

An Exploration of an Individual's Decision-Making Regarding Tax-Deferred Investment Plans

M. Hadi Behzad
Patrick S. Lee
Gautam Vora*

ABSTRACT

Several decision making models are presented in this article for allocating an individual's savings in tax-deferred opportunities. Because the models are general, they can examine any number of tax-deferred opportunities. The resource allocation problem is formulated as a linear program. Historical or forecast values of costs and returns are used as exogenous parameters of the linear program. The model is simulated under different scenarios to demonstrate that the linear programming approach can be fruitful, simple, and insightful in bridging the gap between theoretical findings and actual investment in various tax-deferred opportunities.

The Internal Revenue Code of 1954, as amended, allows certain groups of individuals the opportunity to defer tax liability on contributions to individual retirement plans. Interest income on these plans is also tax-deferred. While statutory limitations on maximum contributions must be satisfied, the result is not only an attractive device for supplemental retirement income but also a potentially important investment vehicle in an individual's investment portfolio.

Employees of educational, religious, charitable, scientific, and certain other tax-exempt organizations, as described in section 501(c)(3) of the Code, are granted special income tax allowances in setting aside retirement funds. The self-employed also are allowed to benefit from a similar tax-deferral opportunity as provided by the Self-Employed Individuals Tax Retirement Act of 1962 (otherwise known as the Keogh Act), as amended. As described in the Employee Retirement Income Security Act (ERISA) of 1974, as amended, eligible individuals may set up a tax-deferred Individual Retirement

* M. Hadi Behzad is Associate Professor of Management and Finance at California State University, Hayward. Patrick S. Lee is Assistant Professor of Management at La salle University. Gautam Vora is Assistant Professor of Management at the University of New Mexico.

The authors thank two anonymous referees of the *JRI* and the editor for their patience and helpful suggestions.

Arrangement (IRA). In addition, employees are permitted to defer taxes by contributing to employer-sponsored savings plans, such as 401(k) plans, through salary reduction.¹ These individuals may choose to fund their supplemental retirement income with before-tax dollars within the limits specified by the legislative acts; these mechanisms are known as tax-sheltered or tax-deferred retirement plans.

One prominent tax-deferred retirement plan is called a tax-deferred annuity (TDA). A TDA is a retirement savings scheme funded by an eligible employee through purchase of an annuity contract or a mutual fund. An employee of a qualified organization is allowed to purchase a nontransferable, non-forfeitable annuity contract, for example, by making periodic contributions. Thus, an individual can set aside money during the working years, *i.e.*, the accumulation period, and receive periodical sums after retirement, *i.e.*, the withdrawal period or liquidation period.

In a pioneering article, Mehr (1968) presents in detail the nature of decisions and issues involved in using a TDA as a funding mechanism for retirement. He argues that the TDA decisions are complex and unique to an individual's circumstances. These decisions involve factors whose projected values are uncertain and "in many cases pure guess work" (Mehr, 1968, p. 210). He concludes that:

the amount that he should commit to the tax-sheltered annuity can be anywhere from zero to the amount that he can afford but in no event more than [the maximum allowable] for the year (Mehr, 1968, p. 215).

But he does not provide a method for determining a desirable level of contributions. Several other studies focus on purchase decisions and describe factors important in evaluating TDA plans. Greene and Copeland (1975) identify the following nine factors for consideration in evaluating different plans: interest rates, investment performance of the insurer, charges associated with the plan, guaranteed cash values, annuity rents, mortality benefits, lump-sum transfers, quality of service, and availability of investment funding options. Doyle (1984), O'Neil, Saftner and Dillaway (1983), Smith (1984), and Jacobs and Holland (1984) examine TDA plans as an interim accumulation vehicle. They analyze the feasibility of withdrawal before retirement. Their numerical results show that the tax-deferred aspects of a particular retirement plan are more advantageous for intermediate-term holding periods.

¹ To encourage individual savings, the Bush administration is planning to propose a new tax-sheltered 'Family Savings Account'. The *Wall Street Journal* (1989) reported that the Bush administration is planning to propose a new tax-incentive for savings by individuals. The current idea, though not firm, is to let individuals contribute as much as \$5,000 per family each year to 'Family Savings Accounts'. According to the *Journal*:

The plan would offer no immediate tax deduction. But if the money were left untouched for 10 years, under the current version of the plan, interest earned on contributions would escape taxation altogether. The accounts would be offered to Americans of all incomes and would be in addition to existing Individual Retirement Accounts.

Other researchers study the benefits of TDA plans by means of a comparison with taxed investments. Morehart and Trennepohl (1979) use a simulation model to compare projected retirement income available from a TDA plan with that of a taxed investment alternative. They conclude that the anticipated rates of return on the alternative plans are the most important variable. Holding other factors constant, they show that investment in a TDA plan will produce 1 percent to 3 percent higher rates of return over a taxed alternative.

Healy (1981) compares the future values of a hypothetical sum under both a TDA plan and a taxed investment plan with the same rate of return. He finds that the accumulated fund under a TDA plan is twice as large as that under the taxed alternative if the rates of return are identical on both the investments and the tax liability on TDA withdrawals is ignored. Todd (1976) presents an analysis of factors that affect the evaluation of TDA plans and concludes that rates of return and load fees are crucial elements in comparing TDA plans. In another article, Todd (1978) uses a simulation model to compare retirement incomes of a TDA plan and a taxed investment assuming several independent variables. Assuming identical rates of return and equal take-home incomes, he concludes that investment in a TDA plan is superior to investment in a taxed alternative, even when the tax rate at retirement is higher.

Adelman and Dorfman (1982) assume a fixed rate of interest in computing the future value of a TDA plan and then calculate the interest rate necessary for a non-TDA investment to accumulate the same future value at the beginning of retirement. They conclude that only under a situation where the marginal tax rate during the withdrawal period is higher than that during the accumulation period may one be better off not participating in the TDA plan. In another study, Dorfman and Adelman (1983) use the rate-of-return approach to analyze the impact of the tax incentive on the attractiveness of the plans and concur with their previous finding: the TDA is a good investment for retirement income purposes. Gahin (1983) provides additional analysis on the financial feasibility of tax-deferred retirement plans. He introduces a cashflow model for measuring the net present value of a TDA plan which then can be compared with a taxed investment opportunity. He favors investment in tax-deferred plans over taxed alternatives only when an individual's marginal tax rate at retirement is expected to be smaller than the corresponding rate during the accumulation period. He concludes that taxed investments are essentially impractical vehicles for retirement funding.

In addition, there are other noteworthy studies that are relevant to the subject under discussion. Burkett (1976) and Stoeber (1979) describe important legal and taxation aspects of TDA plans and discuss regulatory limits on contributions. Belth (1982) provides a method of calculating realized rates of return on retirement accumulations. Duncan, Mitchell and Morgan (1984) develop a framework for assessing the retirement needs of an individual. They suggest a method for planning a consumption level that would adjust savings needs to meet retirement goals. Greene, Neter and Tenny

(1977) study annuity payouts of various TDA plans to compare performance of the plans offered by 47 insurers. They recommend that annuity payouts must be evaluated when comparing TDA plans. Lastly, Schnee and Caldwell (1984) describe the advantages and limitations of plans under Section 403(b) and provide some general guidelines for using these vehicles in retirement planning.

In reviewing the literature, the authors observe four deficiencies in the treatment of the subject. First, investment in a TDA plan is viewed as a static funding decision where an individual is faced with making the decision on a once-and-for-all basis, *i.e.*, the individual must either participate in a plan now or forego the opportunity altogether. The authors argue that the dynamic aspect of decision-making over the planning period must be considered because of the long-term nature of the decision. The annuity decision is not a fixed one and must be reviewed periodically. Second, much of the existing literature conceives of only two choices for the contribution amounts, either the full extent or nothing at all. The authors argue that the contribution amounts depend on the accumulation goals, and an individual must have a method of determining these amounts. Optimal contributions may be amounts between the two extremes and could vary over the planning period. Third, the existing literature has evaluated the decision about tax-deferred opportunities by contrasting it with the decision to invest in taxed opportunities. The authors argue that the tax consideration alone should not be the determining factor in the investment decision. A tax-deferred opportunity plan needs to be evaluated on its own merits and against other tax-deferred opportunities.² A well-known tax-deferred opportunity is the IRA. At the same time, many employers have set up 401(k) plans with salary-reduction agreements for employees. Because these opportunities are not mutually exclusive, individuals must evaluate available opportunities while making their decision. The amount of income tax saved because of investment in TDAs often becomes the sole determining factor for investment. Individuals make comparisons between taxed opportunities and tax-deferred opportunities and invest their funds in randomly chosen opportunities regardless of other parameters of the investment decision.³ The approaches mentioned in the literature provide little

² It seems that the academic literature has followed the trail blazed by the advertisements and prospectuses issued by insurance and investment companies. The brochures of insurers, as well as the advertising booklets of mutual funds, invariably compare 'taxed investments' with 'tax-deferred investments' and then extol the virtues of tax-deferred investments. They never offer any advice to their readers on the selection of tax-deferred opportunities or on the allocation of investible funds among the opportunities.

³ Misleading and incomplete comparisons continue to flourish. For an example, this quote from a recent "information" leaflet of Fidelity Investments, the largest family of mutual funds, touts its variable annuity plan which offers only a partial tax deferral:

When you invest in a Fidelity Variable Annuity, you get in on one of the few tax-advantaged investments left after tax reform.

All of your Variable Annuity dividends and capital gains are entirely tax-deferred during the accumulation phase. Over a period of years, you can earn significantly *more* from the Variable Annuity than you would from a taxable investment of equal performance. This may help make

help in making a decision about tax-deferred opportunities. Fourth, the approaches in the literature ignore the need for considering the decision as part of a carefully devised long-term plan of personal financial management. The authors argue that when an individual is able to evaluate several tax-deferred plans, he or she will have the ability to make more enlightened choices.

This article analyzes the problem of allocating retirement savings in tax-deferred opportunities such as several TDAs and/or IRAs. The problem is formulated as a multi-period, multi-plan mathematical program. The programming problem can be easily solved using linear programming or nonlinear programming methods, depending on the objective functions and constraints used. To provide a better understanding of the objective functions of the models, the focus is placed on problems formulated as linear programs.

The remainder of the article is organized as follows: a statement of the problem is given, the decision-making using linear programming is explored, the results of simulations based on linear programming models are presented, and lastly the concluding remarks are offered.

A Statement of the Problem

The problem considered is that of an individual⁴ faced with the choices of investing in different opportunities under different provisions of the IRS Code, as amended.⁵ The individual needs to allocate his or her savings into several investment opportunities periodically. The concern is not with a static portfolio allocation problem, but with a dynamic allocation of the savings to attain a desired accumulation goal at the time of retirement. This goal would generally represent the present value at the first withdrawal date, which is one period after the date of retirement, or at the current date, of all the periodic

a big difference to the standard of living you enjoy during your retirement years. And, that may make it possible for you to take early retirement if you desire.

Of course, once you start receiving annuity payments, only your tax-deferred income will be subject to taxes (not the principal). And by that time, you may be retired and might be in a lower tax bracket.

⁴In this article the word individual is used to mean a decision-maker or tax-payer. The filing status of this decision-maker would depend on his or her marital status. In the discussion in the text below, the filing status of the tax-payer, *e.g.*, single, joint married, or separate married, would be clear from the context or the assumption.

⁵Note the often-ignored subtle distinction between the provisions of the IRS Code and the actual investment in assets. The provisions of the Code, *e.g.*, 401(k), 403(b), permit an individual to defer taxes on a certain specified sum of money which may be invested in assets (or instruments), *e.g.*, stocks, bonds, bullion, of his or her choice. In popular parlance, however, the phrase "TDA investments" is used to denote an investment in a TDA plan without regard to the assets actually bought for that purpose. Similarly, the phrase "IRA investments" is used to denote an investment in an IRA plan without regard to the assets actually bought for that purpose. Most individuals derive the full meaning of the phrase by the context of its use without getting confused or splitting semantic hairs.

withdrawals over the liquidation period.⁶ Further, the concern is not with the investment in individual types of assets such as stocks, bonds, diamonds, real estate, etc., but with the choices within a particular set of provisions of the Code. This set of provisions, as mentioned in an earlier section, allows deferment of income taxes; the investment, *per se*, could be made in almost any asset. Thus, in defining the feasible set of opportunities for investment, the assumption is that the individual considers only tax-deferred opportunities. Therefore, it is not necessary to compare taxed and non-taxed investments. The individual sets the goal for the first withdrawal date in the liquidation period, and the goal will be the accumulation of specified amounts in different plans.

Without any loss of generality, it is assumed that the individual has identified several tax-deferred opportunities, *e.g.*, IRAs and TDA plans.⁷ Now, he or she faces two immediate decisions: the amount of money that should be invested in these opportunities, and the distribution of the savings among these opportunities. The second part of the problem may seem to have few practical implications, but it must be addressed in order to satisfy legislative restrictions. An individual may have more flexibility on some investment plans than on others. This situation is relatively realistic since most of the relevant, qualified employer-organizations offer choice among only a few insurers or mutual funds; at times, the choice is restricted to only one insurer or mutual fund. The individual is assumed to know the costs of investing in these plans. For example, the costs of IRAs, especially if they are kept with mutual funds, are easy to define.⁸ Most of the IRA custodians charge a flat or proportionate fee for maintaining an account. Admittedly, the costs of some other investment plans, such as TDA plans, are hard to define exactly, and so it is difficult to state which one will be more expensive; but the

⁶It is, of course, possible to set up goals of accumulation in current dollars. How the specification of goals affects the decision is explored in the next section.

⁷By way of an example, consider the faculty in U.S. colleges and universities. Many of them qualify for participation in both the TDAs and IRAs. According to statistics published by the U.S. Department of Education (1987), average salary of all faculty was only \$32,392 in 1985-1986. Accounting for inflation, salary figures projected to the current year would still be under \$40,000. Assuming that the individual is married and the spouse has no significant amount of income, the projected amount implies a full deduction of the IRA under the current Code. By way of another example, consider employees who contribute to their employer-sponsored 401(k) plan through salary reduction as well as to IRAs on their own.

In light of the above, both the TDAs and IRAs permit tax-deferral of the current income and of the earnings on the deposits. A contrasting example would be 'Family Savings Accounts', cited in footnote 1 above, or non-qualified annuity plans, cited in footnote 3 above, which permit tax-deferral only of the earnings on the deposited amounts.

⁸The U.S. Securities & Exchange Commission (SEC) recently adopted new rules requiring mutual funds to disclose uniformly all fees and expenses in a consolidated table near the front of the prospectus. This "fee table" must include a breakdown of all the expenses paid directly or indirectly by shareholders (sales loads, redemption fees, management expenses, and 12b-1 charges). In addition, the table must show the cumulative expenses paid on a hypothetical \$1,000 investment at the end of one-, three-, five-, and 10-year periods, assuming a 5 percent annual return to allow uniform comparison of one fund with another.

individual can use the historical data or reasonably forecast them. Given the annual income and stated goals of accumulation, the individual can allocate the savings optimally among these opportunities.

In the next section, the formulation of making periodic investments in the several investor-identified tax-deferred opportunities is illustrated. The formulations, for obvious reasons, neither include all the provisions of the Code nor reflect all the personal preferences of the individual. The concluding section shows how some of these excluded restrictions may be incorporated in the model.

Problem Formulation

The following is a basic decision-making problem formulated to capture the essential requirements and restrictions.

$$\text{Maximize } \sum_{n=1}^N \left(X_{n,T+1} \prod_{t=1}^{T+1} (1 + \rho_t)^{-1} \right) \quad (1)$$

subject to

$$X_{n,t} = X_{n,t-1}(1 + r_{n,t})(1 - g_{n,t}) + x_{n,t}(1 - k_{n,t}) - C_{n,t}, \quad n = 1, \dots, N, t = 1, \dots, T+1 \quad (2)$$

$$X_{n,0} = B_n, \quad n = 1, \dots, N \quad (3)$$

$$X_{n,T+1} \prod_{t=1}^{T+1} (1 + \rho_t)^{-1} \geq E_n, \quad n = 1, \dots, N \quad (4)$$

$$x_{n,t} \geq V_{n,t}, \quad n = 1, \dots, N, t = 1, \dots, T \quad (5)$$

$$\sum_{n=1}^N x_{n,t} \geq U_t, \quad t = 1, \dots, T \quad (6)$$

$$x_{n,t} \geq 0, \quad \forall n \text{ and } t \quad (7)$$

$$x_{n,T+1} = 0, \quad \forall n \quad (8)$$

where

the decision variables are

$x_{n,t}$ = contribution, in dollars, made to plan n at date t

$X_{n,t}$ = accumulated amount, in dollars, in plan n at date t

and the exogenous parameters are

B_n = beginning dollar amount in plan n at date zero

E_n = minimum desired dollar amount in plan n at the beginning of the liquidation period at date $T+1$

- $V_{n,t}$ = legal or personal maximum dollar amount allowed for plan n for period t . (If such a limit does not exist, then the corresponding constraint is not required.)
- U_t = legal or personal maximum dollar amount allowed for the total tax-deferred investment for period t
- T = planning horizon date or number of investing periods for the individual so that he or she could start the withdrawals beginning at date $T+1$
- $k_{n,t}$ = front-end sales charge, in percent, on plan n at date t
- $g_{n,t}$ = asset charge, in percent, on plan n for period t
- $r_{n,t}$ = rate of return, in percent, on plan n for period t
- $C_{n,t}$ = fixed overhead fee, in dollars, for management of plan n for period t
- ρ_t = opportunity cost rate for period t .

In the above formulation, the individual is assumed to maximize his or her own unique objective function subject to a set of constraints. The objective function, given in Expression (1), is in terms of present value of wealth, represented by the accumulated amounts, at the end of one year after the accumulation period, *i.e.*, at the first withdrawal date.⁹ The individual maximizes this function in order to reach the goal of accumulation of savings in the preferred, independent investment opportunities.

The running totals of the accumulation in the investment plans are presented in Constraint (2) for each period. This constraint states that the accumulations should be equal to the investments plus dividend or interest income less the expenses paid to the financial institutions.

The flexibility of making decisions at any date during the working life of the individual is given in Constraint (3). The beginning balances of the plans, B_n , can be set to any amount to reflect any existing accumulations, such as rollovers.

The goal or target accumulation of each plan is stated in Constraint (4). The optimal decision is expected to result in matching or exceeding the goals. Notice, however, that an unreasonable goal, such as a high accumulation goal with a small number of periods and/or limited funding, would render the solution infeasible. Notice that the accumulation goal is stated in present-value terms since it is multiplied by a discount factor. For example, an individual may wish to set the accumulation goal of the present value of, say \$10,000, accumulated in plan n at the beginning of the withdrawal period, *i.e.*, at date $T+1$. A more straightforward formulation would be to set the accumulation goals in current dollars. In such a case, Constraint (4) would become

⁹The objective function could be a cost function, cash outflow function, end-of-period wealth function, present-value function or net-present-value function. In each of these cases, the objective function has to be tuned to the individual's needs by either maximizing wealth or minimizing cash outflows. The individual's preferences would determine the selection of the objective function.

$$X_{n,T+1} \geq E_n, n = 1, \dots, N, \quad (4')$$

where the goal amount is not multiplied by a discount factor.

The statutory limit on any plan n is stated in Constraint (5) whereas the joint investment maximum limit is stated in Constraint (6). These two types of constraints can be used also as personal savings constraints or preferences for periodic investments in these opportunities.

Constraint (7) specifies that investments in each period may not be negative. In order to keep the model simple, early withdrawals are ruled out so that the withdrawal penalty of 10 percent and tax liability, both imposed by the Code, do not arise.

The individual's planning horizon of T periods is specified in Constraint (8). He or she would make the first decision at date 1 and the last decision at date T . The individual maximizes the plan accumulations at date $T+1$ when the first withdrawal would be made. Thus, at date T allocation decisions for tax-deferred investments are concluded.

Note that a close relationship exists between the objective function and the constraints that give the running totals of the accumulations. If the objective function is stated in terms of cashflows, then constraints should be changed accordingly. Details of such modifications are given in the next section.

Some realistic elements have been left out of the basic model. Nevertheless, it serves as a stepping-stone formulation for later, more complicated scenarios. How this basic model can be extended to reflect different situations in real life is demonstrated in the following section.

Simulation Results

The application of the basic model under four different scenarios is illustrated in this section. To simplify the examples, an individual is assumed to be interested in only two different investments plans. For example, Plan 1 may represent a TDA plan whereas Plan 2 may represent an IRA.¹⁰ Furthermore, the planning horizon for accumulating periods is limited to three years. The starting accumulations are assumed to be zero for both plans. The formulation permits these starting sums to be any amounts, thereby giving the individual the opportunity to rollover the funds and alter the strategy at any time during the planning period. The objective of the individual is to accumulate a certain sum of money at date four in each of the two plans. A schematic representation of the problem is given in Table 1.

It must be emphasized that the simplicity of the example with only two investment plans and three periods is for illustrative purposes only; scenarios with more complexity and/or longer planning periods can be accommodated. In fact, a real-life example of 30 years is incorporated in this article to show the applicability of the model. The real-life example is described in more detail near the end of this section.

¹⁰This labelling allows the use of a numerical value on Constraint (5) because the IRA, in general, has a legal limit of \$2,000 per year.

Table 1
A Schematic Representation of an Individual's Three-Period Decision on Tax-Deferred Investments

Period	0	1	2	3	4
Date					
Decisions	$x_1 = 0$ $x_2 = 0$	$x_1 = ?$ $x_2 = ?$	$x_1 = ?$ $x_2 = ?$	$x_1 = ?$ $x_2 = ?$	$x_1 = 0$ $x_2 = 0$
Rates of Return		$r_1 = 8\%$ $r_2 = 5\%$	$r_1 = 8\%$ $r_2 = 5\%$	$r_1 = 8\%$ $r_2 = 5\%$	$r_1 = 8\%$ $r_2 = 5\%$
Asset Charges		$g_1 = 3\%$ $g_2 = 3\%$	$g_1 = 3\%$ $g_2 = 3\%$	$g_1 = 3\%$ $g_2 = 3\%$	$g_1 = 3\%$ $g_2 = 3\%$
Sales Charges		$k_1 = 5\%$ $k_2 = 0\%$	$k_1 = 5\%$ $k_2 = 0\%$	$k_1 = 5\%$ $k_2 = 0\%$	
Management Fees		$C_1 = \$0$ $C_2 = \$20$	$C_1 = \$0$ $C_2 = \$20$	$C_1 = \$0$ $C_2 = \$20$	$C_1 = \$0$ $C_2 = \$20$
Legal/Budget Constraints		$x_1 \leq \$2,000$ $x_1 + x_2 \leq \$9,500$	$x_1 \leq \$2,000$ $x_1 + x_2 \leq \$9,500$	$x_1 \leq \$2,000$ $x_1 + x_2 \leq \$9,500$	
Accumulation Goals		$X_1 = f1(\cdot)$ $X_2 = f2(\cdot)$	$X_1 = f1(\cdot)$ $X_2 = f2(\cdot)$	$X_1 = f1(\cdot)$ $X_2 = f2(\cdot)$	$X_1 \geq \$10,000$ $X_2 \geq \$5,000$

Note: $f_n(\cdot)$ denotes the transition function.

The differences in the simulated models are stated in the following summary. The mathematical formulations of the respective objective functions are given in the appendix.

Model 1 Maximizes the present value of accumulation after three investing periods.

Model 2 Maximizes the present value of total accumulation and of savings through tax shields.¹¹

Model 3 Minimizes the present value of cash outflows of investments and financial charges to meet the goals of accumulations at the beginning of the liquidation period, *i.e.*, date four.

Model 4 Maximizes the net present value of accumulations, *i.e.*, inflows, and after-tax outflows. Thus, the model calculates the net effect of tax savings, investment incomes, actual new investments, financial charges, and costs of capital.

Model 5 Maximizes the net present value of accumulations, *i.e.*, inflows, and after-tax outflows, with a planning period of 30 years and three investment plans. This real-life model calculates the net effect of tax savings, investment incomes, actual new investments, financial charges, and costs of capital. Here the accumulation targets are stated in current dollars. This model is enriched by the inclusion of a naive diversification scheme that is described later.

In order to compare these formulations efficiently, the same input data are used for each, except for Model 5 in which accumulation goals are different. The complete list of parameters used in these linear programming simulations is given in the appendix, where, in addition to the notation defined in the earlier section, τ denotes the marginal tax rate if a tax-deferred investment is not made at date t .

The results of the simulations are shown in Tables 2 and 3. Note the difference between Table 2 and Table 3. Table 2 shows the simulation results when the accumulation constraint is stated in present value (Constraint (4)), whereas Table 3 shows the results when the accumulation constraint is stated in current dollars (Constraint (4')). These tables show the series of investments in two plans for three years for each model. The last column shows the accumulations in these plans at date four. The tables show that the individual

¹¹ Plan 2 is assumed to be fully deductible for the individual. An IRA would be an example of this. Recall that the deductibility of the IRA depends on the amount of the adjusted gross income. For *married* taxpayers filing a *joint return* the earnings constraint for a full deductibility of the IRA is \$40,000, whereas the similar constraint for *unmarried* taxpayers filing a *single return* is \$25,000.

The tax-shield on interest or dividend income on the tax-deferred plans is not considered because (1) that type of tax-deferral is a common element of all tax-deferred opportunities, and generally, common elements are ignored when doing an incremental analysis, and (2) tax-deferred opportunities are not being compared with taxed opportunities.

To keep the models tractable, the individual's tax-deferred investment is assumed to not create several tax advantages. The formulation of such a problem is more complex and beyond the scope of this article.

Table 2
Results of Simulation of Linear Programming Models of
an Individual's Decision on Tax-Deferred Investments
when the Accumulation Target is Stated in Present Value

Model/Plan	Dollar Investment in			Total
	1st Year	2nd Year	3rd Year	Accumulation
<i>Model 1: Present value of accumulations</i>				
Plan 1	7,777.47	7,500.00	7,500.00	\$23,778.29
Plan 2	1,722.52	2,000.00	2,000.00	5,849.32
Total accumulations				29,627.61
Present value of total accumulations				25,325.68
Optimal objective function value				25,325.68
<i>Model 2: Present value of accumulations and tax-shields</i>				
Plan 1	7,777.47	7,500.00	7,500.00	\$23,778.29
Plan 2	1,722.52	2,000.00	2,000.00	5,849.32
Total accumulations				29,627.61
Present value of total accumulations				25,325.68
Optimal objective function value				29,280.14
<i>Model 3: Present value of cash outflows</i>				
Plan 1	7,855.32	2,991.50	0.00	\$11,698.64
Plan 2	1,644.68	2,000.00	2,000.00	5,849.32
Total accumulations				17,547.96
Present value of total accumulations				15,000.00
Optimal objective function value				15,600.36
<i>Model 4: NPV of accumulations and after-tax outflows</i>				
Plan 1	7,855.32	7,500.00	7,500.00	\$23,863.32
Plan 2	1,644.68	2,000.00	2,000.00	5,849.32
Total accumulations				29,712.64
Present value of total accumulations				25,398.37
Optimal objective function value				2,928.06

is able to accumulate at least \$10,000 in Plan 1 and \$5,000 in Plan 2 as indicated in the appendix in the section on parametric values.

The interpretation of the results is straightforward. The results of Model 1 in Table 2 are used as an example. The interpretation of the results of the other models is similar. The individual maximizes the objective function of the present value of accumulations. The optimal strategy for the individual is to invest in the first, second, and third year \$7,777.47, \$7,500.00, and \$7,500.00, respectively, in Plan 1, and \$1,722.52, \$2,000.00, and \$2,000.00, respectively, in Plan 2. The total accumulation in current dollars is \$29,627.61 and in present value is \$25,325.68, which happens also to be the value of the

Table 3

Results of Simulation of Linear Programming Models of
an Individual's Decision on Tax-Deferred Investments
when the Accumulation Target is Stated in Current Dollars

Model/Plan	Dollar Investment in			Total
	1st Year	2nd Year	3rd Year	Accumulation
<i>Model 1: Present value of accumulations</i>				
Plan 1	8,581.36	7,500.00	7,500.00	\$24,656.31
Plan 2	918.64	2,000.00	2,000.00	5,000.00
Total accumulations				29,656.31
Optimal objective function value				25,350.21
<i>Model 2: Present value of accumulations and tax-shields</i>				
Plan 1	8,581.36	7,500.00	7,500.00	\$24,656.31
Plan 2	918.64	2,000.00	2,000.00	5,000.00
Total accumulations				29,656.31
Optimal objective function value				29,304.67
<i>Model 3: Present value of cash outflows</i>				
Plan 1	8,659.20	520.10	0.00	\$10,000.00
Plan 2	840.80	2,000.00	2,000.00	5,000.00
Total accumulations				15,000.00
Optimal objective function value				13,315.31
<i>Model 4: NPV of accumulations and after-tax outflows</i>				
Plan 1	8,659.20	7,500.00	7,500.00	\$24,741.33
Plan 2	840.80	2,000.00	2,000.00	5,000.00
Total accumulations				29,741.33
Optimal objective function value				2,952.58

objective function. The equality of these two quantities is coincidental and is due to the congruence of the objective function and the constraints on accumulations. In general, however, this equality would not hold.

For illustrative purposes, the above results are compared with those of Model 1 in Table 2 where the accumulation goal is stated in current dollars. Note that in the model there is no inconsistency between a constraint stated in current dollars and the objective function stated in present value. The accumulations, at date four, in the model are given by Constraint (2) and not by the objective function.¹² The optimal strategy for the individual is to invest

¹² The objective function, while being necessary, is merely a small part of the model's machinery; transition functions are pivotal. The model is driven by constraints — especially (2) — to reach a particular "place"; the objective function is just the "steering mechanism". The objective function serves as a means to allocate the investible funds; it should not be considered the ultimate figure of wealth, which would be accumulated in the plans.

This point is obvious from the simulation results presented in Tables 2 and 3. Notice from the mathematical formulations of Models 1 and 2 given in the appendix that both the models have the

in the first, second, and third year, \$8,581.36, \$7,500.00, and \$7,500.00, respectively, in Plan 1, and \$918.64, \$2,000.00, and \$2,000.00, respectively, in Plan 2. The total accumulation in current dollars is \$29,656.31, whereas the optimal value of the objective function is \$25,350.21.

The difference between the accumulations of Model 1 in Tables 2 and 3 is the result of the parametric values selected for Plan 2. Plan 2 is deliberately portrayed as a less profitable opportunity than Plan 1. The use of present values for the constraint on accumulation goals forces the individual to put more dollars into Plan 2 to meet the goal; therefore, the individual ends up with a less desirable total accumulation.

When the individual's accumulations are maximized, the solution as expected calls for investing as much as possible, or up to the statutory or budgetary limits, to attain these goals. In this case, the constraints of having at least \$10,000 in Plan 1 and \$5,000 in Plan 2 are rendered superfluous by the objective function of maximization of accumulations. On the other hand, if the individual's objective is to minimize cash outflows, then it is obvious that the investment would be minimum so that the goals are attained and nothing more is accumulated. This is shown in Tables 2 and 3 for Model 3.

Lastly, a real-life example is provided using Model 5 with three investment plans and 30 years of planning horizon. The choice of using three plans in this example is made to show how investment allocation would differ in multi-plan problems with a longer horizon. Despite the complexity of the problem, the solution remains understandable. The extension of the planning horizon to 30 years is made to simulate the approximate number of working years for a hypothetical individual before retirement. The mathematical programming formulation is given in the appendix under Model 5, whereas the exogenous parameters pertaining to each plan are listed near the end of the appendix. As shown in the appendix, the accumulated sums at retirement are targeted to be at least \$30,000, \$20,000, and \$30,000 in Plan 1, Plan 2, and Plan 3, respectively. In addition, to incorporate a simple scheme of diversification for reducing risk, a constraint is imposed that the Plan 1, Plan 2, and Plan 3 have about 40 percent (35 through 45 percent), 20 percent (15 through 25 percent), and 40 percent (35 through 45 percent) of the total accumulations, respectively. The diversification constraint (Constraint (9)) is shown explicitly in the appendix. The annual contributions to each plan and the final accumulations are shown in Table 4.

same constraint set, but their objective functions are different. Now consider the results of these two models in Table 2: the optimal savings strategy is the same; the total accumulations in the two plans are the same; both the objective functions are to be maximized; only the optimal values of the objective functions are different. This comment applies to the results of Models 1 and 2 in Table 3 as well.

Note that this comment is not applicable to Models 3 and 4. Although they have the same constraint set, their driving paths are different because Model 3 is minimizing a present-value objective function, whereas Model 4 is maximizing a net-present-value objective function.

In light of the above, the only requirement imposed on the overall scheme of solution is that the individual use an objective function consistent with his or her set of constraints.

Table 4

Results of Simulation of a Life-Size Linear Programming Model of an Individual's Decision on Three Tax-Deferred Investment Plans when the Accumulation Target is Stated in Current Dollars

Year	Dollar Investment in		
	Plan 1	Plan 2	Plan 3
1	0.00	2,000.00	7,500.00
2	0.00	2,000.00	7,500.00
3	0.00	2,000.00	7,500.00
4	0.00	2,000.00	7,500.00
5	0.00	2,000.00	7,500.00
6	0.00	2,000.00	7,500.00
7	0.00	2,000.00	7,500.00
8	1,994.37	2,000.00	5,505.63
9	7,500.00	2,000.00	0.00
10	7,500.00	2,000.00	0.00
11	7,500.00	2,000.00	0.00
12	7,500.00	2,000.00	0.00
13	7,500.00	2,000.00	0.00
14	7,500.00	2,000.00	0.00
15	7,500.00	2,000.00	0.00
16	7,500.00	2,000.00	0.00
17	7,500.00	2,000.00	0.00
18	7,500.00	2,000.00	0.00
19	7,500.00	2,000.00	0.00
20	7,500.00	2,000.00	0.00
21	7,500.00	2,000.00	0.00
22	7,500.00	2,000.00	0.00
23	746.32	2,000.00	0.00
24	0.00	2,000.00	0.00
25	0.00	2,000.00	0.00
26	0.00	2,000.00	0.00
27	0.00	2,000.00	0.00
28	0.00	2,000.00	0.00
29	0.00	2,000.00	0.00
30	0.00	2,000.00	0.00
Accumulation at $t = 31$	\$215,260.47	\$80,722.66	\$242,168.00
PV ₀ (Accumulation)	\$63,816.17	\$23,931.06	\$71,793.19

Notes:

1. The minimum accumulation targets are stated in current dollars.
2. Here $t = 1, 2, 3, \dots, 30$; $T = 30$; and $N = 3$.
3. Optimal objective function value = \$36,735.78.
4. Model 5 simulated as indicated in the text.

Note first of all that the individual would not be able to come up with the optimal contribution pattern heuristically. The accumulations at date 31 are \$215,260.47, \$80,722.66, and \$242,168.00 in Plan 1, Plan 2, and Plan 3, respectively, thereby giving a total of \$538,151.13 in current dollars or \$159,540.42 in present value at $t = 0$. Note that the constraint of minimum target accumulations of \$30,000, \$20,000, and \$30,000 is once again rendered superfluous by the objective function of the maximization of the net present value of target accumulations and after-tax outflows. Also, note how the diversification scheme has worked. Out of the total accumulation of \$538,151.13, 40 percent is in Plan 1, 15 percent in Plan 2, and 45 percent in Plan 3. The diversification constraint, Constraint (9), results in the annual contribution of \$0 in Plan 1 for years 24 through 30, and \$0 in Plan 3 for years 23 through 30. Examine the contribution in year 23. The individual contributes to the limit of \$2,000 in Plan 2, only \$746.32 in Plan 1, and \$0 in Plan 3. The amounts show that the requirement of diversification through Constraint (9) restricts further investment in other opportunities in order to achieve a minimum of 15 percent of total accumulation in Plan 2. In addition, the annual ceiling of \$9,500 imposed by Constraint (6) causes a reallocation of investment of contributions in different plans. This shifting of allocations is apparent in Table 4 where the constraint results in the annual contribution of \$0 in Plan 1 for years one through seven, and \$0 in Plan 3 for years eight through 30.

The approach to the decision-making problem through linear programming requires a careful evaluation of the optimal objective function value delivered by the solution of the model. In the normal interpretation of the objective function, the optimal value is of significance by itself. In the formulations presented in this article, however, that is not necessarily the case. An allocation scheme for the individual's investible funds is more important and interesting than the solution-generated optimal value. The optimal values of the objective functions of the models presented in this article are not comparable since objective functions used in the simulations are different. The optimal values are of importance when they are used as a guide to compare two or more simulations of the same problem setup with different parametric values. Similarly, the optimal values are of importance when the function is cast in terms of cashflows.

The simulations demonstrate another benefit of the linear programming approach: the utmost importance and use of commonsense in problem-formulation and decision-making. In particular, if tax savings are not taken into account and/or if the rates of return are negligible, then investment in tax-deferred opportunities would not be attractive. Owing to the administrative charges, sales commissions, and other expenses, the individual's decision to invest in tax-deferred opportunities could be less than optimal, in the sense that the accumulations at the first withdrawal date could be smaller than anticipated. With tax-savings, however, even negligible rates of return would make the tax-deferred investments attractive. In fact, the overall effect of tax-savings could be so dramatic that any mediocre tax-deferred opportunity

would look attractive enough and lure an individual into making a suboptimal investment decision.

The above is the main reason for an individual to resist making a direct comparison between a tax-deferred investment and a taxed investment; a comparative approach similar to that presented in this article is expected to be discussed in the future. At present, however, this article provides a technique to compare the relative merits of any number of tax-deferred plans, not only one TDA or one IRA, so as to derive the inter-temporal decision of optimal allocation of investible funds among them.

Concluding Remarks

The most distinguishing feature of this article is that it presents a workable, yet simple, model of decision-making. In a general setting where an individual is faced with several legal and personal constraints, taxes on income, wealth and bequest decisions, and transaction costs, intuition or heuristic methods would not be sufficient for deriving an optimal allocation of funds among several competing investment opportunities. The application of linear programming to an individual's intertemporal investment decision provides a normative model for savings allocation. When the investment in tax-deferred opportunities is considered a sub-problem in a larger problem of investments, the approach advocated in this article will enable an individual to evaluate tax-deferred opportunities on their own comparative merits over time. Unlike the traditional investment models, *e.g.*, Markowitz (1952), Sharpe (1964), the linear programming model is not only explicitly intertemporal, but also free from the burdensome restrictive assumptions about individual behavior and market conditions. This article provides an illustration with a planning period of 30 years to highlight the salient features of the method. This illustration further incorporates an average individual's idea of diversification. The optimal solution gives an individual a comprehensive set of decisions over his or her entire planning period.

Although completely different in execution, the model presented in this article is somewhat similar in concept to the promising work of Brown (1987). It is important to note that this model permits an individual a very high degree of flexibility in the choice of parametric values and the number of constraints. An individual can choose different values for the parameters of costs and returns for different plans for different periods as well as for different constraints; an individual can choose to impose as many or as few constraints as he or she thinks will be useful. The choice of the parametric values, constraints, and number of opportunities included in the model enables an individual to fine-tune the objectives/goals at the first withdrawal date before committing himself or herself to a program of savings and their allocation.

The model is transparent in formulation and application. Such modelling is insightful because sensitivity analysis can be carried out, at relatively low costs in effort and in computing time, by investigating various scenarios using different objectives and constraints. Because of the low cost, it is advisable for

an individual to evaluate the life-time sequence of decisions every period, typically, a year. Such evaluation would help in assessing the performance of different assets and the progress towards the goal. Depending upon the progress and the changes, if any, in the individual's circumstances, he or she can alter decisions optimally.

Last but not the least, the practicality of the model is to be appreciated. If an individual is going to use the model every year to assess financial progress, the model must be capable of operation quickly and effortlessly. An individual decision-maker can do so with the model presented in this article without extensive expert advice, or extra training, or special equipment. Linear programming software packages are widely available. In fact, however, one may not need such a specific software: one can work out a plan on commonly-used spreadsheet software packages. In more complex situations where linear programming formulations are inappropriate, other techniques, such as integer programming, dynamic programming, and nonlinear programming may be needed. The authors expect to examine these problems and their solution methods in the future and report the findings separately.

References

- Adelman, Saul W. and Mark S. Dorfman, 1982, A Comparison of TDA and non-TDA Investment Returns, *The Journal of Risk and Insurance*, 49: 73-90.
- Belth, Joseph M., 1982, A Note on Disclosure of Realized Rates of Return for Retirement Accumulations, Savings Accounts, and the Savings Component of Universal Life Insurance Policies, *The Journal of Risk and Insurance*, 49: 613-17.
- Brown, David P., 1987, Multiperiod Financial Planning, *Management Science*, 30: 848-75.
- Burkett, Lawrence V., 1976, TSA's Limits on Contributions after ERISA, *CLU Journal*, 30: 34-52.
- Dorfman, Mark S. and Saul W. Adelman, 1983, Comparing the TDA to a Non-TDA Investment Program for Accumulating and Liquidating Wealth, *CLU Journal*, 37: 36-41.
- Doyle, Robert J., 1984, IRAs and the Capital-Gains Tax Effect, *Financial Analysts Journal*, 40: 60-75.
- Duncan, Greg J., Olivia S. Mitchell, and James N. Morgan, 1984, A Framework for Setting Retirement Savings Goal, *Journal of Consumer Affairs*, 18: 22-46.
- Gahin, Fikry S., 1983, The Financial Feasibility of Tax-Sheltered Individual Retirement Plans, *The Journal of Risk and Insurance*, 50: 84-106.
- Greene, Mark R. and J. Paul Copeland, 1975, Factors in Selecting Tax-Deferred Annuities, *CLU Journal*, 29: 34-46.
- Greene, Mark R., John Neter, and Lester I. Tenny, 1977, Annuity Rents and Rates: Guaranteed vs. Current, *The Journal of Risk and Insurance*, 44: 383-402.

- Healy, Richard C., 1981, An Economic Analysis of Tax Sheltered Annuities for Employees of Non-Profit Organizations, *Journal of Insurance Issues and Practices*, January: 43-49.
- Jacobs, Fred A. and Michael L. Holland, 1984, Alternative Investment Strategy: Using the IRA as an Interim Accumulation Instrument, *Review of Taxation of Individuals*, 8: 349-60.
- Markowitz, Harry M., 1952, Portfolio Selection, *The Journal of Finance*, 7: 77-91.
- Mehr, Robert I., 1968, Tax Sheltered Annuities: Purchase Decisions, *The Journal of Risk and Insurance*, 35: 207-26.
- Morehart, Thomas B. and Gary L. Trennepohl, 1979, Evaluating the Tax-Sheltered Annuity vs. the Taxed Investment, *CLU Journal*, 33: 23-30.
- O'Neil, Cherie J., Donald V. Saftner, and Manson P. Dillaway, 1983, Premature Withdrawals from Individual Retirement Accounts: A Breakeven Analysis, *Journal of American Taxation Association*, Spring: 35-43.
- Schnee, Edward J. and Charles W. Caldwell, 1984, Retirement Planning under Section 403(b)(7) - Advantages and Limitations, *Journal of the American Taxation Association*, Fall: 48-56.
- Sharpe, William F., 1964, Capital Asset Prices: A Theory of Market Equilibrium Under Conditions of Risk, *The Journal of Finance*, 19: 425-42.
- Smith, Donald J., 1984, Individual Retirement Accounts and Intermediate Term Holding Periods, *The Financial Review*, 19: 381-87.
- Stoeber, Edward A., 1979, A Review of Tax-Deferred Annuity Plans, *CLU Journal*, 32: 37-52.
- Todd, Jerry D., 1978, The Retirement Funding Decision: Tax-Sheltered Annuities vs. Alternatives, *Journal of Insurance Issues and Practices*, Fall: 69-80.
- Todd, Jerry D., 1976, The Annuity Selection Problem Benefit-Cost Factors, *Risk Management*, November: 21-32.
- U.S. Department of Education, 1987, *Digest of Education Statistics* (Washington, DC: U.S. Department of Education).
- Wall Street Journal*, 1989, New Incentives for Saving is Slated to be Proposed by White House, (New York, NY: Dow Jones & Co., Inc.) December 15: A3.

Appendix

Linear Programming Models of an Individual's Decision on Tax-Deferred Investments

Model 1: Present value of accumulations

$$\text{Max } (X_{1,T+1} + X_{2,T+1}) \prod_{t=1}^{T+1} (1 + \rho_t)^{-1}$$

subject to constraints (2) through (8) for Table 2,

or

subject to constraints (2), (3), (4'), and (5) through (8) for Table 3.

Model 2: Present value of accumulations and tax shields

$$\text{Max } (X_{1,T+1} + X_{2,T+1}) \prod_{t=1}^{T+1} (1 + \rho_t)^{-1} + \sum_{t=1}^T \tau_t (x_{1,t} + x_{2,t}) \prod_{m=1}^t (1 + \rho_m)^{-1}$$

subject to constraints (2) through (8) for Table 2,

or

subject to constraints (2), (3), (4'), and (5) through (8) for Table 3.

Model 3: Present value of cash outflows

$$\text{Min } \sum_{t=1}^{T+1} [x_{1,t} + x_{2,t} + C_{1,t} + C_{2,t}] \prod_{m=1}^t (1 + \rho_m)^{-1}$$

subject to

$$X_{n,t} = X_{n,t-1}(1 + r_{n,t})(1 - g_{n,t}) + x_{n,t}(1 - k_{n,t}), \quad n = 1, 2; \quad t = 1, \dots, T+1 \quad (2')$$

and constraints (3) through (8) for Table 2,

or

and constraints (3), (4') and (5) through (8) for Table 3.

Model 4: NPV of accumulations and after-tax outflows

$$\begin{aligned} \text{Max } & (X_{1,T+1} + X_{2,T+1}) \prod_{t=1}^{T+1} (1 + \rho_t)^{-1} \\ & - \left[\sum_{t=1}^{T+1} (1 - \tau_t) \{x_{1,t} + x_{2,t} + C_{1,t} + C_{2,t}\} \right] \prod_{m=1}^t (1 + \rho_m)^{-1} \end{aligned}$$

subject to constraints (2') and (3) through (8) for Table 2,

or

subject to constraints (2'), (3), (4') and (5) through (8) for Table 3.

Model 5: NPV of accumulations and after-tax outflows for longer horizon

$$\text{Max } (X_{1,T+1} + X_{2,T+1} + X_{3,T+1}) \prod_{t=1}^{T+1} (1 + \rho_t)^{-1} \\ - \left[\sum_{t=1}^{T+1} (1 - \tau_t) \{x_{1,t} + x_{2,t} + x_{3,t} + C_{1,t} + C_{2,t} + C_{3,t}\} \prod_{m=1}^t (1 + \rho_m)^{-1} \right]$$

subject to

constraints (2'), (3), (4') and (5) through (8), and

$$0.35 K \geq X_1 \quad 0.45 K \quad (9A)$$

$$0.15 K \geq X_2 \quad 0.25 K \quad (9B)$$

$$0.35 K \geq X_3 \quad 0.45 K \quad (9C)$$

where $K = \sum_{n=1}^N X_n$; $t = 1, \dots, 30$; $T = 30$; and $N = 3$ for Table 4.

Appendix (Cont'd)

Parametric Values for Simulation of Linear Programming Models of an Individual's Decision on Tax-Deferred Investments

Description	Symbol	Plan 1	Plan 2	Plan 3
Rate of return in percent per period	$r_{n,t}$	8.0	5.0	9.0
Asset charge in percent per period	$g_{n,t}$	3.0	3.0	3.0
Sales charge in percent per period	$k_{n,t}$	5.0	0.0	6.0
Management fee in dollar per period	$C_{n,t}$	0.00	20.00	50.00
Maximum amount in dollar per period	$V_{n,t}$	N.A.	2,000	N.A.
Total maximum amount in all plans in dollar per period	U_t	9,500		
Targeted minimum accumulation at $T+1$ in dollar ^{1,2}	E_n	10,000	5,000	N.A.
Targeted minimum accumulation at $T+1$ in dollar ^{1,3}	E_n	30,000	20,000	30,000
Discount rate in percent per period	ρ_t	4.0		
Tax rate (marginal) in percent per period	τ_t	15.0		

Notes:

1. In current dollars. To state the minimum accumulation targets in present value, the individual must multiply it by an appropriate present-value factor.

2. For Tables 2 and 3 where $t = 1, 2, 3$; $T = 3$; and $N = 2$.

3. For Table 4 where $t = 1, 2, 3, \dots, 30$; $T = 30$; and $N = 3$.

Legend:

N.A. Not applicable.

Copyright of Journal of Risk & Insurance is the property of Blackwell Publishing Limited and its content may not be copied or emailed to multiple sites or posted to a listserv without the copyright holder's express written permission. However, users may print, download, or email articles for individual use.