

Benchmarking Measures of Investment Performance with Perfect-Foresight and Bankrupt Asset Allocation Strategies

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Different measures of investment performance disagree on the absolute and relative rankings of mutual funds (see, for example, Lehmann and Modest [1987] and Ferson and Warther [1996]). Perhaps this is not surprising because even the most basic performance measures employ different measures of risk. For example, Treynor [1965] and Jensen [1968] used beta as the measure of risk, while Sharpe [1966] used standard deviation. And it is well known that studies which benchmark performance measures using passive portfolios find disagreement among the measures. Fama and French [1996] found that the absolute values of the Jensen capital asset pricing model (CAPM) alphas are larger than the absolute values of the Fama–French [1993] three-factor model alphas. Ferson and Harvey [1999], however, used conditional measures to make strong claims against the Fama–French three-factor model. The lack of agreement among the CAPM (and APT) measures of performance prompted Chen and Knez [1996] to examine what elements constitute a performance measure. Chen and Knez proposed measures that can be identified from market data and are independent of any asset pricing model. But they concluded, as have other studies, that the rankings generated by the measures can be very different.

It is also well known that performance measures suffer from a number of conceptual

problems. First, Roll [1978] pointed out that Jensen's alpha is an ambiguous measure of performance because it is unclear whether a non-zero alpha signals abnormal performance in a disequilibrium setting or simply reflects the effects of a market proxy that is not mean–variance efficient.¹ The Grinblatt–Titman [1993] portfolio change measure eliminates the problem of choosing a market proxy by gauging performance using returns and investor portfolio weights at different points in time. Second, expected returns, alphas, and betas almost certainly vary with economic conditions, especially with active asset allocation strategies. Conditional forms of the Jensen [1968], Treynor–Mazuy [1966], and Henriksson–Merton [1981] performance measures that were developed by Ferson and Schadt [1996], Ferson and Warther [1996], and Christopherson, Ferson, and Glassman [1998] attempted to alleviate this problem by making alphas and betas functions of information variables. Third, Fung and Hsieh [1997] and Lo [2001], among others, argued that the use of static performance measures gives misleading performance results for hedge funds and commodity traders who employ dynamic trading strategies, which frequently include short sales, leverage, and derivatives. Moreover, there have been attempts to adjust performance measures to allow for different types of dynamic trading strategies. For example, Fung and Hsieh addressed the problem by generalizing the Sharpe [1992] static asset-class factor model to

deal with the dynamic trading strategies of hedge funds and Solnik [1993] adjusted the Sharpe ratio to accommodate changes in risk exposures in global asset allocation strategies.

However, this article originates from two problems encountered in measuring the performance of asset allocation strategies. First, in accordance with other studies, the performance measures assign different rankings to portfolios generated from portfolio selection models in a variety of asset allocation settings. Perhaps this is to be expected as the common stock/small stock/long-term government bond and industry-rotation strategies examined in the asset allocation literature and in this article are neither the pure selectivity strategies implicit in the Jensen [1968] measure, nor the pure market-timing strategies embodied in the Treynor-Mazuy [1966] and Henriksson-Merton [1981] market-timing measures. However, the lack of agreement in the rankings reported in Grauer [2008] is troubling given the striking differences in the returns earned by the strategies. Second, in frameworks that preclude short sales, Grauer and Hakansson [1998] and Grauer [2007b] found that the Henriksson-Merton test estimates statistically significant negative selectivity and *negative* down-market betas for pure market-timing strategies that combine the value-weighted CRSP Index (or the S&P 500 Index) with either lending or borrowing. Both results, however, are theoretically impossible in a market-timing setting that does not permit short-selling the market.

In light of these difficulties, this article takes a completely different tack and benchmarks the performance measures against two extremes. One extreme questions whether the measures of investment performance recognize the truly abnormal positive performance of perfect-foresight asset allocation strategies. With only the slightest exaggeration, this performance is the best there is, the best there was, and the best there ever will be.² The other extreme questions whether the measures recognize the destructively negative performance of bankrupt asset allocation strategies (e.g., strategies that lose 100% or more of wealth). The results highlight a fundamental shortcoming with the measures—the risk-adjusted performance of some bankrupt strategies exceeds the risk-adjusted performance of *all* the perfect-foresight strategies! This occurs because the measures are based on arithmetic average returns which completely miss the fundamental importance of bankruptcy. This article also examines the ability of the timing measures to distinguish between timing and selectivity,

moving progressively from a pure market-timing strategy to a series of broader asset allocation strategies. The unconditional Henriksson-Merton test alone correctly makes the distinction in a pure market-timing setting.

The article proceeds as follows. The next section describes the perfect-foresight strategies and mean-variance asset allocation strategies, some of which bankrupted out-of-sample. The following two sections describe the data and performance measures, respectively. The results are reported in the fourth section, and the final section concludes.

PERFECT-FORESIGHT AND MEAN-VARIANCE ASSET ALLOCATION STRATEGIES

We first benchmark the performance measures using seven perfect-foresight asset allocation strategies. The quarterly returns of the first three perfect-foresight strategies are the maximum of the return on Treasury bills or the return(s) on 1) the market, defined as the value-weighted CRSP index; 2) common stocks, defined as the S&P 500 Index, small stocks, and long-term government bonds; or 3) 12 value-weighted industry indices. The second three perfect-foresight strategies allow the first three strategies to be levered. The seventh perfect-foresight strategy invests 40% (60%) of wealth in the market and the remainder in T-bills in down (up) markets and is intended to capture the idea that investors do not always act on their forecasts. This is particularly true of professional money managers who almost always constrain their actions so as to shade (rather than plunge) into or out of asset classes.

To be more specific, the unlevered rate of return on perfect-foresight strategy k at time t is

$$r_{kt} = \max(r_{L,t}, \{r_{jt}\})$$

where r_{kt} is the risk-free lending rate and $\{r_{jt}\}$ denotes the vector of rates of return on n risky assets for strategy k . In a pure market-timing strategy, $n = 1$, and the return on the strategy at time t is the maximum of the return on T-bills and the return on the market. In an industry-rotation strategy, $n = 12$, and the return on the strategy is the maximum of the return on T-bills and the returns on each of the 12 value-weighted industries. The levered return on asset j is defined as

$$r_{jt}^{margin} = \frac{1}{m_{jt}} r_{jt} + \left(1 - \frac{1}{m_{jt}}\right) r_{Bt}$$

where m_{jt} is the margin requirement on asset j at time t , and r_{Bt} is the riskless borrowing rate. The margin requirement for stocks at time t is the initial margin requirement as reported in the *Federal Reserve Bulletin*. The margin requirement for bonds is 10%. The levered rate of return on perfect-foresight strategy k at time t is

$$r_{kt}^{\text{margin}} = \max(r_{Lt}, \{r_{jt}^{\text{margin}}\})$$

We then benchmark the performance measures using mean-variance (MV) asset allocation strategies. The returns on these policies are taken from Grauer [2007a]. Grauer compared the policies and performance of MV portfolios with and without short-sales constraints when the means are estimated in different ways in an industry-rotation setting. The portfolio weights and returns are generated as follows. The MV problem that permits short sales at time t is

$$\max T(\mu'_t \mathbf{x}_t + (1+r_{Lt})x_{rt}) - 1/2 \mathbf{x}'_t \Sigma_t \mathbf{x}_t \quad (1)$$

subject to the budget constraint $\Sigma_j x_{jt} + x_{rt} = 1$ where μ_t , $\mathbf{x}_t$, and $\mathbf{1}$ are n -vectors³ containing unity plus the expected rates of return in period t , the portfolio weights in period t , and ones, respectively. Σ_t is an $n \times n$ positive-definite covariance matrix of asset returns in period t , and $x_{rt} > 0$ ($x_{rt} < 0$) is the fraction of wealth lent (borrowed) in period t . The (scalar) risk-tolerance parameter T is a MV approximation to the risk tolerance of an investor with a power utility function, $u(w) = (1/\gamma)w^\gamma$, $\gamma < 1$, defined on wealth. For the power functions, the Pratt-Arrow measure of relative risk aversion, $-wu''/u'$, is equal to $(1-\gamma)$. Hence, the risk-tolerance parameter T that corresponds to a given power function is equal to $1/(1-\gamma)$. The (implied) T of the tangency portfolio is equal to $1/(a_t - r_t c_t)$, where $a_t = \mathbf{1}' \Sigma_t^{-1} \mu_t$ and $c_t = \mathbf{1}' \Sigma_t^{-1} \mathbf{1}$ are two of the three well-known efficient set constants. The global minimum-variance portfolio of risky assets is found by solving Equation (1) without a risk-free asset and with T set equal to zero.

A more realistic formulation of the MV problem that precludes short-selling is

$$\max T(\mu'_t \mathbf{x}_t + r_{Lt} x_{Lt} + r_{Bt}^d x_{Bt}) - 1/2 \mathbf{x}'_t \Sigma_t \mathbf{x}_t \quad (2)$$

subject to a budget constraint, $\Sigma_i x_{it} + x_{Lt} + x_{Bt} = 1$; lower-bound constraints $x_{it} \geq 0$ for all i and $x_{Lt} \geq 0$, which preclude short-selling risky assets and lending at the borrowing rate; upper-bound constraint, $x_{Bt} \leq 0$, which precludes lending at the borrowing rate; and margin constraint, $\Sigma_i m_{it} x_{it} \leq 1$, where x_{Lt} is the amount lent in period t , r_{Lt} is one plus the risk-free lending rate in period t , x_{Bt} is the amount borrowed in period t , r_{Bt}^d is one plus the risk-free borrowing rate at the time of the decision at the beginning of period t , and m_{it} is the initial margin requirement for asset category i in period t expressed as a fraction.

At time t , the covariance matrix is estimated from historical data in a trailing 32-quarter window. The mean vector is estimated first by regressing returns on dividend yields and risk-free interest rates. The resulting means are then "shrunk" to historic means. Portfolios are selected each quarter from the beginning of 1934 to the end of 1999. The time series of portfolio weights and the realized portfolio returns are employed in the performance tests.

DATA

The data have several sources. The returns for common stocks, small stocks, and long-term government bonds are from the Ibbotson Associates database. The returns for the value-weighted industry groups are constructed from the returns on individual New York Stock Exchange (NYSE) firms contained in the Center for Research in Security Prices (CRSP) monthly returns database. The firms are combined into 12 industry groups on the basis of the first two digits of their SIC codes.⁴ The risk-free lending rate is assumed to be the rate of return on 90-day U.S. T-bills maturing at the end of the quarter and is sourced from the *Survey of Current Business* and the *Wall Street Journal*.

PERFORMANCE MEASURES

The Sharpe [1964]–Lintner [1965] CAPM states that, in equilibrium, expected returns are linearly related to beta or systematic risk. All assets (and portfolios) plot on the security market line (SML),

$$E(r_p) = r + (E(r_m) - r)\beta_p \quad (3)$$

where $E(\cdot)$ is the expectations operator, r is the risk-free rate of interest, r_m is the return on the market portfolio, and β_p is the covariance of the return on asset (or portfolio) p with the return on the market portfolio divided by the variance of the return on the market portfolio. The unconditional Jensen [1968] performance measure is based on the regression

$$R_{pt} = \alpha_p + \beta_p R_{mt} + u_{pt} \quad (4)$$

where $R_{pt} = r_{pt} - r_{Lt}$ is the excess return on portfolio p over the T-bill rate, R_{mt} is the excess return on the market, and β_p is the unconditional measure of risk. The intercept α_p is variously known as Jensen's alpha, the unconditional Jensen measure of performance, Jensen's measure of selectivity, and the CAPM pricing error, as it is the sample estimate of a security's deviation from the SML. The SML also serves as the basis of the Treynor [1965] ratio,

$$T_p = \bar{R}_p / \hat{\beta}_p \quad (5)$$

where $\bar{R}_p$ is the average excess return on asset or portfolio p during the sample period, and $\hat{\beta}_p$ is the measure of systematic risk estimated from the time-series regression in Equation (4).

In contrast, the Sharpe [1966] ratio is based on the capital market line (CML) relationship, which states that MV-efficient portfolio returns are linearly related to their standard deviations. The CML is defined as

$$E(r_p) = r + [(E(r_m) - r) / \sigma_m] \sigma_p \quad (6)$$

where, in this case, r_p is the return on MV-efficient portfolio p , and σ_m and σ_p are the standard deviations of the returns on the market portfolio and the MV-efficient portfolio p , respectively. The Sharpe ratio is

$$Sh_p = \bar{R}_p / \hat{\sigma}_p \quad (7)$$

where $\bar{R}_p$ is as previously defined and $\hat{\sigma}_p$ is the sample standard deviation of returns on portfolio p . An alternative is the capital market line (CML) alpha,

$$\alpha_p = \bar{r}_p - (\bar{r}_L + [(\bar{r}_m - \bar{r}_L) / \hat{\sigma}_m] \hat{\sigma}_p) \quad (8)$$

which measures performance as a deviation from the capital market line.

The Fama-French [1993] three-factor model is an empirical alternative to the CAPM. The model postulates that the expected return on an asset is explained by the sensitivity of its return to three factors: 1) the excess return on the market portfolio, 2) the difference between the return on a portfolio of small stocks and the return on a portfolio of large stocks (SMB), and 3) the difference between the return on a portfolio of high book-to-market stocks and the return on a portfolio of low book-to-market stocks (HML). Specifically, the expected return on portfolio p is

$$E(r_p) = r + \beta_p (E(r_m) - r) + s_p E(r_{SMB}) + h_p E(r_{HML}) \quad (9)$$

where $E(r_m) - r$, $E(r_{SMB})$, and $E(r_{HML})$ are expected premiums, and the factor sensitivities or factor loadings— β_p , s_p and h_p —are the slopes in the time-series regression,⁵

$$R_{pt} = \alpha_p + \beta_p R_{mt} + s_p r_{SMBt} + h_p r_{HMLt} + e_{pt} \quad (10)$$

Once again, the intercept α_p is the measure of abnormal performance, or pricing error, calculated relative to the Fama-French equilibrium pricing equation in Equation (9).

It is difficult to measure market timing in a single-period CAPM or three-factor model framework. Consequently, the Treynor-Mazuy [1966] and Henriksson-Merton [1981] models have become popular ways of measuring market-timing ability. The unconditional regression specification for the Treynor-Mazuy market-timing measure is

$$R_{pt} = \alpha_p + \beta_p R_{mt} + \gamma_p R_{mt}^2 + u_{pt} \quad (11)$$

where α_p is a measure of selectivity, β_p is an unconditional beta, and γ_p is the market-timing coefficient. The intuition underlying the model is that an investor will increase (decrease) his market holdings—and hence his portfolio beta—the higher (lower) the return on the market. The unconditional Henriksson-Merton market-timing performance measure is

$$R_{pt} = \alpha_p + \beta_{dp} R_{mt} + \gamma_p \max(0, R_{mt}) + u_{pt} \quad (12)$$

where α_p is a measure of selectivity; β_p is the down-market beta; γ_p is the market-timing coefficient, which in this case is the difference between the up- and down-market beta; and $\max(0, R_{mt})$ is the payoff on a call option on the market with exercise price equal to the risk-free rate of interest.

A major objection to the unconditional models is that expected returns and betas may change over time. Therefore, Ferson and Schadt [1996] and Ferson and Warther [1996], among others, building on the earlier work of Shanken [1990], advocated the use of conditional performance measures. This article follows their suggestion that portfolio risk is related to dividend yields and short-term Treasury yields, postulating that

$$\beta_p = b_{0p} + b_{1p} dy_{t-1} + b_{2p} r_{Lt} \quad (13)$$

where dy_{t-1} is the value-weighted CRSP Index annual dividend yield lagged one month so that it is observable at the beginning of quarter t , and r_{Lt} is the (observable) beginning-of-quarter T-bill rate, both measured as deviations from their estimation-period means. Substituting for β_p from Equation (13) into Equation (4) yields the conditional Jensen performance measure

$$R_{pt} = \alpha_p + b_{0p} R_{mt} + b_{1p} [dy_{t-1} R_{mt}] + b_{2p} [r_{Lt} R_{mt}] + e_{pt} \quad (14)$$

where α_p is the conditional measure of performance, b_{0p} is the unconditional beta, and b_{1p} and b_{2p} measure how the unconditional beta varies with dividend yields and T-bill rates. Substituting for β_p from Equation (13) into Equation (11) yields the conditional regression specification of the Treynor–Mazuy market-timing test,

$$R_{pt} = \alpha_p + b_{0p} R_{mt} + b_{1p} [dy_{t-1} R_{mt}] + b_{2p} [r_{Lt} R_{mt}] + \gamma_p R_{mt}^2 + e_{pt} \quad (15)$$

where α_p , b_{0p} , b_{1p} , and b_{2p} are defined as in Equation (14) and γ_p is the market-timing coefficient. Following Ferson and Schadt [1996], the conditional Henriksson–Merton test is

$$R_{pt} = \alpha_p + b_{0p} R_{mt} + b_{1p} [dy_{t-1} R_{mt}] + b_{2p} [r_{Lt} R_{mt}] + \gamma_p R_{mt}^* + b_{1p}^* [dy_{t-1} R_{mt}^*] + b_{2p}^* [r_{Lt} R_{mt}^*] + e_{pt} \quad (16)$$

where R_{mt}^* is the product of the excess return on the value-weighted CRSP Index and an indicator dummy for positive values of the difference between the excess return on the index and the conditional mean of the excess return. The conditional mean is estimated by a linear regression of the excess return of the value-weighted CRSP Index on dy_{t-1} and r_{Lt} . The most important coefficients are b_{dp} , the conditional down-market beta, and γ_p , the market-timing coefficient, which in this case is the difference between the up- and down-market conditional betas.

Christopherson, Ferson, and Glassman [1998] took the idea of conditional models one step further, postulating that alpha is also a linear function of the information variables,

$$\alpha_p = a_{0p} + a_{1p} dy_{t-1} + a_{2p} r_{Lt} \quad (17)$$

Substituting for α_p from Equation (17) and for β_p from Equation (13) into Equation (4) yields Jensen's measures with conditional alphas and betas. Ferson and Harvey [1999] investigated the Fama–French three-factor model in a conditional setting by making both the slopes and the intercept linear functions of the information variables. In this article, we specify that the slopes and intercepts are linear functions of dy_{t-1} and r_{Lt} , that is,

$$\begin{aligned} \beta_p &= b_{0p} + b_{1p} dy_{t-1} + b_{2p} r_{Lt} \\ s_p &= s_{0p} + s_{1p} dy_{t-1} + s_{2p} r_{Lt} \text{ and} \\ h_p &= h_{0p} + h_{1p} dy_{t-1} + h_{2p} r_{Lt} \end{aligned} \quad (18)$$

Substituting for α_p , β_p , s_p , and h_p from Equations (17) and (18) into Equation (10) gives the conditional Fama–French three-factor model.

In contrast to most other performance measures, the Grinblatt–Titman [1993] portfolio change measure employs portfolio holdings in addition to rates of return, does not require an external benchmark (market) portfolio, and is not related to any asset pricing model. In order to motivate the portfolio change measure, assume uninformed investors perceive that the vector of expected returns is constant, but informed investors are able to predict whether expected returns vary over time. Informed investors can profit from changing expected returns by increasing (decreasing) their holdings of assets whose expected returns have increased (decreased). The holding

of an asset that increases as a result of an increase in the asset's conditional expected rate of return will exhibit a positive unconditional covariance with its return. The portfolio change measure is constructed from an aggregation of these covariances. For evaluation purposes, let

$$PCM_i = \sum_t r_{it} (x_{it} - x_{i,t-j})$$

where r_{it} is the quarterly rate of return on asset i at time t , and x_{it} and $x_{i,t-j}$ are the holdings of asset i at time t and time $t-j$, respectively. This expression provides an estimate of the covariance between returns and weights at a point in time. Alternatively, it may be viewed as the return on the zero-weight portfolio. The portfolio change measure is an average of the PCM_i s,

$$\overline{PCM} = \sum_t \sum_i [r_{it} (x_{it} - x_{i,t-j}) / T] \quad (19)$$

where T is the number of time-series observations. The portfolio change measure test is a simple t -test based on the time series of zero-weight portfolio returns,

$$t = (\overline{PCM} / \sigma(PCM)) \sqrt{T}$$

where $\sigma(PCM)$ is the standard deviation of the time series of PCM_i s. In their empirical analysis of mutual fund performance, Grinblatt and Titman worked with two values of j that represented one- and four-quarter lags. This article employs the same two lags.

The portfolio change measure is particularly appropriate in the present study because the portfolio weights are chosen, according to a pre-specified set of rules, over the same quarterly time interval as performance is measured. Thus, possible gaming or window dressing problems that face researchers trying to gauge the performance of mutual funds are not a concern.

In all the tests, the null hypothesis is that abnormal performance is equal to zero. The alternative hypothesis is that abnormal performance is positive. Thus, the results of one-tailed tests are reported. All the tests, except the Grinblatt–Titman test, are corrected for heteroscedasticity using White's [1980] correction.

RESULTS

This section discusses the results of the perfect-foresight asset allocation strategies and the mean–variance asset allocation strategies.

Perfect-Foresight Asset Allocation Strategies

Exhibit 1 shows summary statistics for returns and cumulative wealth values at the end of 1999—based on a \$1 investment at the beginning of 1934—for the perfect-foresight strategies and selected passive strategies. In Exhibits 1 to 3, the perfect-foresight strategies are listed in order from the strategy that accumulates the least wealth to the strategy that accumulates the most wealth. The passive strategy of investing in T-bills is the least remunerative, accumulating \$14 at the end of 1999. The passive strategy of investing in the market (the value-weighted CRSP Index) accumulates \$2,079. The constrained perfect-foresight market-timing strategy invests 40% (60%) in the market if the return on T-bills exceeds (is less than) the return on the market, and thus does not act completely on the perfect-foresight forecast. In fact, it does not even keep up with the passive strategy of investing in the market, accumulating only \$1,309 over the period from 1934 to 1999.

By contrast, the unconstrained pure perfect-foresight market-timing strategy yields approximately \$1.3 million over the study period. Clearly, it pays to act on perfect foresight without constraint. With margin permitted, timing the market accumulates about \$13 billion, which pales relative to the perfect-foresight industry-rotation strategy, in which a dollar grows to $\$1.4 \times 10^{14}$. However, the real money comes from the industry-rotation and common stock/small stock/bond strategies with margin. The industry-rotation strategy accumulates $\$1.1 \times 10^{24}$ and the common stock/small stock/bond strategy more than doubles that!

None of this information is new, of course. It is not difficult to find these types of numbers in the popular press.⁶ Moreover, it is unlikely that anyone, even someone with perfect foresight, could accumulate such astronomical amounts of money. Transaction costs, taxes, and the price impact of investing such vast sums would undoubtedly mitigate the gains earned from such a strategy. Nevertheless, this article examines whether popular performance measures recognize this truly amazing performance.

EXHIBIT 1

Return and Cumulative Wealth Values of Selected Passive and Perfect-Foresight Strategies

A perfect-foresight strategy's return each quarter is the maximum of the return on Treasury bills or the return(s) on: 1) the market, defined as the value-weighted CRSP index; 2) CSB, composed of common stocks (the S&P 500 index), small stocks, or long-term government bonds; or 3) 12 value-weighted industry indices. The results are presented with and without margin. Market 40–60 is a perfect-foresight strategy that invests 40(60) percent of wealth in the market without margin and the remainder in Treasury bills in down(up) markets. The time period is 1934 to 1999. Returns are in units of percent per quarter. Wealth is the cumulative wealth at the end of 1999 arising from an investment of \$1 at the beginning of 1934.

Strategies	Arithmetic Average	Standard Deviation	Beta	Compound Return	Wealth
Panel A: Passive Strategies					
Treasury Bills	1.02	0.83	0.00	1.01	14
Market	3.30	8.52	1.00	2.94	2,079
Panel B: Perfect-Foresight Strategies					
Market 40–60	2.85	4.38	0.51	2.76	1,309
Market No Margin	5.61	5.41	0.55	5.48	1.3×10^6
Market Margin	9.76	11.62	1.10	9.22	1.3×10^{10}
CSB No Margin	9.69	9.80	0.83	9.30	1.6×10^{10}
Industries No Margin	13.61	11.05	1.02	13.12	1.4×10^{14}
Industries Margin	25.50	26.40	2.13	23.32	1.1×10^{24}
CSB Margin	26.92	33.42	2.00	23.67	2.3×10^{24}

Intuitively, the performance measures would be expected to 1) register positive abnormal performance for each of the perfect-foresight strategies and 2) rank the strategies in accordance with the cumulative-wealth rankings. Although, in general, performance-measure rankings and cumulative-wealth rankings would not be expected to be in accord, it would be truly embarrassing if a performance measure ranked a strategy that accumulated hundreds of thousands of times the initial wealth ahead of a strategy that accumulated more than billions of trillions of times the initial wealth—and it would be profoundly embarrassing if it ranked a bankrupt strategy ahead of a perfect-foresight strategy.

Exhibit 2 shows the results of applying the Sharpe ratio, capital market line (CML) alpha, Treynor ratio, unconditional and conditional Jensen and Fama–French alphas, and Grinblatt–Titman portfolio change measure to the perfect-foresight asset allocation strategies.⁷ Each of these measures correctly identifies positive abnormal performance for each of the strategies. The Sharpe ratio, however, does a poor job of ranking the strategies. For example, it ranks the industry-rotation strategy without

margin, which accumulates $\$1.4 \times 10^{14}$, *first*; the market-timing strategy without margin, which accumulates \$1.3 million, *fourth*; and the common stock/small stock/bond strategy with margin, which accumulates $\$2.3 \times 10^{24}$, *fifth*. The Treynor ratio does somewhat better than the Sharpe ratio. Its mistakes are that it ranks the market-timing strategy without margin above the market-timing strategy with margin and ranks the industry-rotation strategy without margin above the industry-rotation strategy with margin—even though in the latter case the strategy with margin accumulates over 750 million times as much wealth.

Two problems are associated with ratio measures. A ratio assumes that the borrowing rate is equal to the lending rate and that an investor can lever any strategy as much or as little as is desired. The perfect-foresight strategies examined in this article do not conform to either assumption. The borrowing and lending rates differ and margin constraints are adhered to. One way to overcome the second problem (but not the first one) is to measure performance as deviations from either the CML or SML. Exhibit 2 shows that the CML alpha correctly ranks all the strategies except the industry-rotation strategy with

EXHIBIT 2

Selected Performance Measure Results for the Perfect-Foresight Strategies

The perfect-foresight strategies, defined in Exhibit 1, are listed from worst to best according to the accumulated wealth. The Sharpe ratios are calculated from Equation (7), capital market line (CML) alphas from Equation (8), Treynor ratios from Equation (5), unconditional Jensen alphas from Equation (4), unconditional Fama–French (FF) alphas from Equation (10), and Grinblatt–Titman four-quarter portfolio change measures (PCM) from Equation (19). The conditional Jensen and Fama–French alphas are functions of lagged dividend yields and risk-free interest rates. The alphas and portfolio change measures are in units of percent per quarter. The ranks of the strategies (1 is best, 7 is worst) are reported for the Sharpe ratio, CML alpha, and Treynor ratio. The rankings of the other measures match the accumulated wealth rankings.

Perfect-Foresight Strategies	Sharpe Ratio	Rank	CML Alpha	Rank	Treynor Ratio	Rank	Unconditional Alpha		Conditional Alpha		Grinblatt-Titman PCM
							Jensen	Fama-French	Jensen	Fama-French	
Market 40–60	0.42	7	0.66	7	3.60	7	0.67	0.66	0.66	0.65	0.71
Market No Margin	0.85	4	3.15	6	8.42	5	3.35	3.31	3.30	3.25	3.57
Market Margin	0.75	6	5.63	5	7.91	6	6.22	6.13	6.06	5.89	6.71
CSB No Margin	0.88	3	6.05	4	10.35	4	6.76	6.85	6.62	6.28	6.20
Industries No Margin	1.14	1	9.63	3	12.19	2	10.24	10.26	10.12	9.92	9.90
Industries Margin	0.93	2	17.41	1	11.31	3	19.54	19.69	19.18	18.60	18.52
CSB Margin	0.77	5	16.95	2	13.40	1	21.49	21.67	21.60	20.14	19.85

EXHIBIT 3

Market-Timing Results for Perfect-Foresight Strategies

The unconditional and conditional Henriksson–Merton market-timing tests are given in Equations (12) and (16). The unconditional and conditional Treynor–Mazuy market-timing tests are given in Equations (11) and (15). The alphas measure selectivity and the gammas measure market-timing ability. The Henriksson–Merton gamma is the difference between the up- and down-market beta. The Treynor–Mazuy gamma is multiplied by 100. Returns are in units of percent per quarter. A coefficient in boldface type indicates that it was not statistically significant at the 1% level in a one-tailed test.

Perfect-Foresight Strategies	Henriksson–Merton				Treynor–Mazuy		
	Alpha	Down-market Beta	Gamma	Adj-R ²	Alpha	Gamma	Adj-R ²
Unconditional Results							
Market 40–60	0.00	0.40	0.20	1.00	0.39	0.38	1.00
Market No Margin	0.00	0.00	1.00	1.00	1.93	1.90	0.96
Market Margin	-1.13	-0.09	2.19	0.92	2.91	4.44	0.91
CSB No Margin	1.70	0.01	1.51	0.71	4.50	3.02	0.69
Industries No Margin	6.10	0.36	1.23	0.72	8.24	2.68	0.73
Industries Margin	9.55	0.54	2.98	0.58	14.52	6.74	0.60
CSB Margin	9.57	-0.01	3.56	0.33	16.23	7.05	0.32
Conditional Results							
Market 40–60	0.08	0.41	0.18	1.00	0.37	0.41	1.00
Market No Margin	0.39	0.04	0.92	0.99	1.86	2.03	0.96
Market Margin	0.05	0.03	1.91	0.92	2.89	4.41	0.91
CSB No Margin	2.45	0.09	1.33	0.71	4.45	3.03	0.69
Industries No Margin	6.89	0.46	1.02	0.72	8.16	2.69	0.73
Industries Margin	11.96	0.80	2.30	0.60	14.59	6.21	0.61
CSB Margin	10.82	0.07	3.37	0.38	16.71	7.18	0.35

margin, which it ranks above the common stock/small stock/bond strategy with margin. However, as will be discussed in the next paragraph, Jensen's alpha ranks the strategies correctly—although it does not allow for different borrowing and lending rates in measuring performance relative to the SML. So, what explains the difference? Perfect-foresight strategies never lose. Arguably, standard deviation—whatever its merits as a risk measure for real-world investment strategies—does not really measure risk in this context.

Now we turn to the unconditional and conditional Jensen and Fama–French measures. For brevity, only the results for the conditional models, which assume both the intercepts and slopes are functions of the information variables, are reported. Two points stand out. First, all the alphas are positive (and statistically significantly different from zero at much less than the 0.01 level). Second, all four measures correctly rank the performance of the strategies from best to worst.⁸

The Jensen results are somewhat surprising. Although Jensen's alpha is among the most, if not the most, popular academic measure of performance, it has also attracted the most criticism. In this case, in spite of its stellar performance in ranking the strategies, some of its shortcomings are obvious. For example, it correctly recognizes that the perfect-foresight market-timing strategy achieves abnormal performance, but it attributes all of the abnormal performance to selectivity. Moreover, it estimates beta to be 0.51 (see Exhibit 1), although for a pure market timer beta is zero in down markets and one in up markets. In light of the arguments previously noted that alpha might be negative for market timers, it is interesting that the alpha is positive.

In addition, Exhibit 2 shows that the Grinblatt–Titman [1993] four-quarter portfolio change measure correctly identifies positive abnormal performance for each of the strategies and correctly ranks them. Although the results are not reported in Exhibit 2, we also examine the one-quarter portfolio change measure. Three of the seven one-quarter-lag portfolio change measures are larger than their four-quarter-lag counterparts. Typically, however, with data from mutual funds and asset allocation strategies that are generated from a dynamic portfolio selection model, the portfolio change measure indicates that the four-quarter-lag measure assigns higher abnormal returns. See, for example, Grinblatt and Titman [1993], Grauer [2008], Grauer and Hakansson [2001], and Grauer and Shen [2000].

Exhibit 3 shows that the unconditional and conditional Henriksson–Merton and Treynor–Mazuy tests correctly recognize market-timing ability, but they fail to correctly distinguish between market-timing ability and selectivity. The only exception is for the Henriksson–Merton test applied to the pure market-timing strategy without margin and to the constrained 40–60 market-timing strategy. Exhibits 4 and 5 show why the Henriksson–Merton test works in these cases. Exhibit 4 demonstrates that the payoff on a pure market-timing strategy exactly matches the payoff on the call option. Exhibit 5 shows that the payoff on the constrained 40–60 market timing strategy, which lies below the payoff on the call, is piece-wise linear passing through the origin with a slope of 0.4 in down markets and 0.6 in up markets.

But with margin everything falls apart. The alpha is -1.13 , which is theoretically impossible because selectivity is not associated with a pure market-timing strategy. In addition, the down-market beta is -0.09 , which is again theoretically impossible because short-selling the market is not permitted. The primary problem is that margin requirements change over time, as does the difference in the borrowing and lending rates. With constant margin requirements and the borrowing rate equal to the lending rate, the down-market alpha and down-market beta are equal to zero, the up-market alpha is equal to zero, and the up-market beta is equal to one over the margin requirement.

When the margin rate changes over time, there is more than one up-market beta, and the Henriksson–Merton regression is mis-specified. Exhibit 6 confirms that the payoff on the market-timing strategy with margin matches the payoff on the call in down markets and lies above it in up markets. Hence, the negative down-market beta and negative alpha reported in Exhibit 3 are simply artifacts. The unconditional Henriksson–Merton framework is simply not rich enough to capture the effect of these market frictions. In cases of more than one risky asset, the problems worsen. For example, Exhibit 7 shows that the returns of the perfect-foresight industry-rotation and common stock/small stock/bond strategies plot above the payoff on a pure perfect-foresight market-timing strategy in both up and down markets. Hence, different up- and down-market alphas exist, as well as different up- and down-market betas.

EXHIBIT 4

Excess Return on Perfect-Foresight Strategy without Margin

Excess return on the perfect-foresight market-timing strategy without margin plotted against the excess return on the market. The solid line is the payoff on a call option on the market with exercise price equal to the risk-free rate of interest. The market is defined as the value-weighted CRSP Index.

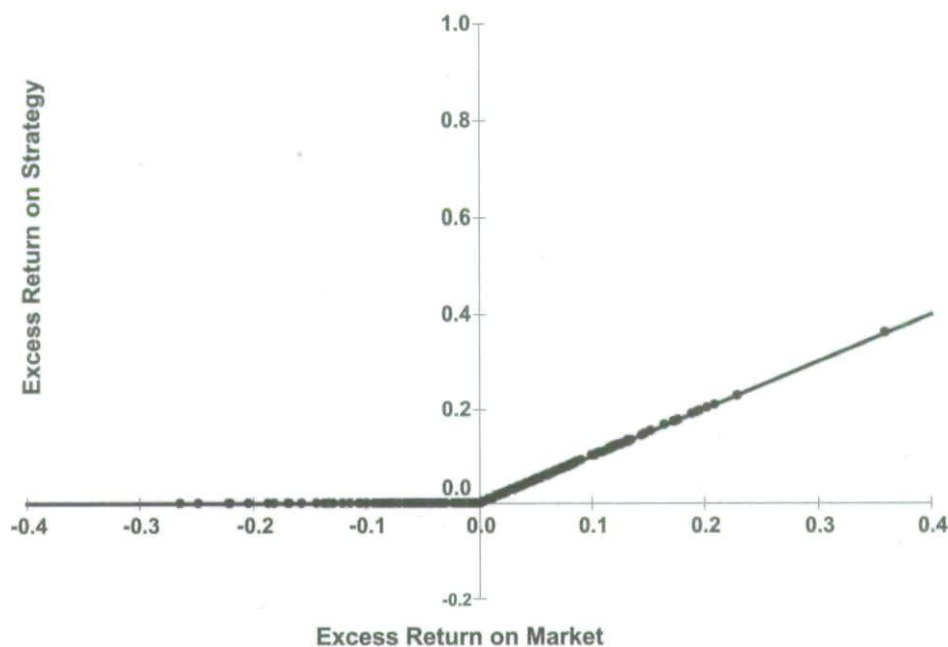


EXHIBIT 5

Excess return on Constrained 40–60 Perfect Foresight Market-Timing Strategy

Excess return on the constrained 40–60 perfect-foresight market-timing strategy plotted against the excess return on the market. The solid line is the payoff on a call option on the market with exercise price equal to the risk-free rate of interest. The market is defined as the value-weighted CRSP Index.

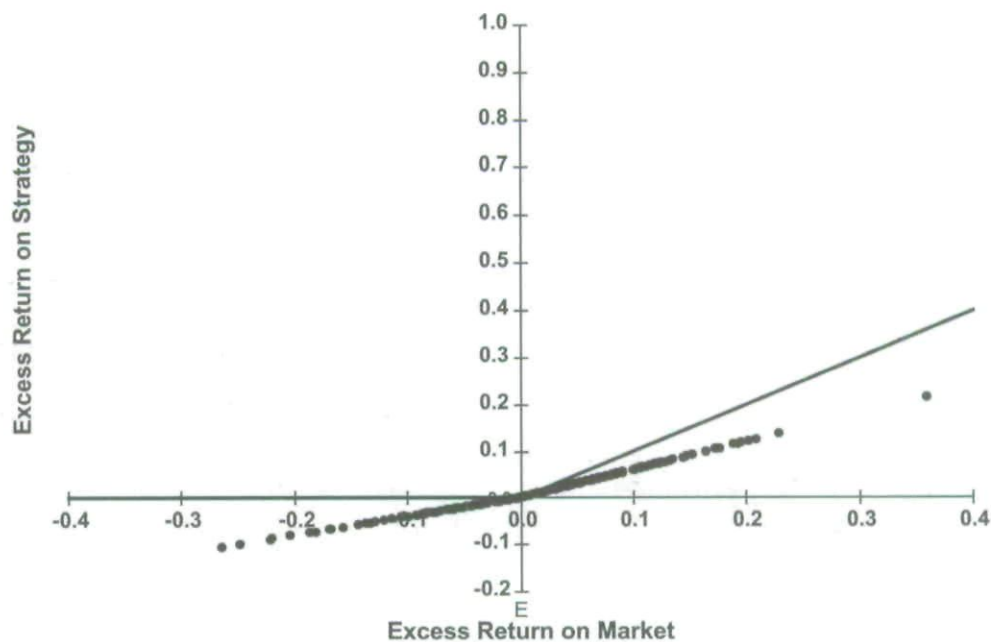
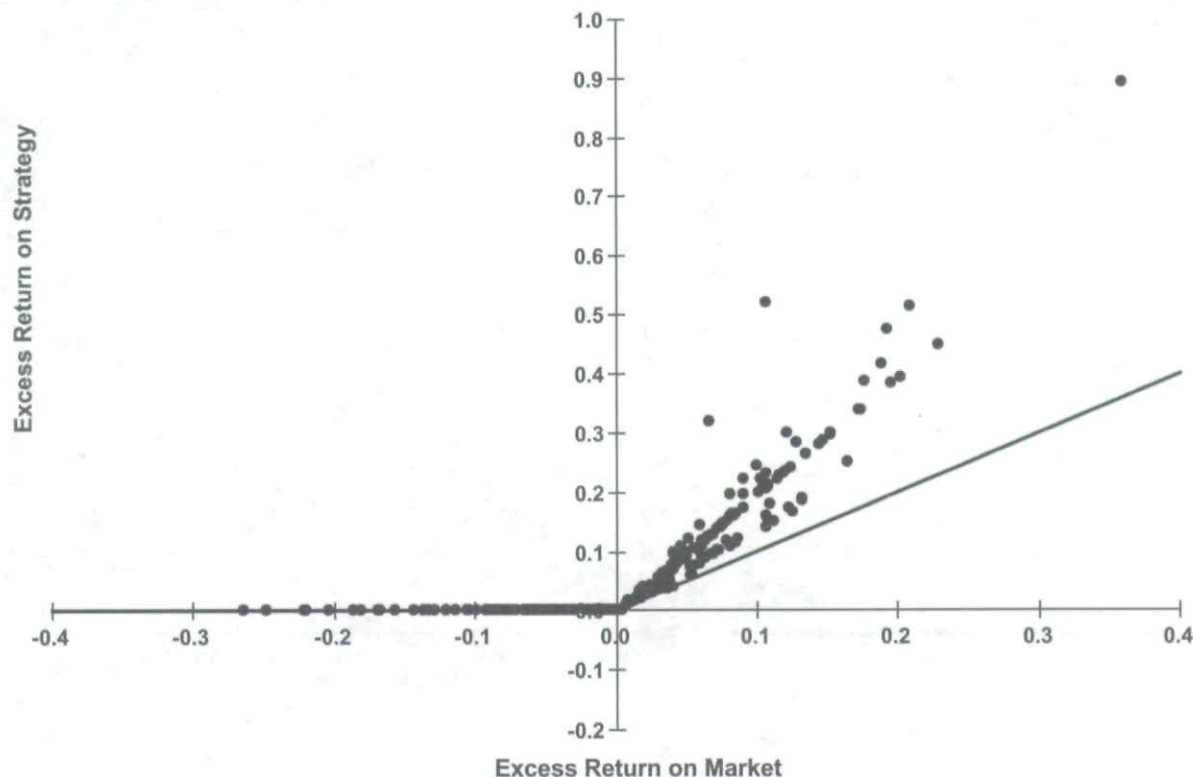


EXHIBIT 6

Excess Return on Perfect-Foresight Market-Timing Strategy with Margin

Excess return on the perfect-foresight market-timing strategy with margin plotted against the excess return on the market. The solid line is the payoff on a call option on the market with exercise price equal to the risk-free rate of interest. The market is defined as the value-weighted CRSP Index.



Mean-Variance Asset Allocation Strategies

Exhibit 8 contains a summary of the accumulated wealth values and performance measures of five MV-efficient asset allocation portfolios. Short sales are permitted for the portfolios in Panel A and are precluded for the portfolios in Panel B. Reading from left to right, the exhibit shows each portfolio's accumulated wealth at the end of 1999 (generated from a \$1 investment at the beginning in 1934), average arithmetic rate of return, standard deviation of return, beta, Sharpe ratio, unconditional and conditional Jensen and Fama-French alphas, and Grinblatt-Titman portfolio change measure.

The results differ dramatically depending on whether short sales are permitted or precluded. When short sales are permitted, four portfolios bankrupted out-of-sample. The MV-efficient portfolio with a γ of 0.5 loses over 100% in 69 of the 264 quarters from the beginning of 1934 to the

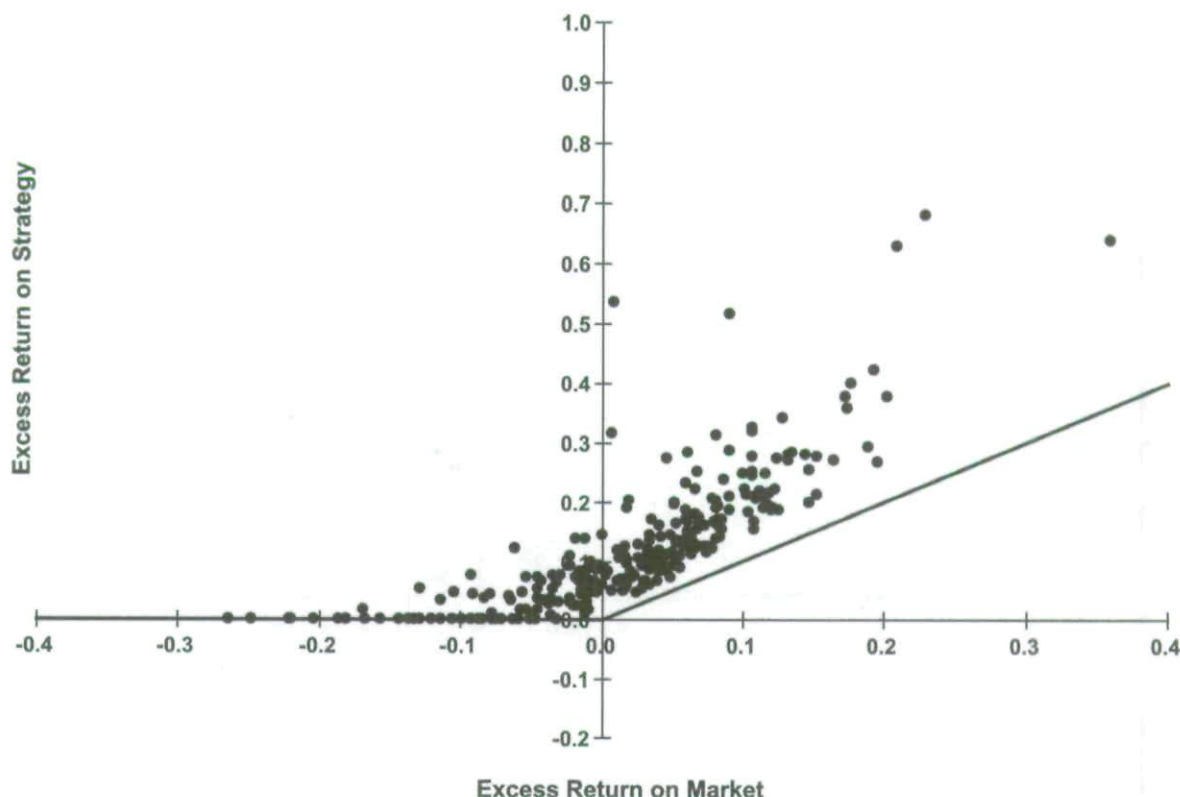
end of 1999. When short sales are precluded, the less risk-averse portfolios accumulate impressive amounts of wealth judged by any real-world standard. The MV-efficient portfolio with a γ of 0.5 accumulates over \$437,000 at the end of 1999 from a \$1 investment at the beginning of 1934.

Unbelievably, the performance measures completely miss the fundamental difference between the bankrupt and extremely profitable strategies. The conditional and unconditional Jensen and Fama-French alphas and Grinblatt-Titman portfolio change measures of the bankrupt portfolios in Panel A are uniformly greater than the corresponding portfolios in Panel B.⁹ Worse, the unconditional and conditional Jensen and Fama-French alphas of two bankrupt portfolios and the Grinblatt-Titman portfolio change measure of three bankrupt portfolios are greater than the alphas and portfolio change measures of *all* the perfect-foresight strategies!

EXHIBIT 7

Excess Return on Perfect-Foresight Industry-Rotation Strategy without Margin

Excess return on the perfect-foresight industry-rotation strategy without margin plotted against the excess return on the market. The solid line is the payoff on a call option on the market with exercise price equal to the risk-free rate of interest. The market is defined as the value-weighted CRSP Index.



CONCLUSION

It is well known that popular measures of investment performance do not agree on the relative performance of passive portfolios, professionally managed funds, or various asset allocation strategies. The measures also suffer from a number of conceptual and empirical shortcomings. This article shows, by benchmarking the performance measures against two extremes, that the problems are more fundamental. Bankruptcy is the ultimate investment risk—bankrupt asset allocation strategies lose everything. And perfect-foresight asset allocation strategies yield returns beyond anyone's wildest dreams. Yet, many popular measures of investment performance cannot differentiate between them.

The Sharpe ratio does not distinguish between some bankrupt and nonbankrupt strategies. The alphas and portfolio change measures of the majority of bankrupt MV-efficient strategies exceed the alphas and portfolio change

measures of MV-efficient strategies that are extremely profitable by any real-world standard. Worse, the unconditional and conditional Jensen and Fama–French alphas of two bankrupt MV-efficient strategies and the Grinblatt–Titman portfolio change measure of three bankrupt MV-efficient strategies exceed the corresponding alphas and portfolio change measures of *all* the perfect-foresight strategies!

It is hard to imagine how these results could be so far off the mark. A bankrupt strategy should never be ranked ahead of a nonbankrupt strategy, a highly profitable strategy, and especially a perfect-foresight strategy. The result can be compared to learning that a battery of medical tests judged cadavers to be more physically fit than hospital patients in an intensive care unit, middle-aged investors, and Olympic gold medalists. In addition, market-timing models are unable to correctly distinguish between selectivity and market-timing ability. The only exception is the unconditional Henriksson–Merton model in a pure market-timing

EXHIBIT 8

Performance Measures for Unconstrained and Constrained Mean-Variance Efficient Portfolios, Global Minimum-Variance Portfolios, and Tangency Portfolios

The performance measures of the unconstrained and constrained mean-variance efficient portfolios, global minimum-variance portfolios, and tangency portfolios are estimated with dividend yield and risk-free rate means. A mean-variance efficient portfolio is defined in terms of the gamma of a power utility function $u(w) = (1/\gamma)w^\gamma$, $\gamma < 1$. The corresponding risk tolerance parameter employed in the mean-variance optimizer is given by $T = 1 / (1 - \gamma)$. The investment universe consists of 12 value-weighted industry groups in the 1934–1999 period. Quarterly portfolio revision with a 32-quarter estimation period is employed. The borrowing rate exceeds the lending rate. The tangency portfolio is constructed using the lending rate. The average and standard deviation of returns are measured in percent per quarter. The ex post standard deviation used in constructing the ex post Sharpe ratio is measured in units of excess return. Wealth is the cumulative wealth at the end of 1999 arising from an investment of \$1 at the beginning of 1934. The Grinblatt–Titman portfolio change measure (PCM) is based on a four-quarter lag. The alphas and PCM are in units of percent per quarter.

Portfolio	Wealth	Arithmetic Average Return	Standard Deviation of Return	Unconditional Alpha			Conditional Alpha			Grinblatt– Titman PCM	
				Beta	Sharpe Ratio	Jensen	Fama– French	Jensen	Fama– French	Jensen	Fama– French
Panel A: Unconstrained, Short Sales Permitted											
$\gamma = -10$	384	4.34	19.78	0.55	0.17	2.07	3.44	2.12	3.48	5.20	5.20
$\gamma = -5$	0	7.10	36.26	1.00	0.17	3.80	6.30	3.89	6.37	9.54	9.54
$\gamma = -1$	0	19.28	108.77	3.01	0.17	11.40	18.90	11.68	19.11	28.61	28.61
$\gamma = 0$	0	37.54	217.54	6.02	0.17	22.80	37.80	23.36	38.23	57.22	57.22
$\gamma = 0.5$	0	74.07	435.08	12.04	0.17	45.60	75.60	46.72	76.45	114.45	114.45
Panel B: Constrained, Short Sales Precluded											
$\gamma = -10$	521	2.55	5.59	0.18	0.27	0.52	0.76	0.57	0.71	0.63	0.63
$\gamma = -5$	2,228	3.30	8.24	0.34	0.28	0.70	1.06	0.80	1.03	0.87	0.87
$\gamma = -1$	63,988	5.20	13.72	0.81	0.31	1.38	2.00	1.57	1.98	1.79	1.79
$\gamma = 0$	186,769	6.17	16.95	1.08	0.30	1.60	2.26	1.75	2.26	2.01	2.01
$\gamma = 0.5$	437,116	7.17	20.87	1.46	0.30	1.80	2.24	1.74	2.29	2.22	2.22

setting. Unfortunately, minor market frictions—changing margin constraints and changing differences in borrowing and lending rates—in a pure market-timing setting cause even the unconditional Henriksson–Merton model to confound selectivity with market-timing ability.

How do we resolve these problems? It is naïve to believe in a magic cure-all. Nonetheless, clear lessons emerge about what might and might not help alleviate the problems. Most measures of performance are characterized by statistical analyses of commonly accepted risk–return tradeoffs. Unfortunately, the analyses are based on arithmetic average returns. This fundamental flaw completely obscures the importance of bankruptcy. The investment industry has marched down this path for forty years. But the results in Exhibit 8 show that no amount of tinkering with measures of risk that go beyond standard deviation or beta, with conditional or unconditional risk measures, with conditional or unconditional risk-adjusted performance measures, or with the portfolio change measure is going to alleviate this fundamental problem.

The last thing the investment industry needs is another arithmetic mean-based performance measure that will only perpetuate the problem. If we go back to basics, however, and supplement analyses of investment performance based on the popular measures with analyses of accumulated wealth, compound returns, or continuously compounded returns—all of which recognize the fundamental importance of bankruptcy¹⁰—it would go a long way toward alleviating the problems associated with ranking the “very good” and the “very bad.” At the descriptive level, compound returns may be coupled with a simple measure

of risk, such as standard deviation, as is done in the financial press and in some academic papers, such as Grauer, Hakansson, and Shen [1990] and Grauer [2008] as well as other power utility-based portfolio selection papers. In addition, paired t -tests may be employed to test for the difference in the compound returns of two investment strategies (see, for example, Fama and MacBeth [1974] and Grauer, Hakansson, and Shen [1990]). At a minimum, the analysis of accumulated wealth, continuously compounded returns, or compound returns would guarantee that a bankrupt strategy would never be ranked ahead of a nonbankrupt, highly profitable, or perfect-foresight strategy.

ENDNOTES

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¹See Dybvig and Ross [1985], Green [1986], and Grauer [1991] for discussion of these points.

²With apologies to former WWE wrestling star Bret "The Hitman" Hart.

³Matrices and vectors are in bold type and scalars in italics. Vectors are column vectors and a prime indicates transposition (e.g., $\mathbf{x}'$ is the row vector corresponding to the column vector $\mathbf{x}$).

⁴A detailed description of the industry data can be found in Grauer, Hakansson, and Shen [1990].

⁵The returns on the SMB and HML portfolios are from Ken French's website.

⁶See, for example, "The Foresight Saga" in the *Economist*, December 18, 1999, pp. 59–61.

⁷I do not provide statistical tests based on either Sharpe or Treynor ratios. Lo [2002] cautioned that the statistical properties of Sharpe ratios depend intimately on the statistical properties of the return series on which they are based.

⁸F-tests indicate that Jensen betas and Fama–French slope coefficients (β_p , s_p and h_p) are functions of dy_{t-1} and $r_{L,t}$ in five of seven instances. However, in spite of what the statistics may indicate, beta is not really a function of the information variables. With perfect-foresight market timing without margin, beta is one in up markets and zero in down markets. The F-tests indicate that Jensen (Fama–French) alphas are functions of dy_{t-1} and $r_{L,t}$ only twice (once). This differs from passive portfolios that exhibit time varying alphas. See Ferson and Harvey [1999].

⁹The Sharpe ratio fares a little better. The Sharpe ratios in Panel B of Exhibit 8 exceed those in Panel A. But the Sharpe ratios in Panel A of Exhibit 8 are equal to each other, even though one portfolio accumulates positive wealth and four bankrupted.

¹⁰Compound returns and continuously compounded returns are not even defined for a time series of returns where there is a loss of 100% or more.

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