

The Purchase of Insurance by a Risk-Neutral Firm for a Risk-Averse Agent

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Abstract

In the purchase of insurance by a risk-neutral firm for a risk-averse prospective employee, the firm must offer the prospect a compensation package consisting of a fee, or salary, and an insurance policy. In designing this policy, the firm is constrained by the utility function of the prospect: the firm seeks the least-cost compensation package that will induce the prospect to accept the firm's offer of employment. Because the risk aversion of the prospect constrains the choices of the firm, this problem is referred to as that of an employee-constrained firm (ECF). The specific ECF situation discussed here concerns a firm that wishes to hire an outside board member. After considering the comparative-static effects of changes in model parameters, the model is extended to permit price to serve as a proxy for risk. A comparison of the ECF-model implications with those obtained from a model of the purchase of insurance by a risk-averse consumer indicates that, even though the ECF is risk neutral, it behaves in some respects as if it were risk averse.

Authors have investigated the purchase of insurance by a risk-averse consumer (e.g., Arrow, 1984; Mossin, 1968; Ehrlich and Becker, 1972). Mayers and Smith (1982) considered the risk-averse corporate buyer of insurance, focussing on improvements in risk-sharing and on potential reductions in transaction costs, contracting costs, tax liabilities, and incentives for improper investment decisions (see also Mayers and Smith, 1986; Main, 1983; Smith and Witt, 1985). In this article a third type of insurance purchase is considered: the purchase of insurance by a risk-neutral firm for a risk-averse prospective employee.

To induce the risk-averse prospective employee to accept its offer of employment, the firm must offer a compensation package consisting of a fee, or salary, and an insurance policy. In designing the compensation package, the firm is constrained by the utility function of the prospect; that is, the firm

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seeks the least-cost compensation package that will induce the prospect to accept the firm's offer of employment. As a result, the risk aversion of the prospect constrains the choices of the firm; thus this problem is referred to as that of an employee-constrained firm (ECF).

One prominent example of the ECF insurance purchase is director and officer (D&O) liability insurance, which is designed to cover the errors and omissions of board members and company officers. As Hancock (1988a) explains,

The director and officer liability insurance policy has two parts. The first part directly insures the directors and officers against liability for errors and omissions. The second part insures the corporation for the amounts of money it may have to pay the directors and officers by reason of indemnification for errors and omissions of the directors and officers under either the state corporation law or the company's charter or bylaws (p. 2007).

D&O insurance is not intended to cover dishonest or intentionally wrongful behavior. Moreover, D&O insurance does not cover the corporation, except for the expense of indemnifying board members.

If the reimbursement of the corporation for indemnification of its officers and directors were the sole purpose of the D&O policy, then the firm's decision to forego self-insurance and purchase insurance might well reflect the determinants discussed by Mayers and Smith (1982). The D&O policy is also designed to cover situations in which the company cannot indemnify the directors and officers. This inability may arise from public-policy constraints, restrictions in the corporation's charter and bylaws, or the financial status of the corporation (Hancock, 1988a). D&O insurance also covers a director when the board of directors cannot satisfy the conditions necessary for indemnification. Most states require that indemnification follow a board finding that the director or officer (1) did not act criminally and (2) did not act contrary to the interests of the corporation. D&O insurance is useful in cases where the board is unable or unwilling to make such a finding (Hancock, 1988a). Thus for certain types of losses, self-insurance is not an option for the firm; D&O insurance is the only way that the firm can protect its directors and officers.

At least four other types of insurance policies protect risk-averse employees, many of whom accept their positions only when the employing entity agrees to insure them against certain types of liability. Professional liability insurance covers lawyers, engineers, accountants, and other professionals. Medical doctors, as employees of hospitals or university facilities, often require professional liability insurance as a condition of employment. Public official liability insurance protects elected and appointed officials serving in city, county, or other public entities. School-board liability insurance covers public-school administrators and board members. Law-enforcement liability insurance protects police personnel, sheriffs, and other law enforcement officials in suits alleging false arrest, police brutality, or other civil rights

violations.¹ In these examples, employment is often contingent on the ability of the employing entity to convince the employee that he or she is protected from certain types of liability.

The remainder of this article considers the characteristics of the least-cost compensation package that will convince a prospect to accept employment with a prospective employer. To focus the discussion, a firm that wishes to hire a prospect to serve as an outside board member (a board member who is not an officer or otherwise employed by the corporation) is considered. Then, the decision of the ECF when the compensation package comprises two elements (a fee and an insurance policy) is modeled and followed by a consideration of the comparative-static effects of changes in various model parameters. Later the price is permitted to serve as a proxy for risk. Finally, the ECF-model implications are compared with those obtained from a model of the purchase of insurance by a risk-averse consumer. This comparison reveals several important commonalities. In both models, an increase in the expected loss or in the perceived probability of loss increases the amount of insurance purchased. Moreover, the effect of an increase in initial wealth on the amount of insurance purchased depends on whether the prospect exhibits increasing, constant, or decreasing absolute risk aversion. Nevertheless, the employee-constrained firm differs from the risk-averse consumer in several respects. The most important difference concerns the likelihood of Giffen-type behavior. In particular, when the prospect's perception of the probability and magnitude of loss are independent of the per-unit premium, insurance purchased by the employee-constrained firm cannot be Giffen.

A Model of the D&O Purchase Decision

The authors abstract from the real-world D&O market in several ways. First, it is assumed that all D&O lawsuits alleging board misconduct result in a fixed judgment against the individual board members. Thus one does not distinguish between filing a D&O lawsuit and winning that lawsuit in court. Second, it is assumed that the only lawsuits that arise are those covered by the D&O policy; i.e., one assumes that no lawsuits arise from willful misconduct by board members. Third, it is assumed that the D&O insurer does not attempt to evade its responsibilities to reimburse either the insured company or the insured board members.²

¹Many insurers have offered public-official liability and law-enforcement liability insurance without ever being a D&O underwriter. D&O insurance was a specialty line introduced by underwriters with no portfolio of public-official or school-board liability protection. Today, however, some large specialty companies offer D&O insurance along with some form of public-entity liability insurance.

²One D&O insurer, on learning that one of its insureds was the target of a hostile-takeover bid, canceled the insured's D&O policy. Moreover, insurers frequently attempt to cancel the D&O policies of firms that file for bankruptcy. A number of insurers also have attempted to avoid liability by claiming that insured firms were guilty of application fraud; i.e., the insurer has claimed that the insured did not disclose all facts that "might give rise to a claim" (Bailey, 1987).

The firm has several motivations to purchase D&O insurance for its officers and directors, as opposed to paying them to purchase their own insurance. Mayers and Smith (1981) have suggested that grouping can lower contracting costs. They argue that "Grouping will be more likely (1) when information asymmetry is greater, (2) when employers' incentive to monitor the employees' purchase of insurance is greater, and (3) when the heterogeneity of demand for insurance within the group is less" (p. 424). D&O insurance appears to satisfy all three conditions. Relative to the insurer, directors and officers have more information about transactions that might lead to a claim. The firm has incentives to monitor the officers and directors, because transactions that generate D&O claims typically accompany claims against the firm. Finally, because the actions of any single director can lead to charges of negligent oversight against the remaining directors, heterogeneity of demand for D&O insurance is expected to be relatively less than the heterogeneity of demand for other types of insurance.

Tax incentives also prompt the firm to purchase D&O insurance for its officers and directors. When the firm purchases D&O insurance, the full purchase price is deductible and the protected employee's taxable income is unaffected. If board members purchased their own insurance, the full purchase price would be deductible only if this expense plus other miscellaneous deductions exceeded 2 percent of adjusted gross income.³

An additional explanation for the firm's purchase of group insurance involves moral hazard. Heimer (1985) argues that moral hazard arises in the presence of reactive risk — risk that is conditional on the behavior of individual actors. One principle guiding insurer management of reactive risk is that "reactivity varies inversely with distance between the policy holder and the person who controls losses" (Heimer 1985, p. 11). To reduce reactivity, then, the D&O insurer will issue the D&O policy to the firm as opposed to individual directors.

Heimer (1985) also argues that, in the presence of reactive risk, insurers with imperfect information will try to make such information irrelevant or arrange for close supervision (p. 154). One method to insure careful monitoring is to contract with the party in the best position to control moral hazards after the contract is issued.⁴ Because D&O policies explicitly exclude claims arising from intentionally wrongful behavior, the primary moral hazard associated with D&O insurance is a potential increase in negligent behavior. Because the firm is in the best position to monitor the negligent behavior of all board members, D&O policies are expected to be issued to firms.

Newer D&O policies contain a contractual provision that reinforces the firm's incentive to monitor its directors and officers. The new provision, called "presumptive indemnification," involves the two parts of the D&O policy. The part directly insuring the directors and officers has a deductible that is much

³ The authors thank Travis Pritchett and Michael Smith for this observation.

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lower than that specified in the part reimbursing the corporation for its indemnification of the directors and officers. Under the presumptive indemnification clause,

the company cannot refuse to indemnify its directors and officers simply to take advantage of the lower deductibles in the director and officer reimbursement clause of the policy. If the company is either required or permitted to indemnify the directors and officers, it will be presumed to have done so, whether or not it has, in fact, really indemnified them. The result is that the loss is covered under the company reimbursement clause with its higher deductible (Hancock, 1988b, p. 2115.04).

Thus the presumptive indemnification clause increases the firm's incentive to monitor its directors and officers.

Historically, D&O insurance has not been available for purchase by individual directors. Because actions by one director can trigger suits involving other directors, insurers offer policies to corporations. This permits the evaluation and acceptance or rejection of coverage for the entire exposure arising from an individual corporation.⁵ In light of current practice and the firm's incentives for grouping, the model that follows assumes that prospective board members cannot purchase their own D&O insurance.

The Outside Director

Consider a single prospective board member with initial wealth $W_0 > 0$. A firm offers the prospect a fee of F dollars to serve as an outside director. The prospect assigns probability p to the event that, after accepting the position, he or she will be found liable for L dollars in a future D&O liability suit. In this event, the firm's D&O policy will reimburse the prospect R dollars.

If the prospect rejects the firm's employment offer, his or her total wealth remains constant at W_0 . If the prospect accepts the offer, his or her wealth depends on the existence of a future D&O liability suit. In the event of a successful suit, prospect wealth becomes

$$W_1 = W_0 + F - L + R. \quad (1)$$

Otherwise, prospect wealth is

$$W_2 = W_0 + F. \quad (2)$$

A Von Neumann-Morgenstern utility function $U(W)$ represents the preferences of the prospect, where $U'(W) > 0$ and $U''(W) < 0$ for every $W > 0$. If the prospect agrees to serve as an outside director, his or her expected utility is $V(W_1, W_2; p) = pU(W_1) + (1 - p)U(W_2)$. The prospect will accept the outside directorship if

$$U(W_0) < V(W_1, W_2; p). \quad (3)$$

⁵ This is quite similar to general liability policies, which clearly state that no coverage applies to any joint venture or to "the insured's exposure with respect to such venture" unless the insurer has had an opportunity to evaluate each such venture.

The Firm's Decision

Let r denote the per-unit D&O insurance premium and let x denote a dummy variable defined as follows:

$$x = \begin{cases} 1 & \text{if the prospect accepts a position on the board,} \\ 0 & \text{otherwise.} \end{cases}$$

Further, let $M(x)$ denote the gross value of the firm, and let $N(x, R, F)$ denote $M(x)$ net of the cost of the prospect's compensation package ($F + rR$). The firm will offer the prospect a board position if a compensation package (R, F) exists such that $N(1, R, F) \equiv M(1) - rR - F > N(0, 0, 0)$ for some $F \geq 0$ and some $R \geq 0$.

The Profit-Maximizing Selection of R and F

Given that the firm wishes to hire the prospect, it must choose R and F to maximize $N(1, R, F)$ subject to the minimum utility constraint (3). Letting μ denote the Lagrangian multiplier, the first-order conditions for a maximum are

$$pU(W_1) + (1-p)U(W_2) - U(W_0) = 0, \quad (4)$$

$$-1 + \mu[pU'(W_1) + (1-p)U'(W_2)] = 0, \text{ and} \quad (5)$$

$$-r + \mu pU'(W_1) = 0. \quad (6)$$

Let (R^*, F^*) denote the compensation package satisfying these conditions. Because the bordered Hessian determinant H is positive (see Appendix), this compensation package is unique. The remaining discussion focuses on the case in which $R^* > 0$, i.e., the case in which the firm purchases a positive amount of insurance for the prospect.

Solving (6) for μ and substituting into (5) yields

$$r = pU'(W_1)/[pU'(W_1) + (1-p)U'(W_2)]. \quad (7)$$

The expression on the right-hand side of (7) is the marginal rate of substitution of F for R ; it is also the absolute value of the slope of the prospect's convex indifference curves (Ehrlich and Becker, 1972). One can rewrite (7) as

$$\delta = [p/(1-p)][U'(W_1)/U'(W_2)], \quad (8)$$

where $\delta \equiv r/(1-r)$. Because $r < p$ implies perverse incentives for the prospective director, it is assumed that the firm only hires prospective directors with $p \leq r$.

To understand the meaning of equation (8), consider the situation in which the firm indemnifies the prospect for a judgment and then receives reimbursement from the D&O insurer. Define states one and two as follows:

State one: the state in which the firm indemnifies the prospect for a D&O judgment, and

State two: the state in which the prospect does not experience a D&O judgment.

Equation (8) states that, if the optimal compensation package includes an insurance policy, then the optimal package will equate δ with the ratio of the prospect's weighted marginal utilities in states one and two. Moreover, if the price of the D&O insurance is actuarially fair, then $\delta = p/(1-p)$ and the prospect receives the same income in both states of the world (Rothschild and Stiglitz, 1976; Ehrlich and Becker, 1972).

Comparative Statics

To explore the influence on F^* and R^* of r , L , W_o , and p , the total derivatives of equations (4)-(6) are taken. The effect of a change in any parameter on F^* and R^* can then be determined by the application of Cramer's rule. For expository convenience, mathematical details are summarized in the Appendix.

Substitution Effects

Economic intuition suggests that an increase in the per-unit insurance premium will alter the least-cost compensation package, increasing F^* and decreasing R^* . For the model described in the previous section, this intuition is correct. The relevant comparative static derivatives are

$$dF^*/dr = (\mu/H)[pU'(W_1)] > 0, \text{ and} \quad (9.1)$$

$$dR^*/dr = -(\mu^2/H) < 0, \quad (9.2)$$

where the Lagrangian multiplier μ and the bordered Hessian determinant H are both positive. Thus an increase in r necessarily increases the optimal fee and lowers the optimal level of coverage.

In general, the effect of a price change can be decomposed into an income effect and a substitution effect (Chiang, 1974; Ehrlich and Becker, 1972), but dR^*/dr cannot be decomposed in this fashion. The reason is that the employee-constrained firm maximizes its value subject to a minimum utility constraint imposed by the prospect. Because the prospect is on the same indifference curve before and after the price change, changes in r induce a pure substitution effect. Thus equation (9.2) implies that when a firm purchases insurance for an employee to induce the employee to accept employment, insurance cannot be a Giffen good. The next section shows that this result does not hold when the prospect's perceived probability of a judgment depends on the per-unit insurance premium.

Income Effects

Analogous to Ehrlich and Becker (1972), the prospect's endowment in each state is defined as that portion of his or her wealth that is independent of the

firm's compensation package. Thus, the prospect's endowments in states one and two are $(W_o - L)$ and W_o , respectively. A change in L affects the prospect's endowment in state one and induces the following changes in prospect compensation:

$$dF^*/dL = 0, \text{ and} \quad (10.1)$$

$$dR^*/dL = 1 > 0. \quad (10.2)$$

An increase in L has no effect on the optimal fee. Instead, the change in L is translated into an equivalent dollar change in the D&O insurance coverage. Thus when L rises, the corresponding adjustment in the prospect's compensation package leaves the prospect's wealth in each state unchanged.

A change in W_o affects the prospect's endowment in both states and alters both components of the prospect's compensation package. The effect of an increase in W_o on F^* is indeterminate, but the effect on R^* depends on the prospect's absolute risk aversion (compare Ehrlich and Becker, 1972). Let RA_i denote the coefficient of absolute risk aversion, where $RA_i = -U''(W_i)/U'(W_i)$. Increasing absolute risk aversion implies $(RA_2 - RA_1) > 0$ and $dR^*/dW_o > 0$, while decreasing absolute risk aversion implies $(RA_2 - RA_1) < 0$ and $dR^*/dW_o < 0$.

Arrow (1984) has argued that decreasing absolute risk aversion is more intuitive, because "it amounts to saying that the willingness to engage in small bets of fixed size increases with wealth" (p. 153). This reasoning suggests that increases in initial wealth will tend to lower the amount of coverage supplied by the firm.

In summary, if the prospect's endowment changes only in state one, the firm should increase R^* , the portion of the compensation package that affects the prospect's wealth in state one only. When the prospect's endowments in both states increase, the firm should decrease R^* as long as the prospect exhibits decreasing absolute risk aversion.

Rare Losses

If, from the prospect's perspective, the per-unit insurance premium is always actuarially fair, then δ always equals $p/(1-p)$, and changes in p have no effect on the optimal compensation package (Ehrlich and Becker, 1972). Moreover, the optimal compensation package is independent of p if δ is proportional to the actuarially fair price—i.e., if the loading imputed by the prospect to the insurer is a constant proportion of the actuarially fair price and is therefore independent of p .⁶

⁶Let β denote the loading imputed by the prospect to the insurer. The authors refer to β as an imputed loading because it is inferred, not from the insurer's expectation of a claim, but from the prospect's expectation. Now β is defined by the identity $\delta \equiv [(1+\beta)p/(1-p)]$. So if β is independent of p , then changes in p translate into proportionate changes in r and the optimal compensation package is therefore independent of p . See Ehrlich and Becker (1972).

It is possible that the prospect's perceived probability of a judgment could rise, even if r , the per-unit premium, were to remain constant. In such a case, the sign of dF^*/dp depends on the magnitude of p . If $p = 1$, $dF^*/dp = -(\mu/H)[U'(W_1)]^2 U'(W_2) < 0$. Moreover, $dF^*/dp < 0$, as long as $p > \{[U'(W_1)]^2/[U'(W_2) - U(W_1)U'(W_2)] + 1$. Intuitively, for high values of p , a rise in p will decrease F^* , while for low values of p , a rise in p will decrease F^* .

The effect of a rise in p on the optimal amount of insurance depends on the prospect's absolute risk aversion. Increasing absolute risk aversion implies $(R_2 - R_1) > 0$ and $dR^*/dp > 0$. In the more intuitive case of decreasing absolute risk aversion, the sign of dR^*/dp is indeterminate.

The Impact of Parameter Changes on Prospect Wealth and Firm Costs

Changes in model parameters affect the prospect's wealth directly and indirectly through their impact on F^* and R^* . It can be shown (see Appendix) that the prospect's wealth in state one is unaffected by changes in L but falls with increases in r and rises with increases in W_0 and p . Similarly, in state two, the prospect's wealth is unaffected by changes in L and rises with increases in W_0 . In the latter state, however, the prospect's wealth rises with increases in r , but the effect of an increase in p depends on the magnitude of p .

Changes in model parameters also affect the cost to the firm of the total compensation package. An increase in L increases this cost, as does an increase in W_0 when the prospect exhibits increasing absolute risk aversion. The effects of a change in r or p are indeterminate, but if the insurance is actuarially fair (from the prospect's perspective), then an increase in r or L increases the cost to the firm of the prospect's compensation package.

Some Simple Extensions

To this point, no distinction has been made between filing a D&O lawsuit and winning that lawsuit in court. Let p_1 = the probability of a lawsuit, and p_2 = the probability of a judgment given a lawsuit. Then if $p = p_1 p_2$, the preceding comparative-static analysis is unaffected.

It has also been assumed that all lawsuits are covered by the D&O policy. By letting p_3 denote the probability that a judgment is covered by the firm's D&O policy, the prospect's wealth in states one and two becomes $W_1 = W_0 - L + F + p_3 R$. Once again, the qualitative results from the preceding comparative-static analysis are unaffected. Other straightforward extensions include multiple lawsuits and claims that are successfully defended but generate heavy legal expenses.

Price as a Proxy for Risk

The model elaborated in the preceding sections assumes that the prospect's perceived probability of a judgment is not affected by changes in r , the per-unit cost of insurance. Recent developments in the D&O insurance market suggest that, when the environment changes rapidly, prospects may adjust p to reflect changes in r . The Wyatt Company's 1984 and 1986 D&O Surveys

(Brockmeier 1984 and 1986) offer the following comparisons of firms purchasing insurance from major D&O insurers and firms participating in D&O insurance pools:

Source of Insurance	1984		1986	
	Premium	Limit	Premium	Limit
Major Insurer	\$24,000	\$30,000,000	\$502,000	\$24,000,000
Insurance Pool	\$7,000	\$17,000,000	\$2,506,000	\$42,000,000

As commercial insurers were reducing limits and increasing premiums, pool participants increased their premiums at a faster rate than commercial insurers and simultaneously increased their limits.

The behavior of pool participants might be explained by environmental events that shifted up the pool-supply curve and the pool-demand curve. Alternatively, if Giffen-type behavior is defined as increasing coverage in response to an increase in the premium rate (Hoy and Robson, 1981), then it is possible that, for some pool participants, D&O insurance was a Giffen good. In particular, Giffen-type behavior might have occurred if the per-unit insurance premium was a measure of risk: when r rose, officers and directors inferred that the probability or size of a D&O claim had risen and demanded greater protection. This scenario would be analogous to the use of price to make inferences about quality (see, for example, Stiglitz, 1987).

This section modifies the model presented earlier by permitting the per-unit premium and environmental events to affect p and L . As noted by Ehrlich and Becker (1972, p. 634), to distinguish between p and L is somewhat artificial, but it is done so here for expository convenience.

The Perceived Magnitude of Loss as a Function of r

Suppose first that the perceived magnitude of loss is a function of r and e , where e denotes environmental factors such as the observed number of D&O judgments. In particular, assume that $L(r,e) > 0$, $\partial L(r,e)/\partial r \geq 0$, and $\partial L(r,e)/\partial e \geq 0$. Under these assumptions, the comparative static effects of a change in the per-unit premium are

$$dF^*/dr = (\mu/H)[pU'(W_1)] > 0, \text{ and} \quad (11.1)$$

$$dR^*/dr = -(\mu^2/H) + \partial L(r,e)/\partial r. \quad (11.2)$$

Because equation (11.1) is identical to (9.1), the assumption that L is a positive function of the per-unit premium has no effect on the optimal adjustment of F^* to changes in r . The sign dR^*/dr depends on the relative magnitudes of two effects induced by an increase in r . On the one hand, the perceived increase in L increases R^* . On the other hand, as a comparison of equations (11.2) and (9.2) reveals, the substitution of F for R in the optimal compensation package decreases R^* . Thus Giffen-type behavior will only occur when the former effect dominates the latter.

If the perceived magnitude of loss is a function of e , the effects of a change in e closely resemble the effects of a change in L reported in equations (10.1) and (10.2):

$$dF^*/de = 0, \text{ and} \quad (12.1)$$

$$dR^*/de = [\partial L(r,e)/\partial r]. \quad (12.2)$$

When e rises, the firm's optimal response is to leave the optimal fee unchanged and increase the amount of insurance purchased.

The Probability of Loss as a Function of r

Now assume that the perceived probability of a loss is a function of r and e , where $p(r,e) > 0$, $\partial p(r,e)/\partial r \geq 0$, and $\partial p(r,e)/\partial e \geq 0$. Under these assumptions, the effects of a rise in r and e on F^* and R^* are indeterminate and depend in part on whether the prospect exhibits increasing or decreasing absolute risk aversion. Under the more intuitive assumption of decreasing absolute risk aversion (Arrow, 1984, p. 153), a necessary condition for Giffen-type behavior is

$$\partial p(r,e)/\partial r > 1/[U'(W_1)U'(W_2)].$$

By assumption, as initial wealth W_0 increases, both W_1 and W_2 increase, and $U'(W_1)$ and $U'(W_2)$ approach zero. Thus for any $\partial p(r,e)/\partial r$, there exists a $W_0^*(\partial p(r,e)/\partial r)$ such that $W_0 > W_0^*$ implies that D&O insurance cannot be a Giffen good.

Discussion

The authors have considered a model in which a risk-neutral firm purchases insurance for a risk-averse employee as part of a compensation package that also includes a fixed fee. In particular, the effect on the optimal compensation package (R^*, F^*) of changes in four model parameters has been examined: the prospect's initial wealth, the probability of loss, the magnitude of that loss, and the per-unit insurance premium. While focussing on D&O insurance, the model has other applications, including professional, public official, and law-enforcement liability insurance.

Comparison with the Risk-Averse Consumer Model

To place the ECF model in perspective, one can compare its implications with those of the Ehrlich-Becker model, in which a risk-averse consumer (RAC) purchases insurance. In this case, W_1 and W_2 reduce to $W_1 = W_0 - L + (1-r)R$, and $W_2 = W_0 - rR$. The consumer chooses R to maximize $V(R; W_0, L, p) = pU(W_0 - L + (1-r)R) + (1-p)U(W_0 - rR)$.

A comparison of the relevant comparative static derivatives (see Appendix) reveals that the ECF and RAC models share the following implications: (1) an increase in the expected loss increases the amount of insurance purchased; (2) an increase in the perceived probability of loss increases the amount of

insurance purchased; and (3) the effect of an increase in initial wealth on the amount of insurance purchased depends on whether the prospect exhibits increasing, constant, or decreasing absolute risk aversion.

Despite these similarities, several differences emerge. An increase in the per-unit insurance premium reduces the amount of insurance purchased by the employee-constrained firm, but the effect on the amount of insurance purchased by a risk averse consumer depends on the consumer's absolute risk aversion. If the consumer exhibits increasing absolute-risk aversion, then a rise in r decreases R^* ; however, if the consumer exhibits decreasing absolute risk aversion, then the effect of a rise in r on R^* is indeterminate (Hoy and Robson, 1981).

Several other differences involve the prospect's wealth in the presence or absence of a claim. First, in the RAC model, the effects of an increase in r on W_1 and W_2 are indeterminate. The ambiguity arises because the consumer must pay more (r rises) for less insurance (R^* falls when r rises). It is easy to show that $dW_2/dr = -R(1 - E_r)$, where

$$\begin{aligned} E_r &= \text{the elasticity of demand for insurance} \\ &= -(dR^*/dr)(r/R^*). \end{aligned}$$

Thus W_2 rises only if E_r is greater than 1 (Ehrlich and Becker, 1972). In the ECF model, however, a rise in r decreases W_1 but increases W_2 . The effect of a change in r on W_2 differs because, in the ECF model, the firm, not the prospect, pays for the insurance. Because the firm responds to an increase in r by increasing F^* , W_2 rises.

Second, in the RAC model, a rise in L decreases W_2 . In addition, if $(1 - r)dR^*/dL < 1$, then $dW_1/dL < 0$; that is, if the increase in the consumer's net coverage is less than the increase in L , W_1 falls. In the ECF model, the firm responds to an increase in L by increasing R^* by an equal amount, so that W_1 is unchanged. Because the firm does not alter F^* , W_2 is unaffected by the decrease in L .

Third, in the RAC model, the effect of a rise in W_0 depends on the consumer's absolute risk aversion. On the one hand, increasing absolute risk aversion implies that $dW_1/dW_0 > 0$, but the sign of dW_2/dW_0 is indeterminate. On the other hand, decreasing absolute risk aversion implies that $dW_2/dW_0 > 0$, but the sign of dW_1/dW_0 is indeterminate. In the ECF model, the firm responds to an increase in W_0 by increasing both W_1 and W_2 . Because the increase in W_0 raises the minimum level of utility that the firm's compensation package must attain, the firm must offer a compensation package that raises prospect wealth in both states.

Finally, in the RAC model, a rise in p increases W_1 but decreases W_2 , because the rise in p induces the consumer to spend more on insurance. In the ECF model, a rise in p also increases W_1 , but the effect on W_2 depends on the magnitude of p . In general, for high values of p , W_2 and p vary directly. For low values of p , W_2 and p vary inversely.

Extensions

Because the desire to avoid a D&O lawsuit might affect board-member decisions, one extension of the model presented in this article would permit the employed prospect to affect the value of the firm. A second extension involves an analysis of the firm's selection of D&O insurance over alternative mechanisms for protecting its board members. When multiple alternatives are available, the firm must first determine the least-cost compensation package under each alternative and then select the alternative that permits the firm to hire the prospect at the minimum cost.

The model can also be applied to the more general problem of employment contracts between risk-neutral firms and risk-averse employees. The labor contract literature (e.g., Hart 1983) suggests an additional extension: the incorporation of asymmetric information. The authors defer a detailed examination of this extension, as well as the moral-hazard and multiple-alternative problems, to future research.

Appendix

This appendix presents (i) the second-order condition for a maximum and (ii) various comparative static derivatives discussed in the text. The second-order condition for a maximum (Chiang, 1974, p. 388) requires that the bordered Hessian determinant H be positive. Letting $A = pU'(W_1) > 0$, $B = (1-p)U'(W_2) > 0$, $C = pU''(W_1) < 0$, and $D = (1-p)U''(W_2) < 0$, it follows that $H = -\mu[(B^2C) + (A^2D)] > 0$.

Comparative Statics

Taking the total derivatives of equations (4) through (6) yields the following system of equations:

$$\begin{bmatrix} 0 & A+B & A \\ A+B & \mu(C+D) & \mu C \\ A & \mu C & \mu C \end{bmatrix} \begin{bmatrix} d\mu \\ dF \\ dR \end{bmatrix} = \begin{bmatrix} 0 & U'(W_0) - (A+B) & A & U(W_2) - U(W_1) \\ 0 & -\mu(C+D) & \mu C & \mu[U'(W_2) - U'(W_1)] \\ 1 & -\mu C & \mu C & -\mu U'(W_1) \end{bmatrix} \begin{bmatrix} dr \\ dW_0 \\ dL \\ dp \end{bmatrix}. \quad (A.1)$$

Letting $RA_i = -U''(W_i)/U'(W_i)$ and applying Cramer's rule to the system of equations in (A.1) yields

$$dF^*/dr = (1/H)[A(A+B)] > 0, \text{ and}$$

$$dR^*/dr = -(1/H)[(A+B)^2] < 0,$$

$$dF^*/dL = 0,$$

$$dR^*/dL = 1 > 0,$$

$$dF^*/dW_o = -1 - (\mu/H)[U'(W_o)BC],$$

$$dR^*/dW_o = (\mu/H)[U'(W_o)AB(RA_2 - RA_1)],$$

$$dF^*/dp = -(\mu/H)U'(W_2)\{(1-p)[U(W_2) - U(W_1)]C + AU'(W_1)\}, \text{ and}$$

$$dR^*/dp = (\mu/H)[U(W_2) - U(W_1)]AB[RA_2 - RA_1] \\ + (\mu/H)(A+B)U'(W_1)U'(W_2).$$

Now let x denote the vector of model parameters,

$$x = [r \ L \ W_o \ p]'$$

The effect of a change in x_i on W_1 and W_2 is given by $dW_1/dx_i = dF/dx_i + dR/dx_i + dW_o/dx_i - dL/dx_i$, and $dW_2/dx_i = dF/dx_i + dW_o/dx_i$.

Making the appropriate substitutions, one has

$$dW_1/dr = -(1/H)[B(A+B)] < 0,$$

$$dW_1/dL = 0 + 1 - 1 = 0,$$

$$dW_1/dW_o = -(\mu/H)U'(W_o)AD > 0,$$

$$dW_1/dp = -(\mu/H)AD[U(W_2) - U(W_1)] + (\mu/H)BU'(W_1)U'(W_2) > 0,$$

$$dW_2/dr = dF/dr > 0,$$

$$dW_2/dL = 0,$$

$$dW_2/dW_o = dF/dW_o + 1 = -(\mu/H)BCU'(W_o) > 0, \text{ and}$$

$$dW_2/dp = dF/dp.$$

Let $CP(F^*, R^*; r)$ denote the cost to the firm of the compensation package (R^*, F^*) . The effect of a change in x_i on $CP(F^*, R^*; r)$ is given by $dCP(F^*, R^*; r)/dx_i = dF/dx_i + r(dR/dx_i) + R(dr/dx_i)$. Making the appropriate substitutions,

$$dCP(F^*, R^*; r)/dr = (1/H)(A+B)\{(1-r)A - rB\} + R^*,$$

$$dCP(F^*, R^*; r)/dL = r > 0,$$

$$dCP(F^*, R^*; r)/dW_o = -1 - (\mu/H)U'(W_o)[(1-r)BC - rAD], \text{ and}$$

$$dCP(F^*, R^*; r)/dp = -(\mu/H)\{[(1-r)BC + rAD][U(W_2) - U(W_1)] \\ + [(1-r)A - rB]U'(W_1)U'(W_2)\}.$$

If, from the prospect's perspective, the insurance is actuarially fair, then

$$dCP(F^*, R^*; r)/dr = \{(\mu/H)r(1-r)(A+B)[U'(W_1) - U'(W_2)]\} + R^* > 0.$$

The Perceived Magnitude of Loss as a Function of r

If the perceived magnitude of loss is a function of r and e , the system of equations in (A.1) becomes

$$\begin{bmatrix} 0 & A+B & A \\ A+B & \mu(C+D) & \mu C \\ A & \mu C & \mu C \end{bmatrix} \begin{bmatrix} d\mu \\ dF \\ dR \end{bmatrix} \\
= \begin{bmatrix} L_r A & U'(W_o) - \mu(A+B) & L_e A \\ \mu L_r C & -\mu(C+D) & \mu L_e C \\ 1 + \mu L_r C - \mu C & & \mu L_e C \end{bmatrix} \begin{bmatrix} dr \\ dW_o \\ de \end{bmatrix} \quad (A.2)$$

where $L_r = \partial L(r,e)/\partial r$ and $L_e = \partial L(r,e)/\partial e$. Repeated applications of Cramer's rule yield

$$dF^*/dr = (1/H)[A(A+B)] > 0,$$

$$\begin{aligned} dR^*/dr &= -(1/H)\{(A+B)^2 + \mu L_r[A^2D + B^2C]\} \\ &= -(1/H)\{(A+B)^2\} + L_r, \end{aligned}$$

$$dF^*/de = 0, \text{ and}$$

$$dR^*/de = L_e.$$

The Probability of Loss as a Function of r

If the perceived probability of a loss is a function of r and e , the system of equations in (A.1) becomes

$$\begin{bmatrix} 0 & A+B & A \\ A+B & \mu(C+D) & \mu C \\ A & \mu C & \mu C \end{bmatrix} \begin{bmatrix} d\mu \\ dF \\ dR \end{bmatrix} \\
= \begin{bmatrix} p_r[U(W_2) - U(W_1)] & U'(W_o) - \mu(A+B) & A & p_e[U(W_2) - U(W_1)] \\ \mu p_r[U(W_2) - U(W_1)] & -\mu(C+D) & \mu C & \mu p_e[U(W_2) - U(W_1)] \\ 1 - [\mu p_r U'(W_1)] & -\mu C & \mu C & -\mu p_e U'(W_1) \end{bmatrix} \begin{bmatrix} dr \\ dW_o \\ dL \\ de \end{bmatrix} \quad (A.3)$$

where $p_r = \partial p(r,e)/\partial r$ and $p_e = \partial p(r,e)/\partial e$. Repeated applications of Cramer's rule yield

$$dF^*/dr = A(A+B)/H - (\mu p_r/H)\{BC[U(W_2) - U(W_1)] + AU'(W_1)U'(W_2)\},$$

$$dR^*/dr = (\mu/H)p_r\{(A+B)U'(W_1)U'(W_2) + AB[RA_2 - RA_1][U(W_2) - U(W_1)]\} - (1/H)(A+B)^2,$$

$$dF^*/de = -(\mu/H)p_e\{BC[U(W_2) - U(W_1)] + AU'(W_1)U'(W_2)\}, \text{ and}$$

$$dR^*/de = (\mu/H)p_e\{(A+B)U'(W_1)U'(W_2) + AB[RA_2 - RA_1][U(W_1) - U(W_2)]\}.$$

If the prospect exhibits decreasing absolute risk aversion, then condition for Giffen-type behavior is $(\mu/H)p_r(A+B)U'(W_1)U'(W_2) > (1/H)(A+B)^2$, which can be rewritten as $p_r > 1/[U'(W_1)U'(W_2)]$.

A Comparison with the Ehrlich-Becker Model of a Risk-Averse Consumer

The consumer chooses R to maximize: $V(R; W_0, L, p) = pU(W_0 - L + (1-r)R) + (1-p)U(W_0 - rR)$. Let $G(R^*, r, L, W_0, p) = [U'(W_1)/U'(W_2)] - [r/(1-r)] = 0$. Applying the implicit-function theorem (Chiang, 1974) yields

$$\partial R^* / \partial r = -(\partial G / \partial r) / (\partial G / \partial R^*) < 0 \text{ if } [RA_2 - RA_1] > 0,$$

$$\partial R^* / \partial L = -(\partial G / \partial L) / (\partial G / \partial R^*) > 0,$$

$$\text{sgn}(\partial R^* / \partial W_0) = \text{sgn}[-(\partial G / \partial W_0) / (\partial G / \partial R^*)] = \text{sgn}[RA_2 - RA_1], \text{ and}$$

$$\partial R^* / \partial p = -(\partial G / \partial p) / (\partial G / \partial R^*) > 0.$$

References

- Arrow, Kenneth J., 1984, The Theory of Risk Aversion, Chapter 9 in *Individual Choice under Certainty and Uncertainty*, (Cambridge, MA: The Belknap Press of Harvard University Press).
- Bailey, Dan A. 1987, D&O Liability Insurance Update—Recent Cases, Paper presented at The Wyatt Company's Directors' and Officers' Liability Symposium, New Orleans, LA, March 19 and 20.
- Brockmeier, Warren G., 1984, *The 1984 Wyatt Directors and Officers fiduciary Liability Survey: Comprehensive Report*, Company report, The Wyatt Company.
- Brockmeier, Warren G., 1986, *The 1986 Wyatt Directors and Officers fiduciary Liability Survey: Comprehensive Report*, Company report, The Wyatt Company.
- Chiang, Alpha C., 1974. *Fundamental Methods of Mathematical Economics* (New York: McGraw-Hill Book Company).
- Ehrlich, Issac and Gary S. Becker, 1972, Market Insurance, Self-Insurance, and Self-Protection, *Journal of Political Economy*, 80: 629-49.
- Hancock, William A., 1988a, Corporate Counsel's Guide to Director and Officer Liability Insurance, in Hancock, William A., ed., *Corporate Counsel's Guide to Director and Officer Liability Insurance and Indemnification*, Chesterfield, OH: Business Laws, Inc.
- Hancock, William A., 1988b, Report of the PLI Directors' and Officers' Liability Insurance 1988 Seminar, May 5, 1988, New York City, in Hancock, William A., ed., *Corporate Counsel's Guide to Director and Officer Liability Insurance and Indemnification*, Chesterfield, OH: Business Laws, Inc.
- Heimer, Carol A., 1985, *Reactive Risk and Rational Action: Managing Moral Hazard in Insurance Contracts*, Berkeley, CA: University of California Press.

- Hoy, Michael and Arthur J. Robson, 1981, Insurance as a Giffen Good, *Economic Letters*, 8: 47-51.
- Main, Brian G. M., 1983, Corporate Insurance Purchases and Taxes, *Journal of Risk and Insurance*, 50: 197-223.
- Mayers, David and Clifford W. Smith, 1981, Contractual Provisions, Organizational Structure, and Conflict Control in Insurance Markets, *Journal of Business*, 54: 407-34.
- Mayers, David and Clifford W. Smith, 1982, On the Corporate Demand for Insurance, *Journal of Business*, 55: 281-96.
- Mayers, David and Clifford W. Smith, 1986, Corporate Insurance and the Underinvestment Problem, *Journal of Risk and Insurance*, 54: 45-54.
- Mossin, Jan, 1968, Aspects of Rational Insurance Purchasing, *Journal of Political Economy*, 76: 553-68.19.
- Parker, Marcia A., 1986, How Corporate Directors Deal with the Boardroom Squeeze, *Buyouts and Acquisitions*, 4: (4), 11-17.
- Rothschild, Michael and Joseph Stiglitz, 1976, Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information, *Quarterly Journal of Economics*, 90: 629-49.
- Smith, Michael L. and Robert C. Witt, 1985, An Economic Analysis of Retroactive Liability Insurance, *Journal of Risk and Insurance*, 50: 379-401.
- Stiglitz, Joseph E, 1987, The Causes and Consequences of the Dependence of Quality on Price, *Journal of Economic Literature*, 25: 1-48.

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