The Law Of Large (Small ?) Numbers And The Demand For Insurance

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ABSTRACT

Using an expected utility approach the effect of "risk subdividing" on insurance demand is analyzed. The results indicate that when the total property at risk is scattered only on a small number of pieces (with independent risks), risk retention becomes very attractive relatively to market insurance with a positive loading.

Introduction

If an individual would reject at any wealth level a favorable bet to be played only once, should he or she accept a large number of independent replica of the same bet? According to P. Samuelson (1963) who first raised the question in a very stimulating yet little used article[1], many people would accept the sequence by reference to the intuition of the law of large numbers. Indeed since each play has a positive expected result, the long sequence of independent tosses would make the player virtually certain of coming out ahead.

Samuelson (1963) shows that this intuition is not valid for an expected utility maximizer. In fact while the sequence of tosses creates a beneficial diversification effect it also involves a greater risk since for instance the amount of the maximum possible loss increases with the length of the sequence. As Samuelson observes, this implies that "it is not so much by

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[1] Three exceptions are interesting articles by Chew and Epstein (1988), Nielsen (1985) and Pratt and Zeckhauser (1987).

adding new risks as by subdividing risks among more people that insurance companies reduce the risk of each."

The purpose of the present article is to examine the relevance of this statement in a risk management framework[2]. When a firm increases the size of its fleet of vehicles (or the number of buildings it owns), its wealth and its total risk of accident (or fire) increase but the risk is also better spread on a larger number of units. How should such changes affect the insurance coverage of each piece of property?

This question has received some attention in the literature mostly when the "probability of regret" is assumed to be the decision-maker's choice criterion (e.g. Mc Worther, (1979), Brockett et al, (1984) and Venezian, (1986)). However, the same question has been much less analyzed in an expected utility framework[3] so that an extension of Samuelson's article to the demand for insurance is not available.

The current article is organized as follows. With the purpose of introducing notation as well as the main ideas of the article, the first section proposes a very short analysis of insurance demand when only one piece of property is at risk. Section 2 presents a basic theorem on the optimal allocation of an insurance budget among N ($N > 1$) identical and independent pieces of property. This theorem is used in the following section in order to isolate a diversification effect in insurance demand. The fourth section extends the preceding one by jointly considering changes in wealth, in total risk and in the degree of diversification for two specific utility functions. In a short conclusion, possible developments are suggested.

Insurance Demand For A Single Piece of Property

Let us consider an individual (or a firm) who owns besides a sure wealth, W_0, a single piece of property of value L with probability of (full) loss, p. If this risk can be covered by market insurance for an amount $Q(0 \leq Q \leq L)$ and if the premium P is linked to Q by a proportional relationship:

$$P = (1+\lambda) \cdot p \cdot Q \tag{1}$$

where $\lambda \geq 0$ is the loading factor, then the optimal value of $Q(Q^*)$ is determined by solving:

$$\text{Max } E[U] = p \cdot U(W_O + Q - P) + (1-p) \cdot U(W_O + L - P) \tag{2}$$

The properties of the optimal solution are well known and it was shown by Mossin (1968) that under decreasing absolute risk aversion (DARA), Q^* should be inversely related to W_O.

An other interesting and less investigated comparative statics question is related to the effect of an increased value of L upon Q^*. When L increases

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[2] Samuelson's concern is about the impact of a better diversification upon the level of E[U] while here the authors discuss its impact upon the optimal level of insurance coverage.

[3] For the articles that consider an expected utility environment, see Brockett, Cox and Witt as well as Cozzolino (1978) who proposed the interesting concept of a risk profile curve.

(e.g. because of changes in relative prices) the decision-maker not only feels richer (wealth effect) but his or her total risk also increases (risk effect). It can easily be shown - see Appendix 1 - that under risk aversion Q^* should be positively related to L.[4] This result indicates the strength of the risk effect since even under DARA it dominates the wealth effect and induces a net increase in Q^*.

In the rest of this article the authors develop essentially - but not exclusively - the analysis of the diversification effect, considering the case where the total value L is spread (e.g. for historical reasons) upon many identical pieces of property.[5]

A Preliminary Result

Instead of being concentrated upon a single piece, the total value at risk, L, is now scattered on N units of equal value (L/N) and equal probability of loss, p, for each. Assuming independence among the losses, the role of the diversification effect can be isolated since, by assumption, W_O and L remain constant.

In order to determine how changes in N[6] - coeteris paribus - affect the optimal use of market insurance, the authors first prove a proposition that will be useful in the rest of the article. This proposition describes how an exogenously given insurance budget, B, should be shared among the N identical and independent goods.

If a risk manager avails upon B and if he or she can insure each of the N pieces for a value Q_i ($i = 1, \ldots, N$ with $0 \leq Q_i \leq L/N$) his or her budget constraint implies:

$$B = (1+\lambda) \cdot p \cdot \sum_{i=1}^{N} Q_i \tag{3}$$

where it is assumed, as in section 1, that the loading is of a proportional nature.[7] In proposition 1, it is shown in fact that the risk averse decision-maker should not "gamble with B" by insuring some goods more than other ones. Instead, one has:

Proposition 1: B should always be evenly allocated on the N goods to be insured.

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[4] The same type of result—for a specific utility function—was recently presented by Beenstock et al (1988).

[5] The ex-ante optimally chosen level of diversification under complete or incomplete insurance markets is a more general but also much more difficult question that is not addressed here.

[6] As always a comparative statics analysis can give rise to two interpretations : either a cross-section one (comparing two individuals or firms at a moment in time) or a time-series one (comparison of the choices of an individual at two points in time). Here one should privilege the cross-section interpretation since by assumption the individual (or the firm) does not choose the diversification level: it arises for everybody from past decisions taken here as given.

[7] The proportionality holds with respect to the total value insured (ΣQ_i) and not with respect to the number of units covered. This implies that the premium does not contain a fixed cost element for each unit that is covered.

To prove this proposition two strategies are contrasted. In strategy I (or I, for short), $n(<N)$ goods are equally insured for a value Q_n satisfying:

$$n(1+\lambda)\,p\,Q_n = B \tag{4}$$

and the $(N-n)$ remaining goods are not insured at all.

In strategy II (or II, for short), all the N goods are insured for an equal value Q_N satisfying:

$$N(1+\lambda)\,p\,Q_N = B \tag{4'}$$

so that of course $Q_N < Q_n$.

The consequences of the two strategies are depicted in the decision tree of figure 1.

Whatever strategy is selected, the number of "fires" or "accidents" is represented by a random variable $\tilde{y}$ which follows a binomial density. Consider then a given number of "fires" denoted y. Under strategy II a sure final wealth, W_F, given by:

$$W_F = W_O + (N-y)\cdot\frac{L}{N} + yQ_N - B \tag{5}$$

is obtained. Of course since $Q_N \leq L/N$, W_F is non increasing in y.

If I has been adopted and if y fires occur, W_F is now a random variable denoted $\tilde{W}_F$. $\tilde{W}_F$ depends upon $\tilde{k}$ which represents the number of damaged units for which insurance has been purchased. More precisely $\tilde{W}_F$ is given by

$$\tilde{W}_F = W_O + (N-y)\cdot\frac{L}{N} + \tilde{k}\,Q_n - B \tag{6}$$

Given the nature of the problem, it is easily seen that $\tilde{k}$ follows an hypergeometric distribution so that under I, $\tilde{W}_F$ has an expectation $E[\tilde{W}_F]$ such that

$$E[\tilde{W}_F] = W_O + (N-y)\cdot\frac{L}{N} + \frac{n}{N}\cdot y\,Q_n - B \tag{7}$$

since $E(\tilde{k}) = \frac{n}{N}\cdot y$

Because of (4) and (4')

$$nQ_n = NQ_N$$

so that (7) can also be written:

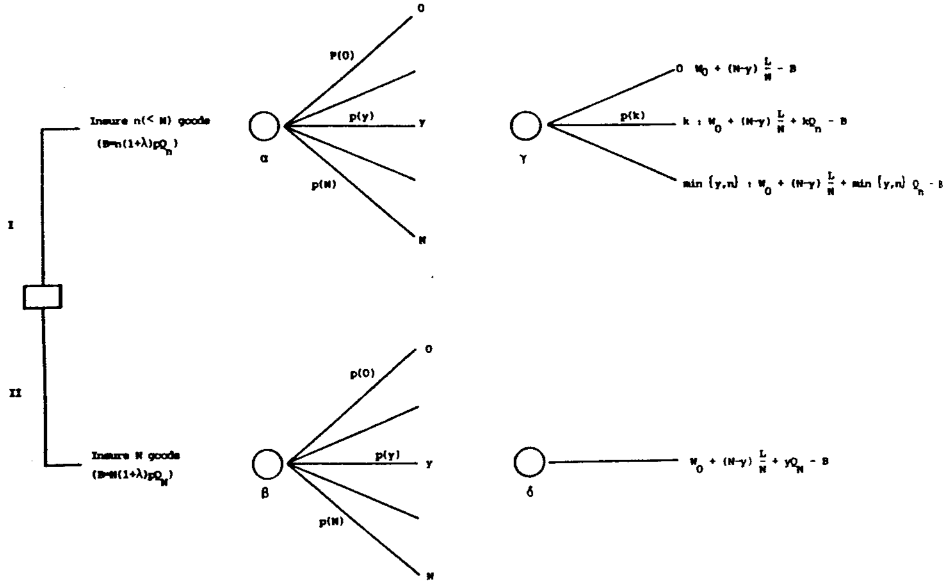
$$E[\tilde{W}_F] = W_o + (N-y)\cdot\frac{L}{N} + y\,Q_N - B \tag{8}$$

Comparing (5) and (8), one obtains an example of a "mean preserving change in risk" as defined by Rothschild and Stiglitz (1971). Indeed for a given y, strategies I and II yield the same expected final wealth[8] but this expected level of wealth is obtained with certainty under II but not under I where $\tilde{W}_F$ is random. Hence, for the value of y that is considered, I is riskier than II which

[8] This result is not surprising since the same total insurance budget is spent under I and II.

Figure 1

The Consequences of Strategies I and II



will henceforth be preferred by all risk averse decision-makers. Since the same type of result can be generated for any y, it necessarily follows that II dominates I globally.

Before using this proposition in the following section, it should be pointed out that it parallels to a wide extent a result presented by Rothschild and Stiglitz (1971) in a portfolio context. Indeed Rothschild and Stiglitz prove that an individual who can purchase shares of N securities whose value next period is described by identical and independent distributions should allocate his or her wealth equally between the N securities. Our proposition 1 leads to the same conclusion for the allocation of an insurance budget among N goods submitted to perils the distribution of which are identical and independent.

The Importance of the Diversification Effect

Consider two individuals, i and j, who avail upon the same sure wealth ($W_{O\,i} = W_{O\,j}$) and own properties at risk of the same value ($L_i = L_j$). These two individuals differ only in the degree of diversification for L. For i(j), L_i (L_j) is scattered on N_i (N_j) pieces and if N_i exceeds N_j, the non-safe component of wealth is better diversified for individual i. Does this imply that he or she should buy less market insurance?

Surprisingly it turns out that no completely general answer can be given to this question. However many partial results indicate that the diversification effect is important and that it shows up even at low values of N.

Ideally one would like to show that for given L insurance demand falls with a better spreading of L on an increased number of pieces of equal value. In more technical terms, one expects that total coverage is monotonically declining in N for a given L. Although such a result does not emerge so far, a less strong version of it is proved in Proposition 2 for any risk averse utility function. Besides for the specific utility curves analyzed later on in this section, the diversification effect is shown to exist already at very low values of N.

Before considering some specific utility functions, Proposition 2 states a result that holds for any concave utility curve.

Proposition 2: The total coverage chosen for N = 1 never falls below that associated with a value of N strictly greater than unity.

The proof given here takes care of the comparison between N = 2 and N = 1.[9]

For N = 1, the first-order condition attached to the problem described in (2) is written:

$$\frac{dE}{dQ_1} = p\,[(1-(1+\lambda)p)\cdot U'(A) - (1-p)\cdot(1+\lambda)\cdot U'(B)] = 0 \qquad (9)$$

where Q_1 = the coverage for one piece (hence the subscript)

$A \equiv W_o + Q_1 - (1+\lambda)p\,Q_1$

$B \equiv W_o + L - (1+\lambda)p\,Q_1$

and $A \leq B$.

For N = 2 and using the result of proposition 1, the objective function can be written:

$$E[U] = p^2\cdot U(W_o + 2Q_2 - 2(1+\lambda)pQ_2) + 2p(1-p)\cdot U(W_o + \frac{L}{2} + Q_2 - 2(1+\lambda)pQ_2)$$
$$+ (1-p)^2\cdot U(W_o + L - 2(1+\lambda)pQ_2) \qquad (10)$$

where Q_2 is the equal coverage allocated to each of the two pieces. For the sake of notational convenience, the levels of final wealth at which U is evaluated in (10) are denoted respectively C, D and E,

with $C \equiv W_o + 2Q_2 - 2(1+\lambda)p\,Q_2$

$D \equiv W_o + L/2 + Q_2 - 2(1+\lambda)p\,Q_2$

$E \equiv W_o + L - 2(1+\lambda)p\,Q_2$

Of course, since more than full insurance is not allowed:

$C \leq D \leq E$.

The first-order condition associated to (10) is

[9] The proof for any N > 2 is lengthy and tedious so that it is not given here. The interested reader may obtain it from L. Eeckhoudt.

$$\frac{dE}{dQ_2} = 2p\,[p\,(1-(1+\lambda)p)U'(C)+(1-p)(1-2(1+\lambda)p)U'(D) - (1-p)^2\cdot(1+\lambda)U'(E)]$$

$$= 0 \tag{11}$$

To prove that $2Q_2^*$ is smaller than Q_1^*,[10] dE/dQ_2 is evaluated at a point such that $2Q_2 = Q_1^*$. If it can be shown that dE/dQ_2 evaluated at Q_1^* is negative, concavity of E[U] guarantees that the optimal solution in (11) is to the left of Q_1^* so that $2Q_2^* < Q_1^*$.[11]

Notice first that if $2Q_2 = Q_1^*$, one has:

$$A = C < D < E = B$$

where the strict inequalities result from the well-known fact that $Q_1^* < L$ under a strictly positive loading. Consequently, by concavity of U

$$U'(A) = U'(C) > U'(D) > U'(E) = U'(B)$$

Using the fact that

$$(1-p)\cdot(1-2(1+\lambda)p) = (1-p)(1-(1+\lambda)p)-(1-p)(1+\lambda)p$$

and the above mentioned results, when dE/dQ_2 is evaluated at $2Q_2 = Q_1^*$, one has:

$$\left.\frac{dE}{dQ_2}\right|_{Q_1^*} = 2\,p\cdot[p(1-(1+\lambda)p)U'(A)+(1-p)\,(1-(1+\lambda)p)U'(D)$$
$$-(1-p)(1+\lambda)p\,U'(D)-(1-p)^2(1+\lambda)U'(B)]$$

Since at $2Q_2 = Q_1^*$, $U'(A)$ exceeds $U'(D)$ it follows that

$$U'(A) > pU'(A)+(1-p)U'(D),$$

and since $U'(D)$ exceeds $U'(B)$, one has

$$U'(B) < pU'(D)+(1-p)U'(B)$$

Consequently:

$$\left.\frac{dE}{dQ_2}\right|_{Q_1^*} < 2p\cdot[(1-(1+\lambda)p)U'(A)-(1-p)(1+\lambda)U'(B)] \tag{12}$$

Since the right side of (12) - divided by 2p - is nothing but the first-order condition for the case where N = 1 (see Mossin, 1968), it is equal to zero and one has:

$$\left.\frac{dE}{dQ_2}\right|_{Q_1^*} < 0$$

so that $2Q_2^*$ is lower than Q_1^*. Q.E.D.

[10] Q_1^* and Q_2^* are the optimal coverages per unit that solve respectively (9) and (11).

[11] This type of proof is currently used in the insurance economics literature. See e.g. Mossin (1968) or Dionne and Eeckhoudt (1985).

Since this type of proof can be extended to any $N > 1$, it follows that for given W_o, L, p and λ the demand for coverage is always smaller for $N > 1$ than for $N = 1$. Although this result is already an illustration of the diversification effect, one would like to show besides that insurance coverage is everywhere declining in N. While such a result does not emerge for any concave utility, it can be obtained for a broad spectrum of specific utility functions. The exponential, quadratic and logarithmic utility curves are considered here.[12] Before turning to the specific cases, it should be stressed that because of proposition 1, the random final wealth $\tilde{W}_F$ can be expressed as:

$$\tilde{W}_F = W_o + (N - \tilde{y}) + \tilde{y} \cdot Q_N - (1+\lambda)pNQ_N \tag{13}$$

where Q_N is the equal level of coverage chosen for each of the N pieces.

The Exponential Utility Curve

In this case $U = -e^{-\gamma W_F}$ where γ is the coefficient of constant absolute risk aversion. Because of (13), the expected utility can be written:

$$E[U] = -e^{-\gamma[W_o + L - (1+\lambda)pNQ_N]} \cdot E\left[e^{-\gamma(Q_N - \frac{L}{N})\tilde{y}}\right] \tag{14}$$

Since the expectation on the right hand side of (14) is the moment generating function of a binomial distribution, (14) can also be written as:

$$E[U] = -e^{-\gamma[W_o + L - (1+\lambda)pNQ_N]} \cdot \left[pe^{-\gamma(Q_N - \frac{L}{N})} + (1-p)\right]^N \tag{14'}$$

Instead of maximizing E[U] with respect to Q_N, $-E[U]$ is minimized so that (14') becomes a positive quantity of which the logarithm can be taken. The problem then is written:

$$\operatorname*{Min}_{Q_N} \ln[-E[U]] = -\gamma[W_o + L - (1+\lambda)pNQ_N] + N \ln[pe^{-\gamma(Q_N - \frac{L}{N})} + (1-p)] \tag{15}$$

After some easy transformations the first-order condition yields:

$$Q^*_N = \frac{L}{N} - \frac{a}{\gamma} \tag{16}$$

$$\text{where } a \equiv \ln \frac{(1-p)(1+\lambda)}{1-(1+\lambda)p}$$

Since for $\lambda > 0$, $(1-p)(1+\lambda)$ exceeds $1-(1+\lambda)p$, a is a strictly positive quantity and less than full coverage is optimal for any N. Besides, the total demand for insurance can be expressed as:

$$NQ^*_N = L - \frac{aN}{\gamma} \tag{16'}$$

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[12] Each of these utility functions exhibits a different behavior of the coefficient of absolute risk aversion with respect to wealth: constant for the exponential, increasing for the quadratic and decreasing for the logarithmic.

It follows from (16′) that with an exponential utility curve the total coverage is everywhere decreasing in N so that for this specific utility the diversification effect negatively affects insurance demand at any value of N.

Notice also that since NQ^*_N is linear in N the value of $N(\hat{N})$ at which the demand for insurance vanishes can easily be found. It follows from (16') that

$$\hat{N} = \frac{L\gamma}{a} \tag{17}$$

To illustrate equations (16′) and (17) an example that also applies for the other two utility functions is used. Let

$W_o = 2$
$L = 20$
$\lambda = .3$
$p = .001$

For a value of γ (taken here as equal to 1/22),[13] it is easy to express total coverage as a function of N. The results are presented in table 1 below. They clearly show the strength of the diversification effects already at low values of N.

Table 1

($L = 20$; $\lambda = .30$) ($W_o = 2$, $p = 001$)

	VALUES OF NQ^*_N		
N	**Exponential** ($\gamma = 1/22$)	**Quadratic** ($\beta = 1/88$)	**Logarithmic**
1	14,221	13,386	14,922
2	8,443	6,774	9,846
3	2,664	0,162	4,771
4	0	0	0

Quadratic Utility Curve

Consider now $U = W_F - \beta W_F^2$ with $\beta > 0$ (risk aversion) and $(1-2\beta\ W_F)$ positive even at the highest possible W so that marginal utility is everywhere positive.

Maximizing expected utility with respect to Q_N yields after some transformations:

$$NQ^*_N = \frac{-\lambda N(1-2\beta(W_o+L))}{2\beta(1 - p(1-\lambda^2 N))} + L \cdot \frac{1-p(1+\lambda N)}{1-p(1-\lambda^2 N)} \tag{18}$$

[13] In appendix 2, it is indicated that this value has been selected in order to make possible a comparison with the other two utility functions especially the logarithmic one.

This expression which is more complex than the one obtained with an exponential utility, exhibits the same desirable properties. For instance, for $\lambda = 0$, $NQ_N^* = L$ so that full insurance prevails. When λ is strictly positive NQ_N^* falls below L and it can be shown that the demand for insurance is everywhere decreasing in N. Indeed after some manipulations, one obtains:

$$\frac{\partial\,(NQ_N^*)}{\partial N} = \frac{-\lambda\,(1-p)[1-2\beta(W_o+L(1-p(1+\lambda)))]}{2\beta[1-p(1-\lambda^2 N)\,]^2}$$

which is everywhere negative for $\lambda > 0$ (and equal to zero for $\lambda = 0$). As a result, under a positive loading, the diversification effect also applies at any value of N for the quadratic utility function.

As for the exponential utility, it is possible to express the value of N at which insurance demand vanishes. From (18) one easily obtains that $Q_N^* = 0$ for a value of N ($\hat{N}$) such that

$$\hat{N} = \frac{2\,\beta L\,(1-p)}{\lambda[1-2\beta(W_o+L(1-p))]}$$

In accordance with intuition, $\hat{N}$ is decreasing in λ.

Table 1 also illustrates the importance of the diversification effect for the quadratic utility function. As for the exponential utility the value of β (β = 1/88) has been selected in order to make easier the comparison with the logarithmic function (see fn 13).

The Logarithmic Utility Curve

With a logarithmic utility, the first order condition for Q_N^* is an equation of the $(N-1)$ degree in Q_N so that a closed form solution is not available for $N > 2$.

Table 1 contains numerical solutions pertaining to the example presented above when $U = \ln W_F$. They confirm the existence of a diversification effect, even for very low values of N.

An Extension

While it has been attempted so far to isolate the diversification effect keeping other things equal, it turns out that in current practice, increases in N are usually accompanied by increases in W_o and/or L. The purpose of this section is to present some results for this case.

Of course, since no completely general result can be obtained for the diversification effect alone, none should be expected in the broader context adopted here. However in the specific cases where an explicit solution for NQ_N^* is obtained, the impact of a simultaneous change in all the explanatory variables on NQ_N^* can be analyzed. For this purpose consider three coefficients of elasticity related to total insurance demand:

$$\eta_N = \frac{\partial\,(NQ_N^*)}{\partial\,W_o}\cdot\frac{W_o}{NQ_N^*}$$

$$\eta_L = \frac{\partial(NQ_N^*)}{\partial L} \cdot \frac{L}{NQ_N^*}$$

$$\eta_N = \frac{\partial\,(NQ_N^*)}{\partial N} \cdot \frac{N}{NQ_N^*} = \frac{\partial\,(NQ_N{}^*)}{\partial N} \cdot \frac{1}{Q_N{}^*}$$

which are respectively the elasticities of (NQ_N^*) with respect to W_o, L and N.

Then, as is well known, if a firm (or an individual) experiences a relative increase simultaneously in W_o ($= d\ln W_o$), $L(= d\ln L)$ and $N(= d\ln N)$, the percentage change in insurance demand is given by:

$$\varepsilon = \eta_{W_o}(d\ln W_o) + \eta_L(d\ln L) + \eta_N(d\ln N) \tag{20}$$

To illustrate the meaning of (20), consider the case of the exponential utility and of an initial situation represented by the previous example with $N = 1$ so that $NQ_N^* = Q_N^* = 14{,}221$ (see table 1). Assume now that the firm experiences an increase in W_o from $W_o = 2$ to $W_o = 4$ and that it acquires a second piece of property the value of which is also equal to 20, so that total property at risk amounts now to 40. Such changes have in general three implications: (1) a wealth effect because of increases in W_o and L; (2) a risk effect since the total value at risk amounts to 40 instead of 20 and (3) a diversification effect since the total wealth at risk is now spread on two pieces instead of one.

For the exponential utility, simple manipulations of equation (16′) lead to:

$$\eta_{W_o} = 0$$

$$\eta_L = \frac{L\gamma}{L\gamma - aN} > 1$$

$$\eta_N = \frac{-aN}{L\gamma - aN} < 0$$

Since the exponential utility exhibits constant absolute risk aversion the wealth effect vanishes in this case so that only the risk and diversification effects are at work. Since in the example the percentage increases in L and N ($d\ln L = d\ln N$) are equal it turns out that

$$\varepsilon = \frac{L\gamma - aN}{L\gamma - aN}(d\ \ln\ N) = d\ln N$$

In fact the diversification effect reduces the positive impact of increased L upon insurance demand but it does not annul it. If L had changed from 20 to 40 with N remaining equal to unity the insurance coverage would have shifted from 14,221 to 34,221. When the same increase in the initial value for L results from a development in the number of pieces, the insurance coverage increases only to 28,442.

Similar results – involving besides a wealth effect – can be generated for the quadratic utility through equation (18) or for the logarithm curve by numerical examples. They all indicate the negative effect of a better diversification on insurance demand even if its impact is mitigated by the presence of the other two effects.

Conclusion

The authors have stressed in this article the importance of the diversification effect ("risk subdividing") in analyzing insurance demand. It turns out that already at low levels of diversification (e.g. N = 5) a moderate loading should induce economic agents not to transfer their risks to insurers and to become their own insurers.

While this conclusion has some relevance for risk management, its robustness should be checked under alternative assumptions. Some ot them are briefly mentioned here:

– How does the possibility of partial loss (instead of the "full loss" - "no loss" case) affect the result? In this more general context, it would be possible to compare the relative attractiveness of the deductibles and of the co-insurance clause (see e.g. Raviv, (1979)).

– If the goods have different values and/or probabilities of loss how should the insurance budget be allocated between them?

– What is the optimal insurance strategy if companies offer a non-proportional loading that is decreasing in the number of pieces to be insured? The suppliers of insurance seem to be aware of this possibility since in some group health plans the loading factor is inversely related to the number of employees.[14] While the three extensions suggested here can be developed in a monoperiodic model as the one proposed in this article, one might also wonder if the consideration of many time periods (for a single piece of property) might become an alternative to "risk subdividing" on many pieces in a single period.

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[14] The authors especially thank Patricia Danzon for drawing their attention upon this fact.

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Appendix 1

The first-order condition associated with (2) can be written:

$$H(Q;L,W_o,p,\lambda) = (1-(1+\lambda)p)\cdot U'(A) - (1-p)(1+\lambda)\cdot U'(B) = 0 \qquad (A.1.1)$$

where $A \equiv W_o + Q - P$ and $B \equiv W_o + L - P$.

Using standard results in calculus, it can be shown that:

$$\text{sign}\,\frac{dQ^*}{dL} = \text{sign}\,\frac{\partial H}{\partial L};$$

but

$$\frac{\partial H}{\partial L} = -(1-p)\,(1+\lambda)\cdot U''(B)$$

Since this expression is positive under risk aversion, this condition is sufficient to create a positive relationship between Q^* and L.

Appendix 2

In all the examples, initial wealth in the absence of risk would amount to 22 (W_o = 2 and L = 20). This information is used to "calibrate" the coefficients of the three utility functions used in terms of the degree of absolute risk aversion (A).

For the logarithmic case, $A = 1/W$ so that the value taken by A is 1/22. Since for the exponential utility A is equal to γ a direct comparison between the logarithmic and the exponential utility in this example is of course obtained at $\gamma = 1/22$.

As far as the quadratic utility is concerned, $A = 2\beta/(1-2\beta W)$ so that to obtain $A = 1/22$ with initial total wealth equal to 22, β must be equal to 1/88.

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