

NOTES AND COMMUNICATIONS

An Insurance Plan to Guarantee Reverse Mortgages: Comment

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In Weinrobe's (1988) article arrangements of reverse mortgage plans (RM) are discussed and analyzed, particularly with respect to the deficit risk of the lender. This comment simplifies the situation by translating the RM concepts into traditional life insurance terms and using actuarial calculations, making the RM commercially accessible for the life insurance industry. Further, the RM possesses interest immunization properties in conjunction with a usual life insurance portfolio."

In its simplest form, an RM is an annuity paid to an individual by a financial institution with the corresponding lump-sum value paid to the institution at the end of the annuity period. The lump-sum payment is secured by a mortgage on residential property owned by the insured. The reversion, in comparison to an annuity investment, applies to the date the lump-sum payment is due, at the beginning of the annuity period in the case of the annuity investment. To extend these concepts to payments depending on the annuitant's being alive, the annuity investment must be translated to an immediate life annuity, purchased by a lump-sum payment at the beginning of the life annuity. The reversion makes the lump sum payment due at the end of the life annuity period, i.e., death.

The size of the payment for a given size of lump sum is here calculated as an expected value by (numerical) integration instead of the corresponding simulation calculation in the Weinrobe's article.

The value of a single life annuity of \$1 yearly, paid continuously, is

$$\bar{a}_x = \int_0^{\infty} (1+i)^{-t} {}_tP_x \, dt. \quad (1)$$

The value of a payment of \$1 at death is

$$\bar{A}_x = \int_0^{\infty} (1+i)^{-t} {}_tP_x \mu_{x+t} \, dt, \quad (2)$$

where

i = annual interest rate

μ_x = force of mortality for an x -year old person

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${}_tP_x = \exp \left\{ -\int_0^t \mu_{x+r} dr \right\}$ = corresponding probability for an x -year-old person to survive a further t years.

The reverse life annuity for an insured aged ξ years at agreement is defined so that the insured receives $\$p_\xi$ yearly (continuously, or approximately equal to 1/12 monthly) from the agreement date to death, and the insurer receives a lump sum of $\$S_\xi$ at death. The connection between p_ξ and s_ξ is

$$p_\xi \cdot \bar{a}_\xi = S_\xi \cdot \bar{A}_\xi. \quad (3)$$

For example: Let $i = 10$ percent p.a. Consider females only with mortality $\mu_x = 0.0005 + 10^{5.728 + 0.038x - 10}$ and let the insured sum be $\$80,000$. For an agreement age of $\xi = 70$ years, the values (1,2) are calculated as

$$\bar{a}_{70} = 7.015282, \quad \bar{A}_{70} = 0.331372$$

and so the yearly payment for an insured sum of $S_{70} = \$80,000$ by rearrangement is (3) becomes $\$3,779$. For the same insured sum, the size of the yearly payment for the different ages at the agreement date is:

Age	Annual Payment
70	\$ 3,779
75	5,300
80	7,467
85	10,591
90	15,151
95	21,896

If the insured vacates the property before death, a surrender debt would have to be paid by the insured. The surrender debt n years after the agreement date can be calculated as:

$$S_\xi \cdot \bar{A}_{\xi+n} - p_\xi \cdot \bar{a}_{\xi+n},$$

but this is only acceptable for the insurer if the health of the insured has not deteriorated (in comparison with that of the age group in general), i.e., if the actual mortality does not exceed the assumed mortality $\mu_{\xi+n}$. If the insured is close to death, a more true surrender debt will be close to S_ξ .

Instead of surrender termination, the insurance agreement is continued after the insured has vacated the property. The property is sold and the insurer retains the whole sum S_ξ , which originally was due at death. As compensation (also as a partial rent compensation), the insured is paid interest on S_ξ of $i \cdot S_\xi$ until death, in addition to the life annuity p_ξ .

Reverse mortgage agreements are net assets for a net insurer, in contrast to usual premium paid insurances, which are liabilities. The payment profiles for these two types are of the same structure, but with opposite signs. Hence the

¹ Mortality used in the Danish G82 valuation basis.

interest risk of the insurer can be eliminated by cash-flow matching. As a consequence, the insurer can guarantee an unchanged future interest rate, fixed at agreement date, for a balanced portfolio of RMs and premium paid insurances. The balance is achieved by calculating expected cash flows and matching these or by a more general duration immunization.

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