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The Equilibrium Insurance Price and Underwriting Return in a Capital Market Setting

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ABSTRACT

This article develops a modified capital asset pricing model, CAPM, which integrates the markets for financial assets, real assets, and insurance. It is shown that the purchase of insurance should not be ignored in determining the equilibrium rate of return on assets, and that the equilibrium insurance premium should adjust for the sources of systematic risk associated with financial assets, real assets, and the insurance market. Consistent with some earlier studies, it was found that the insurer's equilibrium underwriting return could be negative, rather than positive, under certain conditions.

The insurance pricing models based on financial theory differ widely in terms of parameter specifications, computation methods, and underlying assumptions. Models based on the Options Pricing Model (OPM) are based on total variability but are very sensitive to taxation (see Doherty and Garven, 1986). On the other hand, models based on the Capital Asset Pricing Model (CAPM) focus on market risk and ignore other risk factors (see Fairley, 1979). Indeed, unsystematic risk and underwriting risk are also important in determining insurers' rate of return (see Cummins and Harrington, 1988; and Witt and Urrutia, 1983). It has also been suggested that the risk-free rate used to discount loss reserves may be excessive (see D'Arcy, 1988). Insurance pricing models based on financial theory that have gained some prominence as regulatory tools are discrete-time discounted

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The authors are grateful to two associate editors, two anonymous referees and Travis Pritchett for their constructive suggestions. We would also like to thank Bon Sung Koo and James Garven at the University of Texas at Austin for valuable insights. Their helpful comments and suggestions have contributed to the improvement of this paper. Of course, any remaining errors are the sole responsibility of the authors.

cash flow, DCF, models (see Myers and Cohn, 1987; the National Council on Compensation insurance, NCCI, 1987; and Cummins, 1990).

The main objective of this article is the derivation of equilibrium insurance prices and underwriting returns in a capital market setting. The proposed models are based on the CAPM. Even though, the CAPM literature has generally assumed away the importance of risk and return effects of real assets and insurance contracts on an investor's optimal portfolio, several extensions of the standard CAPM have been developed that try to account for the missing assets and expand the concept of a "market portfolio" (see Mayers, 1972; Roll, 1977; Brito, 1978; Breeden, 1979; and Mayers and Smith, 1983). More recently, Ang and Lai (1987) and Turner (1987) have also developed insurance pricing formulas based on the CAPM. The Ang and Lai model is developed from the viewpoint of an insurance firm and does not explicitly incorporate real assets into the ratemaking formulas. The Turner model is derived from the household viewpoint and takes into account real assets, insurance shares, insurance policies and an individual's consumption.

The model proposed in this article differs from the Ang and Lai model in that it is derived from the insured's point of view and explicitly recognizes both real assets and insurance contracts in the pricing formulas. The model presented in this article is similar to the Turner model. It differs from the Turner model in its assumption that insurance is provided by mutual rather than stock insurance companies. The present model was developed independently of the Turner model and has pedagogical value because it uses simpler notation and presents a more straightforward derivation of the pricing formula. The two-fold purpose of this article is to extend the traditional CAPM in order to recognize two important facts: first, most investors invest in real assets as well as financial assets; and, second, most investors also purchase insurance to protect themselves against real asset losses, and, therefore, they are also insureds. It is shown that the equilibrium insurance premium and the fair rate of return on an insurance contract should adjust for three sources of systematic risk: that associated with the markets for financial assets, real assets, and insurance. Even though the magnitude of several covariance terms are unknown, the equilibrium underwriting return for the insurer could be negative. If this were the case, such a result would be consistent with what previous studies have suggested and would imply a positive cost of capital for policyholder supplied funds.

The Assumptions

In the derivation of the proposed model, the usual assumptions underlying the mean-variance capital market theory will be utilized. In addition, the following simplifying assumptions about the insurance market and real assets are also made.

First, individuals are assumed to treat insurance contracts as financial assets along with risky and risk-free assets in determining their portfolios.

That is, it is assumed that investors carry insurance on real assets in order to protect themselves from contingent property or liability related losses. This assumption implies that the insurance purchasing decision must be explicitly considered in the investor's budget constraint.

Second, it is assumed that all insurance contracts are sold by mutual insurers, in order to limit the scope of the article and simplify the analysis. This assumption implies that the investor's wealth constraint must show not only the individual's position with respect to his/her own insurable loss, but also the dividend or retroactive premium adjustment which would accrue to the individual should aggregate losses for the mutual insurer differ from the expected value. This adjustment is incorporated in the model by assuming that the price of insurance is a random variable that includes the premium to the mutual and any retroactive dividend.¹

Third, it is assumed that assets covered by the insurance contracts are real assets subject to accidental property and liability related losses. Accidental losses to real assets of an investor are assumed to be possible to happen at least once. Insurable real assets are assumed to be risky assets and their values are assumed to play a role in the investor's overall portfolio diversification decision. Thus, the stochastic characteristics and correlations of real assets with other financial assets are important parameters in determining the investor's optimal asset portfolio in the model.²

The Notation

The following notation, recognizing the markets for financial assets, real assets, and insurance contracts, will be used in the derivation of the pricing model.

The Financial Asset Market

P^F = the column vector $(p_1^F, p_2^F, \dots, p_h^F)$ represents price per unit, e.g. per share prices of financial assets.

A = the column vector $(a_1, a_2, \dots, a_h)$ represents the number of units, e.g., shares, of investment in the j th financial asset, for $j=1,2,\dots,h$, by the i th investor.

F = the column vector $(\tilde{f}_1, \tilde{f}_2, \dots, \tilde{f}_h)$ of uncertain total dollar cash flow or total return from investment in the j th financial asset. The column vector of the expected cash flow or returns values is $\bar{F} = (f_1, f_2, \dots, f_h)$.

Σ_{ff} = the $h \times h$ variance-covariance matrix of financial asset returns. Its elements are the covariance terms, $\text{cov}(\tilde{f}_i, \tilde{f}_j)$. This matrix measures the riskiness and diversifiability among financial assets.

¹ The authors are grateful to an anonymous referee for suggesting this approach to the market clearing process for the insurance market.

² The authors are grateful to an anonymous referee for identifying the importance of the stochastic characteristics of real assets and for suggesting the incorporation of them in our pricing model.

The Real Asset Market

P^R = the column vector $(p_1^R, p_2^R, \dots, p_k^R)$ represents per unit prices of real assets.

B = the column vector $(b_1, b_2, \dots, b_k)$ represents the number of units of investment in the j th real asset for $j=1,2,\dots,k$, by the i th investor.

S = the column vector $(\tilde{s}_1, \tilde{s}_2, \dots, \tilde{s}_k)$ of uncertain dollar cash flow or total return from investment in the j th real asset. The column vector for expected values is $\bar{S} = (\bar{s}_1, \bar{s}_2, \dots, \bar{s}_k)$. The column vector S contains the end-of-period market values of real assets such as for a home, automobile, and other real property. These market values capture any market appreciation or depreciation of the real asset during the period, as well as any loss of value because of physical or financial damage to the properties due to perils such as fire, liability, or collision. Thus, real asset insurable losses are already reflected in the end-of-period market value of real assets. Any recovery from an insurer for insured losses will compensate the investor for the loss of market value of the real asset.

Σ_{ss} = the $k \times k$ variance-covariance matrix of real asset returns. Its elements are the covariance terms, $\text{cov}(\tilde{s}_i, \tilde{s}_j)$. This matrix measures the riskiness and diversifiability among real assets.

The Insurance market

P^I = the column vector $(p_1^I, p_2^I, \dots, p_n^I)$ represents insurance premiums or prices of insurance policies for insured amounts of $\$X$.

C = the column vector $(c_1, c_2, \dots, c_n)$ represents the number of units of the j th insurance policy type for $j=1,2,\dots,n$, purchased by the i th investor. Each insurance contract is assumed to have an insured amount of $\$X$ which corresponds to the insurable value of the real asset covered or the amount of liability insurance purchased.

L = the column vector $(\tilde{\ell}_1, \tilde{\ell}_2, \dots, \tilde{\ell}_n)$ of uncertain real asset losses paid by the insurer at the end-of-period (insurance coverage) and arising from the i th investor's uncertain claims from policy types $j=1,2,\dots,n$, with expected value $\bar{L} = (\bar{\ell}_1, \bar{\ell}_2, \dots, \bar{\ell}_n)$.

Σ_{ll} = the $n \times n$ variance-covariance matrix of real asset losses. Its elements are the covariance terms, $\text{cov}(\tilde{\ell}_i, \tilde{\ell}_j)$ between the i th and j th type of insurance policies. From the insurer's viewpoint, this matrix provides a measure of the riskiness and diversifiability among insurance contracts.

The Covariances

Σ_{fn} = the $h \times n$ variance-covariance matrix between the financial asset returns and the insurance coverage. Its elements are the covariance terms, $\text{cov}(\tilde{f}_i, \tilde{\ell}_j)$.

Σ_{sl} = the $k \times n$ variance-covariance matrix between the real asset returns and the insurance coverage. Its elements are the covariance terms, $\text{cov}(\tilde{s}_i, \tilde{\ell}_j)$. This variance-covariance matrix incorporates in the model the fact that the

investors hold insurance contracts that are negatively correlated with returns on real assets.

Σ_{fs} = the $h \times k$ variance-covariance matrix between the financial asset returns and the real asset returns. Its elements are the covariance terms $\text{cov}(\tilde{f}_i, \tilde{s}_j)$.

Also define:

W = Initial wealth of the i th investor.

D = Units of net debt of the i th investor in a risk-free asset, whose standardized price is assumed to be \$1.

R_f = The total rate of return on a risk-free asset.

The Model

The asset pricing formulas presented in this article can be seen as extensions of earlier research on extending the concept of a "market portfolio" of the CAPM, conducted by authors such as Mayers (1972), Roll (1977), Brito (1978), and Breeden (1979). Turner's (1987) model is also an extension of the work of Mayers (1972). Several assumptions of the Turner model are also found in our model, mainly because they are assumptions underlying the standard CAPM. However, we assume insurance policies are sold by mutual insurance companies and not by stock insurance firms as assumed under Turner's model. In order to incorporate stock insurers into the model, Turner introduced insurance shares as well as insurance policies. As might be expected, this added generality is achieved at the cost of more complexity in the notation and derivation of the model. The model presented in this article has been developed independently of the Turner model with simpler notation and shorter intuitive derivations. In fact, our mathematical results can also be obtained by using the CAPM with missing assets suggested by Mayers (1972)³. The current model can also be viewed as a special case of the k -fund portfolio separation theorem developed by Ingersoll (1987).

Indeed, the algebraic derivation of the equilibrium price and rate of return for insurance presented in this article follows more closely the approach of Ang and Lai (1987). These authors have developed an insurance pricing formula based on capital market theory that allows for the explicit recognition of the insurance market. Their model, however, is derived from the viewpoint of the insurer and ignores the real asset market. The pricing formulas in the current model, on the other hand, are derived from the insured's viewpoint and take into account both the insurance and the real asset markets. Nevertheless, given the similarities between the derivation of the current model and that of Ang and Lai, several intermediate steps are

³ This alternative derivation of our model is not presented here due to space constraints, but it is available from the authors upon request.

skipped in order to concentrate on the presentation and discussion of the main results.⁴

The model derived is for one-period. At the beginning of the period an investor is characterized as holding a portfolio composed of financial and real assets. To protect himself/herself against real asset losses, the investor purchases insurance from a mutual insurer. The investor can also borrow or lend at the rate of return on the risk-free asset. Both the returns or market values at the end of the period for financial and real assets are uncertain. The magnitude of real asset losses incurred during the period and the corresponding payments by the insurer at the end of the period are also subject to uncertainty. It is assumed that the insurer collects premiums at the beginning of the period and pays losses at the end of the period.

Under this scenario, the investor's problem is to maximize the utility of terminal wealth subject to a budget constraint. The problem is solved by choosing the optimal mix of financial assets, real assets, and insurance contracts in the investor's portfolio. The competitive and fair equilibrium prices for financial assets, real assets, and insurance contracts are obtained by aggregating among all investors and among all suppliers of assets and insurance policies.

Based on the foregoing assumptions and notation, a budget constraint for the *i*th investor can be specified as follows.

$$W + D = A'P^F + B'P^R + C'P^I \quad (1)$$

At the end of the period, the terminal wealth of the *i*th investor is uncertain with expected value *E* and variance *V* given by:

$$\begin{aligned} E &= A'\bar{F} + B'\bar{S} + C'\bar{L} - D(1+R_f) \\ V &= A'\sum_{ff}A + B'\sum_{ss}B + C'\sum_{ll}C + 2A'\sum_{fs}B + 2A'\sum_{fl}C + 2B'\sum_{sl}C \end{aligned} \quad (2)$$

The *i*th investor's portfolio problem is to maximize the utility of the expected end-of-period wealth subject to a budget constraint; that is:

$$\begin{aligned} \text{Max } & U(E, V) \\ & (A, B, C, D) \end{aligned} \quad (3)$$

$$\text{s.t. } W + D = A'P^F + B'P^R + C'P^I$$

by using (3) and the assumption that all investors have homogeneous expectations regarding expected returns and risks, the Lagrangian equation for the *i*th investor can be formed:

$$L = U(E, V) + \Theta [W - A'P^F - B'P^R - C'P^I + D] \quad (4)$$

⁴ A detailed derivation of the model is available from the authors upon request.

Where Θ is the Lagrangian multiplier. By using (2) and differentiating (4) with respect to A, B, C, and D, the optimal mix of financial assets, real assets and insurance can be found. The first order conditions for the i th investor can then be solved. Finally, by using $U_1/U_2 = \partial V/\partial E$ (the marginal rate of substitution between risk and return for the i th investor), by aggregating over all n investors, and by rearranging terms, the following equilibrium prices are obtained for the j th financial asset, p_j^F , the j th real asset, p_j^R , and the j th insurance policy, p_j^I , from the investor's viewpoint:

$$p_j^F = \frac{\bar{f}_j - \lambda [\text{cov}(F_m, f_j) + \text{cov}(S_m, f_j) + \text{cov}(L_m, f_j)]}{1 + R_f} \quad (5)$$

$$p_j^R = \frac{\bar{s}_j - \lambda [\text{cov}(S_m, s_j) + \text{cov}(F_m, s_j) + \text{cov}(L_m, s_j)]}{1 + R_f} \quad (6)$$

$$p_j^I = \frac{\bar{l}_j - \lambda [\text{cov}(L_m, l_j) + \text{cov}(F_m, l_j) + \text{cov}(S_m, l_j)]}{1 + R_f} \quad (7)$$

where: $F_m = \sum_{j=1}^h f_j$, $S_m = \sum_{j=1}^k s_j$, and $L_m = \sum_{j=1}^n l_j$

are the uncertain, end of period, total market values of financial assets, real assets, and insurance losses, respectively. F_m corresponds to the value of the market portfolio as it is used in the traditional CAPM, and λ corresponds to the market price per unit of risk given by:

$$\lambda = \frac{\bar{F}_m - (1 + R_f) P_M^F}{\sigma_{F_m}^2 + \text{cov}(F_m, S_m) + \text{cov}(F_m, L_m)}$$

Note that the above pricing formulas allow for the explicit recognition of the real asset and insurance markets, which are ignored under the standard CAPM. It should also be noted that the equilibrium prices are functions of three potential sources of systematic risk (the covariances of each asset or insurance policy with each of the three market values).

Equations (5), (6), and (7) can be rearranged to display the equilibrium or fair rate of return for financial and real assets and insurance contracts. By using the definition of return $R = (P_1 - P_0)/P_0$, where P_0 and P_1 are the asset's initial and terminal prices, and after some algebraic manipulation, the following results can be obtained:

$$\bar{R}_j^F = R_f + \lambda [P_m^F \text{cov}(\bar{R}_m^F, \bar{R}_j^F) + P_m^R \text{cov}(\bar{R}_m^R, \bar{R}_j^F) + P_m^I \text{cov}(\bar{R}_m^I, \bar{R}_j^F)] \quad (8)$$

$$\bar{R}_j^R = R_f + \lambda [P_m^R \text{cov}(\tilde{R}_m^R, \tilde{R}_j^R) + P_m^F \text{cov}(\tilde{R}_m^F, \tilde{R}_j^R) + P_m^I \text{cov}(\tilde{R}_m^I, \tilde{R}_j^R)] \quad (9)$$

$$\bar{R}_j^I = R_f + \lambda [P_m^I \text{cov}(\tilde{R}_m^I, \tilde{R}_j^I) + P_m^F \text{cov}(\tilde{R}_m^F, \tilde{R}_j^I) + P_m^R \text{cov}(\tilde{R}_m^R, \tilde{R}_j^I)] \quad (10)$$

where $\bar{R}_j^F$, $\bar{R}_j^R$, and $\bar{R}_j^I$ = The expected one-period returns on the *j*th financial asset, real asset, and insurance contract, respectively.

$\tilde{R}_m^F$, $\tilde{R}_m^R$, and $\tilde{R}_m^I$ = The uncertain one-period returns on the financial, real, and insurance markets, respectively.

P_m^F , P_m^R , and P_m^I = The beginning of the period total market value of the financial, real and insurance markets, respectively.

The above formulas show that the required rate of return on an asset is the return on the risk-free asset plus a risk premium. However, the risk premium is a function of the covariance of the return on the asset with the returns on each of the three markets for financial assets, real assets and insurance contracts.

The Equilibrium Price and Return on Insurance

Equations (7) and (10) give the equilibrium price and rate of return for the *j*th insurance contract, respectively. These pricing formulas differ from other insurance pricing models in that they integrate the markets for financial assets, real assets, and insurance. Authors such as Biger and Kahane (1978), Hill (1979), Fairley (1979), Urrutia (1986), and Ang and Lai (1987) have also applied the CAPM to estimate the fair insurance premium. However, these pricing models ignore the existence of the real asset market, or the insurance market, or both. Thus, they tend to underestimate or overestimate the fair insurance return because of the exclusion of the covariance terms between the insurance market and the markets for financial assets and real assets.

The equilibrium price for insurance established in equation (7) is a negative function of three sources of risk: the covariances of the *j*th insurance policy with the insurance market portfolio, the real asset market portfolio, and the financial asset market portfolio, respectively. Real asset losses to be paid by the insurer, like the end-of-period cash flows of financial and real assets, are uncertain and investors require risk premiums for investing in these markets. Thus, the equilibrium prices for financial and real assets and insurance policies must be reduced by their own market risk and their association with other markets. For insurance contracts, however, the equilibrium prices could obviously be increased if the sum of the covariance terms in (7) were negative.

The equilibrium or fair rate of return for the *j*th insurance policy, given by formula (10), corresponds to the return on the risk-free rate asset plus a risk premium. That is, the ratemaking formula (10) treats the insurance

purchasing decision as if it were an investment decision on financial assets. The first covariance term for an individual insurance contract and the insurance market return, $\text{cov}(R_m^I, R_j^I)$, may be positive, for the same reason that most financial assets are positively correlated. The other covariance terms, $\text{cov}(R_m^R, R_j^I)$ and $\text{cov}(R_m^F, R_j^I)$, are expected to be negative because the occurrence of losses tend to reduce the market values of real assets as well as financial assets.

Summary and Conclusion

The asset pricing model presented in this article is based on the CAPM and is developed from the insured's viewpoint and explicitly recognizes both the insurance and real asset markets. In other words the proposed asset pricing model recognizes that investors face not only financial risk from investments in risky assets, but also risk resulting from unpredictable losses on the real assets they hold in their portfolios. The asset pricing formulas can be seen as extensions of earlier work on extending the concept of "Market portfolio" by Mayers (1972), Roll (1977), Brito (1978), and Breeden (1979). The proposed model may also be viewed as a special case of the k-fund portfolio separation theorem developed by Ingersoll (1987). Also, our conclusions about holdings of insurance and pricing of insurance coverages are extensions of work by Mayers and Smith (1983) and Ang and Lai (1987).

It is found that the equilibrium or fair return for insurance should adjust for three sources of systematic risk: the systematic financial asset market risk, the systematic insurance market risk, and the systematic real asset market risk. Even though the sizes and effects of the several covariances are unclear, the fair insurance underwriting return is more likely to be negative for insurers. The result is consistent with conclusions reached by several earlier studies conducted by Biger and Kahane (1978), Fairley (1979), Hill (1979), and Urrutia (1986). The integrative general asset pricing model developed in this article demonstrates that the effect of losses on insurable assets, and the purchase of insurance as a way of protecting investors from real asset losses, cannot be ignored in determining the equilibrium prices of capital assets.

References

- Ang, James S. and Tsong-Yue Lai, 1987, Insurance Premium Pricing and Ratemaking in Competitive Insurance and Capital Assets Markets, *Journal of Risk and Insurance*, 54: 767-79.
- Biger, Nihum, and Yehuda Kahane, 1978, Risk Considerations in Insurance Ratemaking, *Journal of Risk and Insurance*, 45: 121-32.
- Breeden, Douglas, 1979, An Intertemporal Asset Pricing Model with Stochastic Consumption and Investment Opportunities, *Journal of Financial Economics*, 7: 256-96.

- Brito, O. Ney, 1978, Portfolio Selection in an Economy with Marketability and Short Sales Restrictions, *Journal of Finance*, 33: 589-601.
- Cummins, J. David, 1990, Multi-Period Discounted Cash Flow Ratemaking Models in Property-Liability Insurance, *Journal of Risk and Insurance*, 57: 79-109.
- Cummins, J. David and Scott E. Harrington, 1988, The Relationship Between Risk and Return: Evidence for Property-Liability Insurance Stocks, *Journal of Risk and Insurance*, 55: 15-31.
- D'Arcy, Stephen P., 1988, Use of the CAPM to Discount Property-Liability Loss Reserves, *Journal of Risk and Insurance*, 55: 481-91.
- Doherty, Neil A. and James R. Garven, 1986, Price Regulation in Property-Liability Insurance: A Contingent Claims Approach, *Journal of Finance*, 41: 1031-50.
- Fairley, William B., 1979, Investment Income and Profit Margins in Property-Liability Insurance : Theory and Empirical Results, *Bell Journal of Economics*, 10: 192-210.
- Hill, Robert D., 1979, Portfolio Regulation in Property-Liability Insurance, *Bell Journal of Economics*, 10: 172-91.
- Ingersoll, Jonathan, 1987, *Theory of Finance Decision Making*, (Totowa, N.J.: Rowman and Littlefield).
- Mayers, David, 1972, Non-Marketable Assets and Capital Market Equilibrium Under Uncertainty, in M.C. Jensen, ed., *Studies in the Theory of Capital Market*, (New York, Prager Publishers) 223-48.
- Mayers, David, and Clifford W. Smith, Jr., 1983, The Interdependence of Individual Portfolio Decisions and the Demand for Insurance, *Journal of Political Economy*, 91: 304-11.
- Myers, Stewart and Richard Cohn, 1987, Insurance Rate Regulation and the Capital Asset Pricing Model, in J.D. Cummins and S.E. Harrington, eds., *Fair Rate of Return in Property-Liability insurance* (Norwell, MA: Kluwer Academic Publishers).
- Roll, Richard, 1977, A Critique of the Asset Pricing Theory's Tests; Part I: On Past and Potential Testability of the Theory, *Journal of Financial Economics*, 4: 129-76.
- Turner, Andrew I., 1987, Insurance in an Equilibrium Asset-Pricing Model, in J.D. Cummins and S.E. Harrington, eds., *Fair Rate of Return in Property-Liability Insurance*, (Norwell, MA: Kluwer Academic Publishers).
- Urrutia, Jorge L., 1986, The Capital Asset Pricing Model and the Determination of Fair Underwriting Returns for the Property-Liability Insurance Industry, *The Geneva Papers on Risk and Insurance*, 11: 38: 44-60.
- Witt, Robert C. and Jorge L. Urrutia, 1983, Price Competition, Regulation, and Systematic Underwriting Risk in Automobile Insurance, *Geneva Papers on Risk and Insurance*, 8: 403-429.

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