

Karl Borch's Research Contributions to Insurance

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Abstract

The late Karl Borch made pioneering and fundamental contributions to the field of insurance economics. Borch was the first insurance scholar to recognize the significance of the new paradigms for dealing with the economics of uncertainty which were developed in the 1940s and 1950s. He used these ideas to investigate the form of optimal reinsurance contracts. His results not only illuminated the problems in insurance but also provided new insights into the determinants of optimal contracts in the general area of risk sharing. Borch's use of utility theory, game theory, and paradigms from financial economics provided a fresh new approach to insurance problems. This new approach was able to provide a synthesis that was unavailable under the traditional actuarial models which until then had dominated analytical models in the insurance area.

Introduction

The late Karl Borch was an eclectic scholar who made research contributions to several branches of economics. However, his most significant accomplishments were to insurance economics and to the economics of uncertainty. This review will assess some of his major research achievements in the insurance field.

Karl Borch's career was both interesting and unconventional. Born in Norway, he graduated in actuarial science from the University of Oslo in 1947 and spent the next 12 years in research posts with the United Nations and the Organization of European Economic Cooperation. Perhaps this experience away from a traditional academic environment heightened his awareness of the importance of practical considerations. From 1959 to 1962 he studied for

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his doctorate in mathematics at Oslo University and during this period started applying ideas from the economics of uncertainty to problems in insurance. In 1963 Karl Borch was appointed to the Chair in Insurance at Bergen. Although he remained based in Bergen for the remainder of his life, he maintained contact with a broad range of scholars around the world through correspondence, conferences, and academic visits abroad.

When Borch started his doctoral studies in 1959, the importance of the new developments in the economics of uncertainty had been realized by researchers in economics. These new paradigms for modelling decision making under uncertainty had been developed in the 1940s and 1950s. In 1944 von Neumann and Morgenstern had published their seminal book entitled *Theory of Games and Economic Behavior* which provided an axiomatic foundation for the expected utility hypothesis. In the 1950s Arrow and Debreu (1954) laid the foundations of general equilibrium theory. Borch realized the potential of these new paradigms to tackle problems in the insurance area. In a series of articles he showed how the expected utility hypothesis and game theory could be used to formulate and rigorously analyze a variety of important insurance problems. These paradigms are now so deeply embedded in the study of insurance economics that it may be difficult for a modern student to appreciate that this was not always the case.

Although insurance represents an important economic activity there was, until Borch's work, very little use of economic concepts in theoretical work in insurance. For example, in his famous Helsinki lectures in 1963 Arrow (1965) stated:

Insurance is an item of considerable importance in the economies of advanced nations, yet it is fair to say that economic theorists have had little to say about it, and insurance theory has developed with virtually no reference to the basic economic concepts of utility and productivity (p. 45)

In the notes to his Helsinki lectures Arrow recognizes the contributions being made by Karl Borch to apply economic paradigms to the study of insurance problems. In his foreword to Borch's 1974 book, Arrow pays tribute to Borch's achievements:

... Dr. Borch quickly grasped the importance of the work that has been done on the economics of uncertainty and the value of the expected-utility hypothesis. He has made direct contributions to economic analysis outside the field of insurance, but perhaps his most valuable work has been in illuminating the special problems of the field of insurance, for he clearly displays many of the deep issues of uncertainty theory (p. ix).

Prior to Borch's work in the early 1960s, analytical approaches to insurance issues were usually developed within the framework of actuarial science. Some two centuries earlier the actuarial profession had developed a scientific basis for the measurement of risks, the computation of premiums, and the establishment of appropriate reserves. Actuaries placed great emphasis on ensuring that the insurer remained solvent and this emphasis encouraged the use of conservative margins in the assumptions used by actuaries in their valuation bases. Around the turn of the century Lundberg (1909) created the

so-called collective theory of risk which modelled insurer claims as time dependent stochastic processes. This theory provided a rigorous mathematical framework for estimating quantities such as the probability that the insurer would become insolvent or ruined. However, collective risk theory did not permit many important problems in insurance to be formulated in a convincing way.

Borch demonstrated that the expected utility-game theory approach yielded a more economically meaningful framework for the analysis of insurance problems than the existing actuarial models. In addition he participated in research efforts to transform the traditional collective risk theory into a more dynamic and economically interesting approach. This line of research was based on work pioneered by de Finetti in 1957 which generalized collective risk theory from a static model to a dynamic one and obtained a more meaningful objective function for an insurance enterprise.

This survey does not attempt to cover all the research topics that Karl Borch worked on. Instead it focuses on a few of the major themes that represent his most significant contributions to the field of insurance economics. His principal papers dealing with these topics appear together in his 1974 book entitled *The Mathematical Theory of Insurance*. His research articles dealing with other topics in economics and his more recent research which cover quite a broad range of subjects will not be covered.

The next section discusses the first major research theme: the analysis of optimal insurance (and reinsurance) contracts. Borch used the tools of economic analysis to examine existing reinsurance contracts and to explore the optimal structure of reinsurance arrangements. He noted that there are (at least) two parties to a reinsurance treaty and that a reinsurance agreement is unlikely to persist unless the interests of both parties are taken into account. This leads naturally to the formulation of reinsurance negotiations as a bargaining game and Borch consistently stresses the importance of game theoretic approaches to model such negotiations. He uses the concept of Pareto-optimality to characterize the preferred solutions and discusses various solution concepts which lead to a unique contract.

The second major research theme deals with the analysis of optimal risk sharing arrangements involving several risk averse economic agents. Although his analysis is structured in terms of reinsurers, Borch's seminal contributions provided fundamental insights into the nature of optimal risk sharing. Borch's research in this area has provided the basis for important subsequent contributions to the field. For example, the important article by Wilson (1968) on syndicates is based on Borch's pioneering results.

Recent work by Mirless (1976), Holmström (1979), and others examines optimal contract design in the presence of both moral hazard and pure risk sharing. This work can be viewed as a significant extension of Borch's work to incorporate both risk sharing and incentive effects.

Borch's major research contributions on dynamic extensions of collective risk theory are discussed next. de Finetti (1957) had proposed that the

discounted expected value of future stockholder dividends was a more meaningful concept for insurance company management than was the probability of ruin. Borch developed models to explore the implications of managers following a decision rule to maximize the present value of future dividends.

Next, some of Borch's articles which pioneered the application of new paradigms from financial economics to problems in insurance are discussed. This analysis provided a much broader and more economically satisfying approach than that provided by the traditional models of actuarial science. In particular he showed how to construct a financial model of an insurance firm which provided an integrated approach to premium determination, the shareholders' risk return relationship, and government supervision. It was not until a decade later that finance scholars and insurance economists began to construct similar models.

The final Section covers just a few of Borch's other research contributions and provides an overall assessment of his research contributions.

Optimal Insurance and Reinsurance Contracts

The traditional actuarial approach to the analysis of insurance contracts placed heavy emphasis on the calculation of appropriate premiums and the establishment of reserves. European actuaries, in particular, developed sophisticated mathematical models to tackle these problems. However, in the actual operation of insurers, the importance of other factors was recognized. For instance, although there was no specific modelling of the demand for insurance or the impact of the premium level on demand, insurance company managers appreciated the relationship. Although the actuarial models viewed the risk as a mechanical stochastic process, the importance of behavioral considerations was recognized in contract design and claims administration. Often the classical actuarial models took the contract design as given, and attempted to lay down rules for determining the correct premium to be charged for the contract.

In order to focus primarily on pure risk sharing effects, Borch analyzed reinsurance arrangements using the tools of expected utility and game theory to analyze the optimal reinsurance treaty or arrangement. Within this framework he assumed that both the insurer and the reinsurer were risk averse. The concept of Pareto-optimality provides a useful characterization of feasible contract arrangements. A risk sharing arrangement is said to be Pareto-optimal if it is impossible to make one party better off without making the other party worse off. Here better and worse are measured in terms of expected utility. These results are based on pure risk sharing and do not assume any informational asymmetries.¹ It turns out that there will be, in

¹ In a reinsurance arrangement we would expect pure risk sharing effects to be more important than in direct insurance contracts. Borch (1974, p. 27) notes that both parties are likely to have the same beliefs and that moral hazard is not likely to be present: "Reinsurance contracts are based on complete confidence between both parties. If one party has information that may be relevant in the

general, an infinite number of Pareto-optimal risk sharing arrangements between the two parties and that concepts from game theory need to be introduced to determine a unique arrangement.

The emergence of Borch's ideas on this topic can be traced to his 1960 paper to the 10th International Congress of Actuaries (1974, chapter 1) which discusses a particular type of reinsurance contract within the context of the traditional actuarial approach and the limitations of this approach.² It is important to note that the premiums for all contracts are assumed to be computed on the same basis. Intuitively, the reinsurance contract transfers the claims arising from the entire right hand tail of the claims distribution from the direct writer to the reinsurer and this gives rise to the reduction in variance. Having derived this result Borch goes on to criticize the restrictiveness of the underlying assumptions. He notes that there are two parties to the reinsurance arrangements and that an arrangement which is very attractive to one party may be quite unacceptable to the other. Borch concludes this paper with a discussion of how to model the objectives of the insurer using a utility function. He cautions the reader that this step may involve considerable simplification.³

Despite this reservation Borch continued to use the expected utility framework to analyze the issue of contract design in a reinsurance context. In 1960 (reprinted as Chapter 3 of Borch, 1974) he formulated the problem of arranging reciprocal reinsurance treaties in terms of a two-person cooperative game. Borch showed that optimal reinsurance arrangements could be characterized as Pareto-optimal risk sharing arrangements. He noted that in general the Pareto-optimality criterion does not lead to a unique contract or in modern terminology a unique point on the Pareto frontier. Borch suggests the use of the Nash solution concept⁴ to resolve this indeterminacy.

In another 1960 paper (reprinted as Chapter 4 of Borch, 1974) published in the *ASTIN Bulletin*, Borch provided a more general analysis of reciprocal reinsurance treaties between two insurers. This paper contains an important result on the characterization of optimal risk sharing arrangements. For any (Pareto) optimal risk sharing arrangements the ratio of the marginal utilities of the two contracting parties is equal to a constant. Different constants correspond to different risk sharing arrangements (or reinsurance treaties). As Borch noted, it is remarkable that the optimal treaty obtained from this

estimation of the probability distributions, it is considered as fraud, or breach of faith, if he does not make this information available to the other party."

²A stop loss reinsurance contract is a contract under which the reinsurer pays all claims in excess of a certain amount known as the stop loss limit. Borch shows that a stop loss reinsurance type of contract leads to the greatest reduction in the variance of the losses of the direct writer for a given premium.

³"... Some reflection will show almost immediately that the motives which guide the reinsurance operations of most companies are too complex to be represented by a function of simple mathematical form."

⁴In the last few years several new solution concepts have been proposed for cooperative games, but the Nash solution is still viewed as very important.

characterization depends only on the risk attitudes (or utility functions) of the contracting parties and not on the probability distributions of the underlying risks themselves. This result assumes that both parties have homogeneous beliefs and that there are no informational asymmetries.

Borch also noted that the relationship between the respective marginal utilities (i.e., the ratio of the marginal utilities is constant) provided both a necessary and sufficient condition for any pure risk sharing agreement to belong to the Pareto frontier. Although his earlier proofs of this result contained an error which was pointed out and corrected by Du Mouchel (1968), the results themselves were correct. These results provided the basis for important subsequent research by Borch and others into the area of optimal risk sharing involving several parties.

Optimal Risk Sharing Involving Several Parties

In a series of papers in the early 1960s Borch generalized the results obtained for optimal risk sharing between two parties to the situation involving several contracting parties. He showed that the characterization of the optimal risk sharing contract for two parties could be generalized to the situation involving several parties. In addition he was able to derive conditions on preferences which gave rise to linear risk sharing. Borch's research provided the foundations for important subsequent work by Wilson (1968) on syndicates. It also laid the groundwork for subsequent research on aggregation in securities markets.

The key results are contained in his 1960 article in the *Scandinavian Actuarial Journal* entitled "The Safety Loading of Reinsurance Premiums" and his 1962 *Econometrica* article entitled "Equilibrium in a Reinsurance Market."⁵ Although these articles discuss reinsurance treaties, the results apply to any general setting involving pure risk sharing. Below is a brief summary of the basic result.

Assume there are n reinsurers, each with a concave utility function, $U_i(\bullet)$, $1 \leq i \leq n$. Let y_i denote the share of claims to be paid by company i . A risk sharing arrangement will be Pareto-optimal if, and only if, there exist non-negative constants k_i such that

$$U'_i(y_i) = k_i U'_1(y_1) \quad (1)$$

Different values of the k_i correspond to different risk sharing contracts. This means that there is no unique optimal solution and Borch notes that the problem can best be analyzed as an n -person cooperative game.

The conditions which lead to linear risk sharing are of interest. In this case the insurers pool all their losses and each one absorbs a fixed proportion of the total losses. In a more general setting linear risk sharing rules are of interest since they correspond to the combining or aggregation of all the risks. The conditions under which such pooling takes place are important in the

⁵These articles are reprinted as Chapters 9 and 10 of Borch (1974).

context of securities markets. The sharing rule will be linear if all the utility functions belong to the same family of the HARA⁶ (Hyperbolic Absolute Risk Aversion) class and each utility function has the same risk cautiousness parameter.

Borch's results on optimal risk sharing among n risk averse economic agents provided the basis for an important article by Wilson on the theory of syndicates. Wilson analyzed the decision process of a syndicate where the members have different risk tolerances and heterogeneous beliefs. He explored the conditions under which a group utility function could be constructed and his work affirmed the importance of linear risk sharing rules.

The stock market provides another important mechanism for risk transfer and the significance of linear sharing rules in this connection is that they imply investors will hold a percentage of the entire market portfolio. The book by Mossin (1973) shows clearly the key role played by Borch's results on pure risk sharing in the construction of preference based models of investor behaviour. Rubinstein (1974) and Brennan and Kraus (1978) subsequently studied the aggregation problem in securities markets and further clarified the connection between HARA utility functions and linear sharing rules. More recently, Amershi (1985) has provided general conditions for a fully Pareto-efficient allocation of aggregate wealth in the case of arbitrary preferences.

In recent years there has been considerable research interest in developing models which handle both pure risk sharing and incentive effects. Both Mirless (1976) and Holmström (1979) produced seminal articles in this area. Such models are sometimes known as principal agent models. One economic agent, the principal, engages another economic agent, the agent, to perform a certain task. The pay-off is a function of both the random state of nature that ensues and the effort expended by the agent. The agent's effort is assumed to be unknown to the principal. The principal's problem is to design a contract that will provide the appropriate mix of risk sharing and incentives for the agent. The derivation of the optimal sharing rule in this case is much more complicated because the agent is assumed to maximize his or her own welfare. This situation whereby one party to a contract can take a hidden action to improve his or her own welfare at the expense of the other party is known as moral hazard.⁷

However, the necessary conditions for the optimal sharing rule involve a relationship between the marginal utilities of the two parties and the probability density function of the outcome. This relation is a generalization of the one first derived by Borch for pure risk sharing and indeed the borch result can be obtained as a special case. The optimal sharing rule when there is

⁶The HARA utility functions can be classified into three forms depending on the value of the risk cautiousness parameter α . If $\alpha = 0$ there is exponential utility, if $\alpha = 1$ there is logarithmic utility, while if $\alpha \neq 0$ and $\alpha \neq 1$ there is power utility. For a discussion of the explicit forms of these functions see Boyle and Butterworth (1988).

⁷Moral hazard is of course well known in insurance and indeed is most vividly illustrated in an insurance setting. It is now recognized as being very important over a broad range of economic activities.

moral hazard has to take account of both risk sharing and incentive effects. Holmström (1979) provides a good analysis of the basic model and discusses a number of applications and extensions. The principal agent paradigm has been used to model a wide range of applications in recent years.

Contributions to Dynamic Models in Insurance

The classical collective risk theory is a static model. Borch extended the model developed by de Finetti (1957) to develop a dynamic approach. This extension indicates that the management of an insurer can be viewed as a problem in optimal control. A brief review of the classical theory of risk is provided first.

Collective risk theory was created initially by Lundberg (1909) and developed chiefly by European actuaries. It is assumed that the total amount of claims paid by the insurer evolves as a stochastic process. At the end of time T the capital of the company will be the initial reserve plus the accumulated premiums less the claims already paid. Note that investment income has been ignored. Suppose that the reserve capital at time T is denoted by $S(T)$. If the reserve capital at time T is negative the company is insolvent or ruined. Much of the focus of the traditional collective risk theory was in the computation of the probability of ruin. The probability that the company remains solvent during the time period $[0, T]$ is equal to

$$\text{Probability } \{\text{Minimum } S(\tau) \geq 0\} = R(S(0), T) \quad (2)$$

$$0 \leq \tau \leq T$$

While collective risk theory has considerable mathematical elegance, it has serious shortcomings which limit its practical applicability and theoretical usefulness. First, it is a static theory whereas insurers operate in a dynamic environment. Second, the theory is based on a set of mathematical and somewhat mechanical assumptions, not on economic assumptions such as profit maximization or utility maximization. In the real world insurers generally strive to make profits and remaining solvent is an important but secondary objective. The traditional collective risk theory suggests that the insurer's sole objective should be to minimize the probability of ruin over some time horizon. This does not correspond to how insurers actually behave. Third, collective risk theory ignores the existence of markets and their consequences. For example, the fact that an insurer's assets are priced and traded in financial markets is not recognized. The insurer's role as a financial intermediary with financial liabilities is usually ignored.

In order to overcome some of the major defects in the traditional collective risk theory model, Borch built on ideas developed by de Finetti who observed that as T tended to infinity that $R(S(0), T)$ would tend to zero unless the insurer allowed its reserve capital to become infinite. In the long run, ruin was certain unless the reserves could grow without limit. Since insurers do not operate in this way the computation of ruin probabilities does not appear to be a useful

exercise. As the insurer's reserve capital increases, some fraction of it will be paid out as dividends to the shareholders or as bonuses to the participating policyholders. de Finetti suggested that the expected future life of the company or the expected discounted value of future dividends would represent more useful criteria than would the probability of ruin. This formulation goes part of the way towards remedying the first two disadvantages of collective risk theory mentioned earlier. Note that the criterion of maximizing the discounted value of expected future dividends is consistent with current theories in modern financial economics, where it is usually stated that a firm's objective should be to maximize the value of current equity.

Borch developed a number of explicit solutions to the de Finetti model. He assumes that the insurer requires a minimum risk capital of Z to meet its solvency requirements. If the available reserve capital, $S(T)$, exceeds Z the excess is paid out as a dividend equal to

$$S(T) - Z.$$

Should the reserve capital $S(T)$ become negative the company becomes insolvent. Assume that the dividend payable after the i -th underwriting period is $d(i)$, then the expected discounted value of the future dividend payments is

$$V(S(0), Z) = E \sum_{i=0}^{\infty} v^i d(i), \quad (3)$$

where the symbol v denotes the one-period discounting factor. The insurer maximizes the function V .

The function V can be shown to satisfy an integral equation and Borch obtained some particular solutions and discussed their implications. In his 1966⁸ article in the *ASTIN Bulletin* entitled "Control of a Portfolio of Insurance Contracts," Borch derived a simple analytical expression for the value of the function V . He notes that if an insurer's objective is to maximize discounted dividends, this can cause the company, in some situations, to accept an insurance contract where the premium is less than the expected claim amount. In one numerical example he assumes that the premium is 0.5 and that the claim will be either zero with probability one-third or one unit with probability two-thirds. In the context of his numerical example Borch (1974, p. 244) notes:

Any actuary worthy of the name will advise against accepting this offer. If, however, the company accepts the offer in spite of actuarial orthodoxy, the expected discounted value of dividends will increase (from 3.73) to 3.94. Hence it is to the advantage of the company to accept the offer, even if it is grossly unfair.

This example illustrates the possible conflicts between the shareholders of the company and the existing policyholders. In modern finance parlance the insurer is willing to accept a negative net present value project so that the value of the ownership interest is increased. Since the equity holders have a

⁸This article is reprinted as Chapter 16 of Borch's 1974 book.

contingent claim on the assets of the company, the ownership interest can be viewed as an option. It was not until fairly recently that finance theorists first used option pricing ideas to derive similar insights.⁹ The policyholders' claim can be viewed as a risky bond and the shareholders hold a call option on the assets of the firm with a stochastic strike price equal to the amount of the losses. It is interesting to note that Borch's analysis of the problem obtained similar insights.

Of course the formulation of objectives of the insurance company in terms of maximizing expression (3) is much more in the spirit of modern financial economics. The criterion for management becomes value maximization, within the solvency constraints imposed by the regulators.

In 1967 Borch presented a survey of risk theory to the Royal Statistical Society in London. Bather, in discussing this paper, raised the possibility that a bankrupt insurer might be rescued. Borch developed this idea further in a paper entitled "The Rescue of an Insurance Company after Ruin." This article first appeared in the *ASTIN Bulletin* in 1968 and is reproduced as Chapter 20 in *The Mathematical Theory of Insurance* (Borch, 1974). He notes that opening up this possibility has important implication:

To practical insurance men the simple suggestion of Professor Bather may seem next to trivial. Insurance companies get into difficulties fairly regularly, and rescue operations are considered in the insurance world, if not daily, at least annually. The suggestion has, however, far-reaching implications for the theory of risk, and these do not seem to have been fully realized. If ruin does not mean the end of the game, but only the necessity of raising additional money, the current theories of risk may have to be radically revised (p. 313).

Borch indicates how the basic de Finetti model can be modified to incorporate this possibility and he illustrates the consequences of the possibility of rescue using some simple examples. The conclusion is that the company should be rescued if the benefits to the new shareholders exceed the cost of the new financing required. For the models considered, rescue will take place if the deficit is not too large. Borch notes that one of the simplifying assumptions is that if a company gets into financial difficulties it will still be able to attract business of the same quality and he questions the realism of this assumption:

This is a most unrealistic assumption, but it does not seem easy to modify it without constructing a general theory for the insurance market (p. 323).

The rescue operations in these models are those that would occur under the operation of market forces recognizing Borch's comments on the quality of new business. Of course there may be political pressure from policy holder groups on the government to step in and rescue an insolvent insurer.

⁹See Cummins (1988) and Doherty and Garven (1986) who apply option pricing theory to model insurer operations.

Applications of Finance Theory to Insurance Problems

The traditional valuation models of actuarial science focus in detail on the valuation of liabilities. Until recently much less attention was paid to the valuation of assets and the relationship between assets and liabilities. Borch was the first scholar to apply the paradigms of modern finance theory to the valuation and regulation of insurers. Recently, researchers in this field have searched for scientific bases for the regulation of insurance prices.¹⁰ In some cases the insights developed in these papers are already available in Borch's own research writings.

One of the central tenets of modern finance theory is that there is a relationship between the expected return on an investment and its associated risk. Equity holders in an insurance company will require an expected return commensurate with the risk of their investments. Borch noted that this has implications for government regulations on the solvency of insurers:

If the government insists that premiums should be kept at a low and "reasonable" level, profits may become so low that the insurance company is unable to attract the necessary reserve capital in a free competitive market. This leads to the rather trivial conclusion that supervision and regulation cannot secure the public good quality insurance at low premiums - unless, of course, the government itself is prepared to provide the reserve capital, or a guarantee that insurance contracts will be fulfilled (1974, p. 327).

Borch again uses a simple model to illustrate these ideas. He assumes that the regulators insist that insurance contracts meet certain standards of benefit security. One way of doing this is to insist that the probability of solvency lies within some acceptable limit. Equity capital will be invested in the insurer as long as the expected return is comparable with that on other investments of comparable risk. Borch suggests that the Capital Asset Pricing Model provides a useful framework for incorporating this relationship. In Borch's model the connection between premium levels, benefit security, and amount of equity capital are clearly demonstrated. For a given level of security the premium rates and capital requirements are jointly determined by the requirement that the equity holders earn a market return on their investment.

Borch also analyzes the role played by reinsurance within this model. He comments on problems of under capacity in the reinsurance sector:

By reinsurance arrangements the companies reduce the probabilities that they shall be unable to meet their obligations. If the reductions which can be obtained in this way are unsatisfactory, there will be complaints that the reinsurance market has insufficient capacity. This can only mean that the insurance sector as a whole does not have enough equity capital. The obvious remedy is to increase the premiums, so that more capital can be attracted from other sectors of the economy. This is in fact the only remedy in a world with free competitive markets (1974, p. 349).

¹⁰See for example Cummins (1988), Doherty and Garven (1986), Kraus and Ross (1982), Hill (1979), and Fairley (1979).

Other Research Contributions and Overview

Karl Borch continued to produce research until the end of his life. Since it is impossible to cover these in depth, his emphasis on the use of game theory as a powerful tool for the analysis of problems in insurance will be briefly discussed, as will one particular application suggested by Borch. Then an assessment of his research contributions to the insurance field will be made.

Throughout his writings on insurance topics, Borch constantly emphasizes the importance of game theory as a tool for the analysis of insurance problems. In the last few years game theory has been used to tackle a variety of problems in economics (for example, see Friedman, 1986; and Shubik and Levitan, 1980) and also in financial economics. Since insurance economics borrows from economics and finance, it is safe to predict that the promise of Borch's vision will be fulfilled.

Borch used the tools of game theory to analyze a number of insurance problems. For example, he used this approach to analyze the problem of determining the premium loadings for different risk classes. He considers the situation where there are several different groups, finding that though the risk attributes are assumed to be homogeneous within groups, they differ across groups. If the insurance premium consists of the expected claim plus a security loading, the amount of the loading can be reduced if the groups combine. Borch studied how the premium reductions achieved by pooling are to be shared across the different groups. He notes the conflicts that can arise among the groups in this situation and discusses the problems involved in obtaining an equitable solution, suggesting that game theory provides a valuable tool for analyzing this type of problem. The solutions obtained will depend on the assumptions made and Borch suggests that the Shapley solution has some attractive properties in this context.

Karl Borch also wrote articles on a diverse range of topics. For example, in 1976 he wrote an interesting little piece on insuring the monster in Loch Ness. A Scottish whisky firm had offered an award of one million pounds for the capture of the Loch Ness monster. The firm took out an insurance policy with Lloyds of London to cover the risk that the monster would be captured. Borch analyzes this contract with insight (and a nice sense of humor) from a Bayesian perspective and notes that:

Most elementary textbooks on insurance contain a chapter explaining to the student when a risk is insurable. Usually it is required that the risk must be random in nature, and that it must be possible to estimate the relevant probabilities from available statistics. The same question was discussed on a higher level at an international congress of actuaries in 1954. On this occasion learned actuaries presented 20 papers which together included 400 pages laying down different conditions which a risk must meet in order to be insurable. All these sets of conditions make it impossible to insure against the capture of a monster in Loch Ness, but still the insurance was written.

A suitable conclusion seems to be *Ab esse ad posse valet consequentia* although that proverb may be non-Bayesian in spirit.

The Loch Ness article illustrates Borch's ability to develop deep insights by analyzing practical problems from a theoretical perspective. It is vintage Borch.

It should be stressed that this review has dealt only with selected themes within a given area of Borch's research activities. Not discussed, for example, is his important 1968 book entitled *The Economics of Uncertainty*. Borch continued to publish research articles on a wide range of problems in insurance and financial economics until the end of his life. This review does not cover these publications.¹¹

However, the themes that have been covered attest to the seminal character of Borch's contributions to the field of insurance economics. In particular his research on optimal reinsurance treaties is of special importance. This work clarified the nature of optimal risk sharing among risk averse economic agents and paved the way for important subsequent work. Borch used ideas and paradigms from the emerging field of financial economics to analyze problems in the insurance area. As an actuary he introduced to actuarial journals a whole new way of thinking about insurance problems from an economic perspective. Accomplishing this task with the appropriate mixture of zeal, tact, and wry humor, Karl Borch made outstanding contributions to the development of insurance economics as a scientific discipline.

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