Moral Hazard and Moral Imperative: Reply

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Barney [1] argues that our proofs in Appendix A and footnotes 9 and 10 contain a "fundamental error" and therefore, the discussion of partial insurance in Section III of our article [2] is "fatally flawed." He asserts that our equation (A.2) is not correct and consequently our comparative statistics results for partial and full insurance are invalid. To help us understand our error, Barney provides a demand and supply metaphor, where quantity demanded and quantity supplied are denoted by $Q_d = F(P)$ and $Q_s = G(P)$, and P is the price of goods. At the point of intersection of demand and supply, $F(P) = G(P)$, but the derivatives of both curves, $F'(P)$ and $G'(P)$, are unequal.

Barney's comments demonstrate lack of understanding of an important technique called implicit differentiation used in the comparative static analysis. We were not concerned with the first derivatives of the two curves. Rather, we were concerned about the comparative statistics. In other words, we were not interested in the slopes of two curves (i.e., demand and supply), but were interested in the changes in the positions of two curves as a result of changes in exogenous factors (e.g., weather).

Barney agrees that at the intersection of the two marginal benefit curves (MB_p and MB) for partial and full insurance the following equality holds:

$$-\alpha L\pi' - p'(U_1-U_0)/[(1-p)U_1'+pU_0'] = -L\pi'$$

or

$$-p'(U_1-U_0) = -(1-\alpha)L\pi' \cdot [(1-p)U_1'+pU_0'] \qquad (1)$$

We denoted $\bar{s}$ as the safety production level corresponding to the point of intersection and $\bar{p}$ as the associated probability of loss when the amount of safety production is $\bar{s}$. In proposition 1, we tried to determine whether the

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insurance coverage ratio α will affect the position (value) of the marginal benefit curve MB_p. To do so, we evaluated $\frac{\partial MB_p}{\partial \alpha}$ at $\bar{p}$ where MB_p intersects MB (marginal benefit curve under full coverage). Our proof uses standard implicit differentiation. First, we differentiated equation (1) with respect to α and set the derivatives of both sides equal to insure that MB_p equals MB as α changes. This yields

$$\begin{aligned} -p'[U_1'(-\pi L) - U_0'(1-\pi)L] &= L\pi'[(1-p)U_1' + pU_0'] \\ &\quad -(1-\alpha)L\pi'[(1-p)U_1''(-\pi L) + pU_0''(1-\pi)L] \end{aligned} \tag{2}$$

Second, we substituted the relationship in (2) into our original equation (9) to evaluate the impact of α on MB_p in the neighborhood of $\bar{p}$. Note that when $p \lesseqgtr \bar{p}$, $-p'(U_1-U_0)/-(1-\alpha)L\pi' \gtreqless (1-p)U_1' + pU_0'$. Using these relationships and equation (2), we showed that $\frac{\partial MB_p}{\partial \alpha} \lesseqgtr 0$ if $p \gtreqless \bar{p}$ as shown in Figure 4 of our article. Using a similar principle, we evaluated the impact of loss size L on the marginal benefit functions in the neighborhood of $\bar{p}$ in footnotes 9 and 10 and found that $\frac{\partial MB_u}{\partial L} \gtreqless \frac{\partial MB}{\partial L}$ if $p \lesseqgtr \bar{p}$ as shown in Figure 6 of our article. Another way to evaluate $\frac{\partial MB_u}{\partial L}$ is to rewrite $\frac{\partial MB_u}{\partial L} = -p'[B + L\frac{\partial B}{\partial L}]$ where $MB_u = -LBp'$, and B changes as L changes as indicated in our original equation (14). Note that $B \lesseqgtr 1$ if $p \gtreqless \bar{p}$. At the intersection of MB and MB_u (the marginal benefit function in the absence of insurance), $\frac{\partial B}{\partial L} = 0$. Therefore, evaluating the impact of loss size in the neighborhood of $\bar{p}$ yields $\frac{\partial MB_u}{\partial L} \gtreqless \frac{\partial MB}{\partial L} = -p'$, if $p \lesseqgtr \bar{p}$.

Our procedure for obtaining the above comparative statistics is straightforward. The problem with Barney's assertion is that he mistook the meaning of equation (2) and thought that we claimed the equality in equation (2) for all safety production levels. We made no such claim. Instead, we only made use of the condition in equation (2) to evaluate the impact of α on MB_p in the neighborhood of $\bar{p}$.

However, we did find a mistake in Proposition 2 for a reason not mentioned by Barney. In Appendix B, we indicated that

$$\bar{p} = \frac{p'(U_1-U_0)/(1-\alpha)L\pi' - U_1'}{(U_0' - U_1')} \tag{3}$$

We differentiated $\bar{p}$ with respect to α to give the result in (A.4) of our original article. We then substitute equations (1) and (2) into equation (A.4) and concluded that $\frac{\partial \bar{p}}{\partial \alpha} = 0$. This procedure is incorrect. Since $\bar{p}$ in equation (3) was derived from equation (1), using equations (1) and (2) to evaluate the derivative of (3) with respect to α will always make equation (A.4) degenerate to zero. Thus, contrary to our original assertion, $\frac{\partial \bar{p}}{\partial \alpha}$ will depend on α in a way as described in equation (A.4) of our article. Similarly, we cannot claim that the size of the loss does not affect the regions of moral hazard and moral imperative [2, p. 115]. Although it is correct that $\frac{\partial MB_u}{\partial L} \gtreqless \frac{\partial MB}{\partial L}$ when $p \lesseqgtr \bar{p}$, $\bar{p}$ and the intersection point of MB_u and MB may be affected by the loss size L. In fact,

$$\frac{\partial \bar{p}}{\partial L} = \frac{1}{(U_0' - U_1')^2}\left\{(U_0' - U_1')\left\{\frac{p'(U_1 - U_0)}{(1-\alpha)\pi'}\frac{-1}{L^2} + \frac{p'}{(1-\alpha)L\pi'}\right.\right.$$
$$\left.[U_1'(-\alpha\pi) - U_0'(-1+\alpha(1-\pi))] - U_1''(-\alpha\pi)\right\}$$
$$\left. - [p'(U_1 - U_0)/(1-\alpha)L\pi' - U_1'][U_0''(-1+\alpha(1-\pi)) - U_1''(-\alpha\pi)]\right\}$$

We therefore make the following correction for Proposition 2:

Proposition 2: the value of $\bar{p}$ at which MB_p = MB depends on the coverage ratio α and loss size L.

References

1. Barney, L.D., "Moral Hazard and Moral Imperative: Comment," *Journal of Risk and Insurance* (forthcoming).
2. Wu, C. and P. Colwell, "Moral Hazard and Moral Imperative," *Journal of Risk and Insurance* (March 1988), pp. 101–17.

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