The Drift Factor in Biased Futures Index Pricing Models: A New Look

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The presence of bias in index futures prices has been investigated in various research studies. Redfield (1986) asserted that the U.S. Dollar Index (USDX) futures contract traded on the U.S. Cotton Exchange (now the FINEX division of the New York Board of Trade) could be systematically arbitraged for nontrivial returns because it is expressed in so-called "European terms" (foreign currency units/U.S. dollar). Eytan, Harpaz, and Krull (1988) (EHK) developed a theoretical factor using Brownian motion to correct for the European terms and the bias due to the USDX index being expressed as a geometric average. Harpaz, Krull, and Yagil (1990) empirically tested the EHK index. They used the historical volatility to proxy the EHK volatility specification. Since 1990, it has become more commonplace to use option-implied volatility for forecasting future

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volatility. Therefore, we have substituted option implied volatilities into EHK's correction factor and hypothesized that the correction factor is "better" *ex ante* and therefore should lead to better futures model pricing. We tested this conjecture using twelve contracts from 1995 through 1997 and found that the use of implied volatility did not improve the bias correction over the use of historical volatility. Furthermore, no matter which volatility specification we used, the model futures price appeared to be mis-specified. To investigate further, we added a simple naïve δ based on a modification of the adaptive expectations model. Repeating the tests using this naïve "drift" factor, it performed substantially better than the other two specifications. Our conclusion is that there may be a need to take a new look at the drift-factor specification currently in use. © 2002 Wiley Periodicals, Inc. Jrl Fut Mark 22:579–598, 2002

INTRODUCTION

It is important for all participants in a futures market to have an appropriate pricing model of the futures contract. With this in mind, the presence of bias in the USDX currency-index futures prices has been investigated in various research studies. Redfield (1986) asserted that the U.S. Dollar Index (USDX) futures contract traded on the Cotton Exchange (now the New York Board of Trade) systematically could be arbitraged for nontrivial returns because it is expressed in so called "European terms" (foreign currency units/U.S. dollar). Redfield provided a correction factor, called the "volatility multiplier," which Eytan, Harpaz, and Krull (1988), herein called EHK, felt was incomplete. Building upon previous work by Eytan and Harpaz (1986) on the Value Line Stock Index futures, EHK noted that the USDX needed a more comprehensive correction because, in addition to being in European terms, it also was averaged geometrically. Subsequently, Harpaz, Krull, and Yagil (1990), herein called HKY, conducted tests of efficiency of EHK's re-adjusted USDX futures model.

The correction factor specified by EHK, called the "drift factor" on the exchange floor, was designed to adjust the theoretical futures price to be within arbitrage-free bounds.[1] Our investigation of more recent data showed that the EHK's drift factor might be of lower magnitude than necessary to price the contract properly. To investigate this further, we proposed and tested three extensions of the existing literature. As the first two extensions, we provided new proxies for the drift factor on the USDX. The first proxy uses implied volatilities derived from foreign-exchange

[1]For those unfamiliar with the use of a drift factor, we provide in the Appendix a derivation of the USDX futures contract together with a summary showing how volatility factors serve to correct the futures contract.

options contracts and USDX futures options rather than historical volatilities, as employed in HKY's tests. For the second proxy, we formulated a drift factor based on a naïve adaptive-expectations model. Finally, a "rollover" test is presented and demonstrated that might be a more operationally convenient version of the expectations model. Our tests results suggest that while the implied-volatility correction does not improve the futures pricing, the adjusted adaptive-expectations volatility does. Finally, we tested a daily rollover strategy that was specified *ex ante*.

We begin by presenting the general model used in testing for implied profitability or "quasi-arbitrage" under the three specifications of the drift factor. We then show the results of testing these models and the rollover model before concluding the article.

METHODOLOGY

We provide several formal specifications for the drift factor that might serve to "correct" the theoretical USDX futures price. The specification of the drift factor, as well as the methodology for testing, begins with the method developed by HKY.

On each day in the life of a nearby USDX futures contract (the life span averages 71–97 days, with an average of 89), we set up a long–long position. This position has the index futures position as one leg while the other leg is the trade-weighted, arithmetic-averaged forward position on the ten currencies comprising the index.[2] Each position was held until maturity of the futures contract at time T. At that time, all the components of each position taken were liquidated. The index futures positions were liquidated at spot index, $I(T)$, and each of the various forward contracts, i, were liquidated at the terminal spot rate, $S_i(T)$. This strategy required an increasing number of contracts each day until maturity.

If, over the period of the position, the dollar strengthened, the gain in the futures position should be offset by the forward position losses. Likewise, if the dollar weakened, the futures loss should be offset by the forward profits. On the other hand, if the *ex-post* offset were incomplete, the position would result in a profit or loss. It is this potential profit or

[2]A long–long position is unusual for setting up a position. As the dollar strengthens, the USDX futures contract will rise and profit from the long position. Meanwhile, because the component currencies are specified in foreign terms, the forward positions in the various component currencies will lose, hopefully offsetting the USDX gain. We keep the long–long position throughout the testing period. HKY devised a method for switching from long–long to short–short positions in their tests depending on implied quasi-arbitrage opportunities. We feel that the constant position is appropriate because it produces the same results. That is, if the futures contract is priced correctly, there should be no implied profits or losses whether you use our constant position or HKY's shifting-position method.

loss that we call quasi-arbitrage because a perfect pricing model would produce a perfectly hedged, no-arbitrage position. Thus, consistent quasi-arbitrage profit (or loss) implies that the pricing model is misspecified. The objective of these tests was to determine the existence of and the degree of this mispricing, and to see if variations on the pricing model affect this degree of mispricing.

Measuring Quasi-Arbitrage Results

From HKY, the net profit or loss for period (t, T) is

$$\pi(t, T) = (I(T) - F_e(t, T)) + \sum_{i=1}^{10} w_i \frac{I(T)}{f_i(t, T)}(S_i(T) - f_i(t, T)) \quad (1)$$

Where:

$I(T)$ = spot USDX Index value at maturity of the futures contract T
$F_e(t, T)$ = model (theoretical) Index price at time t
w_i = trade weight for the respective currency i
f_i = forward rate for currency i adjusted for the time left until contract maturity (t, T)
$S_i(T)$ = spot price of currency i at contract maturity T.

$F_{e\,\mathrm{in}}$ (1) is specified as:

$$F_e(t, T) = I(t)e^{((\bar{r} - r_{US})_t + \delta_t)(T-t)} \quad (2)$$

where:

$I(t)$ = spot USDX Index at the time of investment, t
$(\bar{r} - r_{US})_t$ = difference between the weighted average annual foreign interest rates and the U.S. rate at the time of investment, t
$(T - t)$ = percentage of a year left until contract maturity
δ = drift or correction factor.

The Historical Drift Factor

HKY tested the EHK model using the historical variance of the various component currencies.

$$\delta_{historical} = \frac{\sum_{i=1}^{10} w_i \sigma^2_{hist,i} + \sigma^2_{USDXspot}}{2} \quad (3)$$

where w_i is the trade weight for the i currency, $\sigma^2_{hist,i}$ is the annualized squared volatility (derived from historical variances data) of the percent movement in spot currency i, and $\sigma^2_{USDXspot}$ is the annualized historical variance volatility of the percent movement in the USDX spot index. HKY went back 40 business days to develop the historical volatilities.

Option Volatility Drift Factor

As we will show later, our tests of the historical volatility drift factor suggest that it might be somewhat understated. We attempted to improve upon the historical drift factor by specifying the drift factor as an implied *option* volatility rather than measured from historical observations. Previous research (see the literature review provided by Mayhew, 1995) has found that implied option volatility surpasses historical variances in forecasting realized volatility on common stock returns. It therefore seemed reasonable to see if it could improve the forecasting ability of the EHK model.

The option drift factor δ is specified as:

$$\delta_{option} = \frac{\sum_{i=1}^{10} w_i \sigma^2_{opt,i} + \sigma^2_{USDXoption}}{2} \tag{4}$$

which is similar to (3) except for the substitution of option implied volatilities.

Adaptive Expectations Drift Factor

To gain insight into the functioning of these models, we adopted a naïve adaptive expectations for comparison.[3] In essence, δ_{adexp}, is the adaptive-expectations-generated expected level of δ for use in setting up today's trading position. We computed δ_{adexp} by iteration to find a value that forces the profit in eq (1), at the end of yesterday's trading day, to be equal zero. Then this *correct* δ is used in valuing F_e for today's trade using today's spot index and interest rates. In effect, the correct δ is being relearned each day, which is the essence of the adaptive expectations model.

Specifically, we altered (2) for yesterday, $t - 1$, as

$$F_e(t-1, T) = I(t-1)e^{((\bar{r} - r_{US})_{t-1} + \delta_{ad\,exp,t-1})[T-(t-1)]} \tag{5}$$

[3]The adaptive expectations specification for δ used herein is a modified form of the adaptative expectations model provided on page 234 of Pyndick and Rubinfeld (1981).

The left side of (5) is substituted into (1) for $t - 1$ as:

$$0 = (I(T) - F_e(t - 1, T)) + \sum_{i=1}^{10} w_i \frac{I(T)}{f_i(t - 1, T)} \times (S_i(T) - f_i(t - 1, T)) \tag{6}$$

Thus, for yesterday, $t - 1$, after the end of trading, an $\delta_{adexp,t-1}$ in (5) is found *ex post*, which reprices the futures index F_e to a value that, when substituted into (6), forces the profit for yesterday to 0. In effect, (5) and (6) are being determined simultaneously where $\delta_{adexp,t-1}$ is adjusted until the first and second terms in (6) offset each other. This *correct* δ for yesterday, $\delta_{adexp,t-1}$, is used in today's trade at time t by computing today's model index value

$$F_{adexp}(t, T) = I(t)e^{((\bar{r}-r_{US})_t + \delta_{adexp,t-1})[T-t]} \tag{7}$$

Equation (1) for today's trading, t, then is modified as follows for use in adaptive expectations.

$$\pi(t, T) = (I(T) - F_{adexp}(t, T)) + \sum_{i=1}^{10} w_i \frac{I(T)}{f_i(t, T)} (S_i(T) - f_i(t, T)) \tag{8}$$

where F_{adexp} is substituted for F_e in eq (1).

Rollover Testing Model

Actually, eq (8) for the adaptive expectations model is not strictly *ex ante* because $I(T)$ and $S_i(T)$, which occur at contract maturity, cannot be known ahead of time. In addition, as a practical matter, the buildup of margin requirements that accumulate from each day's set up of a position might be somewhat onerous, involving margin payments of \$1596 each day for 50–60 days per contract.[4] Therefore, we have provided rollover modifications to eq (1) and (8). The rollover testing model closes out positions daily, and then new positions are repositioned, requiring only the initial margin amount to be paid out for each contract. The rollover model for adaptive expectations is *ex ante* because the δ correction is occurring each day contemporaneously with the theoretical value for the USDX futures contract.

[4]As of January, 2001, the commodity discount broker, Lind-Waldock, required an original margin of \$1596 per contract to trade the USDX futures contract.

Rollover Model

The rollover model involves respecifying (1) for both historical and option volatilities and adaptive expectations. This forms the basis for comparison of the three methods of adjusting for the drift factor. The rollover model we are proposing for historical and option volatilities from time $t - 1$ to time t involves substituting $F_e(t, T)$ for $I(T)$ in eq (1) and $f_i(t, T)$ for $S(T)$ in eq (1). Equation (1) now becomes eq (9):

$$\pi(t-1, t) = (F_e(t, T) - F_e(t-1, T)) + \sum_{i=1}^{10} w_i \frac{F_e(t-1, T)}{f_i(t-1, T)} (f_i(t, T) - f_i(t-1, T)) \tag{9}$$

Thus, the daily rollover profit, π, from time $t - 1$ to t arises from any difference between the futures leg rollover from $F_e(t-1)$ to $F_e(t)$ and the summation of the forward leg rollovers from $f_i(t-1)$ to $f_i(t)$ for currencies i.

For the adaptive expectations rollover, we substituted in $F_{adexp}(t, T)$ for $I(T)$ in eq (1) and $f_i(t, T)$ for $S(T)$ in eq (1).

$$\pi(t-1, t) = (F_{adexp}(t, T) - F_{adexp}(t-1, T)) + \sum_{i=1}^{10} w_i \frac{F_{adexp}(t-1, T)}{f_i(t-1, T)} (f_i(t, T) - f_i(t-1, T)) \tag{10}$$

DATA

For this study, we used annualized, daily, nearest at-the-money implied option volatilities derived from the closing-price option premium using the Black Model, as computed by the Philadelphia Options Exchange (PHLX). We used daily data for the years 1995 through 1997 on PHLX volatilities for the following option contracts: DM, ¥, Ffr, £, CAN$, SWISSfr. Because no options are written on the remaining currencies in the ten-currency USDX index, we computed historical variances on the spot rate for Lire, Guilder, BELGfr, and SWEDkr for the previous forty calendar-day period, in the same manner as that employed by HKY. Because the optioned currencies comprised 72.1% of the total trade weighting of the USDX, we expected that the use of option volatilities would improve the specification of the drift factor, and thus the model price of the futures contract.

We also used the daily option volatility as implied for the USDX futures option premium as of 3:00 pm, as published by FINEX. The

USDX futures prices were as of 3:00 pm, as published by FINEX. Daily spot and forward exchange rates were as of 5:30 pm from the Bloomberg Composite. Daily interest rates were the Bloomberg generic 91-day treasury bill as of 5:15 pm. All prices and rates were at the midpoint between bid and ask.

In addition to the availability of data, there needs to be sufficient liquidity in the contract. Clyman, Allen, and Jaycobs (1997) demonstrated that the USDX contract had sufficient liquidity for market participants to put on their positions and trade. As further evidence of liquidity, an official at the FINEX indicated that the bid–ask spread on the USDX futures contract and the bank forward contracts was about 5 basis points, which was sufficient to demonstrate adequate liquidity.

The fact that the data are nonsynchronous might be a problem. This nonsynchronicity could be producing the results that suggest that the currently traded USDX futures contract is still priced inefficiently. It is true that nonsynchronicity can account for non-zero profit/loss on the positions established, and thus apparent quasi-arbitrage. In our tests, each specification used all of the same data at exactly the same times. The only difference between specifications was the number of contracts bought. Therefore, only a consistent bias produced by nonsynchronicity would show one specification to be different from another. To see if there was a consistent bias in our results, we replicated all of our tests, but aligned the spot, forward, and interest rates to the next day's options and futures data. Thus, the yesterday's 5:15 and 5:30 quotes were aligned with today's 3:00 pm quotes. If there were a systematic bias between 3:00 pm and 5:15, then it should be reversed in this realignment. Because of this nonsynchronous data, we describe our results as quasi-arbitrage profits.

RESULTS

Table I shows the various averaged drift factor δs for each contract period. Note that the adaptive expectations δs in column 4 are larger. If they produce smaller profits *ex post*, this suggests that a larger drift factor was needed than that provided by the historical variance drift factor used by HKY. In addition, most of the δs in column 4 are negative. Negative δs were used to reduce the value of the uncorrected futures contract when the futures leg were larger than the forward leg. This was counter to the theory that showed that the drift factor had to be positive.

Table II shows the results on tests conducted in a manner similar to that of HKY. Notice in column 3 that the nontrivial quasi-arbitrage

TABLE I

The Average of the Computed δs per Contract Life for Each Volatility Specification

Contract Expiration	*Historical Volatility Correction δ_{hist}*	*Option Volatility Correction δ_{opt}*	*Adaptive Expectations Volatility Correction δ_{adexp}*
(1)	(2)	(3)	(4)
15-Mar-1995	0.0084	0.0151	−0.5244
21-Jun-1995	0.0181	0.0186	−0.0822
20-Sep-1995	0.0106	0.0144	−0.1361
18-Dec-1995	0.0082	0.0119	−0.0441
20-Mar-1996	0.0036	0.0082	0.0189
19-Jun-1996	0.0029	0.0048	0.0362
18-Sep-1996	0.0034	0.0040	−0.0681
18-Dec-1996	0.0027	0.0041	0.0149
19-Mar-1997	0.0055	0.0077	−0.0025
18-Jun-1997	0.0067	0.0065	0.0843
17-Sep-1997	0.0080	0.0080	0.0105
17-Dec-1997	0.0083	0.0086	0.0864

Note. Columns 2 and 3 are generated using eqs (3) and (4); column 4 uses eq (5).

TABLE II

Adding π for Daily Trades That Cumulate and Terminate at Time $I(T)$

Contract Expiration	*Days in Contract*	*Actual USDX*	*Uncorrected USDX*	*Historical Volatility Corrected*	*Option Volatility Corrected*	*Adaptive Expectations Volatility Corrected*
(1)	(2)	(3)	(4)	(5)	(6)	(7)
15-Mar-1995	51	−9,420	−8,720	−8,950	−9,150	−1,410
21-Jun-1995	67	−30,840	−22,900	−35,960	−37,250	−1,520
20-Sep-1995	62	−51,170	−42,760	−50,550	−52,860	−2,780
18-Dec-1995	62	−19,070	−11,310	−17,780	−19,890	−1,100
20-Mar-1996	60	−5,150	620	−1,710	−5,210	−170
19-Jun-1996	62	1,180	6,820	4,920	3,250	110
18-Sep-1996	57	−14,970	−12,190	−14,340	−14,860	−930
18-Dec-1996	61	2,480	5,250	3,740	2,720	160
19-Mar-1997	59	−16,810	−11,110	−14,580	−16,020	−420
18-Jun-1997	61	19,390	23,110	18,510	18,210	1,120
17-Sep-1997	58	−8,030	−3,330	−8,400	−8,330	−80
17-Dec-1997	60	23,360	27,331	20,724	20,645	1,310

Note. Columns 3, 4, 5, and 6 use eq (1); column 7 uses eq (8); (π in dollars).
Notice that column 4 uses eq (3) with a δ set = 0 as input into eq (1).

profits for the currently traded (actual) USDX futures contract. Actual futures contract prices appeared to produce no less quasi-arbitrage profits than the uncorrected theoretical futures contracts in column 4. This suggests that either the drift factors currently employed in the markets are not fully offsetting the two legs of the positions or the USDX futures contract is no longer efficiently priced.

Column 5 shows the quasi-arbitrage results from using historical delta. The general contract results are no better than dealing with an uncorrected theoretical futures contract in column 4. Thus, the use of historical variances by HKY as a proxy for the drift factor might not produce a properly priced futures contract, at least in the current environment.

Our first attempt to provide an improved drift factor appears to have fared no better. The option drift factor δ correction results in column 6 do not dominate those previously reported. This is shown in Figure 1, where the historical and option quasi-arbitrage profits superimpose over time. There could be several reasons for this. Because approximately 28% of the option correction is still comprised of historical drift factors (not all of the currencies have options trading on them), the option might not be providing sufficiently new content to improve results over using only historical volatilities into their models. In addition, it is possible that traders of individual currency options are factoring in historical variances into the Black model, using these results to trade, thus the

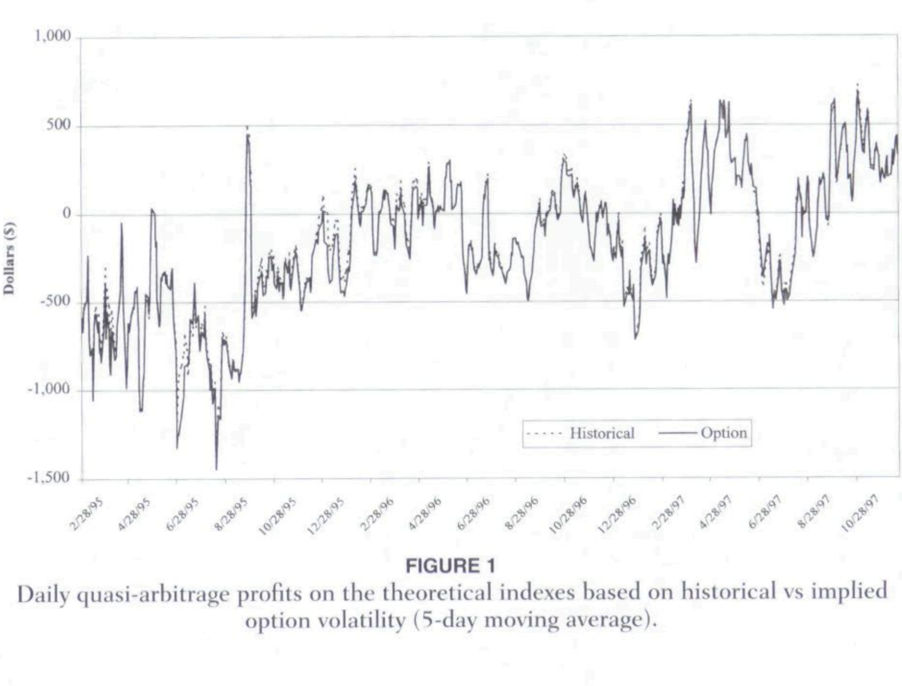

FIGURE 1

Daily quasi-arbitrage profits on the theoretical indexes based on historical vs implied option volatility (5-day moving average).

implied variances we are backing out closely resemble the historical volatility.

Actually, while option volatilities on common stock are useful as predictors of future volatility of common stock, such cannot be said for currency volatilities. Jorion (1995) found that while individual currency volatilities are statistically significant variables in explaining future variability, they are rather poor predictors of future volatility because they can explain only 10 to 15% of distant volatility in terms of R^2. Furthermore, individual currency predictions show upward bias. Wei and Frankel (1991) found that currency option volatilities for one-month contracts were not significantly related to realized volatility in the ensuing one-month period, although they can predict the direction of change. Campa and Chang (1995), studying the term structure of currency volatilities, found that the spread between long-term and short-term volatility did predict at best only the direction of change in volatility of future short-term options. Therefore, we should not be surprised that the option volatility does not improve results.

Column 7 of Table II reports the results using the adaptive expectations drift factor. These results are interesting. The quasi-arbitrage profits from using the adaptive expectations model appear to be greatly reduced. This is shown in Figure 2, which compares quasi-arbitrage profits using adaptive expectations volatility adjusted δs versus those adjusted using historical volatilities.

FIGURE 2
Daily quasi-arbitrage profits on the theoretical indexes based on historical volatility correction vs adaptive expectations (5-day moving average).

TABLE III

Significance of Quasi-Arbitrage Profits Expressed as a *P*-Value Probability from the *t*-Test That Each Value in Table II Has a Mean of 0

Contract Expiration	*Actual USDX*	*Uncorrected USDX*	*Historical Volatility Corrected*	*Option Volatility Corrected*	*Adaptive Expectations Volatility Corrected*
(1)	*(2)*	*(3)*	*(4)*	*(5)*	*(6)*
15-Mar-1995	0.30	0.35	0.33	0.32	0.93
21-Jun-1995	0.42	0.53	0.33	0.32	0.97
20-Sep-1995	0.04	0.09	0.04	0.04	0.92
18-Dec-1995	0.51	0.70	0.54	0.48	0.97
20-Mar-1996	0.76	0.97	0.91	0.76	0.99
19-Jun-1996	0.94	0.66	0.76	0.84	1.00
18-Sep-1996	0.36	0.44	0.36	0.34	0.96
18-Dec-1996	0.87	0.72	0.80	0.85	0.99
19-Mar-1997	0.45	0.60	0.50	0.46	0.99
18-Jun-1997	0.43	0.33	0.44	0.45	0.97
17-Sep-1997	0.74	0.88	0.71	0.72	1.00
17-Dec-1997	0.28	0.20	0.32	0.32	0.96

Table III reports the statistical significance of the quasi-arbitrage profits as p values. Generally, the adaptive expectations δ implied profits have a higher probability of being zero than those provided by other δs tested.

Table IV shows the economic importance of the results using holding-period returns for the various drift factors.[5] Note, for example, that the 51-day return on the March 1995 contract is 12%. This 85% on an annualized basis. Clearly, the returns are nontrivial for all volatility corrections except for adaptive expectations where the maximum return is below 1%. This is further evidence that these models are mis-specified.

Table V shows the results of the daily rollover model computed for the various specifications. Notice that the naïve adaptive expectations produces zero quasi-arbitrage profits. This is precisely what we expected because the position legs are being adjusted daily by the simultaneous adjustment of δ, such that the theoretical USDX index should not exhibit systematic mispricing. Of course, this does not rule out random noise and other inefficiencies in the actual trading markets, but the simultaneous correction does rule out systematic structural imbalances in the theoretical futures contract due its components being stated in foreign terms and geometrically averaged.[6]

[5]We are grateful to Scott Hein for the suggestion of this table.

[6]See the Appendix for a discussion of the bias caused by these characteristics.

TABLE IV

Holding Period Returns per Contract Considering a Daily Margin Requirement of $1596

Contract Expiration	*Days Holding Period*	*Actual USDX*	*Uncorrected USDX*	*Historical Volatility Corrected*	*Option Volatility Corrected*	*Adaptive Expectations Volatility Corrected*
(1)	*(2)*	*(3)*	*(4)*	*(5)*	*(6)*	*(7)*
15-Mar-1995	51	−12%	−11%	−11%	−11%	−2%
21-Jun-1995	67	−29%	−21%	−34%	−35%	−1%
20-Sep-1995	62	−52%	−43%	−51%	−53%	−3%
18-Dec-1995	62	−19%	−11%	−18%	−20%	−1%
20-Mar-1996	60	−5%	0.6%	−2%	−5%	−0.2%
19-Jun-1996	62	1%	7%	5%	3%	0.1%
18-Sep-1996	57	−16%	−17%	−16%	−16%	−1%
18-Dec-1996	61	3%	5%	4%	3%	0.2%
19-Mar-1997	59	−18%	−12%	−16%	−17%	−0.44%
18-Jun-1997	61	20%	24%	16%	19%	1%
17-Sep-1997	58	−9%	−4%	−9%	−9%	0%
17-Dec-1997	60	24%	28%	22%	22%	1%

Note. For example, the actual USDX holding period return for the March 1995 contract is cumulative profits of $9420 from Table II divided by $1596 (margin requirement) × 51 trading days = 11.6% (rounded to 12% in the Table above).

TABLE V

One-Day Rollover Quasi-Arbitrage Profits Added Up per Contract Period

Contract Expiration	*Days in Contract*	*Actual USDX*	*Uncorrected USDX*	*Historical Volatility Corrected*	*Option Volatility Corrected*	*Adaptive Expectations Volatility Corrected*
(1)	*(2)*	*(3)*	*(4)*	*(5)*	*(6)*	*(7)*
15-Mar-1995	51	183	−359	−331	−345	0
21-Jun-1995	67	−601	−595	−920	−1,011	0
20-Sep-1995	62	−1,107	−1,384	−1,789	−1,870	0
18-Dec-1995	62	2,549	2,793	2,611	2,483	0
20-Mar-1996	60	−327	−147	−237	−35	0
19-Jun-1996	62	−265	−186	−250	−323	0
18-Sep-1996	57	−644	−550	−619	−631	0
18-Dec-1996	61	104	191	142	117	0
19-Mar-1997	59	−424	−404	−519	−546	0
18-Jun-1997	61	−575	22	−115	−166	0
17-Sep-1997	58	8	165	−25	32	0
17-Dec-1997	60	−44	236	−43	3	0

Note. Columns 3, 4, 5, and 6 use eq (9); column 7 uses eq (10); (in dollars).

Next, we replicated all of the tests re-aligning the data to test for systematic bias produced by nonsynchronicity; our results did not substantially change.[7] Thus, we concluded that while the magnitude of quasi-arbitrage profits we show might not exactly occur in practice, the size of the *reduction* in quasi-arbitrage profits should occur at least approximately.

CONCLUSION

While option-adjusted drift factors are no improvement over the traditional historical variance δ, the use of adaptive expectations δs shows promise. This should make sense since the naïve adaptive expectations model generates the previous day's corrected δs by forcing the hedge legs to be equal, thus producing no quasi-arbitrage profits after the fact. By taking this "learned," "corrected" δ as the expected δ for the next day's trade, we feel we might be closer to the "true" δ.

APPENDIX

The following is provided for those unfamiliar with the USDX contract and the need for correction.

The USDX Futures Contract

The USDX spot is an index of ten currencies. Each component is scaled by a base exchange rate in existence in March 1973, and the ten currencies are geometric averaged by a trade weight related to the relative importance of each country's foreign trade is with the US. Redfield (1986) showed the specification of the USDX futures contract as:

$$\text{USDX Futures} = 100e^{(\sum_{i=1}^{10} w_i \ln(B_i/C_{it})}$$

where, for currency i:

w_i = the trade weight
B_i = base exchange rate
C_{it} = is the forward rate at time t (current)

Relevant numbers are shown in Table A1.

[7]Results are available from the authors upon request.

TABLE A1
Relevant Numbers for Ten Currencies

Currency (FX/$)	*Trade Weight*	*Base Weight (FX/$)*
DM	0.208	2.8131
¥	0.136	261.8487
Ffr	0.131	4.5063
£	0.119	0.4045
CAN$	0.091	0.9967
Lire	0.090	568.1818
Guilder	0.083	2.8708
BELGfr	0.064	39.4058
SWEDkr	0.042	4.4283
SWISSfr	0.036	3.2171

The Drift Factor

A correction factor, δ, is necessary to reprice the USDX futures contract to avoid systematic bias because it is expressed in foreign terms (foreign currency units/$) and as a geometric average. First, we show the difference between an index expressed in foreign terms and one expressed in American terms. Second, we explain the effects of geometric averaging. Finally, we show how the correction factor causes the quasi-arbitrage profits to disappear.

Foreign Terms

To illustrate the problem of foreign terms, consider a hypothetical USDX index future consisting of one currency, the German DM, quoted at a fictional forward rate of DM 3/$:

$$USDX_{HYPOTHETICAL} = \left[\frac{\text{DM } 3}{\text{DM } 2.8131}\right]^{1.0} \times 100 = 106.64$$

where 2.8131 is the base rate from the previous page. According to the conventions of the contract, the 106.64 is multiplied by $1000 to come to a delivery value of $106,640 index BUY.

To set up a potential arbitrage position, concurrent long positions in the index and the DM forward (a long–long position) are taken.[8] The

[8]It can be shown that a short–short position would produce the same result. In an arbitrage position, one would normally expect the long leg to be offset by a short position. However, because of the way the index is stated, if one expects the DM to fall, the numerator becomes larger, causing the index value to rise. Hence, the index leg is long to make money. With the offsetting DM forward contract also being long, the forward loses money. Such an arbitrage position also would provide profits should the DM rise because the index long position loses while the forward DM long makes money. The arbitrage profit is riskless regardless of the currency movement. The investment is trivial, requiring only a small margin on the future and nothing on the forward contact.

DM value of the forward contract is:

$$\frac{\text{DM3}}{1\$} \times \frac{X}{\$106{,}640}; \quad X = \text{DM } 319{,}920$$

where the DM 319,920 is equal to \$106,640 forward-contract BUY.

If the DM forward drops to DM 3.3/\$, the index makes money and the forward DM loses, but the offsets are not exact. The index after the change is:

$$\left[\frac{\text{DM } 3.3}{\text{DM } 2.8131}\right]^{1.0} \times 100 = 117.31 \times \$1000 = \$117{,}308 \text{ SELL}$$

The index gain is:

$$\begin{aligned} &\$117{,}308 \text{ SELL} \\ &\$106{,}640 \text{ BUY} \\ = {}&\$\ \ 10{,}668 \text{ PROFIT} \end{aligned}$$

If the DM forward rate drops to DM 3.3/\$, the value of the forward contract now becomes:

$$\frac{\text{DM } 3.3}{1\$} \times \frac{\text{DM } 319{,}920}{X}; \quad X = \$96{,}945 \text{ SELL}$$

Thus, the forward deficit is:

$$\begin{aligned} &\$\ \ 96{,}945 \text{ SELL} \\ &\$106{,}640 \text{ BUY} \\ = {}&\$\ \ \ \ 9695 \text{ LOSS} \end{aligned}$$

On net, the \$10,668 index profit less the \$9695 loss produces a \$973 arb profit.

American Terms

Note from the above that the USDX contract is expressed in foreign terms, which produce the non-zero profit. That is, when the U.S. dollar rises, the futures index rises. What if the contract was expressed in "American" terms? In this case, the index falls when the home currency rises. For the USDX index, consider the same single currency index position reflecting such a quotation. The arb position will be short–long.[9]

[9] A long–short position would produce the same outcome.

The initial index position is (note the American terms that are the reciprocal of the European terms):

$$\text{USDX} = \frac{\$0.3333/\text{DM}}{\$0.355479\ \text{DM}} \times 100 = 93.76 \times \$1000 = \$93{,}760\ \text{SELL}$$

The long forward is

$$\frac{\$0.3333/\text{DM}}{\text{DM}} = \frac{\$93{,}760}{X}; \quad X = \text{DM}\ 281{,}308$$

Now assume that the DM drops to \$0.30303/DM (which is the reciprocal of DM 3.3/\$). The index now becomes:

$$\frac{\$0.30303/\text{DM}}{\$0.355479/\text{DM}} \times 100 = 85.25$$

The new value of the index is 85.25 × \$1000 = \$85,250 BUY. The profit on the index position is:

$$\begin{aligned} &\$93{,}760\ \text{SELL} \\ &\$85{,}250\ \text{BUY} \\ = \$\ \ &8510\ \text{PROFIT} \end{aligned}$$

The long forward has moved to:

$$\frac{\$0.30303}{\text{DM}} = \frac{X}{\text{DM}\ 281{,}308}; \quad X = \$85{,}245\ \text{SELL}$$

The loss on the shorted forward is

$$\begin{aligned} &\$85{,}245\ \text{SELL} \\ &\$93{,}760\ \text{BUY} \\ = \$\ \ &8515\ \text{LOSS} \end{aligned}$$

which nearly equals the loss on the index of \$8510 (except for rounding).

Thus, when the index is expressed in American terms, no practical arbitrage is possible and there is no need for adjustment.

Geometric Averaging

As noted above, in valuing the USDX futures contract, the use of forward rates is standardized by base weights. This produces a ratio for each of the ten component currencies that make up the USDX index. These currency ratios must be averaged. The method of geometric averaging is especially adapted for averaging ratios and percentages; as a result, it is used with the USDX index.

Geometric means are always smaller than arithmetic averages. This always makes the USDX futures contract value smaller than the respective forward rates because the forward rates are averaged arithmetically in creating a potential arbitrage position.[10] This imbalance provides the opportunity for systematically arbitraging the uncorrected difference between the futures price and the individual forward rates (if the contract actually were priced according to the pricing model).

A Correction Factor Example

The following illustrates how a simple ad-hoc correction factor (provided *ex post*) equalizes the sides of an arbitrage position when the index is in foreign terms and geometric averaged.

The USDX Index

Consider a futures index with two currencies, the DM comprising 60% and the Belgian franc comprising 40% of the index. The Belgian franc starting forward rate is 47/\$. Using the same numbers, the initial value of the index is:[11]

$$\left(\frac{\text{DM } 3/\$}{\text{DM } 2.8131}\right)^{.60} \times \left(\frac{\text{BELGfr } 47/\$}{\text{BELGfr } 39.4058}\right)^{.40} \times 100 = 111.53$$

If we "corrected" the 111.53 by a factor of 1.01075176, it becomes 112.73 BUY.

Now, if the DM drops to DM 3.3/\$ and the BELGfr drops to BELGfr 53.41053/\$, the index becomes:

$$\left(\frac{\text{DM } 3.3/\$}{\text{DM } 2.8131}\right)^{.60} \times \left(\frac{\text{BELGfr } 53.4105/\$}{\text{BELGfr } 39.4058}\right)^{.40} \times 100 = 124.29 \text{ SELL}$$

The profit on the index is:

$$(124.29 - 112.73) \times \$1000 = \$11{,}560 \text{ PROFIT}$$

[10]As we see in the article's empirical results, the combination of foreign terms with geometric means can produce an index that is either underpriced or overpriced.

[11]Notice that the two-currency index is expressed in foreign terms and is averaged geometrically.

The Deutschmark Leg

The forward legs are also long positions. The initial total investment is 112.73 × \$1000 × 60% = \$67,638 BUY invested in DM forwards. In DM terms, this is:

$$\frac{\text{DM } 3}{\$} = \frac{X}{\$67{,}638}; \quad X = \text{DM } 202{,}914$$

If the DM drops to DM 3.3/\$, its forward rate is:

$$\frac{\text{DM } 3.3}{\$} = \frac{\text{DM } 202{,}914}{X}; \quad X = \$61{,}489 \text{ SELL}$$

Loss on the DM forward is:

$$\begin{aligned} &\$61{,}489 \text{ SELL} \\ &\$67{,}638 \text{ BUY} \\ = &\$\quad 6149 \text{ LOSS} \end{aligned}$$

The Belgian Franc

The BELGfr had an initial value of 112.73 × \$1000 × 0.40 = \$45,092 BUY. The BELGfr value of this is:

$$\frac{\text{BELGfr } 47}{\$} = \frac{X}{\$45{,}092}; \quad X = \text{BELGfr } 2{,}119{,}324$$

If the BELGfr drops to BELGfr 53.4105, the value of the forward is:

$$\frac{\text{BELGfr } 53.4105}{\$} = \frac{\text{BELGfr } 2{,}119{,}324}{X}; \quad X = \$39{,}680 \text{ SELL}$$

The BELGfr forward loss is:

$$\begin{aligned} &\$39{,}680 \text{ SELL} \\ &\$45{,}092 \text{ BUY} \\ = &\$\quad 5412 \text{ LOSS} \end{aligned}$$

The net position is:

$$\begin{aligned} &\$11{,}560 \text{ USDX GAIN} \\ &\$\quad 6149 \text{ DM LOSS} \\ &\$\quad 5412 \text{ BELGfr LOSS} \\ = &\$\qquad 1 \text{ NET GAIN} \end{aligned}$$

Thus, the correction of 1.01075176 worked.

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