Corporate Bond Insurance: Feasibility and Insurer Risk Assessment

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ABSTRACT

Bond insurance for municipalities has been available for more than ten years; however, corporate bond insurance has only recently received attention from potential insurers. This article uses a model to assess the pure risk premium for corporate bond insurance where both nonzero correlation between frequency and severity and nonzero correlation between individual risk units are allowed. The results suggest that corporate bond risk insurance could be offered by a third party insurer at a lower cost than the prevailing market default risk premium.

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Bond insurance for municipalities has been available for more than ten years; however, the introduction of corporate bond insurance underwritten by Financial Security Assurance Corporation introduces an entirely new concept in risk management for the firm and a new area of risk assessment and risk management for the insurer.[1]

At issue is the level of risk underwritten by corporate bond insurers, and whether corporate bond insurance is financially justified.[2] In this article the

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[1] Information concerning Financial Security Assurance Association was obtained from material and verbal information received from that firm.

[2] In addition to the insurance industry, other related types of financial guarantees are also prevalent. Major banks have backed corporate borrowing with standby letters of credit which provide essentially the same guarantees as bond insurance. Thus, whether the insurer is an insurance company providing bond insurance or a bank providing standyby letters of credit, the assessment of this risk to the underwriter is critical.

authors will determine the feasibility of corporate bond guarantee insurance or risk insurance where the insurer will assume the full obligation of the bond issue should the issuer default. A model will be used to assess the pure premium for corporate bond insurance where both nonzero correlation between frequency and severity and where spatial dependencies are allowed. If bond insurance is economically justified, the present value of the interest savings must be greater than or equal to the bond insurance premium. In other words, the question addressed is whether or not the existence of a third party insurer can be a more efficient or effective risk management mechanism than financial markets. Can insurers offer sufficiently low premiums to reduce the overall cost of debt issues or can the existence of bond insurance allow firms that otherwise could not enter the public market to avail themselves of this form of capital?[3]

Studies by Fons (1987) and Altman (1989) have suggested that the actual historical default experience of corporate debt does not justify the observed market default risk premiums. They find that returns exceed actual historical default rates. As a result, holders of well diversified portfolios of corporate bonds appear to be rewarded for bearing default risk. If this is the case, it may be possible for corporate bond insurers to reduce the overall cost of corporate debt for those firms purchasing corporate bond insurance.

In an efficient and competitive market the reduction in agency risk resulting from the signalling effect of a bond insurer's guarantee may explain a possible net benefit from bond insurance. Thakor (1982) argues that even in an efficient and competitive market where we would assume that the premium charged by the insurer should exactly offset any potential interest savings to borrowers, net savings to borrowers may exist due to the signal conveyed by insurance coverage about the default risk of insured debt issues. In a recent study, however, Hsueh and Lie (1990) found that the purchase of bond insurance generates no signalling benefits to municipal borrowers. Thus, if no potential signalling effect can be assumed to exist for corporate bond insurance, justification for the existence of corporate bond insurance must be that the insurance premium is less than the present value of the observed interest default premium.

This article is divided into five sections. The next section reviews the existing literature on bond insurance and is followed by a presentation of the risk assessment model that will be used in estimating the parameters of this model and the related insurer risk. The fourth section will address whether bond insurance is economically feasible or whether the existing market mechanism handles these same risks more effectively. Conclusions follow.

[3]This article uses actuarial methodology for estimating insurer risk exposure which differs significantly from the option pricing approach as proposed by Merton (1977) and used by Cummings (1988), Marcus and Shaked (1984), Gatti and Spahr (1990), and Pennachi (1987).

Bond Insurance Reviewed

Bond insurance may be thought of as a form of risk shifting. A firm purchases bond insurance at the time bonds are issued. The bond insurance protects the investor if the firm defaults on an interest or principle payment. Thus, for the cost of the insurance premium, the firm shifts the liability of the bond obligation to a third party, the insurer. The insurer, for the price of the insurance premium, agrees to indemnify the investor from loss of interest and principle.

Theoretically, if a company purchases bond insurance, the bonds should be rated according to the strength of the insurer rather than the bond issuer. Thus, a company that normally issues BBB rated bonds should be able to issue these same bonds at an AAA rating if they purchase bond insurance from a financially strong insurer. If bond insurance is to be economically justified, the present value of the BBB company's interest rate savings must be greater than or equal to the cost of the bond insurance premium.

Since corporate bond insurance is in its infancy, the existing information and research that is available almost invariably concerns municipal bonds. Municipal bonds will be discussed next.

Municipal Bond Insurance

The first company formed to insure municipal bonds - American Municipal Bond Assurance Corporation (AMBAC) - was formed by Mortgage Guaranty Insurance Company in 1971. Today several companies including AMBAC insure municipal bonds. These include Municipal Bond Insurers Association (MBIA), Financial Guaranty Insurance Company (FGIC) and Bond Investors Guaranty (BIG). Other insurers of municipal bonds include affiliates of the Aetna Life and Casualty Insurance Company and United States Fidelity and Guaranty Company. All municipal bonds insured by any one of these companies receive an AAA rating from Standard and Poor's and most (including AMBAC, MBIA, and FGIC) also receive Aaa ratings from Moody's Investor Service.[4]

Municipal bond insurance grew slowly in the 1970's. By 1979, only 2 percent of all issues were insured. However, because changes in the tax laws made municipal bond investing much less attractive to financial institutions – placing more issues in private hands - and the well known default of the Washington Public Power Supply System (WPPSS), municipal bond insurance became much more attractive (see Hoey and Buerger, 1985). As of January 1, 1987 a total of $179.30 billion of new issue, long-term debt was issued.

[4]This information and data concerning the municipal bond insurance industry was obtained from MBIA Corporation, AMBAC, FGIC, BIG and from discussions with their representatives.

Corporate Financial Guarantees

Since the risks of default on unsecured corporate credit may be substantial, investors may find an incentive to purchase corporate bond insurance. Unfortunately for investors corporate bond insurance is generally unavailable. One potential reason for the absence of corporate bond insurance is that insurers believe that large volumes of corporate bond insurance might raise questions about their credit ratings. However, this argument may be inconsistent with observed behavior since other forms of corporate debt guarantees are prevalent.

Guarantees of other forms on corporate debt are not new and are becoming increasingly common. They come in many forms from surety bonds to standby letters of credit. These are long-standing insurance and banking products that are simply being put to many new uses in today's corporate debt markets. They are being used to guarantee limited partnerships, off-balance sheet loans and leases, commercial paper, bank deposits, lease receivables and mortgaged-backed securities. In 1980, total sales of guarantees were $161 billion. By 1986 they were approaching $600 billion (*Banker & Tradesman*, 1986). The current authors theorize that growth in these related products may indicate a demand for underwritten corporate bond insurance that may provide a more complete form of coverage.

The confusion over the existence of the corporate bond insurance industry, that is addressed in this article's introduction, stems from the premise that there are two basic types of financial guarantees. They are commonly referred to as Security or Credit Enhancement Insurance and Risk Insurance (see Hoey and Buerger, 1985 and *Banker & Tradesman*, 1986).

Security Insurance: Security insurance attempts to enhance the credit rating of an issue while requiring for itself a high degree of protection against loss. For this type of financial guarantee, the insurer requires well-structured financing that results in a high quality issue for the investor. Not only does the insurer require collateral for the underlying issue, but also requires additional collateral to provide back-up protection in the unlikely event of default (*Banker & Tradesman*, 1986).[5]

Security insurance is based on a zero loss standard, thus the premiums for this type of insurance is not based on risk exposure. The premium may be justified by the issuer on the basis of increased marketability and possible interest savings resulting from the insurer's impeachable credit rating.[6]

Financial Security Assurance Corporation offers security insurance. According to Hoey and Buerger (1985), both the investor and issuer may be better off without the insurer absorbing undue risk because: (1) insurance makes the bond transaction easier for investors to understand and reduces the necessity of rigorously assessing the issuer's risk, (2) the insurer, in addition to the bond trustee, is responsible for monitoring collateral and other

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[5] Also see Hoey and Buerger (1985).

[6] This information was obtained from a representative of Financial Security Assurance Corporation. Also see Hoey and Buerger (1985) and *Banker & Tradesman,* (1986).

supplemental protections, (3) insurance creates a broader market particularly with respect to institutional investors with fiduciary obligations to achieve the highest level of safety, (4) financial guarantees often provide a regulatory benefit to an issuer that translates into an economic benefit, and (5) to obtain an AAA rating, principal and interest must be paid on time. Insurers may facilitate these payments by standing ready to advance funds to make timely payments with full recovery for the insurer at a later time. The insurer essentially places its "Good Housekeeping Seal of Approval" on the issue which may reduce the "agency risk" to the investor.

Risk Insurance: The other classification of financial guarantee insurance, risk insurance, transfers real economic risks to the guarantor. Risk insurance like property, liability, or life insurance requires the insurer to accept underwriting risk. As in these types of insurance, claims are statistically forecast from claim experience for risk insurance. The insurer, using actuarial data, must set the premium level to compensate itself for the risk level. Losses are then expected to average less than the premium and in bad years, net losses are to be covered by earlier good years (see *Banker & Tradesman*, 1986).

As with municipal bond insurance, the corporate bond insurer guarantee provides, that in the event of a firm's failure to make payment to the paying agent of insured bonds or notes, the payment will be made promptly by the insurer. The guarantee covers the full amount of principal and interest due on the payment dates scheduled at the time the obligations are issued. The insurer becomes the owner of the defaulted coupons or securities upon making payments to the paying agent. However, the issuing firm is not relieved of its obligations by the insurer's payment. As will become important later in this article, a good estimate of the present value of an insurer's potential loss is the reduction in an identical uninsured bond's market value subsequent to the issuer's default.

For an insured bond, the insurer continues to make all timely payments of principal and interest, thus the market price of this bond would not fall as a result of the issuer's default. On the other hand, for an uninsured bond, the market value will decline to the present value of the uncertain future payments that may be made by the issuer. The insurer of a defaulted bond, thus, has an asset that will have the value of an uninsured bond and a liability equal to the market value of the undefaulted bond. Therefore, the cost of an issuer's default to the insurer may be estimated by observing the actual decline in the market value subsequent to actual corporate bond defaults.

Research Methodology

In order to develop a model for estimating the mean pure premium, the relatively unique nature of corporate bond risk insurance must be considered. From an insurance perspective, risk insurance may be characterized, during most economic periods, as low frequency, possibly high severity, catastrophe loss exposure insurance. However, during periods of severe economic recession, possible spatial dependencies and a positive correlation between frequency and severity of defaults may result in high frequency, high severity, catastrophe loss exposure for the insurer.

The model developed below specifically includes both the possibility of a positive correlation between frequency and severity and a second correlation called spatial dependence. During low risk periods as characterized by periods of economic expansion both the frequency and severity of defaults most likely will be low; however, during periods of high risk (most likely characterized by severe economic recession) both the frequency and severity of default may be high. This relationship between frequency and severity of default most likely will result in a positive correlation. A second correlation defined as spatial dependence allows economic cycles to last for more than one year. Since economic expansions or recessions typically last for more than one year, it is likely that default losses in one year will be positively correlated with losses in a subsequent year. The actuarial model developed below will allow for both of these types of dependencies which establishes a more general model than if independence were assumed.[7]

The model developed to estimate the mean pure premium is similar to the model used by Spahr and Escolas (1986) in estimating risk premiums for mortgage insurance.

Corporate bond risk insurance is designed to provide investors protection during economic conditions other than possibly a complete economic collapse.[8] No type of insurance including corporate risk insurance can reasonably withstand sustained periods of high frequency, high severity losses. An assessment of the mean pure premium for economic conditions other than an economic collapse may be attained using the model developed here.

The proposed model considers the relevant parameters for each risk unit (i.e., a dollar of insured par corporate bond) and the resulting aggregate risk premium.[9] To establish the relationships, let:

$E(z)$ = the expected probability of a default (frequency) for an individual bond,

$Var(z)$ = the variance of the default probability,

$E(y)$ = the expected amount of loss (severity) if a loss occurs, per dollar of bond coverage,

$Var(y)$ = the variance of the loss severity per dollar of bond coverage,

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[7] In developing the acturial model no statement is being made that dependence is present; however, that possibility is recognized.

[8] A complete economic collapse would result in all bond issues defaulting with maximum (100 percent) severity.

[9] Risk insurance usually insures both interest and principal payments. As mentioned earlier, the insurer becomes the owner of the defaulted coupons or securities upon making payments to the paying agent. However, the issuing firm is not relieved of its obligations by the insurer's payment. In the absence of corporate bond insurance, the market would consider the severity of the default in arriving at the market value of the bond. Therefore, the present value of an insurer's losses may be estimated by the decline in the market value of the debt. The individual risk unit, however, will be one dollar of par value.

E(x) = the expected pure premium for an individual risk per dollar of bond coverage,
Var(x) = the variance of pure premium for an individual risk per dollar of bond coverage,
E(T) = the expected total incurred loss for the population of insured corporate bonds,
Var(T) = the variance of total incurred losses for the population,
z = frequency of default,
y = severity of default,
x = pure risk premium, and
T = total incurred losses for the population of insured corporate bonds

The mean and variance for the individual pure premium are:[10]

$$E(x) = E(zy) = E(z)E(y) + Cov(z,y) \tag{1}$$

and

$$Var(x) = (E(z))^2 Var(y) + (E(y))^2 Var(z) + 2E(z)E(y)Cov(z,y) \tag{2}$$

For corporate bond defaults, it is likely that the term Cov(z,y) is positive. This will result if the severity of the loss tends to be affected by the number of bond defaults.

The total expected incurred losses for the population of insured corporate bonds is a function of the par value of bonds insured, N, and the expected pure premium such that

$$E(T) = NE(x) \tag{3}$$

and

$$Var(T) = \sum_{i=1}^{N} \sum_{j=1}^{n} Cov(x_i, x_j). \tag{4}$$

[10]According to Goodman (1960), the mean and variance of the product of two dependent random variables are respectively

$$E(x) = E(z)E(y) + Cov(z,y)$$

and

$$Var(x) = (E(z))^2 Var(y) + (E(y))^2 Var(z) + 2E(z)E(y)Cov(z,y) + 2E(z)E_{12} + 2E(y)E_{21} + E_{22} - EE_{11}^2$$

where

$$E_{11} = Cov(z,y),$$
$$E_{12} = E(z - E(z))^2(y - E(y))^2$$
$$E_{21} = E(z - E(z))^2(y - E(y)), \text{ and}$$
$$E_{22} = E(z - E(z))^2(y - E(y))^2.$$

The approximation

$$Var(x) = (E(z))^2 Var(y) + (E(y))^2 Var(z) + 2E(z)E(ye)Cov(z,y)$$

according to Goodman is a very accurate approximation for determining the variance of the pure premium for an individual risk per dollar of coverage because of the small magnitude of the cross-product terms, E_{11}, E_{12}, E_{21} and E_{22}.

Var(T) may be assumed to lie between the independence case $\zeta_{ij} = 0$ (the correlation between the ith and jth insured risk per dollar of bond coverage is zero) and $\zeta_{ij} = 1$.[11]

If one assumes $\zeta_{ij} = 0$, then

$$N\,\mathrm{Var}(T) = \sum_{i=1}^{N} \mathrm{Var}(x_i) = N\mathrm{Var}(x_i)\ . \tag{5}$$

If, however, one assumes $\zeta_{ij} = 1$, and if $\mathrm{Var}(x_i) = \mathrm{Var}(x_j) = \mathrm{Var}(x)$, then (4) becomes

$$\mathrm{Var}(T) = \sum_{i=1}^{N}\sum_{j=1}^{N} \sigma_i\,\sigma_j = N^2\,\mathrm{Var}(x)\ . \tag{6}$$

or, in general,

$$\mathrm{Var}(T) = N\mathrm{Var}(x)\ [\ 1 + N\zeta_{ij} - \zeta_{ij}\ ]\ . \tag{7}$$

Thus, if ζ_{ij} is assumed to be greater than or equal to zero, the true variance of the total losses incurred will most likely lie between NVar(x) and N^2Var(x).

If the gross premium rate on each dollar of bond coverage is P and the total dollars of bond coverage is N, the pure premium is NP dollars and the expected loss ratio is

$$E(LR) = NE(x)/NP = E(x)/P\ . \tag{8}$$

The loss ratio may be expressed as the total expected losses divided by the total pure premium, or it may be written as the expected individual pure premium per dollar of corporate bond coverage divided by the individual premium or dollar of bond coverage.

If N dollars of bond coverage represent all actuarial data for insured corporate bonds, the population mean pure premium, X, will be identical to the individual mean pure premium for each risk per dollar of premium or

$$E(\overline{X}) = E(zy) = E(x)\ . \tag{9}$$

The variance of the population mean pure premium for NP dollars of pure premium is

$$\mathrm{Var}(\overline{X}) = \mathrm{Var}(T)/N^2. \tag{10}$$

Via the Central Limit Theorem, $\overline{X}$ may be assumed to be normally distributed. The assumption is valid regardless of the underlying distribution of x or the covariance relationships so long as that distribution has a finite variance that is known or which, for practical purposes, can be estimated. Even though the greater the skewness, the larger must be the sample size to obtain an adequate

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[11] It was assumed in the development of the model that the spatial dependence between risk i and j were positive or at minimum zero, subsequent data analysis support this assumption. The serial correlation for annual default, ζ_{ij}, in Table 1 is estimated to be .204.

normal approximation, sample size should not be a problem in this situation since essentially the entire population of publicly traded corporate bonds is being used (see McClave and Benson, 1982, p. 256).

In an insurance context, the degree of confidence (or accuracy) in measuring the population mean pure premium, μ_x for an individual risk, is measured next. A sample of historical losses or paid claims may provide an estimate for the population mean pure premium. This estimate, $\overline{X}$, is the expected or average historical pure premium. To determine the degree of confidence, a general mathematical expression is established that provides the probability that the measured premium will deviate from the true population mean by some multiple of the true population mean. More formally, the degree of confidence may be expressed as:[12]

$$PR (\mid \overline{X} \mu_x \mid < h \mu_x) > 1 - \alpha. \tag{11}$$

Expression (11) holds if $h\mu_x > k\gamma \sigma_x$, where k is the standard normal variate corresponding to the probability α. The standard deviation of $\overline{X}$, σ_x, lies between $(Var(x)/N)^{1/2}$ and $(Var(x))^{1/2}$. If the correlation between risk claims, ζ_{ij} is known, the standard deviation of $\overline{X}$ is

$$\sigma_x = ((Var(x) (1 + N \zeta_{ij} - \zeta_{ij}))/N)^{1/2} \tag{12}$$

and equation (2) can be used to estimate Var(x).

An adequate degree of confidence for many types of insurance is that γ = .95 or α = .05 = h. Thus, the estimate mean pure premium will be within +5 percent of the true population mean pure premium 95 percent of the time and the confidence interval is given as

$$\overline{X} \pm 1.96 ((Var(x)(1 + \zeta_{ij} - \zeta_{ij}))/N)^{1/2} \tag{13}$$

where $\overline{X}$ is used to estimate μ_x.[13] To see how these relationships may be used, consider the following example. Assume that $1 trillion dollars of corporate bonds are insured, the probability of a single risk default is .09 percent with a standard deviation of .09 percent, and the severity of the mean loss per dollar is $.40 with a standard deviation of $.20. Further, assume for illustration purposes that the correlation between frequency and severity, ζ_{zy}, as well as the spatial correlation between risk units, ζ_{ij}, is .20. Then from equations (1) and (2)

$$E(x) = (.009)(.40) + (.20)(.009)(.20) = .000396$$

and

[12] See Witt (1973 and 1974).

[13] A two tailed test with α = .05 was used even though it is realized that the insurer would be primarily concerned with only the upper limit. The benefit of using a two tailed test is that it gives a confidence interval; however, using a two tailed test with α = .05 could be duplicated using a one tail test with α = .025 since only the higher end of the distribution is important.

$$\text{Var}(x) = (.0009)^2(.20)^2 + (.40)^2(.0009)^2 + 2(.0009)(.40)(.2)(.0009)(.20) = 1.87\text{x}10^{-7}$$

substituting these values into equation (13) yields a 95 percent interval of

$$.000396 \pm .000380 = (.000016, .000776).$$

Thus, under the assumed parameters, it is 95 percent certain that the annual mean pure premium for corporate bond insurance is greater than \$.000016 and less than \$.000776 per dollar of insured corporate bond, or between approximately 1/450 of 1 percent and 1/13 of 1 percent.

Estimate of the Pure Premium

The example in the preceding section serves to demonstrate how the model can be used to develop an estimate of the mean pure premium. In the remainder of the article, historical bond failures are used to estimate the relevant parameters necessary for determining the mean pure premium. Estimates of both the frequency and severity of defaults are required as well as the covariance between frequency and severity.

An estimate of the annual frequency of default was found by dividing the par value of the corporate bonds that defaulted during the year by the par value of all corporate bonds outstanding during the year. Thus, for each year, frequency was measured as

$$\text{Frequency of Default} = \text{PVDB/PVAB}\ , \tag{14}$$

where

PVDB = Par Value of Defaulted Bonds, and,

PVAB = Par value of All Bonds.

The annual severity of default was more difficult to estimate. Ideally, for each firm, the severity of default should be

$$\text{Severity of Default} = \text{ALBI/PVDB}\ , \tag{15}$$

where

ALBI = Actual Losses to Bond Investors, and

PVDB = Par Value of Defaulting Bonds.

However, the actual losses to bond investors for some firms are not directly available. This is caused by the fact that some firms went directly from default to bankruptcy status while others underwent mandatory restructuring but avoided total bankruptcy.

In the first case, the actual losses to bond investors were equal to the total market value of bonds outstanding prior to the firm filing for bankruptcy with the bankruptcy court. Bond investors in the restructured firm, on the other hand, sustained a loss only equal to the drop in market value of their bonds. Thus for these firms the decline in market value subsequent to default (controlling for changes in interest rates) is a proxy measure of investor losses. The corporate bond price three month prior to default compared with the same bond price nine months subsequent to default, is used as an estimate of investor losses that would be reimbursed by an insurer.[14]

The default date is indicated by the date when the company's Standard & Poor's rating was changed to a D, DD, or DDD, or its filing date with a bankruptcy court, whichever came first.[15]

Because of the nature of the available data, it is difficult to measure the spatial covariance between the i^{th} and j^{th} insured risk per dollar of bond coverage, $Cov(x_i, x_j)$, thus it will be assumed that

$$Cov(x_i,x_j) \approx Cov(\overline{x}_t,\overline{x}_{t+1}) \tag{16}$$

where, $\overline{X}_t$ is the mean individual pure risk premium calculated for period t as

$$\overline{X} = E(z)E(y) \tag{17}$$

and it is assumed that during any single period $Cov(z,y) = 0$. After obtaining an estimate of $Cov(x_t,x_{t+1})$, $E(\overline{X})$ and $Var(\overline{X})$ may be calculated using equations (9) and (10).

Assessment of Pure Risk Premium

To estimate the pure risk premium had corporate bond guarantee insurance been available, historical data on corporate bond defaults were obtained from Altman (1987), Standard and Poor's Bond Guide (1970-1985) and Moody's Bond Record (1970-1985). The total year end par values of straight and convertible public corporate debt outstanding during each year from 1970 through 1985 were obtained from Altman's Tables 5 through 7. Also, severity of loss was estimated for each defaulting issue as the decline in market value from three months prior to default until nine months subsequent to default (adjusting for interest rate changes). The bonds for which ratings, at the time

[14] The insurer is obligated to either continue the payment of interest or if callable, to pay accrued interest plus principal. In the case of a bond selling at a discount, the insurer would opt to continue interest payments or buy the bond in the market rather than call it at par; however, in the case of a callable bond selling at a premium, they would opt to pay the par value and retire the issue. Thus, it is assumed that the loss incurred by the insurer is the difference between the price of the defaulted bonds and either the prior market price or par whichever is lower.

[15] Unlisted bond data were obtained from Edward I. Altman's (1987) article on Junk Bonds. Where the company's debt was not rated by Standard and Poor's, its filing date with the bankruptcy court was used.

of issue, could not be found were grouped together with the B and lower rated bonds.[16] Approximately 5 percent of the initial bond ratings were not found.

Table 1 displays the annual estimate for both frequency and severity of default for the period 1970 through 1985. The mean and standard deviation for frequency of default were 0.1682 percent and 0.1732 percent, and the mean and standard deviation for severity of default were $0.886 and $0.164. Severity of default estimates indicate that, on average, $0.886 per dollar of bond par value would have been lost on defaulting bonds.

Table 1

Estimated Frequency and Severity of Default

Year	# of Defaulting Companies	Par Value of Straight & Convertable Debt Outstanding ($MM)	Par Value of all Defaults ($MM)	Default Rate	Mean Severity	Annual Default
1970	14	125,500	932.52	0.007430	0.98	.007281
1971	5	143,000	124.90	0.000873	0.80	.000698
1972	4	154,400	272.59	0.001765	0.81	.001430
1973	10	162,900	199.91	0.001227	0.96	.001178
1974	11	183,500	288.69	0.001573	0.98	.001542
1975	8	209,900	319.73	0.001523	0.96	.001462
1976	7	229,100	113.50	0.000495	1.00	.000495
1977	9	248,300	454.78	0.001832	0.45	.000824
1978	5	258,600	192.20	0.000743	1.00	.000743
1979	3	281,700	30.70	0.000109	1.00	.000109
1980	5	311,900	255.71	0.000820	1.00	.000820
1981	3	331,400	79.61	0.000240	1.00	.000240
1982	17	352,300	995.63	0.002826	0.94	.002656
1983	14	372,900	412.63	0.001107	0.98	.001085
1984	12	413,200	624.11	0.001510	0.83	.001253
1985	18	459,300	1302.02	0.002835	0.53	.001503

$E(z) = 0.001682$ $\qquad E(y) = 0.89$

$Var(z) = 0.000003$ $\qquad Var(y) = 0.03$

$\zeta zy = 0.129$

$Cov(z,y) = 0.129(.001732)(.1640) = .00003664$

The correlation between the frequency and severity of bond defaults was found to be 0.129 for the 16 years of data in Table 1. Using these estimates, the mean and variance (E(x) and Var(x)) for the individual pure risk premium may be estimated using equations (1) and (2).

[16] Reasons given by Standard & Poor for absence of rating are: the bond belongs to a group of securities which are not rated as a matter of policy, viz., financial institutions; where ratings have been assigned for privately placed bond issues; and no rating due to minimal public interest or reduced size and limited after-marketability.

The expected value and standard deviation of the pure premium per year per dollar par value of bond insurance is E(x) = .001534 and σ_x = .001603 respectively. Thus, for a bond with a par value of \$1000 (which is also the present value of the initial liability for the insurer) the yearly expected default cost is \$1.53. If a discount rate of 10 percent and a 30 year bond are assumed, the present value of the expected pure premium is \$14.269.[17]

$$E(p) = [E(z)E(y)]\left[\frac{1-\left[\frac{1-E(z)}{1+k}\right]^{N}}{k+E(z)}\right] + \text{cov}\,(z,y)\left[\frac{1-(1+k)^{-N}}{k}\right] \tag{18}$$

$$= 1.497\left[\frac{1-\left[\frac{1-.001682}{1+.1}\right]^{30}}{.1+.001682}\right] + .03664\left[\frac{1-(1.10)^{-30}}{.10}\right]$$

$$= 1.497[9.2987] + .03664[9.4269]$$

$$= \$14.269 \text{ for a } \$1000 \text{ par value bond.}$$

From an insurance perspective, however, \$14.269 is only the mean for a 30 year bond. Equation (13) may be used to obtain a 95 percent confidence interval for the expected annual pure premium. The spatial correlation or serial correlation, ζ_{ij}, as estimated using the concept in equation (16) results in unstable estimates using the data in Table 1. For all 16 years of data, the spatial correlation is -.0659; however, if 1970 is omitted as an outlier, the spatial correlation is .204. Thus, Table 2 estimates the 95 percent confidence interval for annual pure premiums using a range for ζ_{ij} of 0, .01, .05, and .10.

If one assumes that the annual pure premium would be set at the upper 95 percent confidence level by the insurer and the correlation between risk units is .10, \$2.78 would be charged per year of coverage on a \$1000 bond. For a 30 year, \$1000 non-sinking fund bond, the total pure premium assuming a 10 percent discount rate is \$25.85 or 2.585 percent.[18]

Bond Insurance Compared to Default Yield Spread

To determine if the existence of corporate bond risk insurance is justified, the estimated premium for bond insurance must be compared to the market

[17] For the derivation of equation (18), please see the appendix. The expected pure premium is based on the conditional probability that the insured firm may default in any time period until maturity. The conditional probability results from the fact that if the firm has defaulted in a previous period that it will not default a second time. The available data does not allow the authors to test whether the probability of default increases or decreases with time subsequent to issue. The annual frequency of default observed in Table 1 is E (z) = .1682 percent; however, the time since issue for each bond default was not available. Even if these data were collected, because of the small number of bonds defaulting during the sample period, the bonds defaulting in each year or interval of years subsequent to issue would be small and the results unreliable.

[18] The existence of a sinking fund will lower this rate. Also, no operating expenses are included in the pure risk premium estimate.

Table 2

Ninety-Five Percent Confidence Interval for Estimated Pure Premium per Year of Coverage for Corporate Bonds per Dollar Par Value

ζ_{ij}	Confidence Interval	Mean Pure Premium
0.0	(.0007484,.0023196)	0.001534
0.01	(.0006915,.0023765)	0.001534
0.05	(.0004947,.0025733)	0.001534
0.10	(.0002918,.0027762)	0.001534

derived premium for default risk. If the present value of the interest savings on an insured bond is greater than the insurance premium, obviously bond insurance is desirable from the corporation's perspective.

The spread between yields on different rated bonds as reported by Standard & Poor's Bond Survey is shown in Table 3. Since it is estimated that the average Standard and Poor's bond rating for defaulting corporations was "BB" when bonds were initially issued (see Table 4), the implied yield spread between "AAA" and "BB" was greater than 121 basis points. Thus, the estimated pure risk premium for corporate bond insurance of 27.8 basis points is considerably lower than the market determined default risk premium. The 27.8 basis point pure risk premium compares very favorably with the default yield spread between "AAA" and "AA" bonds. This strongly suggests that bond insurance is warranted if corporations, through the use of corporate bond insurance, can issue debt to yield approximately the "AAA" rate.

Analysis by Bond Rating

A natural extension of this investigation is to estimate the expected pure premium by bond default rating rather than using the average rating for all

Table 3

Rating Distribution Yield Spread (1970–1985)

Bond Ratings	Yield Spread
AAA to AAA	0 basis points
AA to AAA	22 to 160 basis points
A to AAA	76 to 201 basis points
BBB to AAA	121 to 245 basis points

Investigation of municipal bond insurers have indicated that AAA rated bond insurer obligations (insured bonds) actually carry a yield rate somewhat above the true AAA rate, thus the values shown in this table may overestimate actual yield spreads. However, the yield spreads reported by Standard and Poor's are so wide that the difference between the insurer's AAA and the natural AAA rate will have little effect on the results.

observations. It is apparent that the frequency of default increases as the Standard & Poor's default rating drops. Only one firm with an initial bond rating of AA defaulted (Johns Manville); whereas, as expected many of the defaulting issues had original ratings of BB or less (see Table 4).

Table 4

Rating Distribution of Defaulting Issues
at Various Points Prior to Default *

	AAA	**AA**	**A**	**BBB**	
Original Rating Number	0	2	7	26	
Percentage**	0%	1.6%	5.6%	20.8%	
Rating One Year Prior Number	0	0	2	11	
Percentage**	0%	0%	1.4%	7.7%	
Rating Six Months Prior Number	0	0	2	2	
Percentage**	0%	0%	1.4%	1.4%	
	BB	**B**	**CCC**	**CC**	**TOTAL**
Original Rating Number	23	47	20	0	125
Percentage**	18.4%	37.6%	16.0%	0	100%
Rating One Year Prior Number	16	60	47	7	143
Percentage**	11.2%	42.0%	32.9%	4.9%	100%
Rating Six Months Prior Number	11	58	58	12	143
Percentage**	7.7%	40.9%	40.3%	8.3%	100%

*Includes convertibles and straight debt issues. Excludes railroad issues. Ratings from *Standard & Poor's Bond Guide*.

** Represents the percent of all defaulted bonds with this S&P rating (i.e., 0% of all defaulting bonds with AAA rating originally or one-year or six months before default had a AAA rating).

To calculate the frequency of default for each bond rating during each of the 16 years of the study, it was necessary to estimate the percentage of par value contained in each rating class for corporate bonds. Thus, a sample of 200 bonds, randomly selected, were used each year to estimate the percentage of par value of bonds in each rating class. The percentage of par value in each class was then used to calculate the relative frequency of default during each year. Table 5 presents these results.

Table 6 displays the mean pure premium and the 95 percent confidence interval where it was assumed that the correlation between frequency and severity was still .129 as was found for the entire sample.

Assuming that the pure risk premium is the upper estimate of the 95 percent confidence interval, even for bonds rated B and lower, the estimated cost of default is 116 basis points per year. This is still significantly different from the default yield spreads observed in the market as reported in Table 3.

Table 5

Estimated Frequency of Default by Standard & Poor's Rating

Year	AA Default Frequency (Severity)	A Default Frequency (Severity)	BBB Default Frequency (Severity)	BB Default Frequency (Severity)	B & Lower Default Frequency (Severity)
1970		0.02332 (1)	0.00341 (1)		0.02393 (0.97)
1971					0.01397 (0.80)
1972				0.021704 (0.63)	0.00257 (1)
1973			0.00623 (1)		0.00803 (0.94)
1974			0.00307 (0.94)	0.010308 (1)	0.00723 (1)
1975		0.00392 (1)	0.00409 (1)	0.000951 (0.70)	0.00064 (1)
1976				0.002170 (1)	0.00564 (1)
1977		0.00248 (1)	0.00580 (0.27)	0.004476 (0.67)	0.00209 (1)
1978			0.00512 (1)	0.000951 (1)	0.00128 (1)
1979		0.00004 (1)		0.000136 (1)	
1980			0.00589 (1)	0.0004056 (1)	0.00161 (1)
1981			0.00051 (1)	0.0004156 (1)	0.00241 (1)
1982	0.00214 (1)		0.00734 (1)	0.005833 (1)	0.01670 (0.92)
1983			0.00102 (1)	0.003526 (1)	0.01173 (0.92)
1984				0.013023 (0.83)	0.00883 (0.85)
1985		0.00093 (0)	0.01501 (0.4)	0.004340 (0.33)	0.00867 (0.77)
E(z)	0.000133	0.001920	0.003594	0.004265	0.007121
E(y)	(1)	(0.80)	(0.874)	(0.858)	(0.94)
STD(z)	0	0.005823	0.008080	0.006033	0.006651
STD(y)	0	(0.4472)	(0.2192)	(0.2150)	(0.080)

Table 6

Ninety-Five Percent Confidence Interval for Estimated Pure Premium Risk per Year of Coverage for One Dollar of Corporate Bond by Standard & Poor's Rating

Bond Rating	Mean Pure Premium	Confidence Interval**
AA*	0.0001330	
A	0.001573	(0 , 0.005252)
BBB	0.003178	(0 , 0.008696)
BB	0.003692	(0 , 0.007788)
B & Lower	0.006730	(0.001836, 0.011624)

* The only default of an AA rated bond was Johns Manville Corporation in 1982, thus insufficient data was available to estimate a confidence interval.

** The assumed spatial correlation between risk units is .10.

As an example of the cost savings possible with corporate bond insurance, assume that a BBB rated firm wants to issue $50 million of bonds. The minimum cost to the firm will be 121 basis points above the AAA bond rate. Assuming a 20 basis point loading fee in addition to the upper 95 percent confidence interval estimate for pure premium of 86 basis points (Table 6), the annual cost savings to the corporation per year is

Cost of Average Default Premium	= .0121 * $50MM	=	$ 605,000
Insurance Cost	= .0106 * $50MM	=	530,000
Annual Savings			$ 75,000

where it is assumed that corporate bond guarantee insurance will allow the issue to yield the AAA rate. The annual savings in this case is $1.50 per $1000 bond.[19] Thus, during the period observed in this study, it was found that insurable losses implied by corporate bond defaults were lower than the current market determined default risks. The results of this study seem to be consistent with Fons (1987) and Altman (1987) in that holders of well diversified portfolios of corporate bonds appear to be more than fairly rewarded for bearing default risk.

Conclusions

This article reports on an attempt to determine the feasibility of corporate bond guarantee insurance or risk insurance where the insurer assumes the full obligation of the bond issuer should they default. For the period between 1970 and 1985, it was found that the underwriting risk of this type of insurance was less than the normal yield spreads associated with various default ratings.

[19] For a BBB 30 year, $50 million, nonsinking fund bond, the present value of the 121 basis point premium being as a minimum for uninsured bonds assuming a 10 percent discount rate is $5,703,283 or a minimum savings of $774,955.

A statistical sampling model that allows both non-zero correlations between frequency and severity of default and non-zero spatial correlations between one risk unit and another risk unit was used to estimate the pure risk premium for default. Even when the upper estimate of the 95 percent confidence interval and when a loading fee is added to the insurance premium, it is demonstrated that significant savings in interest rates may still be attained.

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Appendix

The expected pure premium or the insurer's present value of expected losses that would be charged for corporate bond insurance at the time of issue would be a function of the sum of the conditional probabilities of the firm defaulting in any given future period up to maturity times the severity of default. The conditional probability of default for any future period may be calculated as the probability of default in that period given that the firm has not previously defaulted over N discrete periods:

$$\Pr(D) = \Pr(D_1) + \Pr(D_2|NPD_2) + \Pr(D_3|NPD_3) + \ldots + \Pr(D_N|NPD_N) \quad (1A)$$

where

$\Pr(D)$ = the probability of default over N periods,
$\Pr(D_t)$ = the probability of default in period t,
$\Pr(D_t NPD_t)$ = the conditional probability of default in period t given that no prior default (NPD_t) has occurred up through period t.

Equation (1A) may also be written as

$$\Pr(D) = \Pr(D_1) + \Pr(D_2)\Pr(NPD_2) + \Pr(D_3)\Pr(NPD_3) + \ldots + \Pr(D_N)\Pr(NPD_N) \quad (2A)$$

or

$$\Pr(D) = E(z) + E(z)[1 - E(z)] + E(z)[1 - E(z)]^2 + \ldots + E(z)[1 - E(z)]^{N-1} \quad (3A)$$

where

$E(z)$ = the observed frequency of default or the probability of default during any future period.

By including the severity of loss and a discount rate and initially assuming independence between frequency and severity, the present value of the insurance pure premium may be calculated for each dollar of bond coverage.

$$E(p) = E(z)E(y)/(1 + k) + E(z)[1 - E(z)]E(y)/(1 + k)^2 + \ldots + E(z)[1 - E(z)]^{N-1}E(y)/(1 + k)^N \quad (4a)$$

or

$$E(p) = [E(z)E(y)] \left[\frac{1 - \left[\frac{1 - E(z)}{1 + k}\right]^N}{k + E(z)} \right] \tag{5a}$$

where

E(p) = the present value of premium paid resulting from realized losses,
k = the appropriate discount rate.

If the possibility that frequency and severity may have a positive covariance is included in equation (5A),

$$E(p) = [E(z)E(y)] \left[\frac{1 - \left[\frac{1 - E(z)}{1 + k}\right]^N}{k + E(z)} \right] + \text{cov}\ (z,y) \left[\frac{1 - (1 + k)^{-N}}{k} \right] \tag{6a}$$

Equation (6A) is the same as equation (18) in the text.