The Effect of Stochastic Variables on Estimating the Value of Lost Earnings

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ABSTRACT

Courts are frequently asked to calculate the correct compensation to an individual or family for income lost due to injury or death where others are responsible for the occurrence. The usual procedure is to apply point estimates of all relevant values including the award itself. Such a process ignores, however, the probabilistic nature of actual events. Inflation, wages, and length of work life are all stochastic variables. A Monte Carlo simulation experiment accounting for randomness illustrates that the single value method of calculating awards systematically overstates damages. The experiment shows that most of the bias comes from failing to recognize the stochastic nature of remaining work life in particular.

Courts often are faced with the responsibility of determining appropriate compensation for those injured or killed when others are in some way responsible. Usually, among other things, this means determining the net present value of earnings the plaintiff would have earned if the accident had not occurred. In order to make the calculations it is necessary to select particular values for certain salient variables. In particular, values for expected inflation, remaining work life, and future salary are needed. Having these input values, courts then determine appropriate awards in the interest of justice to both plaintiff and defendant.

That these decisions are important is confirmed by the attention devoted to discussions of issues related to damage awards by both academicians and practioners alike. The pages of this Journal frequently contain articles dealing with various aspects of the matter. Both a good example of the genre and summary of approaches is found in Carpenter, Lange, Shannon, and Stevens (1986). Though the process illustrated here goes beyond Carpenter, et al by including the stochastic element, it certainly maintains the spirit of their recommendations.

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The basic model utilized in the analysis can be stated as:

$$PV = \sum_{t=1}^{w} (E_o * L_t - TX_t)(1-P)\left[\frac{1+g}{1+i(1-T)}\right]^t \tag{1}$$

where: PV = present value of lost earnings, E_o = base earnings, i = nominal discount rate, g = inflation rate, w = remaining work life, P = personal consumption expenditure rate, L_t = cumulative life cycle salary factor, T = marginal tax rate, and TX_t = total income tax

Note that the last term of equation (1) may be restated as $[1/1+r]^t$:

$$\text{where } r = \frac{1+i(1-T)}{1+g} - 1 \tag{2}$$

In this formulation r becomes the infamous "below market discount rate" referred to by the courts in the Slater Boat and Pheifer cases.[1]

Typically fixed input values for E_o, i, g, w, and T are determined and the resulting present value, PV, is calculated. While it is true that i and T are known with a reasonable degree of certainty (i is observable in the financial markets and T is embodied in tax law), the values of E, g, and w are subject to some variation and uncertainty. Hence there is a degree of randomness, a stochastic element, in these values which implies that the PV variable is also stochastic. An examination of this resulting random variation of the present value of lost wages seems to be appropriate.

Randomness has always been embedded in calculations of damage awards, but it is either unrecognized or assumed away. The usual assumption is to treat the means of the random input variables as point estimates. This results in a point estimate for the resulting output variable, PV. Making that assumption ignores the potential variability in PV and assumes that the incorporation of mean *input* values will yield the mean *output* value.

A priori, one would expect even estimates ignoring randomness to be statistically close to the true values. Whatever deviations did occur would be expected to occur randomly, some falling above the true value, others falling below it. The fundamental question is whether ignoring the stochastic nature of the problem systematically biases the estimation of lost earnings. Computer-based Monte Carlo simulation offers a powerful means of investigating that question. The heart of this article is a Monte Carlo simulation experiment testing the validity of employing single-valued variables rather than stochastic inputs. The experiment shows that awards derived when randomness is ignored systematically overstate the true value of lost earnings. Though this overstatement is not large in absolute terms, it is statistically significant. Bias was discovered to result from the interaction of variability in both expected inflation and work life. Ignoring randomness in salary apparently imparts no bias.

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[1] Culver v. Slater Boat Co., 722 *Federal Reporter*, 2d series 114 (1983) and Jones & Laughlin Steel Co. v. Pheifer, 103 *Supreme Court Reporter*, 2541 (1983).

The next section considers the current manner used to estimate lost earnings. Section three briefly reviews the theory and use of Monte Carlo techniques. Section four presents details of the experiment. Conclusions are presented in the final section of the article.

Present Procedures

To demonstrate both the current process of estimating damage awards and the power of the Monte Carlo procedure, a hypothetical example is constructed.[2] Consider the accidental death of a 22 year old white male. According to standard work life tables, this worker could have expected to work another 36 years, had he not been killed. Also assume his salary and nominal market interest rate to have been $28,000 per year and 9 percent, respectively. The marginal tax rate is 28 percent. A 41.1 percent factor for consumption expenditures is assumed. Predicted inflation for 1989 is 3.62 percent, as discussed below. Then, applying equation (2), the appropriate after-tax real rate of interest (discount) rate is 2.76 percent. The present value of lost earnings is $465,960 calculated by applying equation (1). This point estimate is the value that the courts would generally accept as the best estimate of lost earnings. This estimate ignores the stochastic element which may bias damage awards. Awards based upon point estimates differ statistically from those based upon stochastic variables. This suggests there is some danger of imparting bias when randomness is ignored.

The $465,960 value is a point estimate taken from a distribution of continuous values. The same can be said for the estimates of expected inflation, future salary, and remaining work life. In fact, every variable whose value is either observed or estimated comes from a distribution arrayed around its mean value with some dispersion. Hence, the exact values taken on by all variables are subject to random deviations from their means. This fact raises two related questions. First, does ignoring randomness distort damage awards? Second, if there is systematic distortion, does it cause damage awards to exhibit bias in favor of either plaintiff or defendant? Since it would be difficult to directly answer these questions, a simulation experiment is used to shed light on these questions.

Monte Carlo Simulation: A Brief Introduction

For this experiment Monte Carlo simulation is used to calculate the effect of random input elements on the value of lost earnings for the example described above. Monte Carlo techniques are a useful way to discover the properties of a probability distribution that for whatever reason is not subject to straight-forward mathematical manipulation. In many cases the problem is simply that the properties of the relevant distributions are unknown. Monte

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[2] See Deakle vs. John E. Graham & sons, 756 F. 2d 821 (1985) for a detailed description of the court's currently favored method for computing present values upon which to base awards.

Carlo techniques may be used to estimate those properties, including a distribution's shape, its mean, and its variance.

It is always, of course, preferable to compute the exact probability distribution, but many situations are far too complex to make that feasible. As Cragg (1967), Summers (1965), and Wagner (1958) have demonstrated, Monte Carlo techniques have been used to determine the small sample properties of statistical estimation procedures when mathematical analysis is limited to the less relevant large sample properties. More recently, Hamlen et al. (1988) studied the relationship between monetary policy and interest rates using Monte Carlo methods. In addition, Monte Carlo techniques have been used to estimate the properties of complex econometric models as Nager (1969) shows. The use of Monte Carlo techniques is not restricted, however, to the fields of business and economics. In fact, some of the more interesting results have been obtained in other fields of study that are subject to random impacts. See Inman and Conover (1980) and Newendorp (1975) for discussions of the application of Monte Carlo simulations in physics and geology, for example.

A Monte Carlo Simulation Experiment

In order to simulate random effects when a variable's value comes from a probability distribution it is necessary to know the parameters of that distribution. In this case possible stochastic variables are inflation, remaining work life, and starting salary. While other input variables may be stochastically determined, the major random variables are included in the model and serve to illustrate the concept.

The distribution of expected inflation is straight forward but requires a brief explanation. There is considerable divergence of opinion on the correct way to handle the interrelatedness of expected inflation and nominal interest rates in calculating lost wages. The debate has previously centered upon which point estimate is the appropriate discount rate. To some extent that debate misses the point by avoiding the random element. A demonstration of the benefits of Monte Carlo techniques is not dependent upon particular levels of expected inflation. Making that demonstration does, however, depend on the stochastic nature of expected inflation. Though the parameters of the distribution could be simply assumed, it would be preferable to base the numbers on some empirical foundation.

In order to derive a probability function for expected inflation, a regression equation was estimated. Following Carlino (1982), the equation is a distributed lag model which expresses expected inflation[3] as a function of actual inflation and the growth in the money supply in past time periods. Table 1 summarizes both the regression equation and its results.[4]

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[3]Expected inflation is measured by Joseph Livingston's survey results which appear in the Philadelphia *Inquirer*, normally on the last Sunday of June and of December each year.

[4]This equation is described in Brooking and Taylor, (1990).

Table 1

Basic Equation for Expected Inflation

$$INF_t = -.4267 + \sum_{i=1}^{5} a_i CPI_{t-i} + \sum_{i=1}^{2} b_i M1_{t-i}$$

$$\bar{R}^2 = .9615 \qquad SE = .513$$

$a_1 = .4574$	$b_1 = .0903$
(12.45)	(2.53)
$a_2 = .0987$	$b_2 = .0452$
(4.10)	(2.53)
$a_3 = .0266$	
(1.21)	
$a_4 = .0881$	
(4.81)	
$a_5 = .1302$	
(4.82)	

where: INF_t = Expected inflation for year t [Livingston], CPI_t = Actual inflation for year t, $M1_t$ = Rate of change of M1 in year t, and numbers in parentheses are "t" statistics.

Substituting the appropriate historical values into the equation produced a mean forecast value for expected inflation of 3.62 percent for 1989, with a standard prediction error of 0.525. Invoking the usual regression assumption of normally distributed error terms, the distribution of expected inflation for 1989 can thus be approximated by using a normal random variable with mean of 3.62 and a standard deviation of 0.525.

A caveat concerning the preceding discussion is necessary. Anderson and Roberts (1989) reach the conclusion that the appropriate discount rate is fixed at -0.5 percent. This falls generally within the Court's directions as promulgated in the *Culver* and *Pheifer* cases. It seems unlikely, however, that the appropriate interest rate is either fixed or negative. The apparent discrepancy lies in the fact that Anderson and Roberts employ an ex post approach while the procedure relied upon here is an ex ante analysis. That is, Anderson and Roberts investigate the relationship between historical rates of inflation and nominal interest while the method here involves a prospective, rather than retrospective, analysis. While it is true that the estimate of real interest used here varies over a fairly narrow range, it is neither fixed nor negative. Hence it is necessary to make some estimate of expected future real rates which formally acknowledges their stochastic nature. As the simulation experiment suggests, ignoring the variation in real interest rates biases award estimates.

The distributions of work life and salary are less well defined, although quite clearly they are stochastic variables and therefore subject to some uncertainty. Little information is available on the distributions from which these variables come. Those distributions would have to be developed from the estimation procedures. The important aspect of these variables is that they are stochastically determined.

The stochastic nature of these variables is due to possible measurement error of the wages and work life of the hypothetical victim. A priori there is no reason to assume upward or downward bias in these estimates: thus a symmetrical distribution is called for. In fact, the absence of convincing evidence to the contrary and reliance on the central limit theorem leads directly to the assumption of normality which is made here. Specifically work life is assumed to have a mean of 36 years, as suggested by standard mortality tables, and a standard deviation of 2 years. Salary is assumed to have a mean of $28,000 and a standard deviation of $2,500. While perhaps not precisely the true values, these distributions and their parameters are not unreasonable, with standard deviations of less than 10 percent of their means. The resulting distributions effectively illustrate the impact of randomness on lost earnings models.

With these basic data, a Monte Carlo simulation was used to generate a distribution of lost earnings when all three input variables are stochastic. The simulation calculated 4000 iterations using the Latin Hypercube sampling procedure outlined by McKay, et al. (1979).[5] The simulation generated means for remaining work life, future salary, and expected inflation which were practically identical to their point estimates. For example, remaining work life from standard tables for an individual with the characteristics of the employee in the example was 36 years. The simulated value was 36.01 years.

Running the simulation produced a distribution of present values of lost earnings with a mean and standard deviation of $462,910 and $60.16 respectively. The figure $462,910 is the true expected present value of damages which should be awarded to the plaintiff's estate if the court has found for the plaintiff. That is, $462,910 is the mean of the distribution of possible values. The distribution is shown in Figure 1. Notice that it appears to be somewhat skewed to the right. A Chi-Square test for normality permitted rejection of the null hypothesis that the distribution is normal with confidence at the .01 level. Thus, while all of the input values are normally distributed, the resulting distribution of lost earnings is skewed to the right and not normally distributed.

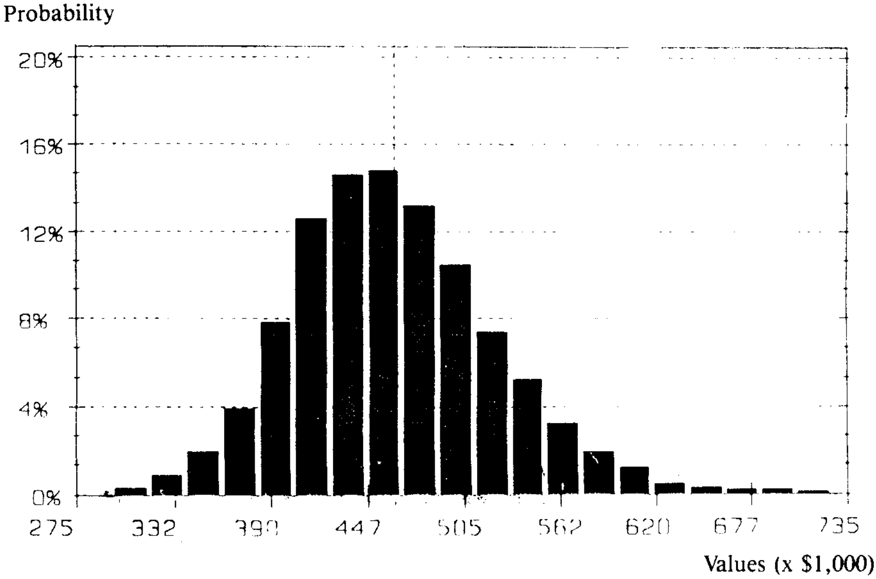
It is easy to see that the value used by courts, $465,960, lies only about 0.7 percent above the true expected value of $462,910. Despite this apparent closeness, however, $465,960 is above the upper limit of a 95 percent confidence interval around $462,910. Thus there is a less than a 5 percent chance that one would observe $465,960 if the true expected value is $462,910. Furthermore, successive trials produced strikingly similar results promoting confidence in the conclusions below. The point estimate of damages was always above the true mean by very nearly the same 0.7 percent and it was

[5] The Latin Hypercube sampling process is a stratified sampling technique which guarantees that all regions of the probability distribution are represented in the sample. McKay, et al show that such a process is more efficient than random sampling.

Figure 1

Distribution of Present Values

After 4000 Interations



always statistically different at the .05 level or better. This is clear evidence of systematic bias in the nonstochastic estimate.

Essentially, this bias is due to the nonlinear nature of the present value calculation. For example, let g(x) represent the function used to calculate present value given some random input x and E[ ] represent the expected value operator. The mean or expected value of PV is denoted E[g(x)]. This is the true expected value of the present value of lost earnings. Taking the mean as a point estimate of the input x, the conventional methodology yields g(E[x]). Any elementary statistics text will indicate that for linear functions E[g(x)] = g(E[x]).[6] This result does not generally hold for nonlinear relationships, however.[7]

In order to examine which input variables might be imparting bias, the experiment was expanded by systematically reverting values to their point estimates. The simulation was rerun and Table 2 summarizes the results. Case 1 represents the randomization of all three input variables, as is recommended here, while Case 5 represents the use of fixed estimates for all inputs, as in the currently accepted estimation process. These two cases therefore represent the polar extremes compared above. For Case 2 a fixed point estimate was used

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[6] See, for example, Pfaffenberger and Patterson, (1987).

[7] For rigorous elaboration on this point see, Hall, (1986).

Table 2
Comparison of Lost Earnings Estimates

	Input Values Used				
	Inflation	**Work life**	**Salary**	**Mean Present Value of Lost Earnings**	**Difference Case n-Case 1**
Case 1	N(3.62,.525)	N(36,2)	N(28,2.5)	$462,910	0
Case 2	N(3.62,.525)	N(36,2)	28	$462,840	- 70
Case 3	3.62	N(36,2)	N(28,2.5)	$460,700	- 2210*
Case 4	N(3.62,.525)	36	N(28,2.5)	$468,070	5160*
Case 5	3.62	36	28	$465,960	3050*

Where N (μ, σ) indicates a normal distribution with a mean of μ and a standard deviation of σ.
* Significant at the .05 level

for salary with the other two input values randomized. The difference between the estimates in Case 1 and 2 was not significant.

Case 3 represents a simulation using a fixed estimate for expected inflation with work life and salary randomized. The resulting mean value for lost earnings was $460,700, significantly below the true value for the fully randomized case at the .05 level. The failure to consider the distribution of expected inflation apparently leads to a significant downward bias.

Case 4 represents a simulation using a fixed estimate for work life with the other two variables being random. This results in a significant upward bias ($468,070 versus $462,910). It can now be seen that the Case 5 estimate, which is calculated much as courts currently do, combines the downward bias of Case 3 with upward bias of Case 4. Though they partially offset, the net result is consistent over estimation of the true value when randomness is ignored. Further, the difference between the point estimates and the true value is statistically significant at the .05 level.

In addition to the elimination of bias, Monte Carlo simulations have the advantage of yielding more information about the estimate than is available when point estimates are used. An estimate of the range of lost earning values is also produced. This may be an important by-product of the Monte Carlo process. As can be seen in Figure 1, the Monte Carlo simulation resulted in values ranging from $295,840 to $725,260. Ninety percent of the values fell between $391,500 and $540,710. This information provides the court with an estimate of the potential variability of the award. It could also provide a basis for the court to set awards at levels other than the mean. Using this information awards could, for example, be set so that the plaintiff would receive at least as much as the 95th percentile. Determining the award in this way, for example, may be appropriate for a worker who at the time of death possessed education and skills which could have been expected to generate earnings well above the mean for the occupation.

Conclusions

The Monte Carlo simulation experiment described here illustrates that ignoring the stochastic nature of inflation, future wages, and remaining work life, produces an award estimate significantly above the stochastic estimate. Most of the bias comes from using a point estimate rather than a probability distribution for remaining work life expectancy. In fact, that impact, working in the direction of overstatement, is so large that it negates the opposite but smaller effect of failing to randomize the inflation input. Using a point estimate for salary while the other two variables were taken from probability distributions does not appear to distort the award from its true value. Experimental results also suggest that the actual net present value of lost earnings comes from an array of values which is not normally distributed but is skewed to the right.

This experiment is not intended to show what is broadly true of the relationship between actual awards and true awards. It will take further study comparing historic awards in specific cases with what would have been the award had randomness been considered in order to make that judgment. The results reported here do raise the suspicion that some systematic over estimation has occurred.

While the award based upon point estimates is only slightly too large in absolute terms, the difference is statistically significant. The tools necessary to eliminate bias by considering the effect of stochastic variables are widely available. As a result, there seems to be no reason why one should settle for a consistent, even if small, overcharge of defendants to the advantage of plaintiffs in cases involving lost earnings.

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