Increases in Risk and the Optimal Deductible

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ABSTRACT

Using Arrow's (1971, 1974) model of the optimal (endogenously determined) deductible, this article examines the impact of increases in risk (Rothschild-Stiglitz mean-preserving spreads and Diamond-Stiglitz mean-utility-preserving spreads) and first-order stochastic dominance shifts in the distribution function of losses on the optimal insurance contract between a risk-averse decision-maker and a risk-neutral insurer. In particular, it is found that a first-order stochastic dominance shift in the distribution of losses has generally an ambiguous impact on the deductible. This result raises some doubt about the asymmetric information (signalling) literature's identification of low (high) risk with smaller (greater) demand for insurance.

Introduction

Borch (1960) and Arrow (1971, 1974) were the first to analyze optimal insurance contracts in which the form of the insurance policy was derived endogenously. In these studies the optimal insurance contract is seen as a Pareto-optimal risk-sharing arrangement between the insurer and the insured.[1]

Consider Arrow's (1974) model of the optimal deductible. A risk averse expected utility maximizing individual faces random losses but has the possibility of insuring against these losses. An optimal insurance contract is characterized by the premium paid by the insured and by a coverage function (indemnity payments schedule) specifying the transfer payment from the insurer to the insured for each possible loss. The indemnity payments are restricted to be non-negative and cannot exceed the size of the loss. If the insurer is risk neutral and if insurance costs are proportional to the indemnity payments (proportional loading), then the premium required by the

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The authors are grateful to Harris Schlesinger, Christian Gilles and to two referees for very helpful and insightful comments. Thanks also are expressed to Edna Loehman and Ronald Warren for useful discussions.

[1] Mossin (1968), Smith (1968) and Gould (1969) have been the precursors in this literature. However, these studies assumed that the insurance policy was exogenously specified.

competitive insurers will be proportional to the expected value of the indemnity payments. As shown by Arrow and subsequently by Raviv (1979), an optimal insurance contract with these characteristics involves full insurance of losses above an optimal deductible which is endogenously determined. The determining factor which underlies the existence of a positive deductible is the non-availability of fair insurance due to loading.[2] Otherwise, in the absence of loading (i.e., under fair insurance) and in the absence of asymmetric information (caused by moral hazard or adverse selection) between insurers and insureds the optimal risk-sharing contract will have a zero deductible.[3] The optimal choice of an insurance contract is affected by several factors, two of the most important being the degree of risk aversion of the insured and the riskiness of the distribution of losses faced by the insured. The first of these two elements has received more attention in the literature. Several studies have analyzed the impact of increased risk aversion on the optimal choice of the deductible. See, for example, Schlesinger (1981), Karni (1983, 1985) and Doherty and Schlesinger (1983). Karni conducts his analysis under the assumption of non-random initial wealth for both state-independent and state-dependent preferences while Doherty and Schlesinger analyze the case of random initial wealth for state-independent preferences.

The objective of this article is to examine the impact of increased riskiness of loss on the optimal insurance contract between a risk averse insured and a risk neutral insurer. The impact of a Rothschild-Stiglitz (1971) mean-preserving spread, of a Diamond-Stiglitz (1974) mean-utility-preserving spread and of a shift in the loss distribution in the sense of first-order stochastic dominance (Hadar and Russell, 1971) are examined.[4] In each case

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[2] Fair (unfair) insurance is a technical term referring to a premium with zero (positive) loading.

[3] See Raviv (1979) who indicates that the deductible is strictly positive if and only if the cost of providing insurance is a function of the insurance coverage. A positive deductible will also be optimal when the insurer is risk averse. In this case the optimal contract will provide for coinsurance above a deductible, that is, only a percentage (and not the totality) of losses above the deductible will be covered by the insurer.

[4] The variance of a distribution function has also been used as a measure of the random variable's riskiness. Only in some special cases will a change in the variance be an adequate representation of an increase in risk for an expected utility maximizer. If the utility function over wealth is quadratic, expected utility maximization requires consideration of only the first two moments in which case the variance is a good indicator of risk. Alternatively, one can postulate a general utility function but restrict the cdf's F and G to a class of cdf's characterized by a location (e) and a scale parameter (s) (i.e. $F(x) = \Psi[(x-e_F)/s_F]$ and $G(x) = \Psi[(x-e_G)/s_G]$ where the scale parameters s_F and s_G are monotone increasing functions of the variance alone and the means are equal. In this case, a larger variance is equivalent to a MPS. This is the case for the normal distribution which belongs to this class of cdf's. However, for the log-normal distribution which does not belong to this class, the relevant criterion of riskiness is the variance of the logarithm and not the variance itself. Therefore, the variance of a random variable is not a sufficient index of riskiness for all two parameter cdf's. (See also Bawa, 1975).

For a general utility function U, say a polynomial of degree n, and for general cdf's F and G, a larger variance is not a satisfactory indicator of greater risk. The mean preserving spread (MPS) is a more general definition of increased riskiness. If G is a MPS of F then $\mu_F = \mu_G$ and $\sigma_G^2 > \sigma_F^2$ where μ is the mean and σ^2 is the variance. (See Hadar and Russell, 1971, Theorem 3). It also follows that $\int U(x)dG(x) < \int U(x)dF(x)$ if U is displays risk aversion (i.e. is concave). However, if the comparative statics effects of increases in risk are shown to be related to the degree of risk aversion of the insured agent. The conditions obtained by Rothschild and Stiglitz and by Diamond and Stiglitz for an increase in risk to have a determinate impact on the control variable are no longer sufficient to determine conclusively the impact of increasing risk on the insurance contract. In their case the impact on the control variable occurs solely through changes in the distribution function of the exogenous random variable, whereas in the present case the increase in risk affects not only the distribution function of the exogenous losses but it also influences the insurance premium which in turn affects the insured's wealth in all states of loss. That is, in the presence of a non-negative optimal deductible, an increase in risk in the distribution function of losses will induce insurers to require a higher premium for a given indemnity payments schedule (that is, for a given deductible). At the same time, this increase in risk will alter the optimal choice of indemnity payments (and thus the deductible) and of the premium on the part of the insured. Hence, an analysis of the comparative statics of increases in risk must take into account the feedback from the insured (the demand for insurance) to the insurer (the supply of insurance) and vice versa.

The present results, in particular, the ones relating to first-order stochastic dominance (FSD), may be relevant to the literature on asymmetric information and signalling where it is argued that low-risk consumers will signal their characteristics by demanding more insurance (see, for example, Rothschild and Stiglitz, 1976). As is demonstrated near the end of this article, a shift in the distribution of losses in the sense of FSD has generally an ambiguous impact on the deductible, thus raising some doubt about the identification of low (high) risk with smaller (greater) demand for insurance.[5]

In the next section Arrow's model of the optimal deductible is presented, followed by an investigation of the impact of a Rothschild-Stiglitz mean-preserving spread including examples. A Diamond-Stiglitz mean-utility-

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$\mu_F = \mu_G$ and $\sigma_G^2 > \sigma_F^2$ this does not imply that G is a MPS of F and hence it does not imply $\int U(x)dG(x) < \int U(x)dF(x)$ even if U is concave. Thus, the ranking by variance and the ranking by expected utility do not coincide. That is, if U is concave (but not quadratic) there exist distributions F, G and H, all with the same mean such the $\sigma_F^2 < \sigma_G^2$ but $\sigma_G^2 > \sigma_H^2$ while $\int U(x)dF(x) < \int U(x)dG(x) < \int U(x)dH(x)$.

[5] The authors are grateful to a referee for emphasizing this point. In the Rothschild-Stiglitz asymmetric information model, the loss distribution has a two-point support (loss-no loss) and the magnitude of the loss is the same for all agents. "High-risk" consumers are those with a higher probability of a loss than "low-risk" consumers. Hence high-risk consumers face a stochastically larger loss (as well as a higher average loss) than low- risk consumers and their loss distribution first-order stochastically dominates that of low-risk consumers. As will be seen below, a stochastic increase in losses (in the sense of FSD) has *generally* an ambiguous impact on the insurance purchase but the purchase of insurance *decreases* if the utility function belongs to the class of constant or increasing absolute risk aversion (CARA or IARA). Thus, a high-risk consumer whose preferences exhibit CARA or IARA would purchase less insurance than the low-risk consumer with the same preferences.

Alternatively, the results pertaining to mean-preserving increases in risk could be relevant to a *generalization* of the Rothschild-Stiglitz asymmetric information model in which high-risk agents would have a riskier loss distribution than low-risk agents in the sense of a mean-preserving spread.

preserving spread and a shift in the distribution of losses in the sense of first-order stochastic dominance are also analyzed.

The Model

Consider a risk averse von Neumann-Morgenstern expected utility maximizer and prospective insurance purchaser who has non-random initial wealth w_o and who faces the risk of property loss. Let the random variable Z with cumulative distribution function (cdf) F(z) denote the random loss.

$F:[0,B]\rightarrow[0,1]$ is assumed to be absolutely continuous and differentiable. Let $F_z(z)$ denote the density function of Z. In order to simplify the exposition, following Diamond and Stiglitz (1974), it is assumed that there exists a family of cdf's of Z parameterized by r, an index of the riskiness of the distribution. F(z,r) is assumed to be differentiable in r.

Suppose that unfair insurance is available from a risk neutral insurer. The insurance contract stipulates state-contingent indemnity payments I(z) to be made by the insurer to the insured for all realizations z of the random loss Z, and a premium P as the price to be paid by the insured. I(z) is constrained to be non-negative for all realizations of Z. That is, the insured does not make indemnity payments to the insurer. It is assumed that the cost of providing insurance is proportional to the indemnity payment, that is, it involves fixed percentage (or proportional) loading. Therefore, the cost of providing the insurance policy is proportional to the expected value of the indemnity payments.[6] In order for the insurer to offer the insurance contract, the payment received from the insured must at least cover the expected value of the indemnity payments and the costs. That is,

$$P \geq (1+k)E(I(Z)) \tag{1}$$

where $k > 0$ is the loading factor and E is the expectations operator. Insurance is unfair since $k > 0$. The insurance industry is assumed to be perfectly competitive, so that (1) holds with equality.

The Pareto-optimal insurance contract will be the solution to the following problem:

$$\max_{\{I(z)\},P} \int_0^B U(w_o - z + I(z) - P)\, F_z(z,r)dz$$

$$\text{subject to: } I(z) \geq 0 \qquad \forall z\in[0,B]$$

$$P = (1+k)\int_0^B I(z)\, F_z(z,r)dz \tag{2}$$

where U, the utility function of the insured, is assumed to be monotonic increasing, three times continuously differentiable and strictly concave ($U' > 0$, $U'' < 0$, $U''' > (=)(<) 0$, where the prime, the double prime and the triple prime denote the first, second and third derivative respectively).

[6]More generally the cost of insurance can be taken as an increasing and convex function of the indemnity payment for each state of loss. With this specification of insurance costs the optimal insurance contract would provide for co-insurance above the deductible. See Raviv (1979).

$w_o - z + I(z) - P$ is the post-insurance wealth of the insured. The maximization problem specifies that the insured will maximize the expected utility of post-insurance wealth by choosing an optimal indemnity payments schedule $\{I^*(z)\}$ and an optimal premium P^*, subject to the non-negativity of $I(z)$ and subject to the constraint that the insurer's expected profits are (no less than) zero.[7] The Lagrangean for this problem can be written as

$$L = \int_0^B U(w_o - z + I(z) - P)\, F_z(z,r)dz + \lambda(P(1+k)^{-1} - \int_0^B I(z)F_z(z,r)dz)$$

The Kuhn-Tucker conditions for this problem reduce to:[8]

$$U'(w_o - z + I^*(z) - P^*) - \lambda^* \le 0 \qquad \forall z \tag{3}$$

$$< 0 \Rightarrow I^*(z) = 0$$

$$-\int_0^B U'(w_o - z + I^*(z) - P^*)\, F_z(z,r)dz + \lambda^*(1+k)^{-1} = 0 \tag{4}$$

$$P^*(1+k)^{-1} - \int_0^B I^*(z)\, F_z(z,r)dz = 0 \tag{5}$$

where an asterisk denotes the optimal value. Since the utility function U is jointly (strictly) concave in $I(z)$ and P, these conditions are both necessary and sufficient. By the continuity of U' in its argument, condition (3) implies the existence of a loss level $\bar{z}$ such that

$$U'(w_o - z + I^*(z) - P^*) = \lambda^* \qquad \text{for } z \ge \bar{z} \Rightarrow I^*(z) \ge 0$$

$$U'(w_o - z + I^*(z) - P^*) < \lambda^* \qquad \text{for } z < \bar{z} \Rightarrow I^*(z) = 0$$

As Arrow (1974) and Raviv (1979,Theorem 3 and Corollary 1) have shown, as long as insurance is unfair ($k > 0$ with proportional loading) and the insurer is risk neutral, the optimal indemnity payments will involve 100 percent insurance of losses above a positive deductible ($\bar{z} > 0$), that is, $I^*(z) = z - \bar{z}$ for

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[7] As shown by Arrow (1974) and Raviv (1979), this problem can be solved in two steps. First the optimal insurance payments are determined as a function of a fixed given premium P. Then, in the second step, the optimal premium is chosen. Alternatively, some authors have regarded the premium in the second step as being a function of the deductible (obtained in the first step) and reformulated the second stage optimization problem as a choice of the deductible $\bar{z}$. Several authors have proceeded in this manner, namely, Pashigian, Schkade and Menefee (1966), Gould (1969), Schlesinger (1981), Karni (1983) and Doherty and Schlesinger (1983). One difficulty with this approach, however, is that the objective function is not jointly concave in $I(z)$ and in the deductible whereas it is jointly concave in $I(z)$ and the premium. See Schlesinger for a discussion of how the second order condition for the choice of the deductible is not trivially satisfied. These difficulties do not arise in the present framework.

[8] The Kuhn-Tucker conditions are:

(i) $\partial L/\partial I(z) = U'(w_o - z + I^*(z) - P^*) - \lambda^* \le 0$ for all z

(ii) $\partial L/\partial P = -\int_0^B U'(w_0 - z + I^*(z) - P^*)\, F_z(z,r)dz + \lambda^*(1+k)^{-1} \le 0$

(iii) $\partial L/\partial \lambda = P^*(1+k)^{-1} - \int_0^B I^*(z)\, F_z(z,r)dz \ge 0$

(iv) $I^*(z) \ge 0$ (v) $P^* \ge 0$ (vi) $\lambda^* \ge 0$

(vii) $I^*(z)(\partial L/\partial I(z)) = 0$ (viii) $P^*(\partial L/\partial P) = 0$ (ix) $\lambda^*(\partial L/\partial \lambda) = 0$

where an asterisk denotes optimal values. Perfect competition implies that

$P^* = (1+k)\int_0^B I^*(z)F_z(z,r)dz \Rightarrow \partial L/\partial \lambda = 0$ and $P^* > 0$. Therefore, by (viii), $\partial L/\partial P = 0$. By (vii), for all z such that $I^*(z) > 0$, $(\partial L/\partial I(z)) = 0$. Also $(\partial L/\partial I(z)) < 0 \Rightarrow I^*(z) = 0$.

all $z \geq \bar{z}$ and $I^* = 0$ for $z < \bar{z}$. Thus, substituting for $I^*(z)$ in (3)-(5) one obtains:

$$U'(w_o - \bar{z} - P^*) = \lambda^* \ \forall z \geq \bar{z} \tag{3'}$$

$$(1+k)\{\int_0^{\bar{z}} U'(w_o - z - P^*)F_z(z,r)dz + [1 - F(\bar{z},r)]U'(w_o - \bar{z} - P^*)\} = \lambda^* \tag{4'}$$

$$P^*(1+k)^{-1} = \int_{\bar{z}}^{B} (z - \bar{z})F_z(z,r)dz \tag{5'}$$

Condition (3') defines the optimal deductible. Condition (4') indicates that the insurance buyer will equate the expected marginal benefit of an extra unit of insurance purchased (λ^*, or equivalently $U'(w_o - \bar{z} - P^*)$, which is the marginal utility of wealth in insured states) to the expected marginal cost of the insurance purchased (the left-hand-side of (4')), that is, $(1+k)EU'$ where

$$EU' = \int_0^{\bar{z}} U'(w_o - z - P^*)F_z(z,r)dz + [1 - F(\bar{z},r)]U'(w_o - \bar{z} - P^*). \tag{6}$$

Equations (3'), (4') and (5') are used in order to conduct a comparative statics analysis of the impact of increases in risk on the choice of the optimal insurance policy.

A Mean Preserving Increase in the Risk of Loss

Theoretical Analysis

First, a mean-preserving spread in the cdf of losses is analyzed. A mean-preserving spread (MPS) is defined as follows (see Diamond and Stiglitz, 1974):

Definition 1 An increase in the index r denotes a mean-preserving increase in the riskiness of the loss distribution $F(z,r)$ in the sense of Rothschild-Stiglitz (1971) if the following conditions are satisfied

$$T(x,r) = \int_0^x F_r(z,r)dz \geq 0 \ \forall x \in [0,B) \tag{7a}$$

$$T(B,r) = \int_0^B F_r(z,r)dz = 0 \tag{7b}$$

Condition (7b) indicates that the increase in risk is such as to keep the mean of the cdf unchanged.

The impact of a mean-preserving increase in risk on the insured agent's expected utility is first examined. After substituting for the optimal values I^*, P^* and λ^* the Lagrangean can be rewritten as

$$\begin{aligned} L = &\int_0^{\bar{z}} U(w_o - z - P^*)F_z(z,r)dz + [1 - F(\bar{z},r)]U(w_o - \bar{z} - P^*) \\ &+ \lambda^*(P^*(1+k)^{-1} - \int_{\bar{z}}^{B}(z - \bar{z})F_z(z,r)dz) \\ = &\int_0^{\bar{z}} U(w_o - z - P^*)\, F_z(z,r)dz + [1 - F(\bar{z},r)]U(w_o - \bar{z} - P^*) = EU \end{aligned}$$

where the second equality follows since $P^*(1+k)^{-1} = \int_{\bar{z}}^{B}(z - \bar{z})F_z(z,r)dz$. Therefore, differentiating L with respect to r, using the envelope theorem and integrating by parts twice:

$dL/dr = \int_0^{\bar z} U''(w_o - z - P^*)T(z,r)dz + U'(w_o - \bar z - P^*)T(\bar z,r) - \lambda^* T(\bar z,r) = \int_0^{\bar z} U''(w_o - z - P^*)T(z,r)dz < 0$ where the last equality follows from (3′) according to which $\lambda^* = U'(w_o - \bar z - P^*)$ and where (7a), (7b) and $T(0,r) = F_r(0,r) = F_r(B,r) = 0$ are used. This analysis indicates that, as in Rothschild and Stiglitz (1971), a MPS reduces the expected utility of a risk averse agent. A MPS shifts probability density away from the center and towards the tails of the distribution and thereby increases the probability of high and low values of Z. Since the individual is not covered by the insurance policy in states of small losses, expected utility falls as a result of the increase in risk.

In order to find the impact of a MPS on the optimal choice of insurance, (3′), (4′) and (5′) are differentiated with respect to $\bar z$, P^*, λ^* and r. The resulting system of three simultaneous equations is then solved for $d\bar z/dr$, dP^*/dr and $d\lambda^*/dr$. Integrating by parts, using (7a) and (7b):

$$\begin{aligned} dz/dr = {} & [D^{-1}(-EU')T(\bar z,r)] \\ & \cdot \{[\int_0^{\bar z} U'''(w_o - z - P^*)T(z,r)dz][U'(w_o - \bar z - P^*)T(\bar z,r)]^{-1} \\ & - [1 + (1+k)\beta]A(\bar z) + (EU')^{-1} \int_0^{\bar z} U'(w_o - z - P)A(z)F_z(z,r)dz\} \end{aligned} \quad (8)$$

$$\begin{aligned} dP^*/dr = {} & [D^{-1}(-EU')T(\bar z,r)] \cdot \{A(\bar z) \\ & - [1 - (1+k)\beta][\int_0^{\bar z} U'''(w_o - z - P^*)T(z,r)dz] \\ & \cdot [U'(w_o - \bar z - P^*)T(\bar z,r)]^{-1}\} \end{aligned} \quad (9)$$

$$\begin{aligned} d\lambda^*/dr = {} & [D^{-1}(U'')EU'T(\bar z,r)(1+k)\beta] \\ & \cdot \{[\int_0^{\bar z} U'''(w_o - z - P^*)T(z,r)dz][U'(w_o - \bar z - P^*)T(\bar z,r)]^{-1} \\ & - A(\bar z) + [(1+k)\beta(EU')]^{-1} \int_0^{\bar z} U'(w_o - z - P)A(z)F_z(z,r)dz\} \end{aligned} \quad (10)$$

where $D^{-1} < 0$ (see the Appendix), $U'' < 0$, $\beta \equiv \{(1+k)^{-1} - [1 - F(\bar z,r)]\}$, where EU' was defined in (6) and where $A(\bar z) \equiv -U''(w_o - \bar z - P^*)/U'(w_o - \bar z - P^*)$ is the Arrow-Pratt measure of absolute risk aversion of the insured agent for those states of nature which are insured, i.e. for all $z \geq \bar z$, while $A(z) \equiv -U''(w_o - z - P^*)/U'(w_o - z - P^*)$ is the Arrow-Pratt measure of absolute risk aversion for all $z < \bar z$. β must be positive for an interior solution,[9] while $T(\bar z,r)$ is positive by condition 7(a). This general result is summarized in the following proposition where (8)-(10) are rewritten more compactly by letting

$V \equiv [(1+k)\beta(EU')]^{-1}[\int_0^{\bar z} U'(w_o - z - P^*)A(z)F_z(z,r)dz] > 0$, and

$Y \equiv [\int_0^{\bar z} U'''(w_o - z - P^*)T(z,r)dz] \cdot [U'(w_o - \bar z - P^*)T(\bar z,r)]^{-1} >(=)(<) 0$ as $U''' >(=)(<) 0$.

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[9] Substituting for λ^* from (3′) into (4′) and rearranging, one obtains $\int_0^{\bar z} U'(w_o - z - P^*) F_z(z,r)dz - \beta U'(w_o - \bar z - P^*) = 0$ where $\beta = \{(1+k)^{-1} - [1 - F(\bar z,r)]\}$. β must be positive for (4′) to hold with equality. As explained in footnote 8, the assumption of perfect competition implies $P^* > 0$. Therefore, by the Kuhn- Tucker condition (viii), $\partial L/\partial P = 0$ which in turn implies that $\beta > 0$. Note that from the above equation, β can be written as $\beta = [\int_0^{\bar z} U'(w_o - z - P^*)F_z(z,r)dz][U'(w_o - \bar z - P^*)]^{-1}$.

Proposition 1 Let an increase in r represent a MPS in the distribution of losses. Then

$$d\bar{z}/dr = [D^{-1}(-EU')T(\bar{z},r)] \cdot \{Y - [1+(1+k)\beta]A(\bar{z}) + (1+k)\beta V\} \qquad (8')$$

$$dP^*/dr = [D^{-1}(-EU')T(\bar{z},r)] \cdot \{A(\bar{z}) - [1-(1+k)\beta]Y\} \qquad (9')$$

$$d\lambda^*/dr = [D^{-1}(U'')EU'T(\bar{z},r)(1+k)\beta] \cdot \{Y - A(\bar{z}) + V\} \qquad (10')$$

The signs of (8′)-(10′) will be determined by the sign of the terms inside the braces since in each case these terms are multiplied by a positive term (in brackets). The signs of $d\bar{z}/dr$, dP^*/dr and $d\lambda^*/dr$ are ambiguous and will in general depend on whether the insured exhibits decreasing, constant or increasing absolute risk aversion, on the magnitude of the degree of risk aversion as well as on the loading factor k, on the properties of the loss distribution F(z,r) and on the MPS.[10]

Let us isolate the effect of the incremental increase in risk on the insurer. The latter faces an increase in the cost of the insurance that it provides keeping the amount of insurance purchased the same as before. Hence, the premium must also increase for the zero-expected-profit constraint to be satisfied. To see this, keeping $\bar{z}$ constant, differentiate (5′) with respect to P* and r. Integrating by parts, using (7a) and (7b) and since $F_r(B,r) = 0$, one obtains $(dP^*/dr)|_{\bar{z}\ \mathrm{const.}} = (1+k)T(\bar{z},r)$. Therefore, the insurer will only be willing to provide *the same amount of insurance as before* at a higher premium, the increase in the premium being given by $(1+k)T(\bar{z},r)$.

In the general case, while the insured faces a higher risk of loss than before and may want to increase the amount of insurance purchased, he or she must also take into account the increase in the premium that will follow the increase in risk as well as any increase in insurance coverage.[11] The increase in the premium has two effects. First, through its impact on wealth, it alters the degree of risk aversion in insured states. This first effect may lead to an increase or decrease in the optimal deductible depending on how risk aversion varies with wealth. Second, there is a price effect: the increase in the premium will reduce the demand for insurance as long as the latter is a non-Giffen

[10] According to Rothschild and Stiglitz (1971), a MPS reduces (increases) the optimal value of the control variable if the marginal utility of the control variable is a concave (convex) function of the random variable. Recall that the control variables in this problem are $\{I(z)\}$ for $\forall z\in[0,B]$ and P, and that $\bar{z}$ appears in the problem after substituting for $\{I^*(z)\}$ which is of the form $I^*(z) = z - \bar{z}$ $\forall z\in[\bar{z},B]$ and $I^*(z) = 0$ for $\forall z\in[0,\bar{z}]$. While the present model differs from the models with a single control variable considered by Rothschild and Stiglitz, it is clear that in the present model, the concavity (convexity) of the marginal utility of the control variables does not suffice to sign the impact of a MPS. Thus the concavity (convexity) of the marginal utility of $\{I(z)\}$ (i.e., of $U'(w_0 - z + I(z) - P)$) in z depends on the sign of U‴. (The same condition applies in the case of P.) As can be seen from (8)-(10), the sign of U‴ is not sufficient to determine the impact of the MPS.

[11] The premium is negatively related to $\bar{z}$. To see this, differentiate (5′) with respect to P and $\bar{z}$: $dP/d\bar{z} = -(1+k)[1-F(\bar{z},r)] < 0$. Therefore, an increase in insurance coverage (that is, a fall in $\bar{z}$) *ceteris paribus* leads to an increase in P.

good.[12] Thus the optimal insurance decision may result in an increase or decrease in insurance coverage depending on which effect dominates given the degree of risk aversion and the distribution of losses. Note that in the case of a MPS, if $d\bar{z}/dr < 0$ then dP^*/dr must be positive since an increase in the amount of insurance purchased will cause P^* to increase, while the increase in risk by itself already causes P^* to increase. As noted above, the insurer would require a higher premium for the initial amount of insurance due to the greater risk of high levels of loss brought about by the MPS. Hence when $d\bar{z}/dr$ is negative, there are two effects both acting positively on P^*. In what follows the impact of a MPS on $\bar{z}$, P^* and λ^* is characterized by considering in turn the cases of decreasing, increasing and constant absolute risk aversion (DARA, IARA and CARA respectively).

First focus on the cases of DARA and IARA. From equations (8′)-(10′) observe:

(i) $d\bar{z}/dr >(<)\ 0$ as $A(\bar{z}) <(>)\ [1+(1+k)\beta]^{-1}\ \{Y+(1+k)\beta V\} \equiv \mathbf{J}_{\bar{z}}$

(ii) $dP^*/dr > (<)\ 0$ as $A(\bar{z}) >(<)\ [1-(1+k)\beta]y \equiv \mathbf{J}_{\mathbf{P}}$

(iii) $d\lambda^*/dr >(<)\ 0$ as $A(\bar{z}) <(>)\ \{Y+V\} \equiv \mathbf{J}_{\lambda}$

Since $1 > [1+(1+k)\beta]^{-1}(1+k)\beta$ and $1 > [1+(1+k)\beta]^{-1} > [1-(1+k)\beta]$, it can be easily shown that $\mathbf{J}_{z} > \mathbf{J}_{\mathbf{P}}$ and that $\mathbf{J}_{\lambda} > \mathbf{J}_{z}$. Hence, the ranking $\mathbf{J}_{\lambda} > \mathbf{J}_{z} > \mathbf{J}_{\mathbf{P}}$ is obtained.

Recalling from (3′) that $\lambda^* = U'(w_o - \bar{z} - P^*)$, if $d\lambda^*/dr > 0$, the marginal utility of wealth in insured states will increase, and therefore, by the strict concavity of the utility function, wealth in insured states will decrease, with a MPS in the loss distribution. (Similarly, wealth in insured states will increase if $d\lambda^*/dr < 0$.) Note that, as implied by (3′), $d\lambda^*/dr = -U''[d\bar{z}/dr + dP^*/dr]$, so that the signs of $d\bar{z}/dr$, dP^*/dr and $d\lambda^*/dr$ must be consistent with this relation. Thus, for example, if $d^*\lambda/dr > 0$, and if, say, $dP^*/dr < 0$ and $d\bar{z}/dr > 0$, then $|dP^*/dr| < |d\bar{z}/dr|$ for this relation to be satisfied.

Under DARA $[A(\bar{z}) - V] > 0$ and any restriction imposed on $A(\bar{z})$ must also satisfy $A(\bar{z}) > V$.[13] That is, the individual is more risk averse in states of low wealth (i.e., of high loss) than in states of high wealth (i.e., of small loss). Furthermore, DARA is only consistent with $U''' > 0$ ($\Rightarrow Y > 0$).

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[12] The authors thank a referee for emphasizing these two effects. The theoretical possibility that insurance be a Giffen good is discussed by Hoy and Robson (1981) and Borch (1986). As Schlesinger pointed out, this is not likely empirically.

[13] To see this, factoring out $[(1+k)\beta]^{-1}$ and using the fact that from the first order conditions, $\beta = [\int_0^{\bar{z}} U'(w_o - z - P^*)F_z(z,r)dz][U'(w_o - \bar{z} - P^*)]^{-1}$ (see footnote 9)
and $(1+k) = [U'(w_o - \bar{z} - P)][EU']^{-1}$,
$A(\bar{z}) - V$
$= [(1+k)\beta]^{-1}\{A(\bar{z})[EU']^{-1} \int_0^{\bar{z}} U'(w_o - z - P^*)\ F_z(z,r)dz - [EU']^{-1} \int_0^{\bar{z}} U'(w_o - z - P^*)\ A(z)F_Z(z,r)dz\}$
$= \{(1+k)\beta EU']^{-1}\ \{\int_0^{\bar{z}} U'(w_o - z - P^*)[A(\bar{z}) - A(z)]F_Z(z,r)dz\} >(<)\ 0$ if DARA (IARA)
where the second equality follows since $A(\bar{z})$ is a constant. Hence, $A(\bar{z}) - V >(<)\ 0$ if absolute risk aversion is decreasing (increasing). Under CARA $A(\bar{z}) - V = 0$.

Corollary 1 If the insured agent exhibits **DARA**, a MPS in the distribution of losses will lead to

(a) an increase in the deductible, a fall in the premium and a decrease in wealth in insured states if the insured agent has low risk aversion in insured states, i.e.,

$d\bar{z}/dr > 0$, $dP^*/dr < 0$, $d\lambda^*/dr > 0$ ($\Rightarrow |dP^*/dr| < |d\bar{z}/dr|$), if $A(\bar{z}) < \mathbf{J_P}$;

(b) an increase in the deductible, an increase in the premium and a decrease in wealth in insured states, if the insured agent's risk aversion is somewhat higher in insured states i.e.,

$d\bar{z}/dr > 0$, $dP^*/dr > 0$, $d\lambda^*/dr > 0$ if $\mathbf{J_P} < A(\bar{z}) < \mathbf{J_{\bar{z}}}$;

(c) a fall in the deductible and an increase in the premium if the insured agent has high risk aversion in insured states i.e.,

$d\bar{z}/dr < 0$ and $dP^*/dr > 0$ if $A(\bar{z}) > \mathbf{J_{\bar{z}}}$, and

(i) a decrease in wealth in insured states, i.e.,

$d\lambda^*/dr > 0$ ($\Rightarrow |dP^*/dr| > |d\bar{z}/dr|$) if $\mathbf{J_{\bar{z}}} < A(\bar{z}) < \mathbf{J_\lambda}$;

(ii) an increase in wealth in insured states if the insured agent's risk aversion in insured states is even higher, i.e.,

$d\lambda^*/dr < 0$ ($\Rightarrow |dP^*/dr| < |d\bar{z}/dr|$) if $A(\bar{z}) > \mathbf{J_\lambda}$;

Corollary 1 indicates that the insured has a tendency to buy more insurance the higher is risk aversion in insured states. If the insured agent is sufficiently risk averse in insured states, the insurance purchase and the premium will increase. If the agent is not significantly risk averse in insured states, the insurance purchase may decrease in spite of the increase in risk. If $A(\bar{z})$ is low enough as in condition 1(a), the insurance purchase may be reduced to such an extent as to even result in a lower premium. This is due to the fact that the increase in risk ceteris paribus already leads to a higher premium given the initial amount of insurance purchased, and that the insured agent must pay an increase in the premium in both insured and "uninsured" states of loss, that is, both in states which are covered by the insurance and in states which fall below the deductible. Any incremental increase in insurance, on the other hand, simply extends the agent's coverage to include "a few more states" of losses. When $A(\bar{z})$ is low enough, the individual prefers to *reduce* the amount of insurance purchased and be incrementally more exposed to risk rather than face the certainty of a higher premium in all states of nature.

Note, however, that as long as $A(\bar{z}) > \mathbf{J_{\bar{z}}}$, the agent buys more insurance and pays a higher premium as indicated by condition (c). Condition (c)(ii) simply indicates that if risk aversion is even higher, the agent's wealth in insured states will increase. That is, recalling that $d\lambda^*/dr = -U''[d\bar{z}/dr + dP^*/dr]$, in case (c)(ii) the agent buys so much additional insurance that the fall in the deductible overpowers the increase in the premium, resulting in an increase in wealth in insured states (i.e.,$d\lambda^*/dr < 0$).[14]

Turning to the case of increasing absolute risk aversion, three possible cases: $U''' > 0$, $U''' < 0$ and $U''' = 0$ need to be distinguished, since IARA is consistent with all of these. First consider the case of IARA with $U''' > 0$. Note that under IARA, $V > A(\bar{z})$ (see footnote 13). Accordingly, when $U''' > 0$, by equation (8′), $d\lambda^*/dr$ is unambiguously positive. Therefore, the results obtained under IARA with $U''' > 0$ are summarized by Corollary 1(a), 1(b) and 1(c)(i). Since $d\lambda^*/dr > 0$, part (c)(ii) of Corollary 1 cannot hold. In other words, under IARA with $U''' > 0$ the insured agent insures more the higher the degree of risk aversion in insured states $A(\bar{z})$, but has a tendency to insure less than in the case of DARA. That is, since $d\lambda^*/dr > 0$, the insurance purchase never increases to the extent of increasing wealth in insured states.

The case of IARA with $U''' < 0$ yields essentially similar results. However, since $Y \equiv [\int_0^{\bar{z}} U'''(w_o - z - P^*)T(z,r)dz][U'(w_o - \bar{z} - P^*)T(\bar{z},r)]^{-1}$ is now negative, it can be seen from equation (9′) that dP^*/dr is unambiguously positive. On the other hand, from (10′), $d\lambda^*/dr$ is no longer of unambiguous sign. Therefore, the results obtained for the case of IARA with $U''' < 0$ are given by Corollary 1(b), 1(c)(i) and 1(c)(ii). Note that the case given in Corollary 1(a) is ruled out since dP^*/dr must be positive. In other words, the insured agent will never reduce the insurance coverage to such an extent as to result in a fall in the premium. In conclusion, an insured agent who exhibits IARA with $U''' < 0$ insures more the higher $A(\bar{z})$ is, but is also more likely to increase the insurance purchase than an agent who exhibits DARA or IARA with $U''' > 0$. To this effect, note also that the terms $\mathbf{J}_{\bar{z}}$ and $\mathbf{J}_\lambda$ are smaller when Y is negative implying that a lower level of risk aversion $A(\bar{z})$ suffices for the deductible to fall ($d\bar{z}/dr < 0$) and for wealth in insured states to increase ($d\lambda^*/dr < 0$).

Now consider the case of $U''' = 0$ which is an important special case, namely, that of a quadratic utility function. Since $U''' = 0$, so is Y. Setting $Y = 0$ in (8′)-(10′), and using the fact that in this case $U''(.)$ is a constant for all values of wealth, more clear-cut results are obtained: the premium unambiguously increases and wealth in insured states unambiguously falls with a MPS. The deductible may increase or decrease depending on the magnitude of the loading factor k. That is, $d\bar{z}/dr >(=)(<) 0$ as $k >(=)(<) 1$. If k is large (i.e., $k > 1$) and the insurance premium is heavily loaded, the insurance purchase will be reduced following a MPS. The following corollary summarizes this discussion.

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[14]Since DARA implies $A(\bar{z}) > V$, cases (a), (b) and (c) of Corollary 1 can occur if $V < [1-(1+k)\beta]Y$. If $[1-(1+k)\beta]Y < V < Y$ then $A(\bar{z}) > V > [1-(1+k)\beta]Y \equiv \mathbf{J}_p$. Hence case (a) will not occur and only cases (b) and (c) of Corollary 1 exist. If $V > Y$ then $A(\bar{z})$ cannot simultaneously be larger than V *and* smaller than $\mathbf{J}_{\bar{z}} \equiv [1+(1+k)\beta]^{-1}[Y+(1+k)\beta V] < V$. Therefore, if $V > Y$, case (b) cannot occur and only case (c) of Corollary 1 is possible.

Corollary 2 Suppose that the insured agent exhibits **IARA**.

(a) If $U''' > 0$, then a MPS in the distribution of losses will lead to a fall in wealth in insured states ($d\lambda^*/dr > 0$). The impact of a MPS on the deductible and the premium is the same as in Corollary 1(a), 1(b) and 1(c)(i) depending on the magnitude of $A(\bar{z})$.[15]

(b) If $U''' < 0$, then a MPS in the distribution of losses will lead to an increase in the premium. The impact of a MPS on the deductible and on wealth in insured states is the same as in Corollary 1(b), 1(c)(i) and 1(c)(ii) depending on the magnitude of $A(\bar{z})$.[16] The insured agent has a greater tendency to increase the purchase of insurance under IARA with $U''' < 0$ than under DARA or under IARA with $U''' > 0$.

(c) If $U''' = 0$ (quadratic utility function) then a MPS in the distribution of losses will lead to an increase in the premium ($dP^*/dr > 0$), to a fall in wealth in insured states ($d\lambda^*/dr > 0$) and to an increase (decrease) in the deductible if the loading factor k is larger (smaller) than 1, that is, $d\bar{z}/dr >(<) 0$ as $k >(<) 1$. If $k = 1$, $d\bar{z}/dr = 0$. If $d\bar{z}/dr < 0$, $d\lambda^*/dr > 0 \Rightarrow |dP^*/dr| > |d\bar{z}/dr|$.

Proof: See the Appendix

In the case of CARA, first note that the measure of absolute risk aversion will be the same at all wealth levels, and hence $A(\bar{z}) = A(z) \equiv A$ is constant over the entire range of final wealth. This also implies that $[A - V] = 0$. (See footnote 13) Furthermore, under CARA $U''' > 0$ ($\Rightarrow Y > 0$). With these simplifications, equations (8′)-(10′) become:

$$d\bar{z}/dr = [D^{-1}(-EU')T(\bar{z},r)] \cdot \{Y - A\} >(<) 0 \text{ as } A <(>) Y \tag{8''}$$

$$dP^*/dr = [D^{-1}(-EU')T(\bar{z},r)] \cdot \{A - [1-(1+k)\beta]Y\} >(<) 0 \text{ as } A >(<) [1-(1+k)\beta]Y \tag{9''}$$

$$d\lambda^*/dr = [D^{-1}(U'')EU'T(\bar{z},r)(1+k)\beta] \cdot \{Y\} > 0 \tag{10''}$$

Equation (10″) indicates that under CARA, wealth in insured states will unambiguously decrease with a MPS in the loss distribution, as was the case under IARA with $U''' > 0$ and with $U''' = 0$. Noting that $[1-(1+k)\beta] = (1+k)[1-F(\bar{z},r)] < 1$ (see footnote 9) the following corollary is obtained:

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[15] IARA implies $A(\bar{z}) < V$. Thus, under IARA with $U''' > 0$, the cases (a), (b) and (c)(i) of Corollary 1 can occur as long as $V > Y$. If $[1-(1+k)\beta]Y < V < Y$ case c(i) cannot occur since $A(\bar{z})$ cannot be simultaneously less than V *and* larger than $[1+(1+k)\beta]^{-1}\ [Y+(1+k)\beta V] \equiv \mathbf{J}_{\bar{z}} > V$. Hence, only (a) and (b) can occur in this case. Finally, if V is even smaller, that is if $V < [1-(1+k)\beta]Y$, then only case (a) of Corollary 1 is possible since $A(\bar{z})$ cannot be simultaneously larger than $\mathbf{J}_p \equiv [1-(1+k)\beta]Y > V$, *and* smaller than V.

[16] IARA implies $A(\bar{z}) < V$. Hence with $U''' < 0$ ($\Rightarrow Y < 0$), cases (b), (c)(i) and (c)(ii) of Corollary 1 can occur as long as $V > -[(1+k)\beta]^{-1}\,Y$. If $-Y < V < -[(1+k)\beta]^{-1}Y$ then $\mathbf{J}_{\bar{z}} \equiv Y + [(1+k)\beta]V < 0$. Since $A(\bar{z})$ is positive, $A(\bar{z})$ cannot be less than $\mathbf{J}_{\bar{z}}$, and hence case (b) of Corollary 1 will not occur. Only cases 1(c)(i) and 1(c)(ii) can occur. Finally, if $0 < V < -Y$, then $\mathbf{J}_\lambda \equiv [Y+V] < 0$ as well. Therefore, $A(\bar{z})$ must be larger than $\mathbf{J}_\lambda$, ruling out case (c)(i). Thus, if $0 < V < -Y$, only case c(ii) of Corollary 1 will occur.

Corollary 3 If the insured agent exhibits **CARA**, a mean- preserving increase in risk in the distribution of losses will lead to a fall in wealth in insured states and to

(a) an increase in the deductible and a fall in the premium if the insured agent has low risk aversion, i.e., $d\bar{z}/dr > 0$, $dP^*/dr < 0$, (and $d\lambda^*/dr > 0 \Rightarrow |dP^*/dr| < |d\bar{z}/dr|$) if $A < [1-(1+k)\beta]Y$;
(b) an increase in the deductible and an increase in the premium if the insured agent is somewhat more risk averse, that is, $d\bar{z}/dr > 0$ and $dP^*/dr > 0$ if $[1-(1+k)\beta]Y < A < Y$;
(c) a fall in the deductible and an increase in the premium if the insured agent is highly risk averse, that is, $d\bar{z}/dr < 0$, $dP^*/dr > 0$, (and $d\lambda^*/dr > 0 \Rightarrow |dP^*/dr| > |d\bar{z}/dr|$) if $A > Y$.

Corollary 3 reveals that under CARA, if $A < [1-(1+k)\beta]Y$, then the insured will prefer to substantially reduce the insurance purchase so as to pay a lower premium in spite of the increase in risk. It is only if $A > Y$ that the insured finds it optimal to purchase more insurance. Moreover, since $d\lambda^*/dr > 0$, the insurance coverage never increases by enough to result in an increase in wealth in insured states.

Now consider three special cases which involve a restriction on the type of increase in risk.

First, suppose that the MPS occurs entirely below the deductible. That is, suppose that $T(z,r) \geq 0$ for all $z \in [0,\bar{z})$ and that $T(\bar{z},r) = 0$. Then multiplying through by $T(\bar{z},r)$ in equations (8)- (10) and setting it equal to zero, one obtains

$$d\bar{z}/dr = [D^{-1}(-EU')] \cdot \{[\int_0^{\bar{z}} U'''(w_o - z - P^*)T(z,r)dz][U'(w_o - \bar{z} - P^*)]^{-1}\} \quad (8''')$$

$$dP^*/dr = [D^{-1}(-EU')] \cdot \{-[1-(1+k)\beta][\int_0^{\bar{z}} U'''(w_o - z - P^*)T(z,r)dz][U'(w_o - \bar{z} - P^*)]^{-1}\} \quad (9''')$$

$$d\lambda^*/dr = [D^{-1}(U'')EU'(1+k)\beta] \cdot \{[\int_0^{\bar{z}} U'''(w_o - z - P^*)T(z,r)dz][U'(w_o - \bar{z} - P^*)]^{-1}\} \quad (10''')$$

Corollary 4 Suppose that the mean preserving spread in the distribution of losses occurs entirely below the deductible. Then $U''' >(<) 0$ implies $d\bar{z}/dr >(<) 0$, $dP^*/dr <(>) 0$ and $d\lambda^*/dr >(<) 0$.

As Corollary 4 indicates, the impact of a MPS on the deductible and the premium is determined by the Rothschild-Stiglitz sufficient condition only in the special case where the MPS occurs entirely below the deductible. As noted before, for a constant deductible, $(dP^*/dr)|_{\bar{z}\ \text{const.}} = (1+k)T(\bar{z},r)$ which equals zero in the present case. Therefore, the change in the premium is entirely due to the change in the deductible in response to the MPS. In fact, (9''') can be rewritten as $dP^*/dr = (dP^*/d\bar{z})(d\bar{z}/dr) = -(1+k)[1- F(\bar{z},r)] (d\bar{z}/dr)$.

Now, suppose instead that the MPS occurs entirely above the deductible, so that $T(z,r) = 0$ for $0 \leq z \leq \bar{z}$.

Corollary 5 When the mean preserving spread in the distribution of losses occurs entirely above the deductible, $d\bar{z}/dr = dP^*/dr = d\lambda^*/dr = 0$.

The intuition behind this result is that the insured individual does not face the increase in risk at all since it only affects the probability of occurrence of losses which are already insured. Since $\bar{z}$ does not change and since the MPS occurs entirely above $\bar{z}$, this particular type of MPS in the distribution of losses corresponds to a MPS in the induced distribution of indemnity payments. As the mean of the indemnity payments remains unchanged by this increase in risk, the risk neutral insurer does not require any compensation for it.[17] Thus, the premium remains unchanged as well.

The next result involves a special type of MPS, namely a simple MPS where the two distribution functions cross only once. To see the implications of restricting the MPS to be of the single-crossing type, integrate the term Y by parts,[18] and rewrite $d\bar{z}/dr$ from (8′) as $d\bar{z}/dr = D^{-1}(-EU')T(\bar{z},r) \{[\int_0^{\bar{z}} U''(w_o - z - P^*)F_r(z,r)dz] \cdot [U'(w_o - \bar{z} - P^*)T(\bar{z})]^{-1} - (1+k)\beta\, [A(\bar{z}) - V]\}$. The term $[A(\bar{z}) - V]$ is positive under DARA and zero under CARA. Therefore, $d\bar{z}/dr$ would be unambiguously negative for these two classes of utility functions if $F_r(z,r) \geq 0$ for all $z \in [0,\bar{z}]$. In general for a MPS, $F_r(z,r)$ could be of either sign and could change signs many times as long as conditions 7(a) and 7(b) are satisfied. However, restricting the analysis to a single-crossing MPS and requiring that $\bar{z}$ be less than the crossing point ($\hat{z}$) will guarantee that $F_r(z,r) \geq 0$ for all $z \in [0,\bar{z}]$. If $\bar{z} = \hat{z}$, $F_r(\bar{z},r) = 0$. If the utility function belongs to the class of IARA, $[A(\bar{z}) - V] < 0$, so that the restrictions to a simple MPS and to $\bar{z} \leq \hat{z}$ do not suffice to obtain an unambiguous result since $F_r(z,r)$ could not be negative for all $z \in [0,\bar{z}]$ (as that would violate 7(a)). The next corollary summarizes this result.

Corollary 6 Consider a simple MPS where the two distribution functions cross only once. Denote the crossing point by $\hat{z}$. If the utility function belongs to the class of CARA or DARA, *and if the deductible is below* $\hat{z}$ (i.e., $\bar{z} \leq \hat{z}$) then $d\bar{z}/dr < 0$, $dP^*/dr > 0$.

It is interesting to note that if the initial distribution of losses is a Bernoulli distribution with two states (loss, no-loss), then a mean-preserving increase in the probability of a loss conforms to a single-crossing MPS and $\bar{z}$ must automatically be below the crossing point if the individual buys insurance. This case is explored in example 6 below.

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[17] Recall that $P^* = (1+k)E\{I^*(Z)\} = (1+k)\int_{\bar{z}}^{B} I^*(z)F_z(z,r)dz$. $dP^*/dr\,|_{\bar{z}\ \mathrm{const.}} = (1+k)T(\bar{z},r)$ which is positive in general. When the MPS occurs entirely above $\bar{z}$, $T(\bar{z}, r) = 0$ and the mean $E\{I^*(Z)\}$ remains unchanged.

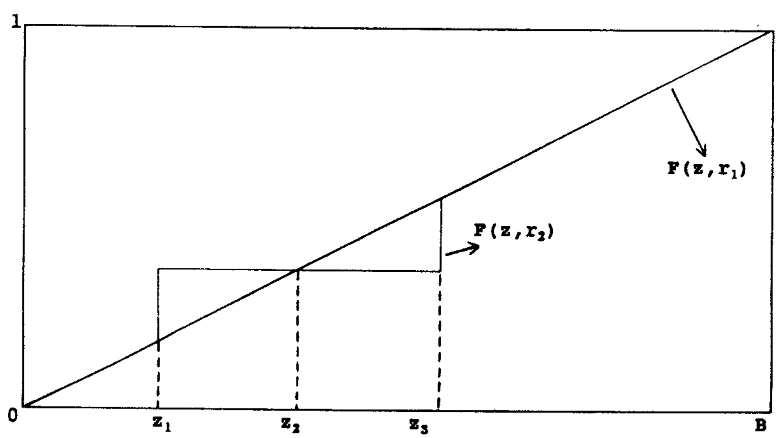
[18] Integrating Y by parts, $Y \equiv [\int_0^{\bar{z}} U'''(w_o - z - P^*)T(z)dz] \cdot [U'(w_o - \bar{z} - P^*)T(\bar{z})]^{-1} = \{\int_0^{\bar{z}} U''(w_o - z - P^*)F_r(z,r)dz\} \cdot [U'(w_o - \bar{z} - P^*)T(\bar{z})]^{-1} + A(\bar{z})$.

Examples

As the above discussion suggests, a mean preserving increase in risk has in general an ambiguous impact on the deductible and on the premium for utility functions which are in the class of DARA, IARA and CARA. Furthermore, it is not possible to provide sufficient conditions which are in closed form. For example, in (8″) $d\bar{z}/dr >(<) 0$ as $A <(>) Y$. Yet, the term Y is itself a function of the measure of absolute risk aversion A. The purpose of this section is to show, by means of examples, that these conditions are meaningful. For utility functions in each class (CARA, DARA and IARA) it is shown that $\bar{z}$ may increase or decrease depending on the magnitude of $A(\bar{z})$, on the loading factor k, and on the particular MPS. The following procedure is adopted in each of the examples: Given a specific utility function, an initial cdf, a specific MPS, and initial values for w_o and k, and given the resulting $A(\bar{z})$, it is first found that $\bar{z}$ increases in response to the MPS. Then one parameter is altered (for instance, A in example 1, w_o in example 2, k in examples 3 and 6) while maintaining all other parameter values constant, as a result of which it is found that the individual exhibits a higher degree of risk aversion at $w_o - \bar{z} - P^*$ and that $\bar{z}$ falls in response to the same MPS. In addition, it is shown that changing the MPS, while maintaining the same degree of risk aversion, affects the response of $\bar{z}$ since the latter depends on the magnitude of $A(\bar{z})$ relative to the particular MPS.

Figure 1

A Simple Mean-Preserving Spread When the Initial Distribution Is Uniform



In examples 1-5, it is assumed that the distribution function of losses is given by the uniform distribution, that is, $F(z,r_1) = z/B$ for $0 \leq z \leq B$. The riskier distribution $F(z,r_2)$, which differs from $F(z,r_1)$ by a MPS, is defined as follows:

$$F(z,r_2) = \begin{cases} F(z,r_1) \text{ for } 0 \leq z \leq z_1 \\ z_2/B \text{ for } z_1 < z < z_3 \\ F(z,r_1) \text{ for } z_3 \leq z \leq B \end{cases}$$

where $0 \leq z_1 < z_2 < z_3 \leq B$, and where $(z_2 - z_1) = (z_3 - z_2)$ insures the equality of means (see Figure 1). Note that this constitutes a discrete rather than a differential increase in risk. (That is, $F(z,r_2)$ differs from $F(z,r_1)$ by a step function.) The results obtained with this MPS will also hold for continuous approximations around $F(z,r_2)$. Note also that the two cdf's cross only once, with z_2 being the crossing point. For this type of single-crossing or simple MPS, it is known from Corollary 6 that if $\bar{z} < z_2$ and if the utility function of the insured exhibits CARA or DARA, then $\bar{z}$ unambiguously falls. However, the impact of the MPS on $\bar{z}$ is ambiguous in the cases of CARA and DARA when $\bar{z} > z_2$, and is always ambiguous for the case of IARA. Therefore, for the cases of CARA and DARA examples with $\bar{z} > z_2$ are considered.

Conditions 7(a) and 7(b) can be expressed as

$$T(x) = \int_0^x [F(z,r_2) - F(z,r_1)]dz = \int_{z_1}^x (1/B)[z_2 - z]dz \geq 0 \ \forall x \in [z_1, z_3) \quad (7'a)$$

$$T(z_3) = \int_{z_1}^{z_3} (1/B)[z_2 - z]dz = 0 \quad (7'b)$$

Here, $T(z) = 0$ for $0 \leq z \leq z_1$ and for $z_3 \leq z \leq B$ (and $T(0) = T(z_1) = T(z_3) = T(B) = 0$). Now, $T(\bar{z}) = \int_{z_1}^{\bar{z}} (1/B)[z_2 - z]dz = (2B)^{-1} [(\bar{z} - z_1)(2z_2 - z_1 - \bar{z})]$ for $z_1 \leq \bar{z} \leq z_3$. Integrating by parts,

$$Y \equiv [\int_0^{\bar{z}} U'''(w_o - z - P^*)T(z)dz] \cdot [U'(w_o - \bar{z} - P^*)T(\bar{z})]^{-1}$$

$$= \int_{z_1}^{\bar{z}} U''(w_o - z - P^*)[F(z,r_2) - F(z,r_1)]dz - U''T(\bar{z},r)\} \cdot [U'(w_o - \bar{z} - P^*)T(\bar{z})]^{-1}$$

$$\begin{aligned} = \{&(z_2/B)[U'(w_o - z_1 - P^*) - U'(w_o - \bar{z} - P^*)] \\ &+ (1/B)[U(w_o - \bar{z} - P^*) - U(w_o - z_1 - P^*)] \\ &+ (\bar{z}/B)U'(w_o - \bar{z} - P^*) - (z_1/B)U'(w_o - z_1 - P^*) \\ &- U''(w_o - \bar{z} - P^*)T(\bar{z})\} \cdot \{U'(w_o - \bar{z} - P^*)T(\bar{z})\}^{-1} \end{aligned}$$

Furthermore $(1 + k)\beta V \equiv [(EU')]^{-1} [\int_0^{\bar{z}} U'(w_o - z - P^*)A(z)F_z(z,r)dz]$

$$\begin{aligned} &= (1/B)[U'(w_o - \bar{z} - P^*) - U'(w_o - P^*)] \cdot \{(1/B)[U(w_o - P^*) \\ &\quad - U(w_o - \bar{z} - P^*)] + [1 - (\bar{z}/B)]U'(w_o - \bar{z} - P^*)\}^{-1}. \end{aligned}$$

Example 1, CARA: Suppose that the utility function of the insured belongs to the class of CARA. Then $U(w) = -e^{-Aw}$ where A denotes the constant absolute risk aversion measure and w denotes wealth. Solving the first order conditions (3')-(5') for $\bar{z}$ yields $\bar{z} + A^{-1} [e^{-A\bar{z}} - 1] - B\{(1 + k)^{-1} - 1\} = 0$ which is approximately solved by $\bar{z} \approx 1.8888888$ when $A = .5$, $k = .5$ and $B = 2$. Let the MPS be as defined in (7′), with $z_1 = .1$, $z_2 = 1$, and $z_3 = 1.9$ so that $z_2 < \bar{z} < z_3$. By (8″), $\bar{z}$ increases (falls) with a MPS as A <(>) Y. Now, $Y = -A[\int_{z_1}^{\bar{z}} e^{-A(\bar{z}-z)}(z_2 - z)dz][BT(\bar{z})]^{-1} + A = \{e^{-A(\bar{z}-z_1)}[(z_2 - z_1) + A^{-1}] + (\bar{z} - z_2 - A^{-1})\} \cdot \{BT(\bar{z})\}^{-1} + A = 7.9972716$, $T(\bar{z}) = .0049691$ and $[1-(1+k)\beta] = (1+k)[1 - F(\bar{z},r_1)] = (1+k)[1-(\bar{z}/B)] = .0833334$. Hence, $A < Y$, and $A < [1-(1+k)\beta]Y = .6664398$ so that $\bar{z}$ *increases* and P* *falls*. This is an example of Corollary 3(a).

Now, consider a more risk averse individual with $A = 1$ facing the same increase in risk ($z_1 = .1$, $z_2 = 1$, $z_3 = 1.9$) and let $k = .5$ and $B = 2$ as above. For this individual, $\bar{z} \approx 1.4265$ and $z_2 < \bar{z} < z_3$. In this case $Y = .7795522$, $T(\bar{z}) = .1570244$, $[1-(1+k)\beta] = .430125$, $[1-(1+k)\beta]Y = .3353049$. Thus, $A > [1-(1+k)+\beta]Y > Y$ so that $\bar{z}$ *falls* and P* *increases*. This is an example of Corollary 3(c). For the same increase in risk, a higher degree of risk aversion ($A = 1$ instead of $A = .5$) leads to an increase in the insurance purchase.

To indicate how this result also depends on the increase in risk, let us consider this last case with alternative values for z_1, z_2 and z_3. Let $z_1 = .5$, $z_2 = 1$ and $z_3 = 1.5$. With $A = 1$, $\bar{z} \approx 1.4265$ and $[1-(1+k)\beta] = .430125$ as above, this MPS yields $Y = 1.5993045$, $T(\bar{z}) = .0170244$ and $[1-(1+k)\beta] \cdot Y = .6879008$ so that now $Y > A > [1-(1+k)\beta]Y$. Hence $\bar{z}$ and P* both *increase*. This is an example of Corollary 3(b).

Example 2, DARA: Suppose that the utility function of the insured belongs to the class of DARA. Let $U(w) = \ln w$. Solving the first order conditions for $\bar{z}$ yields $(w_o - \bar{z} - P^*) \cdot \ln[(w_o - \bar{z} - P^*)/(w_o - P^*)] + B\{(1+k)^{-1} - 1\} + \bar{z} = 0$ where $P^* = (1+k)(\bar{z} - B)^2(2B)^{-1}$ Here.

$$Y = \{(z_2/B)[(w_o - z_1 - P^*)^{-1} - (w_o - \bar{z} - P^*)^{-1}] + (1/B)[\ln(w_o - \bar{z} - P^*) - \ln(w_o - z_1 - P^*)] + (\bar{z}/B)(w_o - \bar{z} - P^*)^{-1} - (\bar{z}_1/B)(w_o - z_1 - P^*)^{-1} + (w_o - \bar{z} - P^*)^{-2}\,T(\bar{z})\} \cdot \{(w_o - \bar{z} - P^*)^{-1}\,T(\bar{z})\}^{-1}.$$

$$(1+k)\beta V = (1/B)[(w_o - \bar{z} - P^*)^{-1} - (w_o - P^*)^{-1}] \cdot \{(1/B)[\ln(w_o - P^*) - \ln(w_o - \bar{z} - P^*)] + (w_o - \bar{z} - P^*)^{-1}[1-(\bar{z}/B)]\}^{-1}$$

while $A(\bar{z}) = (w_o - \bar{z} - P^*)^{-1}$. Let $w_o = 3$, $B = 2$, $k = .5$. Also let the MPS be as defined in (7′) with $z_1 = .1$, $z_2 = 1$ and $z_3 = 1.9$. Then for this individual, $\bar{z} \approx 1.7549$, $P^* \approx .0225277$, $T(\bar{z}) = .0600314$, $A(\bar{z}) = .8179475$, $Y = 1.5744046$, $(1+k)\beta V = .4420445$ and $[1+(1+k)\beta] = 2 - (1+k)[1-(\bar{z}/B)] = 1.816175$, $[1-(1+k)\beta] = .183825$, $\mathbf{J_P} \equiv [1-(1+k)\beta]Y = .2894149$, $\mathbf{J_{\bar{z}}} \equiv [1+(1+k)\beta]^{-1}\{Y + (1+k)\beta V\} = 1.1102724$ and $\mathbf{J_\lambda} \equiv \{Y + V\} = 2.1160096$. Thus, $\mathbf{J_P} < A(\bar{z}) < \mathbf{J_z} < \mathbf{J_\lambda}$ so that $\bar{z}$, P* and λ^* all *increase*. This is an example of Corollary 1(b).

Now consider a lower initial wealth. Let $w_o = 2.5$ while $B = 2$ and $k = .5$ as before, and consider the impact of the same MPS as above ($z_1 = .1$, $z_2 = 1$ and $z_3 = 1.9$) on this individual who exhibits greater risk aversion due to a lower initial wealth. Here $\bar{z} \approx 1.5592$, $P^* \approx .0728642$, $T(\bar{z}) = .1243238$, $A(\bar{z}) = 1.152159$, $Y = 1.308376$, $(1+k)\beta V = .4817997$, $[1+(1+k)\beta] = 1.6694$, $[1-(1+k)\beta] = .3306$, $\mathbf{J_p} = .4325491$, $\mathbf{J_{\bar{z}}} = 1.0723468$ and $\mathbf{J_\lambda} = 2.0281246$.

Therefore, $\mathbf{J_P} < \mathbf{J_{\bar{z}}} < A(\bar{z}) < \mathbf{J_\lambda}$ so that $\bar{z}$ *falls* while P* and λ* *increase*. This is an example of Corollary 1(c)(i).

Now consider a different MPS while keeping all other parameters the same. Suppose that $z_1 = .4$, $z_2 = 1$ and $z_3 = 1.6$. With $w_o = 2.5$, $\bar{z} \approx 1.5592$, $P^* \approx .0728642$, $(1+k)\beta V = .4817997$, $A(\bar{z}) = 1.152159$, $[1+(1+k)\beta] = 1.6694$ and $[1-(1+k)\beta] = .3306$ as before, now less insurance will be purchased since $T(\bar{z}) = .0118238$, $Y = 4.5292377$, $\mathbf{J_P} = 1.497366$, $\mathbf{J_{\bar{z}}} = 3.0016996$ and $\mathbf{J_\lambda} = 5.2489863$, so that $A(\bar{z}) < \mathbf{J_P} < \mathbf{J_{\bar{z}}} < \mathbf{J_\lambda}$. Therefore, $\bar{z}$ and λ* *increase* while P* *falls*. This is an example of Corollary 1(a). Note that for all of the examples above, $z_2 < \bar{z} < z_3$.

Example 3, IARA $U''' > 0$: Suppose that the utility function of the insured belongs to the class of IARA with $U''' > 0$. Let $U(w) = -(b-w)^c$ where $w \le b$ and $c > 2$. Choose $c = 4$ and set $b = w_o$. Solving the first order conditions and rearranging, one obtains $3\bar{z} - P^* + 4B[(1+k)^{-1} - 1] + (P^*)^4/(\bar{z}+P^*)^3 = 0$ where $P^* = (2B)^{-1}(1+k)(\bar{z}-B)^2$. Here $Y = [24\int_{z_1}^{\bar{z}}(z+P^*)T(\bar{z})dz] \cdot [4(\bar{z}+P^*)^3T(\bar{z},r)]^{-1}$. Integrating by parts,

$$Y = 6\{(2B)^{-1}\{(2P^* - z_2)(1/3)[\bar{z}^3 - z_1^3] + (1/4)[\bar{z}^4 - z_1^4] - P^*z_2(\bar{z}^2 - z_1^2)\} + \bar{z}T(\bar{z})[P^* + (1/2)\bar{z}]\} \cdot \{(\bar{z}+P^*)^3\, T(\bar{z})\}^{-1}.$$

$$(1+k)\beta V = (4/B)\{(\bar{z}+P^*)^3 - (P^*)^3\} \cdot \{(1/B)[(\bar{z}+P^*)^4 - (P^*)^4] + [1 - (\bar{z}/B)]4(\bar{z}+P^*)^3\}^{-1}.$$

Also $A(\bar{z}) = (c\text{-}1)(\bar{z}+P^*)^{-1}$. Letting $B = 2$, and $k = .5$, it can be verified that $\bar{z} \approx 1.00925$ and $P^* \approx .3680945$ solve the first order conditions. Let the MPS be as defined in (7′) with $z_1 = .05$, $z_2 = .575$ and $z_3 = 1.1$. As a result, $T(\bar{z}) = .0217629$, $A(\bar{z}) = 2.1781043$, $Y = 4.6491216$, $(1+k)\beta V = .7356863$, $[1+(1+k)\beta] = 1.2569375$, $[1-(1+k)\beta] = .7430625$, $\mathbf{J_P} \equiv [1-(1+k)\beta]Y = 3.4545879$, $\mathbf{J_{\bar{z}}} = 4.2840697$ and $\mathbf{J_\lambda} = 7.5124105$. This is an example of Corollary 2(a) with $A(\bar{z}) < \mathbf{J_P} < \mathbf{J_{\bar{z}}} < \mathbf{J_\lambda}$ so that $\bar{z}$ and λ* *increase* while P* *falls*.

Now suppose that the loading factor is lower than .5. In particular, let $k = .1$. With $B = 2$ as before, the first order conditions now yield $\bar{z} \approx .4166$ and $P^* \approx .6894677$. With the same MPS ($z_1 = .05$, $z_2 = .575$, $z_3 = 1.1$), pjnow $T(\bar{z}) = .0626336$, $A(\bar{z}) = 2.7123114$, $Y = .9337686$, $(1+k)\beta V = .4167863$, $[1+(1+k)\beta] = 1.1288$, $[1-(1+k)\beta] = .8716$, $\mathbf{J_P} = .8138727$, $\mathbf{J_{\bar{z}}} = 1.1964519$ and $\mathbf{J_\lambda} = 4.169687$. As can be seen, $A(\bar{z})$ is now larger. This is an example of Corollary 2(a) with $\mathbf{J_P} < \mathbf{J_{\bar{z}}} < A(\bar{z}) < \mathbf{J_\lambda}$ so that $\bar{z}$ *falls* while P* and λ* both *increase*.

Taking the last example where $\bar{z} \approx .4166$ and $P^* \approx .6894677$, suppose that the MPS is as defined in (7′) with $z_1 = 0$, $z_2 = .22$ and $z_3 = .44$. Now $T(\bar{z}) = .0024371$ and $Y = 5.8486165$. Since $(1+k)\beta V$, $A(\bar{z})$, $[1+(1+k)\beta]$ and $[1-(1+k)\beta]$ remain unchanged, $\mathbf{J_P} = 5.0976541$, $\mathbf{J_{\bar{z}}} = 5.5504986$, and $\mathbf{J_\lambda} = 9.084535$. This is an example of Corollary 2(a) with $A(\bar{z}) < \mathbf{J_P} < \mathbf{J_{\bar{z}}} < \mathbf{J}$ so that z and λ* both increase while P* *falls*.

Example 4, IARA $U''' = 0$: Suppose that the utility function is quadratic and belongs to the class of IARA with $U''' = 0$. From Corollary 2(c), it is known that $\bar{z}$ increases (stays constant) (falls) as $k >(=)(<) 1$. Let $U(w) =$

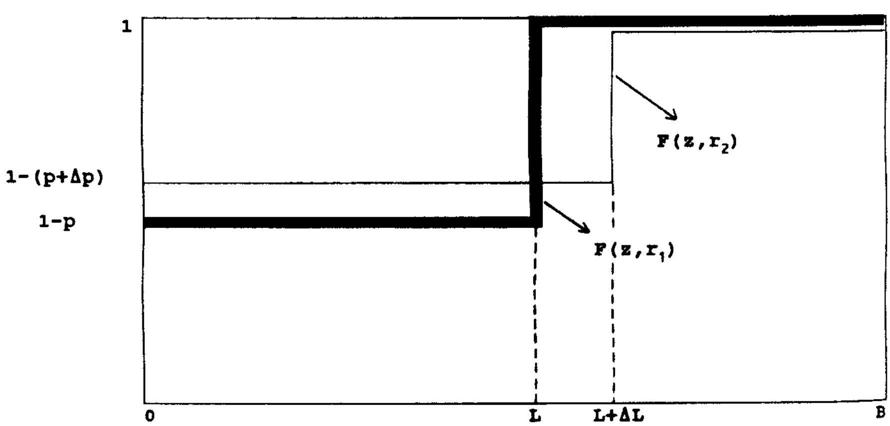
$-(b-w)^c$ where $w \leq b$ and $c = 2$. Letting $b = w_o$ as above, and assuming a uniform distribution of losses, the first order conditions yield $(c-1)\bar{z} - P^* + Bc[(1+k)^{-1} - 1] + (P^*)^c/(\bar{z}+P^*)^{c-1} = 0$. As can be verified, if $k = c-1$, no insurance is purchased (i.e., $\bar{z} = B$ and $P^* = 0$). For insurance to be purchased, the loading factor k must be less than $c-1$. In the case of the quadratic utility function with $c = 2$, this implies that $k < 1$ for insurance to be purchased. Therefore, in this instance, $\bar{z}$ falls.[19]

Example 5, IARA $U''' < 0$: Suppose that the utility function of the insured belongs to the class of IARA with $U''' < 0$. Let $U(w) = -(b-w)^c$ where $w \leq b$ and $1 < c < 2$. Set $b = w_o$ as before. As indicated in example 4, when the initial cdf of losses is uniform, $k < (c-1)$ is required in order for the insured to purchase any insurance, implying in the present case that $k < 1$. However, even though formal proof is unavailable, it seems that $k > 1$ would be required for $\bar{z}$ to increase. In fact, trials with various values of $1 < c < 2$ and of $0 < k < 1$, lead to a fall in $\bar{z}$. As is shown in example 6, changing the initial cdf of losses to a two-state distribution and letting $k > 1$ leads to an increase in $\bar{z}$.

Example 6: Suppose that the insured agent faces the risk of losing an amount L with probability p and of having no loss with probability $1-p$. A MPS is given by $\Delta L > 0$, $\Delta p < 0$ and $\Delta(Lp) = 0$. As Figure 2 below

Figure 2

A Simple Mean-Preserving Spread for the Two-State Case



[19]As indicated in the proof of Corollary 2(c), $\bar{z}$ increases (stays constant)(falls) as $\{(-EU''/EU') - 2A(\bar{z})\} > (=)(<) 0$. Noting that with $U(w) = -(b-w)^2 = -(z+P)^2$, $U''(w) = -2$ for all w, and putting the terms under a common denominator, $\{(-EU''/EU') - 2A(\bar{z})\} = 2(U'EU')^{-1}\ \{4(\bar{z}+P) - 8\int_0^{\bar{z}}(z+P)F_Z(z,r)dz - 8[1 - F(\bar{z},r)](\bar{z}+P)\} = 2(U'EU')^{-1}\ \{-4P - 4\bar{z}[1 - F(\bar{z},r)] - 8\int_0^{\bar{z}} zF_z(z,r)dz + 4\bar{z}F(\bar{z},r)\}$. When F(z,r) is the uniform distribution with density function $F_Z(z,r) = (1/B)$, the last two terms cancel (since $-8\int_0^{\bar{z}} zF_z(z,r)dz = -4(\bar{z}^2/B)$ and $4\bar{z}F(\bar{z},r) = 4(\bar{z}^2/B)$) so that $\bar{z}$ falls.

indicates,[20] this is a simple increase in risk where the two distributions cross at L. Furthermore, since the deductible must be less than L if the individual buys insurance, $\bar{z}$ must be below the crossing point. Therefore, by Corollary 6, $\bar{z}$ *falls and P^* increases* for utility functions which belong to the class of CARA and DARA. However, in the case of IARA, ambiguity remains.

Therefore, a specific example is considered where $U''' < 0$. Let $U(w) = -(b-w)^c$ where $w \leq b$ and $1 < c < 2$ and set $b = w_o$. The first order conditions yield $P^*/(\bar{z}+P^*) = \{[(1+k)^{-1} - p]/(1-p)\}^{1/(c-1)}$ where $P^* = (1+k)p(L - \bar{z})$. Here, $T(\bar{z}) = -\Delta p\bar{z} > 0$ since $\Delta p < 0$. Also, $Y = \bar{z}^{-1} \cdot \{[(P^*)^{c-1}/(\bar{z}+P^*)^{c-1}] - 1\} + A(\bar{z})$. (The term Δp cancels out in the numerator and denominator.) $(1+k)\beta V = (1-p)(c-1)(P^*)^{c-2} \cdot \{(1-p)(P^*)^{c-1} + p(\bar{z}+P^*)^{c-1}\}^{-1}$ Letting $B = 3$, $L = 2$, $p = .25$, $c = 1.9$ and $k = 2$, the first order conditions are solved by $\bar{z} = 1.774432$ and $P^* = .1691759$. For these values $Y = -.0378861$, $A(\bar{z}) = .4630563$, $(1+k)\beta V = 1.3299753$, $[1+(1+k)\beta] = 2-(1+k)p = 1.25$, $\mathbf{J}_{\bar{z}} \equiv [1+(1+k)\beta]^{-1} \cdot \{Y+(1+k)\beta V\} = 1.0336713$, and $\mathbf{J}_\lambda \equiv \{Y+V\} = 5.2820151$. This is an example of Corollary 2(b) with $A(\bar{z}) < \mathbf{J}_{\bar{z}} < \mathbf{J}_\lambda$ so that $\bar{z}$ and λ^* both *increase*. (Recall that with $U''' < 0$, P^* always *increases* with a MPS.)

Alter the value of only the loading factor so that $k < 1$. Choose $k = .5$. The solutions to $\bar{z}$ and P^* are now given by $\bar{z} = .513625$ and $P^* = .5573906$. $Y = -.0249855$, $A(\bar{z}) = .8403238$, $(1+k)\beta V = 1.0091666$, $[1+(1+k)\beta] = 1.625$, $\mathbf{J}_{\bar{z}} = .6056499$ and $\mathbf{J}_\lambda = 1.5896811$. Therefore, $\bar{z}$ *falls* and λ^* *increases*. This is an example of Corollary 2(b) with $\mathbf{J}_{\bar{z}} < A(\bar{z}) < \mathbf{J}_\lambda$.

Mean Utility Preserving Increases in the Risk of Loss

Now turn to the case of a mean-utility preserving increase in risk. A mean-utility preserving spread (MUPS) is defined as follows (see Diamond and Stiglitz, 1974):

Definition 2 An increase in r denotes a mean-utility preserving increase in the riskiness of the loss distribution $F(z,r)$ in the sense of Diamond and Stiglitz if the following conditions are satisfied

$$\hat{T}(x,r) = \int_0^x U' F_r(z,r)dz \geq 0 \quad \forall x \in [0,B) \tag{11a}$$

$$\hat{T}(B,r) = \int_0^B U' F_r(z,r)dz = 0 \tag{11b}$$

Condition (11b) indicates that the increase in risk is such as to keep the mean of utility unchanged.[21] Using (11a) and (11b), and the fact that $\hat{T}(0,r) =$

[20] The authors are grateful to one of the referees for suggesting this distribution and providing the figure.

[21] To verify that expected utility remains constant by imposing these conditions, first substitute the optimal values of $I(z)$, P and λ in the Lagrangean. Then

$$\begin{aligned} L &= \int_0^B U(w_o - z + I^*(z) - P^*) F_z(z,r)dz + \lambda^*(P^*(1+k)^{-1} - \int_0^B I^*(z)F_z(z,r)dz) \\ &= \int_0^{\bar{z}} U(w_o - z - P^*) F_z(z,r)dz + [1-F(\bar{z},r)]U(w_o - \bar{z} - P^*) + \lambda^*(P^*(1+k)^{-1} \\ &\quad - \int_{\bar{z}}^B (z-\bar{z})F_z(z,r)dz) = \int_0^{\bar{z}} U(w_0 - z - P^*) F_z(z,r)dz + [1-F(\bar{z},r)]U(w_o - \bar{z} - P^*) = EU \end{aligned}$$

$F_r(0,r) = F_r(B,r) = 0$, and defining $V \equiv [(1+k)\beta(EU')]^{-1} [\int_0^{\bar{z}} U'(w_o - z - P^*)A(z)F_z(z,r)dz] > 0$ as before, the following result is obtained.

Proposition 2 Let an increase in r represent a MUPS in the distribution of losses. Then

$$d\bar{z}/dr = [-D^{-1}(1+k)^{-1}] \cdot \{-\int_0^{\bar{z}} -[(U'''/U') - (U''/U')^2]\hat{T}(z,r)dz - A(\bar{z})\hat{T}(\bar{z},r) - (1+k)\beta\ [A(\bar{z}) - V]\ \hat{T}(\bar{z},r)\} \quad (12)$$

$$dP^*/dr = [-D^{-1}(1+k)^{-1}] \cdot \{(1-\beta) \int_0^{\bar{z}} -[(U'''/U') - (U''/U')^2]\hat{T}(z,r)dz + A(\bar{z})\hat{T}(\bar{z},r)\} \quad (13)$$

$$d\lambda^*/dr = [-D^{-1}] \cdot \{[\beta U'''(w_o - \bar{z} - P^*)] \int_0^{\bar{z}} -[(U'''/U') - (U''/U')^2]\hat{T}(z,r)dz - (1+k)\beta\ [A(\bar{z}) - V]\ EU'A(\bar{z})\hat{T}(\bar{z},r)\} \quad (14)$$

where $D^{-1} < 0$.

Proof: See the Appendix.

The arguments in the expression $[(U'''/U') - (U''/U')^2]$, which is the derivative of $-U''(w_o - z - P^*)/U'(w_o - z - P^*)$ with respect to z, are omitted.[22] Under CARA, this term will be zero and so will the term $[A(\bar{z}) - V]$ (see footnote 13). With these simplifications, and letting A denote the constant measure of absolute risk aversion, for the case CARA (12)-(14) become:

$$d\bar{z}/dr = D^{-1}\ [(1+k)^{-1}\ A\ \hat{T}(\bar{z},r)] < 0 \quad (12')$$

$$dP^*/dr = D^{-1}\ [-(1+k)^{-1}\ A\ \hat{T}(\bar{z},r)] > 0 \quad (13')$$

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where the first equality follows after substituting for $I^*(z)$ and the second equality follows since $P^*(1+k)^{-1} = \int_{\bar{z}}^{B}(z-\bar{z})F_z(z,r)dz$. Differentiating the Lagrangean with respect to r, where an increase in r now denotes a mean utility preserving spread, using the envelope theorem and integrating by parts *once:*

$$\begin{aligned} dL/dr &= \int_0^{\bar{z}} U'(w_o - z - P^*)F_r(z,r)dz + U(w_o - \bar{z} - P^*)F_r(\bar{z},r) - U(w_o - \bar{z} - P^*)F_r(\bar{z},r) + \lambda^* \int_{\bar{z}}^{B} F_r(z,r)dz \\ &= \int_0^{\bar{z}} U'(w_o - z - P^*)F_r(z,r)dz + U'(w_o - \bar{z} - P^*) \int_{\bar{z}}^{B} F_r(z,r)dz \\ &= \int_0^{\bar{z}} U'(w_o - z - P^*)F_r(z,r)dz + [U'(w_o - \bar{z} - P^*) \int_0^{B} F_r(z,r)dz - U'(w_o - \bar{z} - P^*) \int_0^{\bar{z}} F_r(z,r)dz] \\ &= \hat{T}(\bar{z},r) + [\tilde{T}(B,r) - \hat{T}(\bar{z},r)] = 0 \end{aligned}$$

where two terms cancel out in the first equality and where $\lambda^* = U'(w_o - \bar{z} - P^*)$ by (3') and $\hat{T}(B,r) = 0$ by (11b) have been used. (Recall also that $U'(w_o - \bar{z} - P^*)$ is constant for all values of $z \in [\bar{z}, B]$.) Therefore mean utility remains constant.

[22] Let α^* denote the optimal value of a control variable which maximizes expected utility. According to Diamond and Stiglitz (1974), a MUPS reduces (increases) the optimal value of α if the marginal utility of the control variable α is a concave (convex) function of the random utility level. The concavity (convexity) of the marginal utility of P or $\{I(z)\}$ with respect to the utility level is captured by the term $[(U'''/U') - (U''/U')^2]$ in (12), (13) and (14). However, as can be seen, it is not sufficient to restrict this term in order to sign (12)-(14).

$$d\lambda^*/dr = 0 \tag{14'}$$

The next corollary follows immediately from (12′)-(14′).

Corollary 7 Let an increase in r represent a MUPS in the distribution of losses. If the insured agent exhibits **CARA**, then

(i) $d\bar{z}/dr < 0$ (ii) $dP^*/dr > 0$ (iii) $d\lambda^*/dr = 0$.

By conditions (i) and (ii), an individual with CARA increases the amount of insurance purchased and pays a higher premium when there occurs a MUPS. Hence, in this case the increase in risk leads to a fall in the deductible. However, it is noteworthy that by equations (12′) and (13′), $|dP^*/dr| = |d\bar{z}/dr|$. That is, the deductible falls by exactly the same amount as the increase in the premium thus leaving wealth in insured states unchanged as indicated by (14′) and by condition (iii). To understand this result, recall that a MUPS implies a riskier distribution of losses accompanied by a decrease in the mean of losses (or equivalently an increase in mean wealth), to keep mean utility constant. However, even though the mean of losses has fallen, the truncated mean has increased i.e., $d\{\int_{\bar{z}}^{B} z\, F_z(z,r)dz\}/dr = \hat{T}(\bar{z},r)/U'(w_o - \bar{z} - P^*) > 0$. This increase in the average losses above the deductible implies that the premium must also increase for the insurer's expected profits to be no less than zero. Therefore, the new insurance policy will have a higher premium and a lower deductible reflecting the insurer's adjustment and the insured agent's response to this adjustment.

The cases of DARA and IARA are somewhat ambiguous. In the interest of brevity they are not discussed.

A Change in the Distribution of Losses in the Sense of First Order Stochastic Dominance

A shift in the distribution of losses in the sense of first-order stochastic dominance (FSD) implies a stochastic increase or decrease in the random variable Z. The random loss Z_2 with cdf $F(z,r_2)$ is stochastically larger than Z_1 with cdf $F(z,r_1)$ in the sense of FSD if $F(z,r_2)$ lies entirely below $F(z,r_1)$. Following Hadar and Russel (1971), such a FSD shift in F(z,r) is defined as:

Definition 3 The distribution of losses $F(z,r_2)$ is said to (strictly) dominate the distribution $F(z,r_1)$ in the sense of first-order stochastic dominance (FSD) if $F(z,r_2) - F(z,r_1) < 0 \; \forall z \in (0,B)$.

Differential changes in risk are examined. Let a change in r denote a shift in the distribution in the sense of FSD with $F_r(z,r) < 0 \; \forall z \in (0,B)$. Note that $F_r(0) = F_r(B) = 0$. For ease of notation, let $X \equiv [\int_0^{\bar{z}} -U''(w_o - z - P^*)\, F_r(z,r)dz] \cdot [U'(w_o - \bar{z} - P^*) \int_{\bar{z}}^{B} F_r(z,r)dz]^{-1} > 0$, and $V \equiv [(1+k)\beta(EU')]^{-1}[\int_0^{\bar{z}} U'(w_o - \bar{z} - P^*) A(z)F_z(z,r)dz] > 0$ as before. The following result is obtained.

Proposition 3 Let an increase in r represent a shift in the cdf of losses in the sense of FSD such that $F_r(z,r) < 0$ for all $z \in (0,B)$. Then

$$d\bar{z}/dr = [D^{-1}(EU') \int_{\bar{z}}^{B} F_r(z,r)dz] \cdot \{X - (1+k)\beta\, [A(\bar{z}) - V]\} \tag{15}$$

$$dP^*/dr = [D^{-1}\,(EU')\beta(1+k) \int_{\bar{z}}^{B} F_r(z,r)dz] \cdot \{A(\bar{z}) - \beta^{-1}[1 - F(\bar{z},r)]X\} \tag{16}$$

$$\begin{aligned} d\lambda^*/dr = {} & [-D^{-1}U''(w_o - \bar{z} - P^*) \int_{\bar{z}}^{B} F_r(z,r)dz] \cdot \{\beta U'(w_o - \bar{z} - P^*)X \\ & + \int_0^{\bar{z}} -U''(w_o - z - P^*)F_z(z,r)dz\} > 0 \end{aligned} \tag{17}$$

where $D^{-1} < 0$.

Proof: See the Appendix.

Note that the first term in brackets in each equation in Proposition 3 is positive since $F_r(z,r)$ is negative for all values of $z \in (0,B)$. As can be seen from equation (17), $d\lambda^*/dr$ is unambiguously positive. This implies that wealth in insured states decreases irrespective of the magnitude of risk aversion and irrespective of how risk aversion varies with wealth. The signs of $d\bar{z}/dr$ and of dP^*/dr, however, will generally depend on the magnitude of risk aversion and on how the latter changes with wealth. The cases of CARA and IARA are considered together since the results are similar. However, as the case of DARA is somewhat more ambiguous, it is not analyzed in the interest of brevity.

As seen earlier, in the case of CARA, $A(\bar{z}) = A(z) = A$, and the term $[A(\bar{z}) - V] = 0$. Under IARA, $[A(\bar{z}) - V] < 0$. It can be immediately noted from equation (15) that $d\bar{z}/dr$ is positive both under CARA and IARA. dP^*/dr, however, remains of ambiguous sign. Since under CARA $-U''(\cdot) = U'(\cdot)A$, the term X can be written as $X = A \bullet X'$ where

$X' \equiv [\int_0^{\bar{z}} U'(w_o - z - P^*)\, F_r(z,r)dz] \cdot [U'(w_o - \bar{z} - P^*) \int_{\bar{z}}^{B} F_r(z,r)dz]^{-1} > 0.$

Corollary 8 If the insured agent exhibits **CARA** or **IARA** then a stochastic increase in random losses through a shift in F(z,r) in the sense of FSD implies

(i) $d\bar{z}/dr > 0$

(ii) $dP^*/dr >(<)\ 0$ as $X' <(>)\ [1 - F(\bar{z},r)]^{-1}\,\beta$ (for **CARA**)

as $A(\bar{z}) >(<)\ [1 - F(\bar{z},r)]\beta^{-1}\,X$ (for **IARA**)

(iii) $d\lambda^*/dr > 0$.

Under CARA or IARA the deductible unambiguously increases implying that the insured agent purchases less insurance even though losses are stochastically larger. A shift in F(z,r) in the sense of FSD with $F_r(z,r) < 0$ means that Prob($Z \le \bar{z}$) has decreased and that losses below the (initial) deductible are now less probable. The increase in the deductible is thus a response to this change. Furthermore, the FSD shift implies that the change in the premium *for a fixed* *deductible* is unambiguously positive.[23] Therefore, given that $\text{Prob}(Z \leq \bar{z})$ has decreased, the insured agent is more concerned about paying too high a premium than about getting more coverage. The agent prefers to reduce the insurance purchase to palliate for the increase in P^* brought about by the stochastic increase in losses. As can be seen from (15), under IARA, the smaller $A(\bar{z})$ is, the larger is the increase in the deductible.[24] The more the deductible increases, the smaller the increase in P^* and if $A(\bar{z})$ is small enough, the premium may even decrease.[25] Condition (ii) of Corollary 8 indicates the total effect of the increase in risk on the premium. Since the increase in the deductible has a negative impact on the premium, which of the two effects dominates will depend on the magnitude of the increase in the deductible. Nevertheless, since $d\lambda^*/dr$ is positive and since wealth in insured states unambiguously decreases, if $dP^*/dr < 0$, it must be the case that $|dP^*/dr| < |d\bar{z}/dr|$.[26]

Conclusions

In this article the comparative statics effect of increases in risk on the optimal insurance contract between a risk averse insured and a risk neutral insurer have been analyzed. A mean preserving spread and a mean-utility preserving spread as well as a shift in the distribution of losses in the sense of first-order stochastic dominance have alternatively been considered. The conditions obtained by Rothschild and Stiglitz (1971) and by Diamond and Stiglitz (1974) for an increase in risk to have a determinate impact on the control variable are no longer sufficient to determine conclusively the impact of increasing risk on the insurance contract. As demonstrated, additional restrictions are required to sign the effect of the increase in risk on the Pareto-optimal contract. For each type of increase in risk, the cases of decreasing, increasing and constant absolute risk aversion were distinguished and the effect of the increase in risk was related to the magnitude of the measure of risk aversion of the insured.

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[23] To see this, differentiate the constraint equation (2) with respect to P and r keeping $\bar{z}$ constant. $dP/dr|_{\bar{z}\,\text{const.}} = (1+k)\int_{\bar{z}}^{B}(z-\bar{z})\,)F_{zr}(z,r)dz = -(1+k)\int_{\bar{z}}^{B}F_r(z,r)dz > 0$ since $F_r(z,r) < 0\ \forall z \in (0,B)$. Note that this FSD shift also implies an increase in the mean of losses.

[24] To see that the change in $\bar{z}$ has a tendency to vary inversely with $A(\bar{z})$ rewrite D as:
$D = \beta^2 U''(w_o - \bar{z} - P) + [1 - F(\bar{z},r)]\int_0^{\bar{z}} U''(w_o - z - P)F_z(z,r)dz = [\int_0^{\bar{z}} U'(w_o - z - P)F_z(z,r)dz] \bullet \{-\beta A(\bar{z}) + [1 - F(\bar{z},r)]\int_0^{\bar{z}} U''(w_o - z - P)F_z(z,r)dz / [\int_0^{\bar{z}} U'(w_o - z - P)F_z(z,r)dz]\}$. D is negative (see the Appendix). Now differentiating the right-hand-side of (15) with respect to $A(\bar{z})$, it can be seen that $d\{d\bar{z}/dr\}/dA(\bar{z})|_{\bar{z}\,\text{const.}} < 0$ for IARA.

[25] From the constraint equation (2), differentiating with respect ot P, $\bar{z}$ and r, one has $dP^*/dr = -(1+k)[1 - F(\bar{z}, r)]d\bar{z}/dr - (1+k)\int_{\bar{z}}^{B} F_r(z,r)dz$ where, given that $F_r(z,r)dz < 0\ \forall z \in (0,B)$, the second term on the right-hand-side is positive. Since $d\bar{z}/dr$ is multiplied by a negative term, with $d\bar{z}/dr > 0$, the larger $d\bar{z}/dr$ is, the smaller the increase in P^*. If $d\bar{z}/dr$ is large enough, P^* may even decrease.

[26] The individual is even more concerned about the increase in P^* under IARA than he or she is under CARA. In fact, from (15), the increase in $\bar{z}$ is larger under IARA than it is under CARA since the term $(A(\bar{z}) - V)$ is positive under IARA and zero under CARA.

A Rothschild-Stiglitz mean preserving increase in risk was first considered. The theoretical analysis as well as the examples point to the conclusion that following the increase in risk, the insured agent has a greater tendency to increase the purchase of insurance if $A(\bar{z})$ is large. Furthermore, an insured agent who exhibits IARA with $U''' < 0$ is more likely to increase the insurance coverage than an agent who exhibits DARA or IARA with $U''' > 0$. In the special case of the quadratic utility function the premium unambiguously increases and wealth in insured states unambiguously falls but the deductible may increase, stay the same or decrease depending on whether $k >(=)(<) 1$. The insured agent will increase the insurance purchase if the loading factor is not too large. The impact of restricting the type of MPS (as opposed to restricting the class of the utility function) was also investigated. If the MPS occurs entirely in the interval of losses below the deductible, then the impact of the MPS on the deductible is determined by the sign of U'''. Thus, only in this special case is the impact of a MPS on the deductible and the premium determined by the Rothschild-Stiglitz sufficient condition. If the MPS occurs entirely above the deductible, it has no impact on either the deductible or the premium. Finally, a simple (single-crossing) MPS will lead to a fall in the deductible and an increase in the premium provided that the deductible is below (or at) the crossing point and that the utility function belongs to the class of DARA or CARA.

A Diamond-Stiglitz mean utility preserving spread was also examined. An individual with CARA increases the amount of insurance purchased and pays a higher premium when there occurs a mean-utility preserving spread. Hence, in this case the increase in risk leads to a fall in the deductible. The deductible falls by exactly the same amount as the increase in the premium thus leaving wealth in insured states unchanged.

Finally the comparative statics of stochastic shifts in the distribution of losses in the sense of first-order stochastic dominance were investigated. If losses stochastically increase, under CARA or IARA the deductible unambiguously increases implying that the insured agent purchases less insurance. Such a shift in $F(z,r)$ in the sense of FSD means that $\text{Prob}(Z \le \bar{z})$ has decreased and that losses below the (initial) deductible are now less probable. The increase in the deductible is thus a response to this change.

When the loss distribution has a two-point support (loss-no loss), an increase in risk in the sense of FSD amounts to an increase in the probability of the loss. Consider two individuals whose preferences conform to CARA or IARA, but who face a different probability of loss. The results of the present analysis imply that the individual with the higher probability of loss would purchase less insurance. This result casts some doubt on the identification of low (high) risk with smaller (greater) demand for insurance which is made in the asymmetric information (signalling) literature (as, for example, in Rothschild and Stiglitz, (1976)).

Two interesting generalizations suggest themselves. First, one could obtain alternative sets of sufficient conditions to sign the comparative statics effects of a MPS, for example, by using the concept of prudence: "the propensity to prepare and forearm oneself in the face of uncertainty" (see Kimball (1990)). In a recent paper, the concept of prudence, and in particular, of decreasing prudence, was used by Eeckhoudt and Kimball (1990) to demonstrate that background risk increases the demand for an insurable risk.[27] In the present context, it would be necessary to impose restrictions on the magnitude of prudence at $\bar{z}$, $\eta(\bar{z}) = -U'''(\bar{z})/U''(\bar{z})$, as well as on whether prudence decreases or increases with wealth in order to sign the comparative statics effects of increases in risk on the optimal insurance contract.

Second, the impact on the optimal insurance contract of alternative changes in the distribution of losses could be investigated. For example, the consequences of a strong increase in risk in the sense of Meyer and Ormiston (1985) (a special case of a MPS) could be analyzed.[28]

Appendix

Derivation of equations (8)-(10): Substituting for I^*, the first order conditions (3),(4) and (5) become:

$$U'(w_o-\bar{z}-P^*)-\lambda^* = 0 \tag{A1}$$

$$-\int_0^{\bar{z}}U'(w_o-z-P^*)\,F_z(z,r)dz-[1-F(\bar{z},r)]U'(w_o-\bar{z}-P^*)+\lambda^*(1+k)^{-1} = 0 \tag{A2}$$

$$P^*(1+k)^{-1}-\int_{\bar{z}}^{B}(z-\bar{z})\,F_z(z,r)dz = 0 \tag{A3}$$

Differentiate (A1), (A2) and (A3) with respect to $\bar{z}$, P^*, λ^* and r to obtain:

$$-U''(w_o-\bar{z}-P^*)\,d\bar{z}-U''(w_o-\bar{z}-P^*)\,dP^*-d\lambda^* = 0 \tag{A4}$$

$$\begin{aligned} &U''(w_o-\bar{z}-P^*)[1-F(\bar{z},r)]\,d\bar{z}+\{U''(w_o-\bar{z}-P^*)[1-F(\bar{z},r)] \\ &\quad+\int_0^{\bar{z}}U''(w_o-z-P^*)F_z(z,r)dz\}dP^*+(1+k)^{-1}d\lambda^* \\ &\quad=[-U'(w_o-\bar{z}-P^*)F_r(\bar{z},r)+\int_0^{\bar{z}}U'(w_o-z-P^*)F_{zr}(z,r)dz]dr \end{aligned} \tag{A5}$$

$$[1-F(\bar{z},r)]\,d\bar{z}+(1+k)^{-1}\,dP^* = \int_{\bar{z}}^{B}(z-\bar{z})F_{zr}(z,r)dz]dr \tag{A6}$$

Integrating by parts twice and using (7a), (7b) and the fact that $T(0,r)=F_r(0,r) = F_r(B,r) = 0$, the expressions on the right-hand-side (r.h.s.) of (A5) and (A6) become

$$\begin{aligned} &[-U'(w_o-\bar{z}-P^*)F_r(\bar{z},r)+\int_0^{\bar{z}}U'(w_o-z-P^*)\,F_{zr}(z,r)dz] \\ &\quad=\int_0^{\bar{z}}U'''(w_o-z-P^*)\,F_r(z,r)dz+U'(w_o-\bar{z}-P^*)F_r(\bar{z},r) \\ &\qquad-U'(w_o-\bar{z}-P^*)F_r(\bar{z},r) \\ &\quad=\int_0^{\bar{z}}U'''(w_o-z-P^*)T(z,r)dz+U''(w_o-\bar{z}-P^*)T(\bar{z},r) \end{aligned} \tag{A7}$$

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[27] The authors thank a referee and Harris Schlesinger for bringing this article and this literature to their attention.

[28] $F(z,r_2)$ is a strong increase in risk of $F(z,r_1)$ if it is a MPS and satisfies an additional condition: $F(z,r_2)-F(z,r_1)$ is nonincreasing on (z_2,z_3) where the support of $F(z,r_2)$ is contained in $[z_1,z_4]$ and the support of $F(z,r_1)$ is contained in $[z_2,z_3]$ where $z_1 < z_2 < z_3 < z_4$.

$$[\int_{\bar{z}}^{B}(z-\bar{z})F_{zr}(z,r)dz] = -\int_{\bar{z}}^{B}F_r(z,r)dz = -[\int_0^B F_r(z,r)dz - \int_0^{\bar{z}} F_r(z,r)dz]$$
$$= -[T(B,r) - T(\bar{z},r)] = T(\bar{z},r) \quad (A8)$$

(Note that $dU'(w_o - z - P^*)/dz = -U''(w_o - z - P^*)$.) Substituting (A7) and (A8) into (A5)and (A6), the system of three simultaneous equations (A4)-(A6) is then solved for $d\bar{z}/dr$, dP^*/dr and $d\lambda^*/dr$ by using Cramer's rule. Since from (A1) and (A2), $(1+k)^{-1} = EU'/U'(w_o - \bar{z} - P^*)$ and $\beta \equiv$

$$\{(1+k)^{-1} - [1-F(\bar{z},r)]\} = [\int_0^{\bar{z}}U'(w_o - z - P^*)\, F_z(z,r)dz] \bullet [U'(w_o - \bar{z} - P^*)]^{-1} > 0$$

(see footnote 9), the following are obtained

$$d\bar{z}/dr = D^{-1}(-EU')T(\bar{z},r)\{[\int_0^{\bar{z}} U'''(w_o - z - P^*)\, T(z,r)dz]$$
$$[U'(w_o - \bar{z} - P^*)T(\bar{z},r)]^{-1} + (-EU''/EU') - 2A(\bar{z})\} \quad (A9)$$

$$dP^*/dr = D^{-1}(-EU')T(\bar{z},r)\{A(\bar{z}) - [1-(1+k)\beta][\int_0^{\bar{z}}U'''(w_o - z - P^*)T(z,r)dz]$$
$$[U'(w_o - \bar{z} - P^*)T(\bar{z},r)]^{-1}\} \quad (A10)$$

$$d\lambda^*/dr^{-1}(U'')EU'T(\bar{z},r)\{(1+k)\beta[\int_0^{\bar{z}} U'''(w_o - z - P^*)T(z,r)dz] \cdot$$
$$[U'(w_o - \bar{z} - P^*)T(\bar{z},r)]^{-1} + (-EU''/EU') - A(\bar{z})\} \quad (A11)$$

where $D = \{\beta^2 U''(w_o - \bar{z} - P^*) + [1-F(\bar{z},r)] \int_0^{\bar{z}}U''(w_o - z - P^*)F_z(z,r)dz\} < 0$ since $U'' < 0$ by the strict concavity of the utility function, where

$A(\bar{z}) \equiv -U''(w_o - \bar{z} - P^*)/U'(w_o - \bar{z} - P^*)$

and where $EU'' = \int_0^{\bar{z}}U''(w_o - \bar{z} - P^*)F_z(z,r)dz + [1-F(\bar{z},r)]U''(w_o - \bar{z} - P^*)$.

Also, $[1-(1+k)\beta] = (1+k)[1-F(\bar{z},r)] > 0$. Now note that $[A(\bar{z}) - (-EU''/EU')]$ =

$$-U''(w_o - \bar{z} - P^*)/U'(w_o - \bar{z} - P^*) - (-EU''/EU') = \{-EU' \bullet U''(w_o - \bar{z} - P^*)$$
$$+ EU'' \cdot U'(w_o - \bar{z} - P^*)\}[U'(w_o - \bar{z} - P^*)EU']^{-1}$$

where the terms have been put under a common denominator. The term $-U''(w_o - \bar{z} - P^*)U'(w_o - \bar{z} - P^*)[1-F(\bar{z},r)]$ cancels out in the numerator. Factoring out $U'(w_o - \bar{z} - P^*)$, regrouping and letting

$A(z) = -U''(w_o - z - P^*)/U'(w_o - z - P^*)$ this expression can be rewritten as $\int_0^{\bar{z}} U'(w_o - z - P^*)[A(\bar{z}) - A(z)]F_z(z,r)dz[EU']^{-1} = (1+k)\beta A(\bar{z}) - [EU']^{-1} \cdot$ $[\int_0^{\bar{z}} U'(w_o - z - P^*)A(z)F_z(z,r)dz]$ since $A(\bar{z})$ is a constant. Substituting for this expression in place of $[A(\bar{z}) - (-EU''/EU')]$ in equations (A9)-(A11), yields (8)-(10) in the text. Note that in the short-hand notation of (8′)-(10′), $[A(\bar{z}) - (-EU''/EU')] = (1+k)\beta[A(\bar{z}) - V]$ where $V \equiv [(1+k)\beta EU']^{-1}$ $[\int_0^{\bar{z}}U'(w_o - z - P^*)A(z)F_z(z,r)dz]$ (See also footnote 13).

Proof of Corollary 2: The proofs of parts (a) and (b) are contained in the discussion preceding the corollary. A proof of part (c) is provided here. Since $U''' = 0$ for quadratic utility functions, from (A9) $d\bar{z}/dr >(=)(<) 0$ as $\{(EU''/EU') - 2A(\bar{z})\} >(=)(<) 0$. Now since $U''(\cdot) \equiv M$ is constant for all wealth levels, the term in braces can be rewritten as $(M/U')\{(U'/EU') - 2\}$ where $[M/U'(w_o - \bar{z} - P^*)]$ is factored out. Recalling that from the first order conditions, $(1+k)^{-1} = EU'/U'(w_o - \bar{z} - P^*)$ it follows that $d\bar{z}/dr > (=)(<) 0$ as $\{(1+k)-2\} >(=)(<) 0$ or as $k >(=)(<) 1$. An inspection of (A10) or of (9) in the text reveals that $dP^*/dr > 0$. Also, from (A11), $d\lambda^*/dr > 0$ since $\{(-EU''/EU') - A(\bar{z})\}$ is positive under IARA.

Proof of Proposition 2: In the case where an increase in r denotes a mean utility preserving spread, after integrating by parts once and using the fact that $F_r(0,r) = F_r(B,r) = 0$, the r.h.s. of (A5) becomes:

$$[-U'(w_o - \bar{z} - P^*)F_r(\bar{z},r) + \int_0^{\bar{z}} U'(w_o - z - P^*)F_{zr}(z,r)dz]$$
$$= \int_0^{\bar{z}} U''(w_o - z - P^*)F_r(z,r)dz = \int_0^{\bar{z}} [U''(w_o - z - P^*)/U'(w_o - z - P^*)] \cdot U'(w_o - z - P^*)F_r(z,r)dz$$

where the second equality follows from multiplying and dividing by $U'(w_o - z - P^*)$. Integrating by parts once more making use of conditions (11a), (11b) and $\hat{T}(0,r) = F_r(0,r) = F_r(B,r) = 0$, the following is obtained

$$[-U'(w_o - \bar{z} - P^*)F_r(\bar{z},r) + \int_0^{\bar{z}} U'(w_o - z - P^*)F_{zr}(z,r)dz]$$
$$= \{\int_0^{\bar{z}} [(U'''/U') - (U''/U')^2]\, \hat{T}(z,r)dz + U''/U'\, \hat{T}(\bar{z},r)\} \tag{A12}$$

With similar operations, this time multiplying and dividing by $U'(w_o - \bar{z} - P^*)$, the r.h.s. of (A6) becomes

$$[\int_{\bar{z}}^{B} (z-\bar{z})F_{zr}(z,r)dz] = -\int_{\bar{z}}^{B} F_r(z,r)dz = -(U')^{-1} \int_{\bar{z}}^{B} U'F_r(z,r)dz$$
$$= -(U')^{-1} [\int_0^B U'F_r(z,r)dz - \int_0^{\bar{z}} U'F_r(z,r)dz]$$
$$= -(U')^{-1}[\hat{T}(B,r) - \hat{T}(\bar{z},r)] = (U')^{-1}\, \hat{T}(\bar{z},r) \tag{A13}$$

where the constancy of the insured agent's wealth for all loss levels between $\bar{z}$ and B (since these are insured) is used. Expressions (A12) and (A13) must be substituted into (A5) and (A6) and the resulting system of simultaneous equations must be solved again as before in order to obtain equations (12)-(14) in the text where again $(1+k)^{-1} = EU'/U'(w_o - \bar{z} - P^*)$ and $\beta \equiv \{(1+k)^{-1} - [1 - F(\bar{z},r)]\} = [\int_0^{\bar{z}} U'(w_o - z - P^*)F_z(z,r)dz][U'(w_o - \bar{z} - P^*)]^{-1}$ are used in regrouping terms. Note that $D < 0$ is the same as in (8)-(10).

Proof of Proposition 3: In the case where an increase in r denotes a shift in the initial loss distribution in the sense of first-order stochastic dominance such that $F_r(z,r) < 0$, after integrating by parts once, the r.h.s. of (A5) and of (A6) become respectively:

$$[-U'(w_o - \bar{z} - P^*)F_r(\bar{z},r) + \int_0^{\bar{z}} U'(w_o - z - P^*)F_{zr}(z,r)dz]$$
$$= \int_0^{\bar{z}} U''(w_o - z - P^*)F_r(z,r)dz > 0 \tag{A14}$$

$$[\int_{\bar{z}}^{B} (z-\bar{z})F_{zr}(z,r)dz] = -\int_{\bar{z}}^{B} F_r(z,r)dz > 0 \tag{A15}$$

Both expressions are positive since $F_r(z,r) < 0\ \forall z \in (0,B)$. Expressions (A14) and (A15) must be substituted into (A5) and (A6) and the resulting system of simultaneous equations must be solved for $d\bar{z}/dr$, dP^*/dr and $d\lambda^*/dr$.

Regrouping terms and using $(1+k)^{-1} = EU'/U'(w_o - \bar{z} - P^*)$ and $\beta \equiv \{(1+k)^{-1} - [1 - F(\bar{z},r)]\}$ as before, equations (15)-(17) in the text are obtained where $D < 0$ is the same as in equations (8)-(10).

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