A Note on the Relationships Between Some Risk-Adjusted Performance Measures

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Assuming portfolio returns are normally distributed, it is shown that both Sortino ratio (SR) and upside potential ratio (UPR) are monotonically increasing functions of the Sharpe ratio. As a result, all three risk-adjusted performance measures provide identical ranking among investment alternatives. The effects of skewness and kurtosis are then evaluated within the Edgeworth-Sargan density family. For the Sortino ratio, the above conclusion remains valid in the presence of negative skewness or excessive kurtosis. Similar results apply to the UPR with modifications. For all other cases, both SR and UPR provide exactly opposite ranking among investment alternatives to that suggested by the Sharpe ratio when the Sharpe ratio is large. Applications to futures hedging are discussed. Specifically, it is found that the Sharpe ratio may frequently lead to a smaller futures position than the other two ratios. © 2002 Wiley Periodicals, Inc. Jrl Fut Mark 22:483–495, 2002

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INTRODUCTION

Despite some well-known limitations, the Sharpe ratio (i.e., the excess return per standard deviation) has been widely adopted as a performance measure to evaluate and select investment alternatives. Dissatisfaction with the variance as a risk measure coupled with other behavioral evidence has led some researchers to propose alternative risk-adjusted performance measures. This note examines two alternative measures of investment performance—the Sortino ratio and the upside potential ratio—and investigates whether the difference in performance reported by Plantinga, van der Meer, and Sortino (2001) is attributable to the skewness of the return distribution. Specifically, this note examines the relationships between the three ratios in terms of portfolio evaluation under a normality assumption and an Edgeworth-Sargan density assumption. The implications of the results for a futures hedging problem are discussed.

Variance is a two-sided measure, implying the individual dislikes any deviation from the mean regardless of the direction of the deviation. This is hardly the notion of risk perceived by an individual. In a recent survey, Adams and Montesi (1995) found that corporate managers are mostly concerned with the occurrence of bad outcomes compared to a reference point referred to as the "downside risk." Similar evidence was provided in Sortino and Price (1994). Indeed, the prospect theory of Kahneman and Tversky (1979) suggested that an individual weighs losses much more than gains. Empirical evidence of the so-called "loss aversion" has been established in recent literature; for example, see Benartzi and Thaler (1995), Thaler, Tversky, Kahneman, and Schwartz (1997), and Ordean (1998).

The Sortino ratio constructs a risk-adjusted performance measure by replacing the standard deviation with the downside risk measure. Sortino, van der Meer, and Plantinga (1999) further suggest that the return should be replaced with the upside potential. In fact, following the prospect theory of Kahneman and Tversky (1979), an individual is risk averse when the return exceeds the reference point but risk seeking otherwise. For a risk-return tradeoff to take place, we should consider only the upside potential. The ratio of the upside potential to the downside risk is termed as the "upside potential ratio."

Plantinga, van der Meer, and Sortino (2001) apply the Sharpe ratio, the Sortino ratio, and the upside potential ratio to evaluate mutual fund performance. They demonstrate that the upside potential ratio is a better measure than the Sharpe ratio. Moreover, they attribute the difference to the skewness of the return distribution. This note attempts to investigate the above conjecture. Specifically, I examine the relationships between the three ratios under a normality assumption. I then extend the analysis to the Edgeworth-Sragan density family to evaluate the effects of skewness and excessive kurtosis, two properties commonly found in financial returns. Finally, I discuss the implications of the results for the futures hedging problem.

SHARPE RATIO, SORTINO RATIO, AND UPSIDE POTENTIAL RATIO

Let X denote the portfolio return with a probability density function $f(\cdot)$. Denote the mean of X by μ and denote the standard deviation of X by σ. The Sharpe ratio is defined as the excess return over the standard deviation of X. That is,

$$Sh = (\mu - r)/\sigma \tag{1}$$

where r is an appropriate return rate. In this note, r is taken to be the minimally acceptable rate. The Sortino ratio replaces the standard deviation with the downside risk measure δ, where

$$\delta^2 = \int_{-\infty}^{r} (r - x)^2 f(x)\,dx \tag{2}$$

Consequently, the Sortino ratio is

$$So = (\mu - r)/\delta \tag{3}$$

Suppose that X is normally distributed. Let $\phi(\cdot)$ denote the probability density function of a standard normal random variable and let $\Phi(\cdot)$ denote the corresponding distribution function. It can be shown that

$$\delta^2 = (r - \mu)\sigma\phi\left(\frac{r - \mu}{\sigma}\right) + [(r - \mu)^2 + \sigma^2]\Phi\left(\frac{r - \mu}{\sigma}\right) \tag{4}$$

Define $z = (r - \mu)/\sigma$, i.e., the negative Sharpe ratio. The Sortino ratio can be written as follows:

$$So = \frac{-z}{\sqrt{z\phi(z) + (z^2 + 1)\Phi(z)}} \tag{5}$$

Upon taking derivative of the Sortino ratio with respect to z, we have

$$\frac{d(So)}{dz} = -m(z)\Phi(z) < 0 \tag{6}$$

where $m(z) = [z\phi(z) + (z^2 + 1)\Phi(z)]^{-3/2}$. That is, the Sortino ratio increases with increasing Sharpe ratio (i.e., decreasing z). Consequently, the two ratios rank investment portfolios identically and choose the same optimal portfolio.

To further explore the above result, define adjusted downside risk as follows: $\Delta = \delta/\sigma$. Then

$$\frac{d\Delta}{dz} = (1/2\Delta)(\phi(z) + z\Phi(z)) > 0 \tag{7}$$

The adjusted downside risk decreases as the Sharpe ratio increases. Because the Sortino ratio is defined as the Sharpe ratio divided by the adjusted downside risk, it increases as the Sharpe ratio increases.

The upside potential ratio, advocated by Sortino et al. (1999), refines the Sortino ratio by replacing the excess return with the upside potential, θ, defined as follows:

$$\theta = \int_r^\infty (x - r)\, f(x)\, dx \tag{8}$$

Consequently, the upside potential ratio (UPR) can be written as:

$$UPR = \theta/\delta \tag{9}$$

Given the normality assumption, we can derive the following:

$$\theta = \sigma\left[\phi\left(\frac{r-\mu}{\sigma}\right) - \left(\frac{r-\mu}{\sigma}\right)\Phi\left(-\frac{r-\mu}{\sigma}\right)\right] \tag{10}$$

Use the previous notation $z = (r - \mu)/\sigma$, from equations (4) and (10) we have

$$UPR = \frac{\phi(z) - z\Phi(-z)}{\sqrt{z\phi(z) + (z^2 + 1)\Phi(z)}} \tag{11}$$

The first derivative of the upside potential ratio with respect to z is

$$\frac{d(UPR)}{dz} = -m(z)\{[z\phi(z) + \Phi(-z)]\Phi(z) + \phi^2(z)\} < 0 \tag{12}$$

because $z\phi(z) + \Phi(-z)$ is always positive. That is, the upside potential ratio increases as the Sharpe ratio increases. Once again, the two ratios rank investment portfolios identically and choose the same optimal portfolio.

The result can be explained as follows. Define the adjusted upside potential as follows: $\Theta = \theta/\sigma$. Then

$$\frac{d\Theta}{dz} = -\Phi(-z) < 0 \tag{13}$$

The adjusted upside potential is an increasing function of the Sharpe ratio. Equation (7) indicates that the adjusted downside risk is a decreasing function of the Sharpe ratio. By definition, the upside potential ratio equals to the ratio of the adjusted upside potential to the adjusted downside risk, and therefore is an increasing function of the Sharpe ratio.

Theorem 1 Suppose that all the portfolios are normally distributed. The Sharpe ratio, the Sortino ratio, and the upside potential ratio generates identical ranking among alternatives. Consequently, the three ratios choose the same optimal portfolio.

EFFECTS OF SKEWNESS

To evaluate the effects of skewness on the relationships between Sharpe ratio and the two other ratios, we consider the Edgeworth-Sargan density; see, for example, Perote and Del Brio (2001). Let $W = (X - \mu)/\sigma$. To allow for skewness, the density function of W is assumed to be as follows:

$$g(w) = \phi(w) + \alpha(w^3 - 3w)\phi(w) \tag{14}$$

Under the new density, $E(W) = 0$, $E(W^2) = 1$, $E(W^3) = 6\alpha$, and $E(W^4) = 3$. Thus, when $\alpha = 0$, W is a standard normal random variable, as discussed in the previous section. A positive α corresponds to positive skewness whereas a negative α indicates negative skewness of the portfolio return.

After algebraic manipulation, it can be shown that the downside risk becomes

$$\delta^2 = (r - \mu)\sigma\phi\left(\frac{r - \mu}{\sigma}\right) + [(r - \mu)^2 + \sigma^2]\Phi\left(\frac{r - \mu}{\sigma}\right) - 2\alpha\sigma^2\phi\left(\frac{r - \mu}{\sigma}\right) \tag{15}$$

As expected, negative skewness promotes the downside risk while positive skewness reduces the downside risk. The Sortino ratio can be written as follows:

$$So = \frac{-z}{\sqrt{z\phi(z) + (z^2 + 1)\Phi(z) - 2\alpha\phi(z)}} \tag{16}$$

Upon taking derivative of the Sortino ratio with respect to z, we have

$$\frac{d(So)}{dz} = n(z)[-\Phi(z) + \alpha(z^2 + 2)\phi(z)] \tag{17}$$

where $n(z) = [z\phi(z) + (z^2 + 1)\Phi(z) - 2\alpha\phi(z)]^{-3/2}$. When $\alpha < 0$, $d(So)/dz < 0$. Thus, the Sortino ratio is an increasing function of the Sharpe ratio when the portfolio return is negatively skewed.

In the presence of positive skewness, we have a different result. Specifically, let $p(z)$ denote the term within the bracket in equation (17). Then its first derivative is

$$p'(z) = -(\alpha z^3 + 1)\phi(z) \tag{18}$$

Note that $p(z)$ approaches zero as z becomes very large. Also, $p'(z) > 0$ when $z < -\alpha^{-1/3}$. Consequently, there is a z^* such that $d(So)/dz > 0$ whenever $z \leq z^*$. The critical value, z^*, solves the following equation:

$$-\Phi(z) + \alpha(z^2 + 2)\phi(z) = 0 \tag{19}$$

Since z is the negative Sharpe ratio, we conclude that, when the Sharpe ratio is sufficiently large, the Sortino ratio decreases with an increasing Sharpe ratio.

Theorem 2 (a) Suppose that the portfolio return is negatively skewed. The Sortino ratio and the Sharpe ratio provide identical ranking among investment alternatives and choose the same optimal portfolio. (b) Suppose that, instead, the portfolio return is positively skewed. If the Sharpe ratio generated by every investment alternative is less than a critical value ($-z^*$), then the Sharpe ratio and the Sortino ratio provide identical ranking for all investment alternatives. When some alternatives have Sharpe ratios larger than the critical value, the Sortino ratio provides exactly opposite ranking to the Sharpe ratio among these alternatives. As a result, the two ratios will not choose the same optimal portfolio when the maximum Sharpe ratio attainable is large. Indeed, following the Sortino ratio an investor will not choose any portfolio with a Sharpe ratio greater than $-z^*$.

Note that negative skewness promotes the downside risk and, therefore, strengthens the relationship between downside risk and the Sharpe ratio. On the other hand, positive skewness reduces the downside risk, leading to a non-monotonic relationship between the Sortino ratio and the Sharpe ratio.

With the new density function, the upside potential is calculated as follows:

$$\theta = \sigma\left[\phi\left(\frac{r-\mu}{\sigma}\right) - \left(\frac{r-\mu}{\sigma}\right)\Phi\left(-\frac{r-\mu}{\sigma}\right) + \alpha\left(\frac{r-\mu}{\sigma}\right)\phi\left(\frac{r-\mu}{\sigma}\right)\right] \tag{20}$$

The upside potential ratio becomes

$$UPR = \frac{\phi(z) - z\Phi(-z) + \alpha z\phi(z)}{\sqrt{z\phi(z) + (z^2+1)\Phi(z) - 2\alpha\phi(z)}} \tag{21}$$

Upon taking the first derivative, we have

$$\frac{d(UPR)}{dz} = n(z)\{-[z\phi(z) + \Phi(-z)]\Phi(z) - \phi^2(z) + \alpha q(z) + \alpha^2 Q(z)\} \tag{22}$$

where

$$q(z) = (1+z^2)[-z(\phi(z) + z\Phi(z)) + 1 - \Phi(z)] + 1 \tag{23}$$

$$Q(z) = (z^2 - 2)\phi^2(z) \tag{24}$$

We consider only the case that $z < 0$, i.e., the Sharpe ratio is positive. Equation (23) leads to $q(z) > 0$. Suppose that $|\alpha|$ is small such that α is sufficiently larger than α^2. When $\alpha < 0$, once again $d(UPR)/dz < 0$. That is, in case of negative skewness, the upside potential ratio is an increasing function of the Sharpe ratio provided the latter is positive; a similar conclusion to that of the Sortino ratio.

Moreover, $q(z)$ approaches infinity as $|z|$ becomes very large. If the portfolio return is positively skewed, there is a z^{**} such that $d(UPR)/dz > 0$ whenever $z < z^{**}$. Specifically, z^{**} solves the following equation:

$$-[z\phi(z) + \Phi(-z)]\Phi(z) - \phi^2(z) + \alpha q(z) = 0 \tag{25}$$

The upside potential ratio contradicts to the Sharpe ratio in ranking investment alternatives when the Sharpe ratio is sufficiently large.

Theorem 3 Assume that $|\alpha|$ is small such that α is sufficiently larger than α^2. (a) If the portfolio return is negatively skewed, the upside potential ratio and the Sharpe ratio rank investment alternatives identically and choose the same optimal portfolio. (b) Suppose that, instead, the portfolio return is positively skewed. If the Sharpe ratio generated by every investment alternative is less than a critical value ($-z^{**}$), then the Sharpe ratio and the upside potential ratio provide identical ranking. When some alternatives have Sharpe ratios larger than the critical value, the ranking by the upside potential ratio is exactly opposite to that by the Sharpe ratio. The two methods therefore choose different portfolios when the maximum Sharpe ratio attainable is large. In fact, following the upside potential ratio an investor will not choose any portfolio with a Sharpe ratio greater than $-z^{**}$.

The interpretation of the above results is similar to that for Theorem 2. From equation (20), negative skewness promotes upside potential when the Sharpe ratio is positive. Consequently, it reinforces the relationship between the upside potential ratio and the Sharpe ratio. Positive skewness, on the other hand, reduces the upside potential. It results a non-monotonic relationship between the two ratios.

When $|\alpha|$ is large, α^2 becomes the dominating term. From equation (24), $Q(z)$ becomes very large as $|z|$ approaches infinity. As a consequently, there is a $\hat{z}$ such that $d(UPR)/dz > 0$ whenever $z < \hat{z}$. Specifically, $\hat{z}$ solves the following equation:

$$-[z\phi(z) + \Phi(-z)]\Phi(z) - \phi^2(z) + \alpha^2 Q(z) = 0 \tag{25}$$

The result is similar to the case described in Theorem 3(b).

Theorem 4 Assume that $|\alpha|$ is sufficiently large. If the Sharpe ratio generated by every investment alternative is less than a critical value ($-\hat{z}$), then the Sharpe ratio and the upside potential ratio provide the same ranking among investment alternatives. When some alternatives have Sharpe ratios larger than the critical value, the ranking by the upside potential ratio is exactly opposite to that by the Sharpe ratio. The two methods will choose different portfolios when the maximum Sharpe ratio attainable is large. In fact, following the upside potential ratio an investor will not choose any portfolio with a Sharpe ratio greater than $-\hat{z}$. The above conclusions apply to both positively and negatively skewed portfolios.

EFFECTS OF KURTOSIS

To allow for kurtosis, we consider the following density function:

$$k(w) = \phi(w) + \beta(w^4 - 6w^2 + 3)\phi(w) \tag{26}$$

Thus, $E(W) = 0$, $E(W^2) = 1$, $E(W^3) = 0$, and $E(W^4) = 3 + 18\beta$. We consider only the excessive kurtosis case: $\beta > 0$. It can be shown that the downside risk is as follows:

$$\delta^2 = (r - \mu)\sigma\phi\left(\frac{r-\mu}{\sigma}\right) + [(r-\mu)^2 + \sigma^2]\Phi\left(\frac{r-\mu}{\sigma}\right)$$
$$- 2\beta(r-\mu)\sigma\phi\left(\frac{r-\mu}{\sigma}\right) \tag{27}$$

The Sortino ratio is then

$$So = \frac{-z}{\sqrt{z\phi(z) + (z^2+1)\Phi(z) - 2\beta z\phi(z)}} \tag{28}$$

Let $t(z) = (z\phi(z) + (z^2+1)\Phi(z) - 2\beta z\phi(z))^{-3/2}$. We have the following:

$$\frac{d(So)}{dz} = t(z)[-\Phi(z) + \beta(1+z^2)z\phi(z)] < 0 \tag{29}$$

when $z < 0$. In spite of the excessive kurtosis, the Sortino ratio remains an increasing function of the Sharpe ratio when the latter is positive.

Theorem 5 Suppose that the portfolio return is characterized by excessive kurtosis. The Sortino ratio provides the same ranking to the Sharpe ratio whenever the latter is positive. Thus, as long as there is a portfolio with a positive Sharpe ratio, both methods will choose the same optimal portfolio.

After algebraic manipulation, the upside potential is calculated as follows:

$$\theta = \sigma\left[\phi\left(\frac{r-\mu}{\sigma}\right) - \left(\frac{r-\mu}{\sigma}\right)\Phi\left(-\frac{r-\mu}{\sigma}\right) + \beta\left(\left[\frac{r-\mu}{\sigma}\right]^2 - 1\right)\phi\left(\frac{r-\mu}{\sigma}\right)\right] \tag{30}$$

The upside potential ratio is

$$UPR = \frac{\phi(z) - z\Phi(-z) + \beta(z^2-1)\phi(z)}{\sqrt{z\phi(z) + (z^2+1)\Phi(z) - 2\beta z\phi(z)}} \tag{31}$$

Upon taking the first derivative, we have

$$\frac{d(UPR)}{dz} = t(z)\{-[z\phi(z) + \Phi(-z)]\Phi(z) - \phi^2(z) + \beta l(z) + \beta^2 L(z)\} \tag{32}$$

where

$$l(z) = \phi(z)\lfloor (z^2 - z^4 + 2)\phi(z) + (7z - z^5)\Phi(z) - z(3 - z^2)\rfloor \quad (33)$$

$$L(z) = \phi^2(z)(z^4 - 4z^2 - 1) \quad (34)$$

Note that $l(0) > 0$ and $L(0) < 0$. Also, when $|z|$ is very large, $L(z)$ is positive and $l(z)$ is negative. Adopting a similar analysis to the previous section, we have the following results.

Theorem 6 Assume that $|\beta|$ is small such that β is sufficiently larger than β^2. The upside potential ratio and the Sharpe ratio provide identical ranking among investment alternatives when the latter is large. The two methods lead to opposite ranking when the Sharpe ratio is moderate. Consequently, they will very likely choose different portfolios.

Theorem 7 Suppose that β is sufficiently large. If the Sharpe ratio generated by every investment alternative is less than a critical value, then the Sharpe ratio and the upside potential ratio provide the same ranking among investment alternatives. When some alternatives have Sharpe ratios larger than the critical value, the ranking by the upside potential ratio is exactly opposite to that by the Sharpe ratio. The two methods will choose different portfolios when the maximum Sharpe ratio attainable is large. In fact, following the upside potential ratio an investor will not choose any portfolio with a Sharpe ratio greater than the critical value.

FURTHER DISCUSSIONS

The above results indicate that, when the portfolio return is characterized by skewness or excessive kurtosis, it is more likely that the Sortino ratio agrees with the Sharpe ratio in ranking investment alternatives as compared to the upside potential ratio. Consequently, the rank correlation coefficient between the Sortino ratio and the Sharpe ratio tends to be larger than the rank correlation coefficient between the upside potential ratio and the Sharpe ratio.

More generally, the rank correlation coefficient between the Sortino ratio (or the upside potential ratio) and the Sharpe ratio depends on the proportion of investment alternatives that have large Sharpe ratios. The smaller the proportion is, the larger the rank correlation. As a corollary, if the minimally acceptable return rate increases, the rank correlation coefficient will increase.

APPLICATIONS TO FUTURES HEDGING

Consider a short hedger who assumes a futures position to reduce the risk associated with a non-tradable spot position. Let R_s and R_f denote respectively the spot and futures returns and let h denote the futures position. The portfolio return is then

$$R_p = R_s + hR_f \tag{35}$$

In fact, each different value of h corresponds to a different portfolio. The conventional approach assumes the hedger chooses the optimal h corresponding to the portfolio with the minimum variance (Ederington, 1979). That is, the optimal futures position, h^*, satisfies the condition: $Var(R_s + h^*R_f) \leq Var(R_s + hR_f)$ for all h. An alternative method assumes the hedger chooses the optimal futures position to maximize $E(R_p) - (\lambda/2)Var(R_p)$, where λ is a subjectively determined risk aversion coefficient.

Howard and D'Antonio (1984) adopted the Sharpe ratio as a criterion to determine the optimal futures position. Subsequent modifications are discussed in Chang and Shanker (1987) and Howard and D'Antonio (1987). Lien (1993) extends the analysis to the case of multiple assets with multiple futures contracts. Accordingly, the hedger chooses the optimal futures position, h_{Sh}, such that

$$\frac{E(R_s + h_{Sh}R_f)}{\sqrt{Var(R_s + h_{Sh}R_f)}} \geq \frac{E(R_s + hR_f)}{\sqrt{Var(R_s + h_{Sh}R_f)}} \tag{36}$$

for every h. On the other hand, several recent papers, including de Jong, de Roon, and Veld (1997) and Lien and Tse (1998), propose the downside risk hedge strategy. Tradeoffs between the return and the downside risk will give rise to the Sortino-type hedge ratio denoted by h_{So}. If we further substitute the upside potential for the return, the result leads to the upside potential hedge ratio denoted by h_{UPR}.

To derive the Sortino hedge ratio, note that h_{So} must satisfy the following first order and second order conditions:

$$\frac{d(So)}{dh} = \frac{d(So)}{d(Sh)}\frac{d(Sh)}{dh} = 0 \tag{37}$$

$$\frac{d^2(So)}{dh^2} = \frac{d^2(So)}{d(Sh)^2}\left[\frac{d(Sh)}{dh}\right]^2 + \frac{d(So)}{d(Sh)}\frac{d^2(Sh)}{dh^2} < 0 \tag{38}$$

Note that $h_{So} = h_{Sh}$ if the portfolio return is normally distributed. The same result holds in the presence of negative skewness or excessive kurtosis. When the portfolio return is positively skewed, we have two cases, either $h_{So} = h_{Sh}$ or h_{So}, corresponding to the futures position with a Sharpe ratio of $-z^*$. The former solution applies when the Sharpe ratio at h_{Sh} is smaller than $-z^*$. Otherwise, we have the latter case. Herein the Sortino ratio begins to decline before the maximum Sharpe ratio is reached. Because the optimal futures position is usually negative (i.e., a long futures position to cope with a short spot position), $h_{So} < h_{Sh}$. That is, the Sortino hedger establishes a larger futures position than the Sharpe hedger.

The upside potential ratio is less likely to agree with the Sharpe ratio. In case of positive skewness, $h_{UPR} = h_{Sh}$ when the Sharpe ratio at h_{Sh} is smaller than $-z^{**}$. Otherwise, the UPR hedger will choose a larger futures position than the Sharpe hedger: $|h_{UPR}| > |h_{Sh}|$. Similar conclusions apply to the case of excessive kurtosis.

To sum up, if both spot and futures returns are normally distributed, the UPR hedger and the Sortino hedger will choose the same optimal futures position as the Sharpe hedger. The same result applies to the Sortino hedger when the portfolio return is characterized by negative skewness or excessive kurtosis. In case of positive skewness, optimal futures positions between the Sortino hedger and the Sharpe hedger are the same provided the resulting Sharpe ratio is below a critical value. Otherwise, the Sortino hedger will establish a larger futures position. The results for the UPR hedger are similar except that the optimal futures position is more likely to be larger than that for the Sharpe hedger.

CONCLUSIONS

Assuming portfolio returns are normally distributed, I show that both the upside potential ratio (UPR) and the Sortino ratio are monotonically increasing functions of the Sharpe ratio. Consequently, all three ratios choose the same optimal portfolio. In the presence of negative skewness, the result remains valid. With positive skewness, both Sortino and upside potential ratios are inverted U-shaped functions of the Sharpe ratio. When the Sharpe ratios are sufficiently large for some portfolios, the two ratios provide exactly opposite ranking to the Sharpe ratio among these portfolios. The Sharpe ratio corresponding to the optimal Sortino ratio or the optimal upside potential ratio is bounded from above. Excessive kurtosis does not affect the monotonic relationship between Sortino and Sharpe ratios, although it leads to a complex relationship between the upside potential and Sharpe ratios. When applied to the future hedging problem, I find that the Sharpe ratio frequently leads the hedger to establish a smaller futures position when compared to Sortino and upside potential ratios when the portfolio return is positively skewed.

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