevertheless yield a very useful and widely accepted formula for pricing short-term options on assets or indices that do not vary much with interest rates. This is very similar to the insurance futures environment, with the exception of the statistical nature of the price index. The usual assumption for the Black-Scholes model is that the prices of the underlying asset are lognormally distributed. We defer for the moment our discussion of the probability distribution of claims. However, we adopt the other assumptions underlying the Black-Scholes model. For example, we assume the existence of a constant risk-free rate of interest, the absence of transactions costs and information asymmetries among investors, and the

absence of arbitrage opportunities (additional assumptions are discussed in Ingersoll, 1987). Some of these assumptions are unlikely to be realized in the real insurance futures market, especially the assumptions about the absence of transactions costs and information asymmetries among investors. However, the Black-Scholes model has gained wide acceptance in the context of financial markets even though financial markets also do not satisfy all of the assumptions. This framework appears to be a reasonable starting point for the study of insurance futures.

A final assumption allows us to specify the statistical nature of the insurance index. We defer making a specific assumption as to the statistical nature of the aggregate claims while continuing the discussion in general terms. Let $S(t)$ denote the aggregate claims reported (or paid) during the interval $[0, t]$, the first t years of the policies in the market portfolio. The final settlement value of the futures contract is

$$\frac{S(T)}{Q} 100,000,$$

where Q is the aggregate premium paid for the market portfolio. Let $Y(t)$ denote the market portfolio losses which are reported after the current time t but included in the settlement value: $Y(t) = S(T) - S(t)$. Various statistical distributions are used in actuarial models of insurance claims processes to describe $Y(t)$. At time t , the exchange announces the current loss ratio on the market portfolio (or, equivalently, the value of $S(t)$). Rather than specify a particular distribution for the claims process, at this point we assume only that $Y(t)$ has finite mean and variance and that increments in $S(t)$ (or $\log S(t)$) are independent. These assumptions are commonly used in actuarial work. Estimates of the distribution would be made from announced values of $S(u)$ for various times $u \leq t$. Estimates of these parameters will depend upon the distribution assumption for the claims process. For example, if a normal distribution is assumed, then the usual sample mean and sample variance are the maximum likelihood estimators.

We denote the final settlement index by $F(T)$. Dropping the 100,000 multiplier, we define it as

$$F(T) = \frac{S(T)}{Q}.$$

At times $t < T$, the futures price should be the expectation of $F(T)$, conditional on the information available at time t . We assume that all market participants have the same information.⁵ This information includes all of the prior announcements about aggregate claims. Let J_t denote the information available

⁵ As noted above, this assumption will not hold in the proposed market, because reporting companies will know paid losses before they are reported to the pool manager and announced to the public.

at time t . The information sets are increasing in the sense that $J_t \supseteq J_u$ for $t \geq u$. That is, we assume that J_t includes the information generated by the loss process, $S(u)$, for times $u \leq t$. This leads to the following formula for the futures price:

$$\begin{aligned} F(t) &= E[F(T) | J_t] \\ &= \frac{1}{Q} E[S(T) | J_t] \\ &= \frac{1}{Q} E[S(t) + Y(t) | J_t] \\ &= \frac{1}{Q} (S + E[Y(t) | J_t]), \end{aligned}$$

where $S(t) = S$. Note that the index $F(t)$ is always nonnegative.

Bowers et al. (1986) describe the collective risk model that is used to describe claims for insurance policies such as health, automobile physical damage, and property damage, as well as many other lines. According to their model, $Y(t)$ would have a compound Poisson distribution. The aggregate amount of claims incurred under the insurance contracts is the sum of a random number of individual losses which are independent and identically distributed. The number of losses in the interval $(t, T]$ is a Poisson random variable. Denote its parameter by $(T-t)\lambda$, where λ is a positive constant. According to this model, the aggregate losses would sum to

$$Y(t) = X_1 + X_2 + \dots + X_N,$$

where $X_1, X_2, \dots, X_N$ are identically distributed random variables, and the random variables $N, X_1, X_2, \dots, X_N$ are mutually independent. The moments of the claim size distribution are $p_k = E[X^k]$ for $k = 1, 2, \dots$. Traders would have to estimate the parameters λ and p_k to use the model for trading purposes. These parameters are related to the parameters of the aggregate distribution by the equations

$$E[Y(t) | J_t] = (T-t)\lambda p_1 \quad \text{and} \quad \text{Var}[Y(t) | J_t] = (T-t)\lambda p_2.$$

The futures price according to this model would be

$$F(t) = \frac{1}{Q} [S + (T-t)\lambda p_1].$$

A second popular aggregate claims model uses the lognormal distribution for $S(T)$. This is the same distributional assumption that is used for the spot price in the Black-Scholes model. However, the parameters here are derived from the insurance portfolio, not financial markets. Sherman (1991) discusses a similar method of pricing futures contracts. In this setting, the correct distributional assumption specifies that the logarithm of $S(T)/S(t)$, conditional on $S(t) = S$, is

normal with mean $\mu\tau$ and variance $\sigma^2\tau$, where $\tau = T - t$. In this case, the futures price is

$$F(t) = \frac{1}{Q} \left(S(t) E \left[\frac{S(T)}{S(t)} \middle| J_t \right] \right)$$

$$F(t) = \frac{S(t)}{Q} \exp \left(\mu\tau + \frac{\sigma^2\tau}{2} \right).$$

However, we do not need to specify a particular distribution for $S(T)$; our results on the relationship between prices of futures options and stop-loss reinsurance are valid for any aggregate claims distribution.

Options on Futures

Now we consider European options on futures. The options are being valued at time t . The following notation is used:

$S = S(t)$ is the current announced value of aggregate insurance losses,

$F = F(t)$ is the current futures price,

$\tau = T - t$,

r = the risk free interest rate,

x = the contractually specified exercise price of the option,

C = the current price of the call option, and

P = the current price of the put option.

The absence of arbitrage opportunities implies that the price of an option is equal to the discounted expected value of its exercise value, with the expectation conditional upon the information J_t available at time t . Consider the call option: If the settlement value $F(T)$ is less than the exercise price x , then the value of the call option is zero; if the settlement value $F(T)$ is greater than the exercise price x , then the option is worth $F(T) - x$. Hence, the option's exercise value at time T is $\max\{0, F(T) - x\}$. The price of a futures call option is

$$C = e^{-r\tau} E[\max\{0, F(T) - x\} | J_t],$$

where $F(T) = S(T)/Q$. This simplifies to

$$C = e^{-r\tau} \int_x^\infty (f - x) dG(f),$$

where $G(f)$ is the cumulative probability distribution of $F(T)$, conditional on J_t . That is, $G(f) = \Pr[F(T) \leq f | J_t]$.

Changing variables by substitution of $f = s/Q$ leads to an equivalent expression in terms of a stop-loss insurance premium:

$$C = e^{-r\tau} \int_{xQ}^\infty \left(\frac{s}{Q} - x \right) dH(s),$$

where $H(s) = \Pr[S(T) \leq s \mid J_1] = \Pr[F(T) \leq f \mid J_1] = G(f)$. This simplifies to

$$C = \frac{e^{-r\tau}}{Q} \int_d^{\infty} (s-d) dH(s),$$

where $d = xQ$. Thus, the price of a call option on the entire market portfolio, CQ , is equal to the discounted expected value of claims paid at time T under a stop-loss reinsurance on the market portfolio with retention $d = xQ$, where x is the exercise price of the call.⁶

The mathematics of stop-loss premiums has been developed by Gerber (1979), Bowers et al. (1986), and many others. In the case where $S(T)$ is assumed to have a compound Poisson distribution, Bowers et al. show how to calculate the stop-loss premiums recursively. And when $S(T)$ is lognormal, the call formula becomes

$$C = \frac{e^{-r\tau}}{Q} [S e^{(\mu + \frac{\sigma^2}{2})\tau} \Phi(y_1) - d\Phi(y_2)],$$

where $\mu\tau$ and $\sigma^2\tau$ are the mean and variance of the logarithm of $S(T)/S(t)$, the notation S , Q , r , x , d , and τ are defined as before, $\Phi(y)$ denotes the standard normal cumulative distribution function, and

$$y_1 = \frac{\log(\frac{S}{d}) + \mu\tau + \frac{\sigma^2\tau}{2}}{\sigma\sqrt{\tau}} \quad \text{and} \quad y_2 = y_1 - \sigma\sqrt{\tau}.$$

Because $F(t) = \frac{S(t)}{Q} e^{(\mu\tau + \frac{\sigma^2\tau}{2})}$, the call formula can be written as follows:

$$C = \frac{e^{-r\tau}}{Q} [QF \Phi(y_1) - d\Phi(y_2)]$$

$$C = e^{-r\tau} [F \Phi(y_1) - x\Phi(y_2)].$$

This last expression is similar to Black's (1976) formula for the price of a call option on a futures price.

The European put option on the same futures contract is related to the call option by the put-call parity relation:

$$C - P = e^{-r\tau} \{E[F(T)|J_1] - x\},$$

⁶ The relationship between nonproportional reinsurance and options pricing is also shown in Cummins (1990).

where C is the price of the call option, and P is the price of the put option. Using this relation and the call price formula, we can determine the put option formula. After some manipulations, the put formula can be stated as

$$P = \frac{e^{-r\tau}}{Q} \int_0^d (d-s) dH(s).$$

Thus, the price of a put option on the entire market portfolio, PQ , is equal to the discounted expected value of retained losses under a stop-loss reinsurance on the market portfolio with retention $d = xQ$, where x is the exercise price of the put.

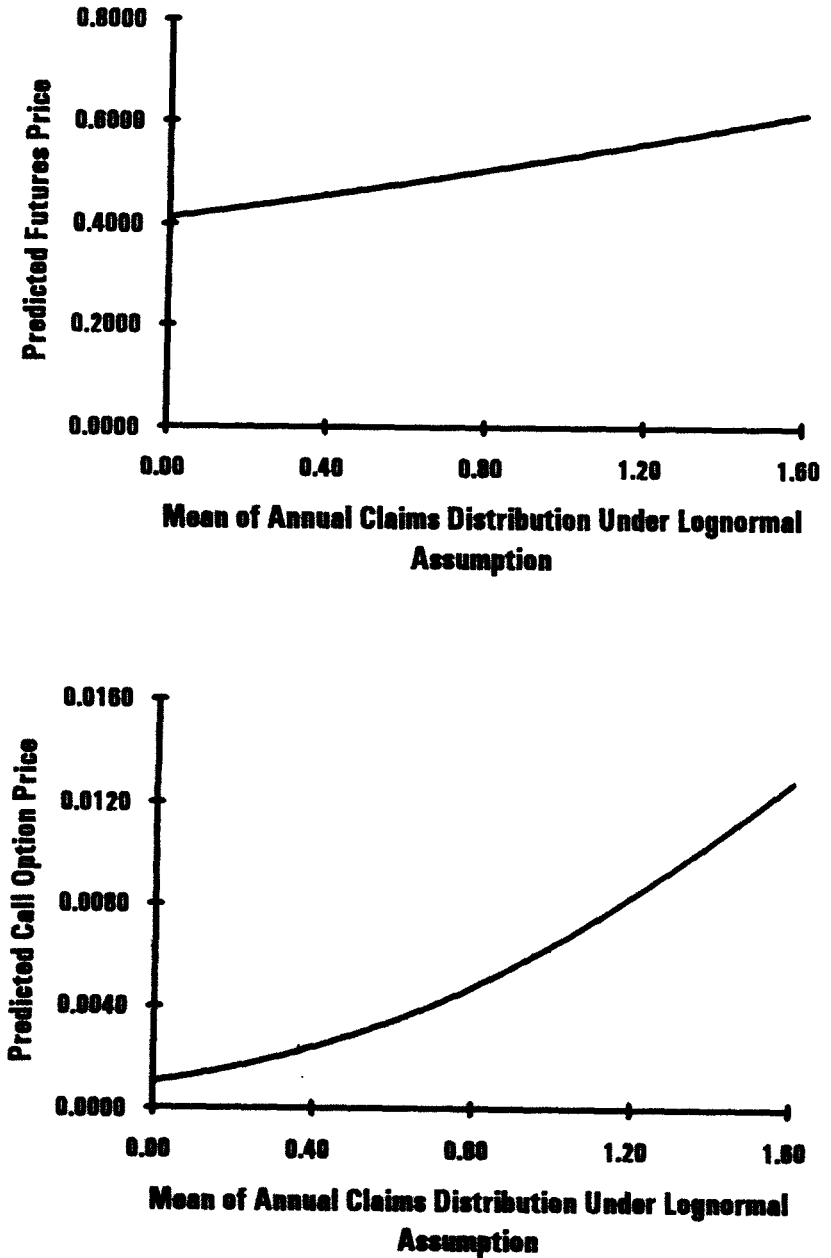
These models show how futures traders might determine speculative positions. For example, a trader may have information (public and private) which it uses to estimate the mean and variance of the logarithm of $S(T)/S(t)$. These can be used to estimate $\mu\tau$ and $\sigma^2\tau$, the parameters of the lognormal model. Then the trader could calculate the value of the put option and call option using the formulas above and use the results to formulate trading strategies.

Our formulas are valid for European options. Because the proposed insurance futures options are American options, their theoretical values may differ. Nevertheless, in other markets, the Black-Scholes formulas for European options are widely used as approximations to prices of American options. Our adaptations to insurance futures markets should be equally useful and successful.

As an example we calculate the value of a call option. We set $Q = 1$, $x = 0.62$, $r = 0.05$, $\tau = 0.25$, $\sigma = 0.5$, and $S = 0.40$. We computed the implied futures price F and the call option price C using the lognormal model and various values of μ (see Figure 2).

If the actual market price (F or C) were different from the computed value, the trader would be tempted to take a speculative position in the futures or options market based on the implied value of μ . For example, a market value of $F = 0.60$ implies a value of μ in the neighborhood of 1.50. If the trader believes that 1.00 is more likely correct, then futures contracts will appear overpriced in the current market and the trader would be tempted to sell futures.

Figure 2
Numerical Example: Pricing of Futures and Futures Options



Conclusions

Our results show the relations that insurance futures contracts have with traditional stop-loss reinsurance contracts. We have identified strong reasons why insurers may choose not to use insurance futures and options. The relationship between insurance futures and reinsurance is imperfect: The “reinsurance” obtained through a futures put option is on the market portfolio rather than on the buyer’s own portfolio. None of the services that reinsurers provide their customers is available through futures options. The potential for manipulation and fraud is greater with futures and options than it is with traditional reinsurance. Insurers may not have the expertise required to engage in futures markets. The accounting and financial reporting of “reinsurance” obtained through a futures option is uncertain. Finally, insurers are often criticized for being poor managers of risk. (Executive Life’s failure was attributable to inappropriate investments and pricing, and other firms have made poor investment decisions or mispriced economic risks.) This criticism and the public perception that futures markets are risky and speculative may discourage many insurers from participating in the insurance futures market.

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