

The Present Value of Future Earnings: Contemporaneous Differentials and the Performance of Dedicated Portfolios

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Abstract

Recent articles in this *Journal* provide evidence that the present value of a stream of future earnings can be estimated with reasonable certainty. Such certainty emerges by virtue of the predictable covariance between the realized rates of growth of workers' earnings and interest rates reflecting the time value of money.

This article presents an investigation of the predictive power of the contemporaneous differential, or the differential between interest rates and the rate of growth in wages at the time the present value estimate is made. The question investigated is whether the contemporaneous differential has a meaningful effect on the realized differential (that is, the differential between the actual investment yield achieved and the actual rate of growth in wages during the subsequent loss period). Empirical results of the study show that the contemporaneous differential does not have a meaningful impact on the realized differential.

Introduction

Anderson and Roberts (A-R) (1989) offer evidence regarding the estimation of the present value of a worker's stream of future earnings. A-R addressed the issue of identifying the appropriate differential between the rate (r) at which the earnings stream should be discounted and the rate of growth (g) of that earnings stream. They concluded "... that a benchmark after-tax net discount rate [i.e., $(r-g)$] of -0.5 percent would be appropriate for assessing awards for lost earnings for most cases." (p. 64).

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Bryan and Linke (B-L) (1988) reached a similar conclusion. Specifically, B-L investigated the use of portfolios of alternative composition, but with each dedicated to the replacement of an earnings stream. The earnings streams to be replaced were of varying duration over the 1953 through 1984 period. Each of these special-purpose portfolios is referred to as a dedicated portfolio. It turned out that dedicated portfolios consisting of U.S. Treasury securities of one-year constant maturity constituted the least cost means of providing for future earnings streams. The averages of the *ex post* or realized yields on dedicated portfolios and the *ex post* rates of growth in wages were approximately the same.

The atypical differentials between interest rates and wage growth rates during the 1981 through 1988 period have raised questions regarding the robustness of A-R's and B-L's evidence that the appropriate differential is approximately zero. At first blush, it may seem plausible for litigants, juries and judges to review and compare recent data relating to the differential between interest rates and wages growth rates. We refer to such data as contemporaneous; that is, year-by-year, *actual* interest rates and *actual* rates of growth in wages as they move through time.

The focus of this article is on a narrow, but timely, issue: namely, does information regarding interest rates and rates of growth in wages just prior to a loss period provide useful information regarding the present value of the future stream of wages during the loss period? The answer to this question is especially important during periods when the contemporaneous differential differs markedly from zero.¹

The Model

In this section there is a discussion of the adjustment of the economy to discrepancies between the actual level of interest rates and rates of growth in wages. Within the context of such adjustments, a model is presented of the impact of contemporaneous differentials on the relation between returns on dedicated portfolios and growth rates in wages.

Contemporaneous Interest Rates and Wages Growth

It is common to define the nominal rate of interest (r) as the sum of the expected real rate (i) and an inflation premium (ρ).² Similarly, the rate of growth in wages (g) may be defined as the sum of the rate of growth in the marginal productivity of labor (ℓ), and the rate of change in the general price level (p). The differential between i and ℓ depends upon an economic relationship between the real rate of interest and the productivity of labor.

¹ As pointed out in Bryan and Linke (1988), the focus of the literature relating to the measurement of pecuniary loss in lawsuits has been on the contemporaneous relationships between the growth in wages and rates of return on alternative investment media (i.e., $r_t - g_t$).

² The nominal rate of interest emerges as follows: $(1+r) = (1+i)(1+\rho)$. Expanding the right-hand side then subtracting one from both sides, we get $r = i + \rho + i\rho$. Following convention, cross-products are ignored. The rate of growth in wages is developed in a similar fashion.

That relationship rests, in turn, on a relation between the productivity of real capital and the productivity of labor.

Relationships among the real rate of interest, the marginal productivity of labor, and the marginal productivity of real capital emerge from steady-state equilibrium conditions of the economy. The marginal productivity of capital is defined as $\pi = mp_K P_O / P_K$ (where $mp_K P_O$ = capital's marginal revenue product in constant prices, and P_K = the price of capital).³ The productivity of labor is defined as $L = mp_L P_O / (W/i)$ (where $mp_L P_O$ = labor's marginal revenue product in constant prices, and W = the nominal wage for labor).⁴ Using K to denote the capital stock, N for the number employed and T to denote the state of technology along with other factors affecting the productive process (e.g., natural resources and education), it is traditional to assert that

$$(1) \pi = u(K, N, T)$$

$$\text{where } \frac{\partial \pi}{\partial K} < 0, \frac{\partial \pi}{\partial N} > 0, \frac{\partial \pi}{\partial T} > 0, \text{ and}$$

$$(2) L = v(K, N, T)$$

$$\text{where } \frac{\partial L}{\partial K} > 0, \frac{\partial L}{\partial N} < 0, \text{ and } \frac{\partial L}{\partial T} > 0.$$

In equilibrium, $(mp_K / P_O / i) = P_K$ and $mp_L P_O = W$; hence,

$$(3) \pi = L = i.$$

Equation (1) says that the productivity of capital declines as additional units of capital are added to an unchanged quantity of labor and an unchanged state of technology. In contrast, capital becomes more productive as units of labor are added and/or as technology improves. Equation (2) states that the productivity of labor declines as additional units are added to an unchanged capital stock, given a state of technology. But the productivity of labor improves as a consequence of increases in the capital stock and/or improvements in technology. Equation (3) is an assertion that, given the state of technology, adjustments occur in the capital stock and in the quantity of labor until there is equality among the marginal productivity of capital, the marginal productivity of labor and the real rate of interest.

The flow of real investment is given, *ceteris paribus*, by the real rate of interest, i . Assumptions regarding the pace of steady-state growth (presumably specified in terms of $\frac{\partial \pi}{\partial T}$ and $\frac{\partial L}{\partial T}$) have implications, at length, for the

³ mp_K is a physical quantity; multiplication by P_O converts the marginal product into dollar terms (i.e., into marginal revenue product). What we are referring to as the marginal productivity of capital is the ratio between its marginal revenue product (in constant prices) and its own price.

⁴ The nominal wage, which is paid periodically, is divided by i in order to express the price of labor as a present value. With this conversion it becomes meaningful to make a comparison between the productivity of labor and the productivity of capital.

flow of real investment net of depreciation and, hence, for the needed level of i . The adjustments defined in (1) and (2) continue until (3) obtains. These adjustments also bring about the steady-state relation between ℓ and i .

To assert a relationship between ρ and p is merely to assert that expectations are determined, at length, by experience. Thus, for example, it is plausible to imagine that ρ is a weighted average of past changes in the price level. If so, the means of ρ and p would be equal, but their variances would differ ($\sigma_\rho^2 < \sigma_p^2$).

As a point of departure, suppose that the economic system has equilibrium properties that serve to drive r and g toward equality over long periods of time.⁵ At the beginning of a loss period the economy finds itself with an existing actual, or contemporaneous, differential, $(r-g)_{t=0}$. A model is specified such that the economy acts as though it compares its actual position at $t=0$ with its desired position, the equilibrium differential, at the end of period k , $(r^*-g)_{t=k}$. Adjustments during t (from $t=0$ to $t=k$), $\Delta(r-g)_t$, serve to move the differential toward the desired position. The size of that adjustment is given by λ . These thoughts are imbedded in the familiar stock-adjustment model as follows:⁶

$$(4) \Delta(r-g)_t = \lambda[(r^*-g)_{t=k} - (r-g)_{t=0}].$$

Within the framework of the stock-adjustment model, λ is the portion of a discrepancy between the economy's equilibrium and actual $(r-g)$ position to be removed during t ($0 < \lambda \leq 1$). The more rapid the equilibrium adjustment the closer is λ to 1.

Return on Dedicated Portfolios and Wage Growth

Equation (4) is specified in terms of contemporaneous variables. A one-to-one mapping of adjustments relating to contemporaneous variables onto adjustments in realized variables is not expected. Changes in contemporaneous interest rates typically have two types of effects on yields of dedicated portfolios. If interest rates decline, for example, there will be capital gains (tending to increase the value of the dedicated portfolio); at the same time, however, there will be reductions in the interest income on the portfolio as reinvestment rates decline. Adjustments estimated below are affected by how, on balance, interest rate shifts impact the realized yield on dedicated portfolios.

Using the stock-adjustment framework specified above, but moving away from first differences of the dependent variable (i.e., adding $(r'-g')_{t=0}$ to both sides of the equation), (4) is rewritten as

$$(5) (r'-g')_{t=k} = \lambda(r'^*-g')_{t=k} + (1-\lambda)(r'-g')_{t=0}$$

⁵In point of fact we do not offer this as a theoretical matter. But as an empirical matter over the periods we and others have investigated, the $(r-g)$ differential has been approximately zero.

⁶For discussion, see [3, pp. 217-221] and [5, pp. 360-364].

where r' = realized yield on dedicated portfolios during period between $t=0$ and $t=k$; g' = realized rate of wages growth during period between $t=0$ and $t=k$; $(r'-g')_{t=k}$ = realized differential; $(r'-g')_{t=0}$ = contemporaneous differential;⁷ λ' = coefficient of adjustment for dedicated portfolios; and k = length of period.

Recasting (5) into a regression format, it is specified that

$$(6) (r'-g')_{t=k} = a_0 + a_1 (r'-g')_{t=0} + \epsilon_t$$

where $a_0 = \lambda'(r'^* - g')_{t=k}$; $a_1 = (1 - \lambda')$; and ϵ = a stochastic term.

The adjustment coefficient, $\lambda'(\hat{\lambda}' = 1 - \hat{a}_1)$, measures the extent to which the initial or contemporaneous differential (i.e., $(r'-g')_{t=0}$) is eroded during the period of adjustment defined by k . Thus, for example, if $\hat{a}_1$ were .25 it would mean that λ' would be .75. In turn, this would imply that 75 percent of any discrepancy between $(r'^* - g')_{t=k}$ (i.e., an equilibrium $(r'-g')$ for a period of k years) and the initial position would be removed by the end of k years.

Data

A data set of artificial experience with dedicated portfolios was developed for the 1953 through 1988 period. The data problem involved identifying the endowment required in a dedicated portfolio at $t=0$ to provide exactly for the earnings stream of the average U.S. worker over alternative time periods. Given that initial endowment with the initial annual wages and the *ex post* growth in wages (g') over the time period, it was possible to solve for the implied *ex post* return (r') on each dedicated portfolio. With these data it was possible to calculate $(r'-g')$, the *ex post* or realized differential between the discount rate and the growth rate.

The input data for the identification of dedicated portfolios consisted of interest rates on various maturities of U.S. Treasury securities and rates of growth in wages for the 1953 through 1988 period. Dedicated portfolios were constructed, alternatively, of one-year, five-year, ten-year, and 20-year U.S. Treasury securities. Each portfolio consisted wholly, at least initially, of securities of a specific maturity; as time passed, any additions to the portfolio consisted of securities of that same maturity, or the longest maturity within the remainder of the loss period.

Results

Regressions were fitted to data consisting of differentials between *ex post* yields of dedicated portfolios of alternative composition and *ex post* rates of wages growth, and differentials between contemporaneous interest rates and growth rates of wages. Each regression with a given type of dedicated portfolio contained all loss period of k length. Alternative time periods of

⁷In terms of actual measurement, the realized differential at time zero, $(r'-g')_{t=0}$, is the contemporaneous differential at time zero.

Table 1: Regression Results: $[(r' - g')_{t-k} = a_0 + a_1(r' - g')_{t-0} + \epsilon_t]$

Dedicated Portfolios Consisting of:	Five Year Loss (n=32)				Ten Year Loss (n=27)				Fifteen Year Loss (n=22)				Twenty Year Loss (n=17)			
	a_0	a_1	R^2	D-W	a_0	a_1	R^2	D-W	a_0	a_1	R^2	D-W	a_0	a_1	R^2	D-W
1-Year Securities	-1.57 (-5.99)	.243 (6.85) [21.34]	.98	1.95 ⁴	-1.17 (-5.95)	.155 (4.52) [24.71]	.97	1.84 ⁴	-.19 (-1.09)	.107 (2.39) [19.99]	.82	1.25 ⁴	-.16 (-2.41) [18.06]	.132 (3.09)	.72	1.75 ³
5-Year Securities	-.51 (-1.71)	1.248 (12.29) [-2.44]	.87	1.97 ¹	.11 (0.14)	.606 (2.43) [1.58]	.56	1.72 ¹	-.31 (-1.22)	.347 (1.63) [3.08]	.25	1.85 ³	-.44 (-4.46) [8.54]	.156 (1.57)	.29	1.79 ³
10-Year Securities	-1.73 (-3.06)	1.66 (9.40) [-3.74]	.87	1.98 ²	-.80 (-1.82)	.936 (3.20) [0.22]	.44	1.84 ²	-.73 (-1.84)	.388 (1.92) [3.04]	.50	1.91 ²	-.95 (-3.55) [3.32]	.286 (1.33)	.25	1.96 ²
20-Year Securities	-3.12 (-3.88)	2.03 (8.08) [-4.09]	.83	1.93 ²	-2.00 (-2.85)	.799 (2.01) [0.50]	.47	1.80 ²	-2.13 (-4.16)	.384 (1.25) [2.00]	.41	1.80 ¹	-1.38 (-2.80) [2.89]	.182 (0.64)	.61	1.98 ²

NOTE: t statistic for parameters as estimated (i.e., $(1-\lambda)$) are in parenthesis; t statistics in brackets are for λ . Superscripts of Durbin-Watson ratios indicate the order of the adjustment for autocorrelation of residuals (1 = first order; 2 = second order; etc.).

investigation, defined by k , included loss periods of five, ten, 15, and 20 years.⁸ Results are shown in Table 1. Chief interest attaches to estimates of a_1 , for these estimates permit the determination of the size of the adjustment coefficient ($\lambda' = 1 - \tilde{a}_1$). The specification of the equilibrium differential is neglected; hence, its average effect ends up in the constant term. To compensate for the misspecification, various autocorrelation adjustments are used. The order of the adjustments is shown as a superscript on the Durbin-Watson statistic for each regression.⁹

According to the estimates in which $0 < \hat{a}_1 \leq 1$ during loss periods of k length, the economy operated in such a fashion that dedicated portfolios were driven from 6.4 percent (for ten-year loss period and ten-year securities) to 89.3 percent (for 15-year loss period and one-year securities) of the way toward their equilibrium values. In several instances (with five-year, ten-year and 20-year securities for five-year loss periods) there were significant estimates of λ' that were negative. The adjustment periods are restated in terms of what they imply about the length of time required to achieve 95 percent closure of any discrepancy between an initial position, $(r' - g)_{t=0}$, and a desired position, $(r' - g)_{t=k}$. Those data are shown in Table 2.

Table 2: Estimates of Adjustment Periods

Dedicated Portfolios Consisting of:	Number of Years to Equilibrium* for Loss Periods of:			
	5 Years	10 Years	15 Years	20 Years
1-Year Securities	10.6	16.1	20.1	29.6
5-Year Securities	—	10.0**	42.5	32.2
10-Year Securities	—	10.0**	47.5	47.9
20-Year Securities	—	10.0**	46.9	35.2

*95 percent closure

**Adjustment coefficient is not significantly different from zero at 99 percent confidence level.

Considering only coefficients that reach acceptable levels of statistical significance, data in Table 2 indicate that the persistence of the effects of differentials depends upon the composition of the dedicated portfolio. With portfolios consisting of one-year securities, the initial differentials are substantially removed during the loss periods. For example, for a 15-year loss period the estimate of $1 - \lambda'$ is .107, meaning that by the end of 15 years only 10.7 percent of the discrepancy would remain; to achieve 95 percent closure

⁸For a detailed discussion of the estimation of the $(r' - g)$ realized differentials, see 2, (pp. 277-278).

⁹The Durbin-Watson statistic is a measure of first-order autocorrelation. The results presented here embody data with K -period lags. We presume that model misspecification extends throughout the K -periods, and do not really know whether the autocorrelation adjustments succeeds in freeing the coefficient estimates from the mischievous consequences of that misspecification.

requires 20.1 years.¹⁰ But with dedicated portfolios consisting of longer-term securities, the effects of pre-existing differentials continue to be prominent throughout the loss periods.

The estimates of $(1 - \lambda')$ for the five-year loss periods are larger than one with dedicated portfolios consisting of securities with maturities equal to or greater than five years. Such estimates imply that $\lambda' < 0$, indicating that the adjustment process enlarges upon any initial discrepancy. Results such as these are inconsistent with the stock-adjustment framework. For, as indicated, a dominant feature of the stock-adjustment model is that the initial position of the dependent variable is driven toward some central position.¹¹ No adjustment periods are shown in Table 2 for $\hat{a}_1 > 1$, although the common sense meaning of such an estimate is that the differential at the beginning of the loss period will persist throughout the period.

Up to this point attention has been directed toward the size and significance of estimates of λ' . The basic statistical question has been whether estimates of λ' differ significantly from zero. Now a somewhat different question is asked. Namely, does knowledge of the contemporaneous differential at the beginning of a loss period permit the formation of a better estimate of the realized differential than is provided by its mean?

One approach to addressing this question is to consider whether the regression equations are of significant explanatory value. The R^2 s range from a high of .98 to lows of .25. The mean R^2 of the 16 regressions is .62. Results such as these suggest that the model possesses significant explanatory power.

Another approach to thinking about the predictive power of a regression equation requires an analysis of the standard errors around the regression line. For these purposes, realized differentials for 20-year loss periods are considered, but with regressions fitted to data from one-year, five-year, ten-year and 20-year securities. In Table 3 there is a display of realized differentials corresponding to hypothetical initial contemporaneous differentials ranging from 0 to 5 percent; also shown (in parentheses) are the standard errors of realized differentials corresponding to alternative predicted values.

As an independent variable departs from its mean there are increases in the standard errors corresponding to those values.¹² For the matter at hand, if contemporaneous differentials prevailing at the beginning of a loss period depart markedly from their means there are substantial increases in the standard errors of realized differentials from their predicted values. A review of Table 3 indicates that standard errors are so large relative to the predicted differential that, in general, the zero differential hypothesis cannot be rejected.

¹⁰ By construction, full closure is not attained until $k = \infty$. We limit the adjustment to 95 percent closure, leaving 5 percent of the initial discrepancy. We find n as follows: $.05 = (1 - \lambda)^{n/15} = (.107)^{n/15}$; $n/15 = 1.34042$; $n = 20.1$ years.

¹¹ The central position need not be a constant. Rather, it may be functionally related to other variables.

¹² For discussion see any standard statistics text. For example, an excellent discussion is given in [5, pp. 81-82].

Table 3: Predicted Realized ($r' - g'$) Differentials for Twenty-Year Loss Periods Given Alternative Beginning-of-Period Contemporaneous $(r - g)_{t=0}$ Differentials

Dedicated Portfolios Consisting of:	Alternative Beginning-of-Period Contemporaneous Differentials					
	0%	1%	2%	3%	4%	5%
1-Year Securities	-.16 (.18)	-.03 (.19)	.10 (.20)	.24 (.22)	.37 (.24)	.50 (.27)
5-Year Securities	-.44 (.39)	-.28 (.41)	-.13 (.48)	.03 (.54)	.18 (.61)	.34 (.61)
10-Year Securities	-.95 (.59)	-.66 (.61)	-.38 (.66)	-.09 (.73)	.19 (.82)	.48 (.92)
20-Year Securities	-1.38 (.61)	-1.20 (.63)	-1.02 (.69)	-.83 (.77)	-.65 (.86)	-.47 (.97)

Shown in parentheses are standard errors of differentials around predictions.

Dedicated portfolios consisting of one-year securities are least-cost portfolios.¹³ That is, these portfolios have the smallest negative ($r' - g'$) differential or the largest positive ($r' - g'$) differential. Consistently, their realized differentials are larger than realized differentials corresponding to dedicated portfolios consisting of longer-term Treasury securities (see Table 3).

None of the estimated realized differentials with dedicated portfolios consisting of one-year securities falls beyond two standard deviations from zero. With other portfolios, most of the predicted differentials falls within one standard error of zero. Only one predicted value, corresponding to a beginning contemporaneous differential of zero, departs from zero with a 95 percent confidence level. But, as it turns out, that differential would create the highest cost dedicated portfolio. In short, in just those circumstances in which it would be most tempting to take the existing contemporaneous differential into account—i.e., when the differential is large—the error terms associated with it become prohibitive.¹⁴

Concluding Remarks

This article is addressed explicitly toward a single issue. Namely, does the contemporaneous ($r - g$) differential at the beginning of a loss period provide useful information regarding the realized ($r' - g'$) differential for the loss period considered as a whole? There are two answers to that question: yes and no. On

¹³ The U.S. Supreme Court recently ruled that awards should be based on the assumption that plaintiffs will invest in the "... best and safest investments ... [and that] ... the discount rate should not reflect the market's premium for investors who are willing to accept some risk of default." [4, p. 2250].

¹⁴ This finding may explain why the ($r - g$) differentials employed in defined-benefit pension plans have not, on average, increased significantly during the decade of the eighties. See [6] for year-by-year data relating to the ($r - g$) differentials used by defined-benefit pension plans.

the one hand, most of the regression tests showed that the regression coefficient attached to the $(r-g)_{t=0}$ differential was statistically significant. The larger the contemporaneous differential at the beginning of the loss period, the larger the realized differential. For most of the tests, estimates of $1-\lambda'$ were less than one and greater than zero, consistent with the stock-adjustment framework. On the other hand, predictions made on the basis of knowledge of the contemporaneous differential are not particularly valuable. They are not better than are provided by knowledge of the mean.

The article also provides insights into other issues. It reinforces views relating to the equilibrating properties of the economy, and it provides practical advice regarding the appropriate $(r'-g')$ differential for valuation purposes. The appropriate differential is approximately zero unless the loss period is very short.

Bibliography

1. Anderson, Gary A. and David L. Roberts, 1989, "Stability in the Present Value Assessment of Lost Earnings," *Journal of Risk and Insurance*, 56, 50-66.
2. Bryan, William R. and Charles M. Linke, 1988, "Estimating Present Value of Future Earnings: Experience with Dedicated Portfolios," *Journal of Risk and Insurance*, 55, 273-86.
3. Johnston, J., 1963, *Econometric Methods*, McGraw-Hill Book Company, Inc., New York.
4. Jones & Laughlin Steel Corp. v. Pfeifer, 103 S. Ct. 2541.
5. Maddala, G. S., 1977, *Econometrics*, McGraw-Hill Book Company, Inc., New York.
6. Wyatt Company, *Survey of Actuarial Assumptions and Funding of Pension Plans with 1000 or More Active Participants*, Washington, D.C., annual.

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