

INFLATION INSURANCE

Zvi Bodie

ABSTRACT

This article uses the concepts of option theory to explore the possibility of providing annuities that offer partial protection against inflation at a cost acceptable to retirees. A contract to insure a future payment against inflation is equivalent to a European call option on the consumer price index. This equivalence makes it possible to apply the techniques of modern contingent claims analysis to the production and pricing of inflation-protected annuities that have deductibles and caps. The method synthesizes a CPI call option through a dynamic trading strategy of borrowing and investing in CPI-linked bonds. The price of the insurance is the cost of implementing the strategy. The article also considers the question of who can provide the CPI-linked bonds that are the ultimate basis for the inflation insurance.

Introduction

While Social Security benefits and pension benefits under some public plans are indexed to the cost of living, the vast majority of private pension plans and annuity contracts in the United States offer no automatic inflation protection. Under most defined benefit pension plans, accrued pension benefits are partially protected against inflation through formulas that tie the retirement benefit to average earnings during the last few years of employment. However, this form of wage indexation stops at retirement. Virtually no private pension plans in the U.S. offer automatic inflation protection after retirement.

This article explores the idea of offering inflation insurance with deductibles and caps. It uses the concepts of option theory to quantify the costs of providing this insurance. It is based on the fact that a contract to insure a future payment against inflation is equivalent to a European call option on the consumer price index (CPI).

The first part of the article shows how to synthesize a CPI call option through a trading strategy involving nominal borrowing and investment in CPI-linked bonds. The second part explores the implications of this

Zvi Bodie is Professor of Finance and Economics at Boston University.

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production process for the pricing of inflation insurance. The next section considers the important question of who can provide the CPI-linked bonds that are the basis for the inflation insurance. The article concludes with a discussion of the implications of the analysis for public and private pension policy.

Synthesizing a CPI Call Option

A call option (or call, for short) is the right to buy a security at a preset exercise price at some specified date in the future. Active markets for options on a wide variety of financial instruments - stocks, bonds, currencies, and commodities - have developed in recent years both in the U.S. and abroad. While options on some market indexes exist, nowhere are options on the CPI traded.

Unlike conventional calls on individual stocks, which can be settled by the delivery of the underlying stock to the holder of the call upon exercise, index options are always settled in cash. This means that if, at the expiration date, the value of the underlying index exceeds the exercise price, then the holder of the call option receives from the seller an amount of cash equal to the difference times some standardized "notional" amount of principal determined by the exchange. At expiration, if the value of the index is less than the exercise price, the call expires without value.

An insurance policy against inflation is equivalent to a call option on the CPI. To see this equivalence, consider the following example. You expect to receive \$10,000 one year from now and want to insure it against inflation in excess of 6 percent. The 6 percent is like a deductible in a fire or theft insurance policy.

With the inflation insurance policy, if the rate of inflation exceeds 6 percent, you receive $P - 1.06$ dollars a year from now for every dollar insured, where P is the ratio of the CPI a year from now to its value now. This is the same as the payoff to a European call option on the CPI with an exercise price of 1.06. The cash received from the call will exactly offset any inflation in excess of 6 percent. Thus if the rate of inflation turns out to be 10 percent, the call will pay 4 cents, enough to compensate you for the 4 percent inflation above the 6 percent deductible.

The amount of the deductible can be changed by changing the exercise price of the CPI call option. Thus, if full inflation insurance is desired with no deductible, the exercise price would be 1. While CPI call options are not traded on any exchanges, bonds linked to the CPI are. Let us now show how one could synthesize such a CPI call option from a CPI-linked bond and a conventional bond, both free of default risk.

Let $P(T)$ be the consumer price level at time T . For convenience and with no loss of generality, let the current price level be 1 (that is, $P(0) = 1$).¹ Let $r(T)$

¹Since what matters is the ratio of the CPI at time T to its value now, there is no loss of generality in setting its current value to 1.

be the riskless real rate of interest on a default-free CPI-linked zero coupon bond maturing T years from now. By definition $(1+r)^{-T}$ dollars invested in such a bond now will pay $P(T)$ dollars at time T .

Let $R(T)$ be the riskless nominal rate of interest on a default-free zero coupon bond of the conventional kind (e.g., a U.S. Treasury bill). By definition $(1+R)^{-T}$ dollars invested in such a bond now will pay \$1 at time T .

The option pricing theory developed by Black and Scholes (1973) and Merton (1973) gives us a way of synthesizing the call option through a dynamic replication strategy. If the movement of the prices of both conventional and CPI-linked bonds can be approximated by diffusion processes, then a modified form of the Black-Scholes option pricing formula can be applied:²

$$C = N(d_1) e^{-rT} - N(d_2) e^{(i-R)T} \quad (1)$$

$$d_1 = \frac{[R - (r+i) + \frac{\sigma^2}{2}] T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

where C is the price of a CPI call (the cost of synthesizing it), i is the deductible rate of inflation, σ is the instantaneous standard deviation of P , and $N(d)$ is the cumulative normal distribution function. The interest rates in the Black-Scholes formula are continuously compounded rates.

The first term on the right-hand side of equation 1 is the amount to be invested in CPI-linked bonds and the second term is the amount to be borrowed. The cost of synthesizing the CPI call option (C) - the break-even price of inflation insurance - is the difference between them.

The replicating portfolio has to be rebalanced over time to maintain the correct amounts of borrowing and investment in CPI-linked bonds. The key feature of the Black-Scholes method is that the provider of insurance will not have to add any money after the initial amount has been put in. The strategy is self-financing from that point on.

Table 1 and figure 1 illustrate both the production process and the pricing of inflation insurance. They present C as a function of the deductible (i), assuming the riskless real interest rate (r) is 3 percent per year, the riskless nominal rate (R) is 9 percent per year, the maturity (T) is 10 years, and the standard deviation of the rate of inflation is 3 percent per year. The last two columns of table 1 show the amounts that have to be borrowed and invested in CPI-linked bonds in order to synthesize the insurance.

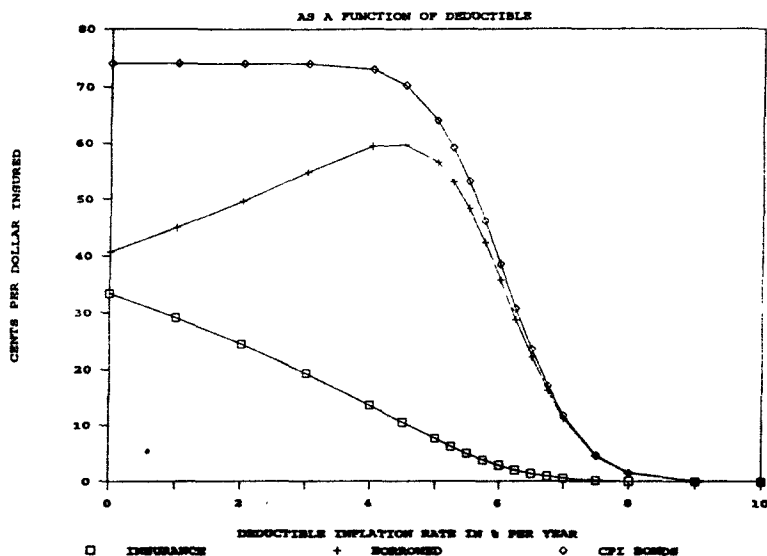
²To be more precise, Merton's (1973) more general version of the Black-Scholes model that allows for stochastic interest rates is used. Merton showed that the Black-Scholes formula is valid if the prices of the securities used to synthesize the option follow diffusion processes of the form: $dP/P = \alpha dt + \sigma dz$ where dz is an increment of a standard Wiener process with a zero mean and variance of 1, α is the instantaneous mean rate of inflation per unit of time, and σ^2 is the variance per unit of time.

Table 1
Price of Insurance with a Deductible

Deductible Rate of Inflation (% per year)	Price of Insurance (cents)	Amount Borrowed (cents)	Investment in CPI-Linked Bonds
0% per year	33.425	40.657	74.082
1	29.149	44.933	74.082
2	24.423	49.659	74.082
3	19.202	54.828	74.030
4	13.469	59.457	72.926
5	7.550	56.501	64.051
6	2.811	35.638	38.449
7	0.553	11.085	11.638
8	0.049	1.410	1.459
9	0.002	.065	.067
10	0.000	0	0

Assumptions: The price is in cents per dollar insured. Maturity is 10 years; the risk-free real interest rate is 3 percent per year; the risk-free nominal rate is 9 percent per year; and σ is 3 percent per year.

Figure 1
Price of Insurance with a Deductible
PRICE OF INFLATION INSURANCE



Assumptions: Maturity is 10 years; the risk-free real interest rate is 3 percent per year; the risk-free nominal rate is 9 percent per year; and σ is 3 percent per year.

When the deductible is very low, the way to synthesize the insurance is to buy a CPI bond for 74 cents, borrowing an amount which rises as the deductible rises. Once the deductible passes a certain point, however, the production process involves a smaller investment in CPI bonds and a smaller amount of borrowing. The mix of borrowing and investment in CPI bonds changes quickly and dramatically when the deductible is in the vicinity of 6 percent per year, the spread between the nominal and the real risk-free interest rates.

While the cost of full inflation insurance with no deductible is 33 cents per dollar insured, this cost falls off rapidly as the deductible is raised. If the deductible is set equal to the spread between the nominal and the real risk-free interest rates, or 6 percent per year, then the cost of the insurance is only 2.8 cents per dollar insured.

The Pricing of Inflation Insurance

Having established the principle that a break-even price for inflation insurance can be found as the net cost of synthesizing a CPI call option by borrowing and investing in CPI-linked bonds, consider how that price will vary as a function of the underlying parameters. For those familiar with the Black-Scholes formula as applied to stock options, there are some similarities and some surprises.

Equation 1 shows that the price of inflation insurance (C) is a function of only five variables: i , r , R , T , and σ . The expected rate of inflation does not appear explicitly as one of the variables, but its effect is felt through its impact on the risk-free nominal rate of interest, R , as explained below.

The price of inflation insurance increases with volatility. This is a well-known result in option pricing theory. It reflects the asymmetric payoff structure of the call option. Increasing the volatility increases the upside potential without increasing the downside risk.

The effect of the maturity of the contract in the present model is strikingly different from the standard Black-Scholes model applied to stocks. In the Black-Scholes stock option model, the value of the call increases monotonically with maturity. In the present model, it first rises and then falls with maturity. The difference is that in the stock option model the price of the underlying security is held fixed when maturity is increased, whereas in the present model it is the real interest rate that is fixed.

It is easiest to understand the effect of maturity on the price of inflation insurance where there is a zero deductible. Then, the inflation insurance is equivalent to a CPI call that is "way in-the-money," and the standard deviation plays virtually no role. The way to synthesize it is to borrow the present value of \$1 at the nominal rate and to invest in a zero coupon CPI bond. The net cost of this strategy is given by:

$$C = e^{-rT} - e^{-RT} \quad (2)$$

Note that the inflation insurance premium first rises and then falls with the maturity of the contract.³ It is at its maximum when the maturity is $\frac{\ln(R/r)}{R-r}$.

For the shorter maturities, the premium rises because the amount of borrowing needed declines more rapidly than the prices of the corresponding CPI bonds. Eventually, however, this is reversed. The premium approaches the price of the CPI bond asymptotically as an upper bound and must, therefore, decline with it (provided that the real interest rate is positive).

Now let us consider the effect of interest rates and expected inflation. By definition the relationship between the risk-free nominal and real interest rates is:

$$R(T) = r(T) + \text{expected inflation rate} + \text{risk premium} \quad (3)$$

$$\text{or } R(T) = r(T) + \pi(T) + \phi(T)$$

Let us define the expected real rate of interest on a nominal bond as the risk-free nominal rate minus the expected rate of inflation: $E(r_N) = R - \pi$. Then it follows that this expected real rate on the nominal bond will exceed the risk-free real rate of interest (r) by the risk premium (ϕ).⁴

Let us maintain the assumption that both r and R are constant across maturities and consider the effect of an increase in the risk-free real rate, holding constant expected inflation and the risk premium on nominal bonds. Under this assumption, any increase in r will be matched by an equal increase in R . An increase in the real interest rate causes a downward shift in inflation insurance premiums of all maturities.⁵

This result is contrary to the effect of an increase in interest rates in the Black-Scholes model applied to European call options on stocks. As with the effect of maturity, the present model does not hold the price of the underlying security fixed while increasing the interest rate.

Greater realism could be added by allowing real and nominal interest rates to vary by maturity, but the analysis would not be affected in any essential way. As long as the zero coupon CPI bonds and the borrowing used to

³ The first and second derivatives of the inflation insurance premium with respect to T are given by the formulas:

$$\frac{\partial C}{\partial T} = -re^{-rT} + Re^{-RT}$$

$$\frac{\partial^2 C}{\partial T^2} = r^2e^{-rT} - R^2e^{-RT}$$

⁴ For a discussion of the size of this risk premium see Bodie (1982).

⁵ The first and second derivatives of the inflation insurance premium with respect to the real interest rate are:

$$\frac{\partial C}{\partial r} = -Te^{-rT} + Te^{-RT} = -TC$$

$$\frac{\partial^2 C}{\partial r^2} = T^2C$$

synthesize the inflation insurance have the same expiration date as the insurance policy, the formula and the method used are valid.⁶

Now consider the effect of an increase in the expected rate of inflation, holding constant r and ϕ . An increase in the expected inflation rate causes the riskless nominal rate to rise. This causes the prices of nominal bonds of all maturities to fall, while leaving the prices of CPI-linked bonds unaffected. The net result is that inflation insurance premiums rise.⁷

The effect of an increase in the expected rate of inflation in the present model is analogous to the effect of an increase in interest rates in the Black-Scholes stock option model. The reason is that in the present model a higher expected inflation rate means a higher nominal interest rate with an unchanged real interest rate.

Inflation-Protected Annuities

If inflation insurance became available, it is likely that the major demand for it would be to insure pension benefits.⁸ The cost of insuring a stream of nominal payments against inflation is the sum of the costs of insuring each individual payment.

Table 2 and figure 2 present the cost of insuring a 20 year nominal annuity of \$1 per year against inflation as a function of the deductible. Thus, with a zero deductible, the cost of inflation insurance is \$5.95. Since the price of the nominal annuity is \$8.86, this means that the cost of insuring it fully against inflation is 67 percent of its value.

The cost of inflation insurance with a deductible equal to 6 percent per year is only \$.52 or roughly 6 percent of the value of the nominal annuity. And the cost of catastrophic inflation insurance, defined as a policy with a deductible equal to 10 percent per year, is only \$.002 or .02 percent of the value of the annuity.

Inflation Insurance with a Cap

Often the cost-of-living adjustments that are promised under certain pension plans and life annuities are subject to a cap. The previous analysis can easily be modified to price such an inflation insurance policy.

The only adjustment needed to the model presented in the preceding section is to subtract from the price of an inflation insurance policy with no deductible the price of a policy that has a deductible equal to the specified cap rate of inflation. The price of an inflation insurance policy with a cap is

⁶See Merton (1973), p. 168.

⁷The first and second derivatives of the inflation insurance premium with respect to π are:

$$\frac{\partial C}{\partial \pi} = T e^{-RT}$$

$$\frac{\partial^2 C}{\partial \pi^2} = -T^2 e^{-RT}$$

⁸For a discussion of this issue see Bodie (1990).

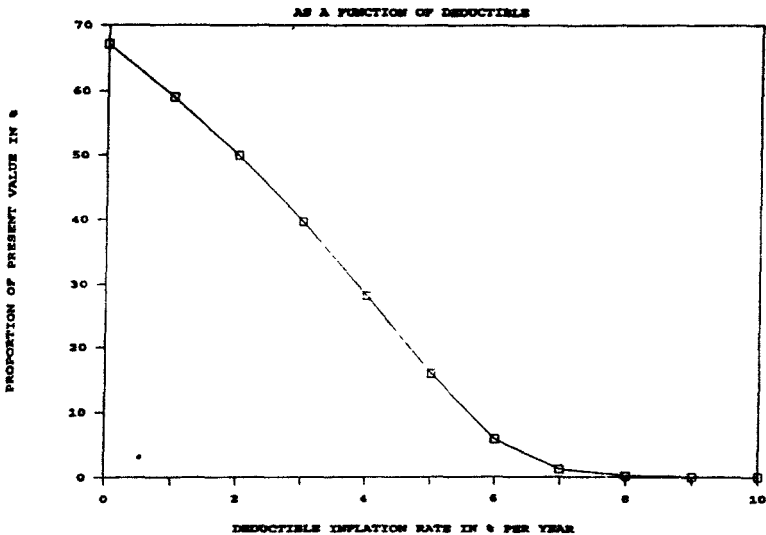
Table 2
Cost of Insuring a 20-Year Annuity

Deductible Rate of Inflation	Cost as a Proportion of Present Value of Annuity
0%	67.15%
1	59.04
2	49.94
3	39.71
4	28.27
5	16.04
6	5.84
7	1.22
8	.24
9	.07
10	.02

Assumptions: The risk-free real interest rate is 3 percent per year; the risk-free nominal rate is 9 percent per year; and σ is 3 percent per year.

Figure 2

Cost of Insuring a 20 Year Nominal Annuity
COST OF INSURING A 20 YEAR ANNUITY



Assumptions: The risk-free real interest rate is 3 percent per year; the risk-free nominal rate is 9 percent per year; and σ is 3 percent per year.

therefore equal to the price of a CPI call option with an exercise price of 1 minus the value of a CPI call option with exercise price e^{-cT} , where c is the cap on the inflation rate.

For example, consider an inflation insurance policy that is capped at 5 percent per year. Assume that $r = 3$ percent, $R = 9$ percent, $T = 10$ years, and $\sigma = 3$ percent per year. The price of the CPI call option with no deductible is 33.42 cents. The price of the CPI call option with a deductible equal to 5 percent per year is 7.55 cents. The price of the capped inflation insurance policy is therefore 25.87 cents, the difference between the prices of the two options.

The Role of the Government and Private Insurers

Economists consider it desirable, if not essential, for the Federal government to issue CPI-linked bonds in order to lay the foundation for inflation insurance. Economists like Milton Friedman, Franco Modigliani, and James Tobin, who hold very different opinions on other issues, are united in their enthusiastic support for the idea of the U.S. Treasury's issuing CPI-linked bonds. They think that the only entity that can truly guarantee default-free inflation insurance is the government.

In some countries, the government has played an active role in providing default-free bonds linked to the consumer price index for pension funds to use as the basis for inflation-protected retirement annuities. In the U.K., for example, the government has issued bonds tied to the retail price index (the U.K. equivalent of the CPI).⁹ Indexation is even more common in Latin America and Israel.

While it is strictly speaking true that only the government can issue 100 percent default-free CPI-linked bonds, it is equally true that private CPI bonds can be almost free of default risk. A private issuer can offer bonds that are virtually free of default risk through a combination of two elements: (1) hedging the risk of its liabilities through appropriate investment strategies, and (2) maintaining adequate equity capital so that the residual risk that is not diversified away or hedged away by the company's investment strategy is fully absorbed by the company's shareholders.

The first method appears difficult without the introduction of new financial instruments. Existing assets, such as common stock, real estate, commodities, and foreign securities, are only imperfectly correlated with the consumer price level and therefore unsuitable as inflation hedges.

Private inflation insurance requires that someone in the economy be willing to bear some part of the risk of inflation at a fair market price. The natural candidates for doing this would be people or institutions who are "over-indexed" for inflation. Feldstein (1983) and Summers (1983) have maintained that substantial numbers of households at all stages of the life cycle may find themselves in this position. During their working years,

⁹See Munnell and Grolnic (1986).

households have their earning power (or human capital) and often own their own homes. While these assets are not risk-free, they certainly seem to be protected against inflation risk. Wages tend to keep pace with inflation, and residential real estate often does especially well in times of inflation.

For these two reasons, a promising source of CPI-linked investments for an inflation insurance intermediary is CPI-linked home mortgages. The U.S. Department of Housing and Urban Development is seriously considering certifying a variety of price-level-adjusted mortgages (PLAMs) for Federal Housing Administration approval (FHA).¹⁰ PLAMs have often been discussed as a simultaneous solution to the problems of young people seeking affordable mortgage financing and to the problems of old people on money-fixed incomes seeking inflation protection.

For the young, PLAMs address important problems associated with both the conventional fixed-rate and the standard adjustable-rate mortgage (ARM) designs. Under both of these, the payment schedule is a level nominal stream rather than a level real stream. As a result, the monthly payment is often too high a fraction of initial monthly income for the young to qualify for a home mortgage loan. With an ARM, when the monthly mortgage payment is recalculated periodically at the new adjustable interest rate, the borrower can be subject to large fluctuations in the monthly payment.

PLAMs set a monthly payment that is fixed in real terms. The nominal payment is adjusted monthly according to the realized rate of inflation. This implies a graduated schedule of nominal payments with a much lower starting value. Assuming that a home-buyer's earnings adjust for inflation, the monthly mortgage payment is a relatively stable proportion of income.

Once FHA mortgage insurance is available and the tax status of PLAMs is clarified, they could account for a significant portion of new lending in the home mortgage market. Financial intermediaries, such as insurers, could then issue inflation-protected retirement annuities using PLAMs as the base.

Fischer (1986) maintains that many business firms may want to issue price-indexed debt in order to reduce their risk. Nonfinancial businesses have shown some willingness to issue debt securities that are indexed to the prices of their output. A financial intermediary could pool such bonds in order to synthesize an investment that hedges annuities indexed to broader price indexes.¹¹

In 1988 several financial institutions issued securities linked to the U.S. consumer price level. The new securities were issued first by the Franklin Savings Association of Ottawa, Kansas, in January 1988 in two different forms.

The first is certificates of deposit, called Inflation-Plus CDs.¹² Interest is paid monthly and is equal to a stated real rate plus the proportional increase

¹⁰See Modigliani and Lessard (1975) for a discussion of these mortgage designs.

¹¹See Blinder (1976).

¹²These CDs were originally insured by the Federal Savings and Loan Insurance Corporation, but now the insurance has been absorbed by the Federal Deposit Insurance Corporation.

in the CPI during the previous month. The second form is 20-year noncallable collateralized bonds, called Real Yield Securities or REALs. These offer a floating coupon rate equal to a stated real rate plus the previous year's proportional change in the CPI, adjusted and payable quarterly. Two other financial institutions followed the lead of Franklin Savings.¹³

Apparently, these institutions were willing to try the second method of providing default-free CPI-linked bonds to the market, by absorbing the inflation risk through their own capital. Federal deposit insurance also makes the Inflation-Plus CDs default-free up to \$100,000.

Recently, however, Federal regulators seized the assets of Franklin Savings Association on technical grounds having nothing to do with Franklin's issuance of CPI-linked securities. This action has made clear that while they were free of default risk, these securities were not free of regulatory risk. At the time this paper is being written, the future status of Franklin's liabilities is uncertain.

While CPI-linked bonds have been described as the basis for synthesizing inflation insurance, they are not really essential. What is required to produce inflation insurance through the process of dynamic hedging is any security whose return is perfectly correlated with the CPI. The history of portfolio insurance suggests that index futures contracts might be used for this purpose.

In this connection it is worth mentioning the failure of the Coffee, Sugar and Cocoa Exchange's recent attempt to establish a CPI futures market.¹⁴ Futures markets require heavy trading in short-term contracts in order to maintain their financial viability. The active participation of speculators is usually essential for this. But the CPI does not have much short-term volatility, and this makes a futures contract unattractive to speculators.

It is conceivable that a market for CPI options could emerge. Inflation insurance could then be accomplished by directly buying CPI calls of the desired maturity and with the desired exercise price.

Policy Implications

Proposals to index pension benefits and other nominal annuities in both the private and public sectors have a long history.¹⁵ In the U.K. the government has gone so far as to mandate the indexation of the minimum level of employer-provided pension benefits, and the government of the Province of Ontario, Canada is on the verge of adopting similar measures.¹⁶ The approach presented in this article permits fairly precise quantification of the cost of implementing such proposals.

¹³ In August 1988 Anchor Savings Bank became the second U.S. institution to issue REALs, and in September 1988 JHM Acceptance Corporation issued modified index-linked bonds subject to a nominal interest rate cap of 14 percent per annum. Morgan Stanley and Company is the underwriter and market maker for REALs.

¹⁴ See Bodie, Kane, and Marcus (1989), p. 683.

¹⁵ See Bodie and Pesando (1983).

¹⁶ See Friedland (1988) for the Canadian situation and Hemming and Kay (1982) for the U.K.

This approach also provides a way of estimating the savings to the Social Security system that would result from introducing a deductible. Some people have advocated limiting the Social Security cost-of-living adjustment to the excess of the actual inflation rate over some deductible. The proposed approach can help to quantify the savings that would result from any deductible rate of inflation.

References

- Black, Fischer, and Myron Scholes, 1973, The Pricing of Options and Corporate Liabilities, *Journal of Political Economy*, 81: 637-54.
- Blinder, Alan, S., 1986, Indexing the Economy Through Financial Intermediation, Princeton University Econometric Research Program Research Memorandum No. 196.
- Bodie, Zvi, 1982, Inflation Risk and Capital Market Equilibrium, *The Financial Review*, 17 (1): 1-25.
- , 1990, Inflation Protection for Pension Plans, *Compensation and Benefits Management*, 6 (2): 105-110.
- , Alex Kane, and Alan Marcus, 1989, *Investments*, (Homewood, IL: Richard D. Irwin.)
- , and James Pesando, 1983, Retirement Annuity Design in an Inflationary Climate, in: Zvi Bodie and John B. Shoven, eds., *Financial Aspects of the U.S. Pension System*, (Chicago: The University of Chicago Press) chapter 11.
- Feldstein, Martin, 1983, Should Private Pensions Be Indexed, in *Financial Aspects of the U.S. Pension System*, op. cit.: chapter 8.
- Fischer, Stanley, 1986, On the Nonexistence of Privately Issued Index Bonds in the U.S. Capital Market, in *Indexing, Inflation, and Economic Policy*, (Cambridge, MA: MIT Press) chapter 10.
- Friedland, Martin, 1988, *Report of the Task Force on Inflation Protection for Employment Pension Plans*, Ontario Government Publication.
- Hemming, Richard, and John Kay, 1982, The Costs of the State Earnings Related Pension Scheme, *Economic Journal*, 92, 366.
- Merton, Robert C., 1973, Theory of Rational Option Pricing, *Bell Journal of Economics and Management Science*, 4: 141-83.
- Modigliani, Franco and Donald Lessard, eds., 1975, *New Mortgage Designs for Stable Housing in an Inflationary Environment*, Federal Reserve Bank of Boston, Conference Series No. 14.
- Munnell, Alicia, and J. Grolnic, 1986, Should the U.S. Government Issue Index Bonds? *New England Economic Review*, September/October: 3-21.
- Summers, Lawrence, 1983, Observations on the Indexation of Old Age Pensions, in *Financial Aspects of the U.S. Pension System*, op. cit.: chapter 9.

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