# Tie-in Arrangements of Life Insurance and Savings: an Economic Rationale

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## ABSTRACT

Cash-value life insurance contracts can be viewed as tying life insurance with savings. The main purpose of this article is to provide an economic rationale for this widespread practice. It is assumed that there is an asymmetry of information between the insurers and insureds, where the latter know better their probability of death, and hence adverse selection may cause a market failure. To overcome this problem, a multiperiod (for exposition purposes a three-period model is used) contract and a single-period contract are offered. The multiperiod contract specifies a savings element which may be received by the insured at his or her discretion if the insured lives at the end of the second period. It is shown that by the right choice of the savings element and the return on this element, the insurers can induce insureds to self select. Insureds with lower probabilities of death will choose the multiperiod contract whereas insureds with higher mortality rates will choose successive single-period contracts. It is therefore shown that tie-in arrangements of savings and insurance may help solve the problem of adverse selection and hence an economic rationale is provided for their existence.

Cash-value life insurance contracts tie savings and risk of death insurance. Many researchers tried to explain the popularity of such tie-in arrangements, and whether they can dominate a strategy of buying risk insurance (i.e., term insurance) and savings separately (see, e.g., Ferrari, 1968; Fortune, 1973; Moffet, 1979; and Winter, 1981). In most cases it was found that if insurance and savings were separately sold in a competitive market, then consumers would be better off without the tie-in arrangements. An equivalent facet of this phenomenon is that the return on savings in the open market is higher than that on the savings part of cash-value life insurance.[1] Various explanations have been given for the tie-in arrangement (see, e.g., Williams and Heins, 1985; Dorfman, 1982; McGill, 1959). Most of these explanations

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[1] See also *Consumer Reports* (1980), (1986).

however, rely on psychological, institutional, or tax arguments.[2] In this article an alternative, informationally-based economic rationale, is provided for tying savings and life insurance.

Life insurance markets are often inflicted by the problem of asymmetry of information between insurers and insureds. The latter usually know better their state of health (and hence their probability of death). In such cases adverse selection may cause the market to fail (see e.g., Akerlof, 1970; Rothschild and Stiglitz, 1976; and Wilson, 1977). It has been shown however, in property-liability insurance markets, that insurers can overcome this problem by offering a limited number of contracts with different amounts of deductible. The insurers in this case induce the insureds to sort themselves according to their claims probability. In life insurance, where deductibles are not available, self-selection (or sorting) by using deductibles is impossible, but may be achieved by the insurers offering contracts with quantity dependent prices.[3] Another method that has been shown in property-liability insurance to induce individuals to self-select, is offering both multiperiod contracts and single-period contracts in such a way that persons with low (high) probability of claiming will select multiperiod (single-period) insurance ( see, e.g., Cooper and Hayes, 1984). Several differences, however exist between life insurance and property-liability insurance which require a special analysis for life insurance. In property-liability insurance, deductibles are used, a practice which is impossible in life insurance. Moreover, property-liability insurance does not tie savings and insurance, whereas cash-value life contracts do so because of either convention (as in the United Kingdom) or law (as in the United States). It will hence be interesting to show that such tie-in arrangements may be helpful in allowing insurers to sort their clients according to their mortality rates.

In the present model, two types of contracts are offered, a multiperiod (actually, three-period) and a single-period contract. The multiperiod contract specifies a savings element which can be collected by the insured if he or she lives at the end of the second period and if he or she chooses to do so. It is shown that by the right choice of the savings element and its yield, insurers may induce individuals with lower mortality rates (that have a higher chance of collecting the savings part) to choose the multiperiod contract, and individuals with higher probability of death to prefer successive single-period

[2] For a comprehensive list of these arguments see Greene and Trieschmann (1984). Berkovitch and Venezia (1991) and Babbel and Ohtsuka (1985) consider the advantages of whole life contracts while neglecting the tie-in arrangements. They showed that the guaranteed renewability feature of such contracts may make them superior to term insurance. Smith (1982) regards cash-value life insurance contracts as a package of options, and Walden (1985) empirically tested this theory. The differential tax treatment that exists between savings through permanent life insurance and other savings vehicles may also explain the preference for the former type of insurance.

[3] See, e.g. Jaynes (1978), and Beliveau (1984). This method may not be feasible since it requires overt price discrimination, and since it requires all insurers to share information with each other about the insurance purchases of every individual.

contracts. [4] It is further shown that such arrangement can work only if savings are tied to life insurance.[5]

The article is organized as follows. The basic model is presented in the next section. A simplifying assumption is made that the insured's strategies are limited to only two: either buy cash-value life insurance for the amount specified or adopt a sequential strategy of buying short term and savings which yields similar bequests. The main motivation for making this assumption is to allow the analysis to concentrate on the choice of contract (cash-value life vs. short term), while holding other decisions (consumption, level of insurance, and savings) fixed. It is noted however that this assumption could be relaxed without altering the results. Concluding remarks follow the analysis.

## The Model

There are only two types of individuals in this market, those with a probability $P_H$ of dying and those with probability $P_L$ of dying, where $P_H > P_L$. (For brevity these individuals will be referred to as "High Probability" and "Low Probability," respectively.) The insurers know of the existence of these two groups of individuals but cannot determine which individual belongs to each group. Furthermore, life insurance is sold in a monopolistic competition market where each insurer breaks even in expectation on each contract. The insurers provide either a multiperiod contract (that will be described below) or a single-period contract. A three period time horizon is assumed, which implies that all die for sure at the end of period 3. This agrees

[4] Many insurers provide individuals who buy cash value life insurance credit points which result in lower prices than those paid by purchasers of term insurance (see, e.g., McGill (1959), p. 384, Black and Skipper 1987, p. 422).

[5] The economic theory of tie-in arrangements (or commodity bundling) in markets other than insurance, provides several reasons for this practice. The most dominant reason for commodity bundling is the cost savings in production, transactions, and marketing associated with package selling. Another important reason is the complementarily in consumption of bundled components. Some authors, however, have shown that in addition to the above obvious reasons for tie-in arrangements, such practices can stem from the following more subtle reason. Adams and Yellen (1976) have shown that the profitability of a monopolist can increase by commodity bundling since this practice may help a monopolist in sorting customers into groups with different reservation prices. This view was first suggested by Stigler (1966) and later elaborated by Paroush and Peles (1981), Schmalensee (1984), and Fisher (1977). The model presented herein has the following advantages over previous models explaining the tie-in practice: First, it explains the tie-in practice by the existence of multiperiod contracts, whereas previous models are all one-period models. This is important because of the multiperiod nature of the demand for insurance, and since the use of multiperiod contracts may enrich the possibilities of sorting the customers. Second, this article postulates a monopolistic competition market whereas in the extant literature the seller is usually a monopolist. Because of the different form of market organization, the nature of equilibrium and the implications of the current analysis differ considerably from those of previous models. The latter stressed that tie-in practices are introduced by monopolists in order to extract as much consumer surplus from the buyers as possible. Such a practice hence seems an undesirable measure. The present analysis, however, shows that in a monopolistic competition market where adverse selection is possible, tie-in practices may be necessary in order to preserve the market.

with the usual practice of sellers of cash value policies to pay the amount covered if the insured reaches the age of 100.

If an individual buys a multiperiod contract this entitles him or her to receive \$1 in case of death either at the end of period t, $t = 1, 2$ or at the end of period three if the insured does not previously die. In addition, the multiperiod contract gives the insured the option to terminate the contract prior to its expiration, at the end of period two, should the insured live at that time.[6] In this case, the insured collects the cash value $R^2S$, where R denotes the return (ignoring mortality) on S, the savings element. Because of mortality the actual return on the savings part is lower than R, and is a decreasing function of the probability of death.

Should the insured live at the end of the second period (i.e. the beginning of period three) and should he or she decline the option to cash the savings element, the insured remains covered until the end of period three. In this case the insured is certain to receive \$1 at the end of that period. From this feature of the multiperiod contract it follows that the value of the life insurance contract at the beginning of period three is $1/R_3$, where $R_3$, is one plus the interest rate for period three. The insured will hence cash the savings element if $R^2S > 1/R_3$, and hold on to the insurance contract otherwise. $R_3$ is unknown at the origin, but becomes known at the end of period two. It is assumed that $R_3$ may obtain either the values $R_{3u}$ or $R_{3d}$ with probabilities and $(1-\alpha)$, respectively. These values are assumed to be such that $(1/R_{3u}) < R^2S < (1/R_{3d})$, and therefore the probability that the insured will cash the savings part is $\alpha$, and the probability that he or she will hold on to the contract is $(1-\alpha)$.[7] The above may be summarized by noting that the insured obtains at the end of period two the maximum between $1/R_3$ and $R^2S$. The insured must pay $\pi_1 + S$ upon purchase of the contract and $\pi_2$ at the beginning of period two in order to keep the contract. The sequence of payments and receipts by the insured if he or she buys a multiperiod contract is described in a tree diagram in Figure 1.

Instead of buying a multiperiod contract that ties savings and insurance the individual could also purchase successive term insurance (one-period) contracts at a price of k per period per unit coverage, and to save in the open market for a return of $R_o$. It is assumed that $R_o$ pertains for periods one and two and is known at the origin . $R_3$ on the other hand is unknown at the origin and will be revealed only at the end of period two.

In what follows, the strategy of an insured who wishes to guarantee \$1 in case of death and $R^2S$ at the end of period two if he or she lives, by means of savings at the open market, and buying single period insurance, will be

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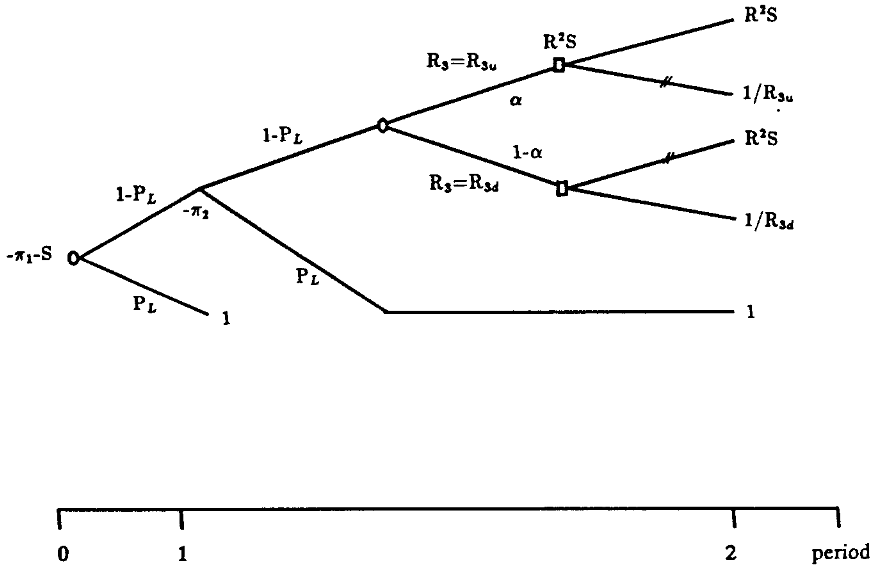
[6] In this model it is assumed there is no savings element at period 1. This assumption was made for analytical convenience and has no qualitative consequences. This also agrees with the usual practice of insurers to provide little or no savings element during the early periods. The lengths of period 1 and 2 need not be the same. To accomplish this, one needs to adjust the discount factor appropriate for each period and make mortality rates functions of time. This generalization will unjustifiably complicate the analysis and notation without adding insight.

[7] If $R^2S > 1/R_{3d}$ then $\alpha = 1$, and if $R^2S < 1/R_{3u}$, $\alpha = 0$.

Figure 1

Sequence of Payments and Receipts of a Low Probability Insured Buying a Multiperiod Contract *, **, ***




* Cash receipts at the end of each period are presented.

** A chance point is designated by a small circle, a decision point by a small square; and an action that will not be taken by a double slash.

***The sequence of payments and receipts of a High Probability insured buying a multiperiod contract is similar to the above, with $P_H$ replacing $P_L$.

investigated. This objective is analyzed since it provides a pattern of cash flows similar to that obtained by an insured who buys the multiperiod contract described above, the only difference being the option of the holder of a multiperiod contract to collect the maximum between $R^2S$ and $1/R_3$, at the end of period two. The buyer of successive single period contracts may also buy life insurance at the beginning of period three. Since by assumption death is certain at the end of this period and since $R_3$ is known at the beginning of the period, buying life insurance at that point is the same as savings, and can be ignored.

The above-stated objective may be achieved as follows. The insured must purchase at the beginning of period one savings for $R^2S/R_o^2$ and insurance coverage of $[1-(R^2S/R_o)]$; at the beginning of period two (if he or she lives) savings for $R^2S/R_o$ and insurance for $(1-R^2S)$. To verify that such a strategy indeed achieves its goals, note that at the end of period one, if the insured dies he or she receives $R^2S/R_o$ from savings and $[1-(R^2S/R_o)]$ from insurance, i.e., a total of \$1. If the insured lives at the beginning of period two, the receipts $R^2S/R_o$ from savings are reinvested, thus guaranteeing at least $R^2S$ at the end of period two. In case of death at the end of the second period, the insured will also receive $(1-R^2S)$ in life insurance, i.e. a total of \$1. The sequence of events and cash flows of the above strategy is described in a tree diagram in Figure 2.

The relative advantages of these two strategies will next be analyzed by comparing the expected utilities they yield. Suppose the insured's preferences are represented by a separable additive utility function defined on consumption if the insured lives, and on bequest in case of death. This function can be written as:

$$U(c_1,c_2,c_3,b_2,b_3) = u(c_1)+\beta w(c_2,b_2)+\beta^2 w(c_3,b_3) \tag{1}$$

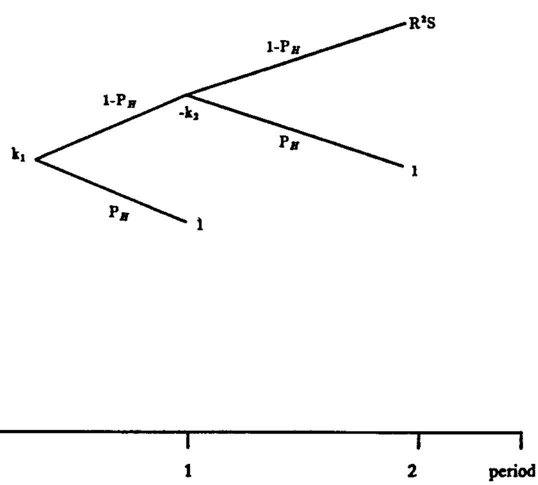
where

$$\begin{aligned} w(c_t,\ b_t) &= u(c_t) \text{ if the insured lives} \\ &= v(b_t) \text{ if the insured lives,} \end{aligned} \tag{2}$$

$\beta$ is a parameter denoting the individual's time preference, and $c_t$, and $b_t$ denote period's t consumption and bequest, respectively. The function u(.) denotes the one-period utility function from consumption, and $\nu$(.) its analogue from bequest. The insured obtains income $I_t$ at period t if he or she lives at that period. All income not saved or spent on insurance is spent on consumption. It is further assumed that the insured is constrained to buy

Figure 2

Sequence of Payments and Receipts of a High Probability Insured Buying a Successive Single-period Contracts




$$\begin{aligned} k &= P_H/R_O \\ k_1 &= (R^2S/R_O{}^2)+k(1-R^2S/R_O) \\ k_2 &= k[1-R^2S] \end{aligned}$$

either the MP or the SP contract. This is a strong assumption. It has been shown however (see Venezia, 1990) that the results of this article hold even when this assumption is relaxed. The model is then cast in a dynamic programming framework which can be solved for quadratic utility functions. The mathematical complexity of the alternative model however obscures the economic intuition explaining the tie-in arrangement. The simpler and clearer model has therefore been chosen in order to better convey the ideas of this article. From the above assumption it follows that the expected utility of an insured of type i, i = L, H buying the multiperiod contract described in the former section is given by

$$\begin{aligned} E\,[U_i|\,MP] &= u(I_1 - S - \pi_1) + \beta[(1-P_i)u(I_2 - \pi_2) + P_i v(1)] \\ &\quad + \beta^2\{(1-P_i)^2\,[\alpha u\,(I_3 + R^2S) + (1-\alpha)u(I_3 + 1/R_{3d})] \\ &\quad + P_i(1-P_1)v(1)]\} \quad i = L,H \end{aligned} \tag{3}$$

The above can be verified by noting that if the insured buys the multiperiod contract then he or she pays $(S + \pi_1)$ at period 1 and has for consumption at that period $(I_1 - S - \pi_1)$. At period two if the insured lives he or she has $(I_2 - \pi_2)$ for consumption. In case of death the amount bequeathed is 1, and the corresponding utility is (1). The probabilities of death and life are $P_i$ and$(1 - P_i)$, respectively. At the end of period two if the insured lives, he or she will either cash the savings and receive $R^2S$, or receive the value of the life insurance contract $1/R_3$ (whichever is higher). The insured will then have at period three for consumption either $R^2S$ with probability $\alpha$ or $1/R_{3d}$ with probability $1-\alpha$. The probabilities of life and death at the end of period 2 are $(1-P_i)^2$ and $P_i(1-P_i)$, respectively. Similarly the expected utility if the individual buys successive single-period contracts and uses the same strategy described in Figure 2 is given by

$$\begin{aligned} E\,[U_i\mid SP] &= u(I_1 - k_1) + \beta[(1-P_i)u(I_2 - k_2) + P_i v(1)] \\ &\quad + \beta^2\,\{\,(1-P_i)^2\,u(I_3 + R^2S) + P_i(1-P_i)\,v\,(1)] \\ &\qquad i = L,H \end{aligned} \tag{4}$$

where

$$\begin{aligned} k_1 &= k[1 - R^2S/R_o] + R^2S/R_o^2 \\ k_2 &= k[1 - R^2S] \end{aligned} \tag{5}$$

The insurer's expected profits from each contract are next analyzed. These expected profits depend on the relative proportions of high and low mortality individuals who buy MP and SP contracts. Anticipating a self separating equilibrium where low (high) mortality individuals buy only MP(SP) contracts, the insurer's profits are given by[8]

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[8] If proportions $\gamma_H$ and $\gamma_L$ of H and L types respectively buy MP, then $P_L$ should be replaced in (6) by $(\gamma_L P_L + \gamma_H P_H)$, and $P_H$ should be replaced in (7) by $((1-\gamma_L) P_L + (1\text{-}\gamma_H) P_H)$. In a separating equilibrium $\gamma_H = 0$ and $\gamma_L = 1$.

$$F(MP) = \pi_1 + (1-P_L)\,\pi_2/R_0 + S - \{P_L/R_0 + (1-P_L)P_L/R_o^2 + (1/R_o^2)\,[\alpha(1-P_L)^2\,SR_o^2 + (1-\alpha)\,(1-P_L)^2/R_{3d}]\,\} \quad (6)$$

$$F(SP) = [k - P_H/R_0] \quad (7)$$

The above analysis provides grounds for developing the necessary conditions for self selection. Before proceeding with these conditions, however, the alternatives of the insured and insurer will be discussed. The insured is constrained by assumption, to buying either the SP or the MP contracts. The amount of savings specified under the MP contract is not a decision parameter in the hands of the insured. Rather, the insurer determines, S, (together with other parameters) so as to achieve separation. The insured will then have to decide whether to buy the specified MP contract or to adopt the SP strategy.

The insurers have to consider the following two points when determining the pricing and structure of the MP contract. First, the contract must induce separation. That is, the contract must attract low mortality individuals to buy MP while discouraging high mortality ones from doing so. Second, given that the contract MP induces separation, it should be "best" for the insured, or else some other insurer will take over the client.

The equilibrium concept used in this article is the reactive equilibrium concept developed by Riley (1979). The motivation for choosing this equilibrium concept is the following. Rothschild and Stiglitz (1976) have shown that a Nash equilibrium may not exist in one-period adverse selection models. In their model, the separating equilibrium contracts may not be stable because they may be countered by some pooling contract offered by other insurers which will attract both the high and the low mortality insureds. The pooling equilibrium may not be viable since yet other insurers could offer other contracts to lure the low mortality insureds from the pool. No Nash equilibrium will exist in this case[9]. Given this negative result, the question arises how may such an insurance market behave? Presumably in the absence of collusion each insured will eventually learn to expect some reaction by other insurers to his or her offers. Suppose the insurer offering the pooling contract suspect that his or her offer will be counteracted by yet another insurer offering a contract that will attract low mortality individuals from the pool. In this case the former insurer will not offer the pooling contract at all, and the equilibrium will be stable. The reactive equilibrium concept provides an equilibrium solution when expectations about competitors reactions fit the above description. This equilibrium concept has thus been chosen in this article since the present model fits within the framework of the models of Riley

[9] The possibility of a pooling contract being offered depends on the relative proportions of high and low mortality individuals in the population. If the proportion of high mortality (i.e., low quality) individuals is high, then a pooling equilibrium is not profitable and a Nash equilibrium will exist (see Riley, 1979, p.346).

(1979) and Rothschild and Stiglitz (1976).[10] A reactive equilibrium contract c requires that for any contract not in equilibrium, say c' (i.e., a defection from equilibrium) there exists a reaction c'' such that c' incurs losses when only these three contracts are tendered, and c'' does not incur losses when these three contracts are tendered, whether or not other contracts are also offered. Riley (1979) has shown that no reactive pooling equilibrium may exist, since any such pooling contract may be reacted against. It hence follows that a reactive separating equilibrium will exist if the insurer will specify the parameters S, $\pi_1$, $\pi_2$, and R so that: 1) only low (high) mortality individuals buy the MP (SP) contract; 2) insurers will earn zero expected profits on both MP and SP contracts; and, 3) the parameters S, $\pi_1$, $\pi_2$, and R maximize the expected utility of low probability individuals subject to the satisfaction of constraints 1 and 2 above. The equilibrium conditions may be stated as:

$$E[U_L \mid MP] > E[U_L \mid SP] \tag{8}$$

$$E[U_H \mid MP] < E[U_H \mid SP] \tag{9}$$

$$k = P_H/R_o \tag{10}$$

$$\begin{aligned}\pi_1 + S + (1-P_L)\,\pi_2\,R_o &= P_L/R_o + (1-P_L)\,P_L/R_o^2 \\ &\quad + (1/R_o^2)\,[a(1-P_L)^2R_o^2S + (1-\alpha)\,(1-P_L)^2/R_{3d}]\end{aligned} \tag{11}$$

$$E[U_L \mid MP, S, \pi_1, \pi_2, R] \geq E[U_L \mid MP, S', \pi'_1, \pi'_2, R'] \tag{12}$$

for all $S_1$, $\pi'_1$, $\pi'_2$, $R'$ satisfying (8) through (11)

Note that (8) and (9) ensure that low (high) probability individuals buy MP (SP) contracts and (10) and (11) express the zero profits conditions for SP and MP contracts, respectively.

Based on the above it can be shown that:

*Proposition 1* A reactive equilibrium exists if parameters c = (S, $\pi_1$, $\pi_2$, R) exist such that inequalities (13) and (14) below hold, as well as equalities (10) and (11). In this case, an equilibrium contract c* must maximize $E[U_L|MP]$ subject to (8) through (11).

*Proof*: Subtracting (3) from (4) while plugging $P_L$ for $P_i$, and rearranging terms one obtains the following equivalent expression for (8):

$$\begin{aligned}&(1-\alpha)\,(1-P_L)^2\,\beta^2\,\{\,u[I_3 + 1/R_{3d}] - u[I_3 + R^2S]\,\} \\ &\quad > u(I_1 - k_1) - u(I_1 - \pi_1 - S) + \beta\,(1-P_L)\,[u(I_2 - k_2) - u(I_2 - \pi_2)]\end{aligned} \tag{13}$$

From (5) and (10) it follows that $k_1$ and $k_2$ are given by:

$$\begin{aligned}k_1 &= (P_H/R_0)\,[1 - R^2S/R_0] + R^2{}_o \\ k_2 &= (P_H/R_0)\,[1 - R^2S]\end{aligned} \tag{14}$$

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[10] Since insureds are constrained to buy either MP or SP contracts, this implicitly rules out other offers. It hence seems that a Nash equilibrium concept (instead of the more complex Riley's reactive equilibrium) may be sufficient. However, as mentioned above, the present model could also be extended to the more general case where the insured can choose other insurance and savings contracts, Riley's equilibrium is needed for these cases. It is also noted that the Reactive Equilibrium concept has been used in many other applications (see e.g., Miller and Rock, 1985).

An explicit expression for (9) is obtained in a similar manner, namely: subtracting (3) from (4) while plugging $P_H$ for $P_i$, obtaining

$$(1-\alpha)\,(1-P_H)^2\,\beta^2\,\{u[I_3+1/R_{3d}]-u[I_3+R^2S]$$
$$< u(I_1-k_1)-u(I_1-\pi_1-S)+\beta(1-P_H)\,[u(I_2-k_2)-u(I_2-\pi_2)] \tag{15}$$

If a set of parameters c = ($\pi_1$, $\pi_2$, R, S) is found such that (10), (11), (13), (14) and (15) are satisfied, then a Nash equilibrium exists. For a reactive equilibrium, a contract c must be found that is Nash and also maximizes the expected utility of low mortality individuals. This is necessary so that the equilibrium cannot be reacted against. The maximization of the expected utility of only the low mortality individuals is required, because only these individuals purchase MP contracts. Since the set c of contracts which satisfy the separation requirements is closed, and since $E[U_L|MP]$ is bounded, it follows that if c is not empty, then there exists a contract that maximizes $E[U_L|MP]$. A reactive equilibrium will then exist. QED

Note that the above proposition does not imply that there always exists a reactive equilibrium. The existence depends on the particular parameters (e.g., if $P_H$ is very close to $P_L$ a separating equilibrium may not exist).

Having shown what the conditions for self-selection are, the following proposition explains the importance of tying savings and life insurance. It is shown that:

*Proposition 2*: Self-selection equilibrium cannot be achieved with zero savings and positive insurance prices.

*Proof*: If S = 0, the possibility that the insured will use the cash value at the end of period two becomes irrelevant. In this case the wealth of the insured at the end of period two is the same under the MP and the SP contracts. The conditions for separation (13) and (15) therefore become, after some rearrangement of terms:

$$u(I_1-\pi_1)-u(I_1-k_1) > \beta(1-P_L)[u(I_2-k_2)-u(I_2-\pi_2)] \tag{16}$$

and

$$u(I_1-\pi_1)-u(I_1-k_1) < \beta(1-P_H)\,[u(I_2-k_2)-u(I_2-\pi_2)] \tag{17}$$

respectively. Subtracting (16) from (17) one obtains

$$\beta\,(P_H-P_L)\,[u(I_2-k_2)-u(I_2-\pi_2)] > 0 \tag{18}$$

which requires that $\pi_2 > k_2$. Since S = 0, $k_2 \equiv P_H/R_O$, and hence $\pi_2 > k_2$ implies $\pi_2 > P_H/R_0$). In this case the price of one unit of insurance in period two under the MP contract is higher than that for one period. The low mortality person will therefore discontinue the MP contract. It thus follows that a self separating equilibrium cannot be reached unless S > 0. QED

To better understand the importance of the tie-in arrangement for the self selection equilibrium the expected monetary benefits and costs to both types of insureds from buying MP are next analyzed. In the absence of major differences in risk attitudes of high mortality and low mortality individuals, differences in cost/benefit of the MP contract for the two groups may explain preference (aversion) of the L (H) individuals for this contract. The conditions of zero expected profits for the insurer imply that the expected costs equal the expected benefits for low mortality insureds. The following equation (19) must therefore hold.

Abstracting from risk preference arguments, expected costs for H type insureds must exceed expected benefits in order to deter them from buying MP, and therefore inequality (20) must hold. That is

$$\pi_1 + \pi_2(1 - P_L) / R_o = P_L / R_o + P_L(1 - P_L) / R_o^2 + [(1 - P_L)^2 / R_o^2] [\alpha/R_{3u} + (1 - \alpha) / R_{3d}] \tag{19}$$

$$\pi_1 + \pi_2(1 - P_H) / R_o > P_H / R_o + P_H(1 - P_H)/R_o^2 + [ (1 - P_H)^2 / R_o^2] [\alpha / R_{3u} + (1 - a)R_{3d}] \tag{20}$$

The left sides (right sides) of equations (19) and (20) represent the expected discounted costs (receipts) of buying a multiperiod contract by a low mortality and high mortality individuals, respectively. The expected costs of the higher mortality person are lower than those of the lower mortality person, since there are lower chances that the higher mortality person will have to pay $\pi_2$ at the beginning of period two. Obviously, the former's expected benefits are higher. Since in equilibrium with no monopolistic profits to the insurer expected costs and benefits of low mortality persons are the same, it follows that when S = 0 the insurer will lose in expectation on the MP contract purchased by the H type person. This contradicts equilibrium in a monopolistic competition market. The introduction of the savings element is tantamount to providing a discount rate option to the insured. Since there are higher chances, $\alpha(1 - P_L)^2 > \alpha(1 - P_H)^2$, that a lower mortality individual will exercise this option, the option is more valuable for a low mortality person than for a high mortality individual. Consequently, by a right choice of the cash value, the expected discounted revenues of the insured can be adjusted so that those of the Low Probability person will be higher than those of the High Probability person, and a separating equilibrium could be achieved.

In property-liability insurance, multiperiod contracts can be used to sort the insureds without a tie-in arrangement by using a bonus-malus (merit-demerit) pricing method. That is, (see, e.g., Venezia, 1983) should the insured claim at some period, his or her future premium rates will rise. A less risky individual is more likely to choose such a contract since his or her chances to claim, and hence be penalized by future higher premia, are lower than those of a riskier individual. In life insurance this method is impossible, but tie-in arrangements of savings and insurance closely resemble it. The merit to the low mortality individual is the higher chance of using the option to cash the savings element, should future changes in interest rate warrant such action.

Having shown what the conditions for self-separating equilibrium are, a numerical example is provided showing that indeed such equilibrium can be obtained. Suppose $I_3 = 0$, $I_2 = 16$, $I_1 = 23$. Suppose further that the one period utility function of the insured is exponential with a measure of a risk aversion $\Gamma = 0.1$. The bequest utility function $\nu(.)$ is not specified since it adds the same value to E[U|MP] and E[U|SP] and therefore does not affect the choice between a multiperiod contract and successive single-period contracts. The risk-free discount factor is $\beta = 0.9$ and $R_3$ may assume the values $R_{3u} = 1.055$, and $R_{3d} = 1.0074958$, each with probability .5. $P_L$ is assumed to be .1, and $P_H = .15$.

A separating equilibrium may be achieved in this case by the insurer offering a multiperiod contract with the following parameters: $R^2 = 1.14$, $S = .8333$ and $\pi \equiv \pi_1 \equiv \pi_2 = .0693082$. That is, the insured pays $(\pi + S) = .9026415$ at the origin and $\pi = .0693082$ at the beginning of period 2. If the insured lives at the end of period 2, he or she receives $R^2S = .95$ if $R_3$ turns out 1.055 or $(1/R_{3d}) = .99256$ if $R_3$ turns out $R_{3d}$. In case of death the insured receives 1. The parameters $k_1$ and $k_2$ are found from (23) and (24) to attain the values .9401039 and .0074257, respectively.

Based on the above it can be shown that the expected utility of a low mortality individual if he or she buys the multiperiod contract (successive single-period contracts) is -.9824 (-.9926). The expected utility of a high mortality individual if he or she buys the multiperiod contract (successive single-period contracts) is -.9927 (-.9923). It hence follows that the Low Probability individual will choose the multiperiod contract and the High Probability individual, successive single period contracts. The insurers will break even on each contract, and a separating equilibrium is thus achieved.

The existence of a separating equilibrium crucially depends on the volatility of interest rates. The tie-in arrangement provides a put option on future interest rates. This option is more likely to be exercised by low mortality individuals, and hence it is more valuable to this type of insureds. The higher the variability of interest rates the higher the value of the option, and hence the better the chances for a separation to be achieved. The level of interest rate has a negative effect on separation possibilities. The higher the interest rates the lower the expected present value of the option, and hence the lower the chances for a separation to be achieved.

Differences in mortality rates are necessary for a separation to exist. Too large a difference however between $P_H$ and $P_L$ may prove obstructive to finding a separating equilibrium. In such a case it will be difficult to find a contract that will break even for the L type individuals, and such that H types will pass in favor of the more expensive SP contracts.

Separation also depends on the nondivisibility assumption, i.e. on the assumption that the individual is constrained to buying either the MP or the SP contracts. This assumption however can be relaxed (see Venezia, 1990) without altering the results. Further discussion on the implications of the model is provided in the following section.

## Conclusion

It has been shown that in a world of asymmetric information, tying life insurance with savings can induce insureds to self-select. Insureds with lower mortality rates will prefer to buy multiperiod contracts (i.e., whole life contracts) tying savings and risk of death insurance, whereas higher mortality individuals will buy short-term life insurance and savings separately. This type of behavior may explain findings showing that returns on the savings part of cash-value life contracts are lower than those provided in the open market. According to the present model, the mortality rates of insureds buying cash-value contracts are lower than those of insureds buying short-term contracts and hence lower than the population average. The estimated yields on the savings part of cash-value contracts are therefore downward biased, since they assume the mortality rates of their buyers are the same as those of the entire population. It may well be that if the mortality rates appropriate for cash-value insurance buyers were used, the estimated differences between the yields on the savings parts of cash-value insurance contracts, and on savings in the open market would only reflect the tax differences between these two forms of savings.

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