DELIVERY RISK AND THE HEDGING ROLE OF OPTIONS

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Multiple delivery specifications exist on nearly all commodity futures contracts. Sellers typically are allowed to deliver any of several grades of the underlying commodity and at any of several locations. On the delivery day, the futures price as such needs not converge to the spot price of the par-delivery grade at the par-delivery location, thereby imposing an additional delivery risk on hedgers. This article derives the optimal hedging strategy for a risk-averse hedger in the presence of delivery risk. In particular, it is shown that the hedger optimally uses options on futures for hedging purposes. This article provides a rationale for the hedging role of options when futures markets allow for multiple delivery specifications. © 2002 Wiley Periodicals, Inc. Jrl Fut Mark 22:339–354, 2002

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INTRODUCTION

To reduce the likelihood and severity of price squeezes derived from shortages in the supply of a certain variety, many commodity futures contracts allow sellers to deliver any of several grades of the underlying commodity and at any of several locations.[1] Although commodity futures contracts specify premiums and discounts for delivering a non-par grade or delivering at a non-par location, the realized spot price differences can deviate significantly from these premiums and discounts. On the delivery day, the futures price as such converges to the spot price of the cheapest-to-deliver grade rather than the spot price of the par-delivery grade.[2] Because it is uncertain which grade would be the cheapest to deliver, hedgers are exposed to an additional delivery risk vis-à-vis the underlying price risk. Both the pricing of delivery options and the hedging of delivery risk with futures have been examined extensively in the literature.[3]

It is believed widely that deliveries on futures contracts rarely are made and taken and that sizable deliveries hint at possible imbalance between the shorts and the longs (see Hieronymous, 1977). Peck and Williams (1991, 1992), however, documented that deliveries on the Chicago Board of Trade (CBOT) wheat, corn, and soybean futures markets and the New York Commodity Exchange copper futures market averaged approximately 15% of the peak open interest in each delivery month and 50% of the open interest on the delivery day in the 1970s and 1980s. These percentages are significant and contradicting to the common perception. Although data disaggregated by varieties are unavailable, using estimated parameter values from a rational expectations-restricted model, Kamara (1990) computed the average values of the expected delivery basis (the difference between the spot and futures prices on the delivery day) and expected delivery risk premiums embedded in the futures prices on the CBOT soybean market to be 17 cents per bushel, with a standard deviation of the mean of less than 1 cent. The ratio of the estimated delivery risk components to the futures price was, on average, 2.85%, with a

[1]Many futures contracts also allow their holders to deliver on any given business day in the delivery month, and the delivery process itself lasts for three days (position day, notice day, and delivery day). However, as empirically documented by Gay and Manaster (1984, 1986), this timing option seems to have little value on commodity and financial futures contracts.

[2]The cheapest-to-deliver grade is the grade with the minimum delivery-adjusted spot price among all deliverable grades.

[3]Evidence on the importance of delivery specifications in the pricing of futures contracts are provided by Garbade and Silber (1983a, 1983b), Gay and Manaster (1984, 1986), Kamara (1990), Kane and Marcus (1986), and Kilcollin (1982). Optimal hedging strategies with futures in the presence of delivery risk are analyzed by Kamara and Siegel (1987), Lien (1988, 1991), and Viswanath and Chatterjee (1992). See also Chance and Hemler (1993) for a rather complete survey on the pricing issues.

standard deviation of the mean of 0.02%.[4] Kamara (1990) showed that these significant expected delivery basis and delivery risk premiums have important implications for market efficiency tests.

Today, most commodity futures contracts also have available options on futures. Despite their good trading volume, the hedging role of these option contracts is lacking analytically. In the absence of any basis risk, it is well known that the optimal futures position is a full hedge for a general strictly concave utility function (see Danthine, 1978; Feder, Just, & Schmitz, 1980; Holthausen, 1979). Allowing a regression relationship to capture the basis risk, the optimal futures position becomes a beta hedge (see Benninga, Eldor, & Zilcha, 1984; Lence, 1995). In either case, option contracts play no role as a hedging instrument, as there is no residual hedgeable risk against which the options can hedge.[5]

There is a growing list of literature that relax various assumptions of the standard hedging model to yield a theoretical benefit from using options for hedging purposes. For example, Moschini and Lapan (1992) showed that production flexibility leads to *ex post* convex profit functions, thereby making options a useful hedging instrument. Sakong, Hayes, and Hallam (1993) and Moschini and Lapan (1995) showed that production uncertainty, which affects the curvature of profit functions, provides another rationale for the hedging role of options. Lence, Sakong, and Hayes (1994) showed that, in a multiperiod setting, options are used because future output prices affect profits from future production cycles. Broll, Chow, and Wong (2001) showed that when the underlying uncertainty is nonlinear in nature, the asymmetric payoff profiles of options are more suitable for hedging purposes.

This article adds to this scant literature on the hedging role of options. In a simple expected utility model of a hedger, there are futures contracts and options on futures that allow the hedger to deliver any of two grades of the underlying commodity. It is shown that the presence of delivery risk is attributable to a special kind of truncation to the price distributions. This truncation makes delivery risk different from the standard basis risk specified in the literature by a regression relationship. The prevalence of delivery risk induces the risk-averse hedger to deviate from a full hedge via the unbiased futures contracts. This gives rise to some

[4]As pointed out by Kamara (1990), an estimated value of 2.85% is substantial given the high leverage of futures contracts. For example, the initial margin levels for hedgers on the trading days in his sample in 1981 were, on average, 2.7% of the futures price.

[5]Indeed, Lapan, Moschini, and Hanson (1991) showed that options are optimally used by a risk-averse hedger only when futures prices and/or option premiums are perceived as biased. In this regard, options are appealing more as a speculative device to exploit private information on price distributions, and less so as a hedging instrument.

residual price risk, which is hedgeable by the unbiased options on futures. Because there are two distinct sources of uncertainty, the hedger requires at least two hedging instruments to meet his/her hedging need. Thus, this study justifies the hedging role of options with multiple delivery specifications on futures markets.

THE MODEL

Consider a one-period model with two dates, indexed by $t = 0$ and 1. At $t = 0$, an individual (hereafter referred to as the hedger) acquires a cash position, Q, on a specific grade (say grade A) of a commodity. Let P_{At} be the spot price of grade A at time t. While P_{A0} is known at $t = 0$, $\tilde{P}_{A1}$ is revealed only at $t = 1$.[6] To hedge against this price risk, the hedger can trade infinitely divisible futures contracts that call for delivery of two possible grades at $t = 1$: grade A and an alternative grade (say grade B). Let F_t be the price of the futures contracts at time t and P_{Bt} be the spot price of grade B at time t. The hedger also avails himself to infinitely divisible call options on futures as an additional hedging instrument.[7] Let K and C be the strike price of and the premium on the call options on futures, respectively. To simplify notation, P_{A0}, P_{B0}, and C are all compounded to $t = 1$.

Following the convention, sellers have the right to choose the deliverable grade. Thus, at $t = 1$, the price of the futures contracts, $\tilde{F}_1$, must equal the minimum of $\tilde{P}_{A1}$ and $\tilde{P}_{B1}$ (see Kamara & Siegel, 1987; Lien, 1988, 1991). Unless $\tilde{P}_{A1} \leq \tilde{P}_{B1}$ so that $\min(\tilde{P}_{A1}, \tilde{P}_{B1}) = \tilde{P}_{A1}$ with probability one, hedging with the futures contracts and/or the call options on futures generates an additional source of risk arising from the multiple grades of delivery. This is referred to as the delivery risk vis-à-vis the price risk associated with the cash position.[8]

The hedger's end-of-period wealth is given by

$$\tilde{W} = (\tilde{P}_{A1} - P_{A0})Q + (F_0 - \tilde{F}_1)H + [C - \max(\tilde{F}_1 - K, 0)]Z \quad (1)$$

where $\tilde{F}_1 = \min(\tilde{P}_{A1}, \tilde{P}_{B1})$, and H and Z are the short (long if negative) positions held by the hedger for the futures contracts and the call options on futures, respectively. The hedger possesses a von Neumann–Morgenstern utility function, U, defined over his/her end-of-period

[6] Throughout the article, random variables have a tilde (~), whereas their realizations do not.

[7] Because payoffs of any combinations of futures, calls, and puts can be replicated by any two of these three financial instruments (see Cox & Rubinstein, 1985), one of them is redundant. Therefore, it is no loss of generality to restrict the hedger to use futures and call options on futures only.

[8] If a commodity deliverable at different locations is treated as different *types* of commodities, the analysis for multiple delivery grades extends to the case of multiple delivery locations.

wealth, W, with $U' > 0$ and $U'' < 0$, indicating the presence of risk aversion.

The hedger's decision problem at $t = 0$ is to choose a hedge position, (H, Z), so as to maximize the expected utility of his/her end-of-period wealth:

$$\max_{H,Z} \mathrm{E}[U(\tilde{W})]$$

where E is the expectation operator with respect to the joint-probability distribution function of $\tilde{P}_{A1}$ and $\tilde{P}_{B1}$, and $\tilde{W}$ is defined in eq. (1). The first-order conditions for an optimum are given by

$$\mathrm{E}[U'(\tilde{W}^*)(F_0 - \tilde{F}_1)] = 0 \tag{2}$$

$$\mathrm{E}\{U'(\tilde{W}^*)[C - \max(\tilde{F}_1 - K, 0)]\} = 0 \tag{3}$$

where an asterisk (*) indicates an optimal level. The second-order conditions for a maximum are satisfied given risk aversion.

The hedger's optimal hedge position, (H^*, Z^*), is defined implicitly by eqs. (2) and (3). Among other things, (H^*, Z^*) would depend on whether the hedger perceives the prices of the futures contracts and the call options on futures as unbiased or biased. In the former unbiased case, (H^*, Z^*) should reflect solely the hedging motive. In the latter biased case, (H^*, Z^*) also reflects the speculative motive. To focus on the hedging role of the futures contracts and the call options on futures, attention is restricted to the case where the prices of these two financial instruments are perceived as unbiased by the hedger. Succinctly, assume hereafter that $F_0 = \mathrm{E}[\min(\tilde{P}_{A1}, \tilde{P}_{B1})]$ and $C = \mathrm{E}\{\max[\min(\tilde{P}_{A1}, \tilde{P}_{B1}) - K, 0]\}$.[9]

HEDGING ROLE OF OPTIONS

Because the futures contracts and the call options on futures are jointly unbiased, eqs. (2) and (3) can be written as

$$\mathrm{Cov}[U'(\tilde{W}^*), \min(\tilde{P}_{A1}, \tilde{P}_{B1})] = 0 \tag{4}$$

$$\mathrm{Cov}\{U'(\tilde{W}^*), \max[\min(\tilde{P}_{A1}, \tilde{P}_{B1}) - K, 0]\} = 0 \tag{5}$$

[9]In equilibrium, the futures price, F_0, should be lower than $\mathrm{E}[\min(\tilde{P}_{A1}, \tilde{P}_{B1})]$ by the amount that long hedgers charge, and short hedgers pay, for the sellers' delivery options. The assumption of unbiased futures price, i.e., $F_0 = \mathrm{E}[\min(\tilde{P}_{A1}, \tilde{P}_{B1})]$, would imply a zero value for the delivery options. Since this paper does not deal with equilibrium, the unbiasedness assumption is made to separate between pure hedging and speculative motives.

where Cov is the covariance operator with respect to the joint-probability distribution function of $\tilde{P}_{A1}$ and $\tilde{P}_{B1}$.

If $\tilde{P}_{A1} \leq \tilde{P}_{B1}$ so that $\min(\tilde{P}_{A1}, \tilde{P}_{B1}) = \tilde{P}_{A1}$ with probability one, the hedger faces only the price risk but not the delivery risk. In this case, it follows from eq. (1) that the hedger's end-of-period wealth is $(F_0 - P_{A0})Q$ when $H = Q$ and $Z = 0$, which is nonstochastic. In other words, this hedge position satisfies eqs. (4) and (5) simultaneously, thereby invoking the following proposition.

Proposition 1. *Suppose that the prices of the futures contracts and the call options on futures are jointly unbiased. If the par-delivery grade is always cheaper than the alternative grade, it is optimal for the hedger to hedge fully via the futures contracts and not to use the call options on futures.*

Proposition 1 is simply the celebrated full-hedging theorem derived by Danthine (1978), Holthausen (1979), Feder et al. (1980), and many others. In the absence of the delivery risk, the call options on futures are redundant because the hedger's end-of-period wealth is linear in $\tilde{P}_{A1}$, as evident from eq. (1). It follows that the linear-payoff futures contracts strictly dominate the nonlinear-payoff call options on futures for hedging purposes (see Lapan et al., 1991). The futures contracts, being unbiased, offer actuarially fair *insurance* to the hedger. Thus, it is optimal for the hedger to eliminate completely the price risk by adopting a full hedge via the unbiased futures contracts.[10]

To show the hedging role of the call options on futures, the condition that $\tilde{P}_{A1} \leq \tilde{P}_{B1}$ must be discarded to resume the delivery risk. For tractability, some structure of the joint-probability distribution function of $\tilde{P}_{A1}$ and $\tilde{P}_{B1}$ is needed. Suppose that $\tilde{P}_{B1} = \tilde{P}_{A1} - \delta$ with probability p or $\tilde{P}_{B1} = \tilde{P}_{A1} + \epsilon$ with probability $1 - p$, where δ and ϵ are positive constants. Thus, p is the probability that grade A is more expensive than grade B at $t = 1$. Given this joint-probability distribution function of $\tilde{P}_{A1}$ and $\tilde{P}_{B1}$, eqs. (2) and (3) can be written as

$$p\mathrm{E}\{U'(\tilde{W}_B^*)[F_0 - (\tilde{P}_{A1} - \delta)]\} + (1 - p)\mathrm{E}[U'(\tilde{W}_A^*)(F_0 - \tilde{P}_{A1})] = 0 \tag{6}$$

$$p\mathrm{E}\{U'(\tilde{W}_B^*)[C - \max(\tilde{P}_{A1} - \delta - K, 0)]\} + (1 - p)\mathrm{E}\{U'(\tilde{W}_A)[C - \max(\tilde{P}_{A1} - K, 0)]\} = 0 \tag{7}$$

[10]The full-hedging theorem is analogous to a well-known result in the insurance literature that a risk-averse individual fully insures at an actuarially fair price (see Mossin, 1968).

where E is the expectation operator with respect to the marginal probability distribution function of $\tilde{P}_{A1}$,

$$\begin{aligned}\tilde{W}_B^* = {} & (\tilde{P}_{A1} - P_{A0})Q + [F_0 - (\tilde{P}_{A1} - \delta)]H^* \\ & + [C - \max(\tilde{P}_{A1} - \delta - K, 0)]Z^* \end{aligned} \tag{8}$$

$$\begin{aligned}\tilde{W}_A^* = {} & (\tilde{P}_{A1} - P_{A0})Q + (F_0 - \tilde{P}_{A1})H^* \\ & + [C - \max(\tilde{P}_{A1} - K, 0)]Z^* \end{aligned} \tag{9}$$

Note that when $p = 0$, the delivery risk disappears because $\tilde{P}_{B1} = \tilde{P}_{A1} + \epsilon$ for sure. As previously shown, the optimal hedge position in this case is given by $H^* = Q$ and $Z^* = 0$. To show the hedging role of the call options on futures because of the presence of the delivery risk, it suffices to perform the comparative statics exercise with respect to p, evaluating at $p = 0$.

Totally differentiating eqs. (6) and (7) with respect to p and evaluating at $p = 0$ yields (see the appendix for the derivation)

$$\begin{aligned}\frac{\mathrm{d}H^*}{\mathrm{d}p} = {} & \frac{U'[(F_0 + \delta - P_{A0})Q] - U'[(F_0 - P_{A0})Q]}{U''[(F_0 - P_{A0})Q]} \\ & \times \frac{\{C - \mathrm{E}[\max(\tilde{P}_{A1} - \delta - K, 0)]\}\,\mathrm{Cov}[\tilde{P}_{A1}, \max(\tilde{P}_{A1} - K, 0)] - \delta \mathrm{Var}[\max(\tilde{P}_{A1} - K, 0)]}{\mathrm{Var}(\tilde{P}_{A1})\,\mathrm{Var}[\max(\tilde{P}_{A1} - K, 0)] - \mathrm{Cov}[\tilde{P}_{A1}, \max(\tilde{P}_{A1} - K, 0)]^2}\end{aligned} \tag{10}$$

$$\begin{aligned}\frac{\mathrm{d}Z^*}{\mathrm{d}p} = {} & \frac{U'[(F_0 + \delta - P_{A0})Q] - U'[(F_0 - P_{A0})Q]}{U''[(F_0 - P_{A0})Q]} \\ & \times \frac{\delta \mathrm{Cov}[\tilde{P}_{A1}, \max(\tilde{P}_{A1} - K, 0)] - \{C - \mathrm{E}[\max(\tilde{P}_{A1} - \delta - K, 0)]\}\,\mathrm{Var}(\tilde{P}_{A1})}{\mathrm{Var}(\tilde{P}_{A1})\,\mathrm{Var}[\max(\tilde{P}_{A1} - K, 0)] - \mathrm{Cov}[\tilde{P}_{A1}, \max(\tilde{P}_{A1} - K, 0)]^2}\end{aligned} \tag{11}$$

where $\{U'[(F_0 + \delta - P_{A0})Q] - U'[(F_0 - P_{A0})Q]\}/U''[(F_0 - P_{A0})Q] > 0$ by risk aversion. The denominator of the second term on the right-hand side of eq. (10) or eq. (11) is clearly positive because

$$\mathrm{Cov}[\tilde{P}_{A1}, \max(\tilde{P}_{A1} - K, 0)] = \rho \times \sqrt{\mathrm{Var}(\tilde{P}_{A1})\,\mathrm{Var}[\max(\tilde{P}_{A1} - K, 0)]}$$

where $\rho \in (0, 1)$ is the correlation coefficient between $\tilde{P}_{A1}$ and $\max(\tilde{P}_{A1} - K, 0)$. Thus, the sign of $\mathrm{d}H^*/\mathrm{d}p$ or that of $\mathrm{d}Z^*/\mathrm{d}p$ is the same as the sign of the numerator of the second term on the right-hand side of eq. (10) or eq. (11), respectively.

If the hedger is not allowed to trade the call options on futures so that $Z \equiv 0$, it is easily verified that

$$\left.\frac{dH^*}{dp}\right|_{Z\equiv 0} = -\frac{U'[(F_0 + \delta - P_{A0})Q] - U'[(F_0 - P_{A0})Q]}{U''[(F_0 - P_{A0})Q]} \times \frac{\delta}{\mathrm{Var}(\tilde{P}_{A1})} < 0 \tag{12}$$

That is, the hedger opts for an under hedge, $H^* < Q$, via the unbiased futures contracts in response to the presence of the delivery risk, when the call options on futures are not available. The intuition behind this result is as follows. Since covariances can be interpreted as marginal variances, eq. (4) implies that the hedger's optimal futures position, given $Z \equiv 0$, minimizes the variance of his/her marginal utility. Suppose that the hedger starts with a full hedge, $H = Q$, so that his/her end-of-period wealth is either $(F_0 + \delta - P_{A0})Q$ with probability p or $(F_0 - P_{A0})Q$ with probability $1 - p$. Given risk aversion, $U'[(F_0 + \delta - P_{A0})Q] < U'[(F_0 - P_{A0})Q]$. Thus, the hedger can reduce further the variability of his/her marginal utility by shifting some wealth in the state where grade B is cheaper to that in the state where grade A is cheaper. As evident from eqs. (8) and (9), $\tilde{W}_B = \tilde{W}_A + \delta H$ given $Z \equiv 0$. Thus, to reduce $\tilde{W}_B$ relative to $\tilde{W}_A$, the only choice is to decrease H, that is, to buy the futures contracts. This long position, starting from a full hedge, is precisely what makes an under hedge optimal for the hedger in response to the presence of the delivery risk.

Using eqs. (11) and (12), eq. (10) can be written as

$$\frac{dH^*}{dp} = \left.\frac{dH^*}{dp}\right|_{Z\equiv 0} - \frac{\mathrm{Cov}[\tilde{P}_{A1}, \max(\tilde{P}_{A1} - K, 0)]}{\mathrm{Var}(\tilde{P}_{A1})} \times \frac{dZ^*}{dp} \tag{13}$$

Equation (13) has the following intuitive interpretation. When $p = 0$, it follows from eq. (1) that the hedger's end-of-period wealth can be written as

$$\begin{aligned}\tilde{W} = {} & [(\tilde{P}_{A1} - P_{A0})Q + (F_0 - \tilde{P}_{A1})H_1] \\ & + \{[C - \max(\tilde{P}_{A1} - K, 0)]Z + (F_0 - \tilde{P}_{A1})H_2\}\end{aligned} \tag{14}$$

where $H = H_1 + H_2$. By the well-known results in the hedging literature (see Danthine, 1978; Feder et al., 1980; Holthausen, 1979), the optimal hedge ratio for the first risk component on the right-hand side of eq. (14) is $H_1/Q = 1$. If the hedger uses the call options on futures, the optimal hedge ratio for the second risk component on the right-hand side of eq. (14) equals the slope of the regression of $\max(\tilde{P}_{A1} - K, 0)$ on $\tilde{P}_{A1}$, and thus, $H_2/Z = -\text{Cov}[\tilde{P}_{A1}, \max(\tilde{P}_{A1} - K, 0)]/\text{Var}(\tilde{P}_{A1})$. Because Q is fixed, the first term on the right-hand side of eq. (13) captures the change in the futures position due entirely to the presence of the delivery risk when the call options on futures are not used. On the other hand, the second term on the right-hand side of eq. (13), captures the additional change in the futures position to account for the hedger's use of the call options on futures in response to the presence of the delivery risk.

To sign dZ^*/dp, let N be the numerator of the second term on the right-hand side of eq. (11). Note that $N = 0$ when $\delta = 0$. In addition,

$$\frac{\partial N}{\partial \delta} = \text{Cov}[\tilde{P}_{A1}, \max(\tilde{P}_{A1} - K, 0)] - [1 - G(K + \delta)]\text{Var}(\tilde{P}_{A1}) \quad (15)$$

$$\frac{\partial^2 N}{\partial \delta^2} = g(K + \delta)\text{Var}(\tilde{P}_{A1}) > 0 \quad (16)$$

where G and g are the marginal probability distribution function and density function of $\tilde{P}_{A1}$, respectively. Inspection of eq. (16) shows that N is a strictly convex function of δ starting at zero. If $\text{Cov}[\tilde{P}_{A1}, \max(\tilde{P}_{A1} - K, 0)] \geq [1 - G(K)]\text{Var}(\tilde{P}_{A1})$, eq. (15) implies that $\partial N/\partial \delta \geq 0$ at $\delta = 0$. It then follows from eq. (16) that $\partial N/\partial \delta > 0$ or all $\delta > 0$, and thus, $N > 0$. If $\text{Cov}[\tilde{P}_{A1}, \max(\tilde{P}_{A1} - K, 0)] < [1 - G(K)]\text{Var}(\tilde{P}_{A1})$, eq. (15) implies that $\partial N/\partial \delta < 0$ at $\delta = 0$. Because N is strictly convex in δ, there must exist a unique $\hat{\delta} > 0$ at which $N = 0$. In this case, $N < (=, >)\ 0$ if $0 < \delta < (=, >)\ \hat{\delta}$. The following proposition is established from the fact that dZ^*/dp has the same sign as N.[11]

Proposition 2. *Suppose that the prices of the futures contracts and the call options on futures are jointly unbiased. If* $\text{Cov}[\tilde{P}_{A1}, \max(\tilde{P}_{A1} - K, 0)] \geq [1 - G(K)]\text{Var}(\tilde{P}_{A1})$, *the presence of the delivery risk induces the hedger to write the call options on futures. Otherwise, the presence of the delivery risk induces the hedger to write or buy the call options on futures, depending on whether* δ *is above or below a critical level,* $\hat{\delta}$, *respectively.*

[11]The results of Proposition 2 can be extended easily to put options on futures. Under the same conditions as specified in Proposition 2, if the hedger optimally opts for a short (long) position of the call options on futures, he/she will adopt a long (short) position of the put options on futures.

The intuition of Proposition 2 is as follows. The hedger's demand for the call options on futures comes from two sources. First, from eq. (12), the hedger would opt for an under hedge via the futures contracts in response to the presence of the delivery risk should the call options on futures be unavailable. This gives rise to some positive residual price risk that is hedgeable by writing the call options on futures. This is referred to as the indirect demand. As evident from eq. (12), this demand for the call options on futures increases with δ.

The second source of demand for the call options on futures is created by the skewness of the price distribution. Intuitively, $\mathrm{Cov}[\tilde{P}_{A1}, \max(\tilde{P}_{A1} - K, 0)] - [1 - G(K)]\mathrm{Var}(\tilde{P}_{A1})$ can be interpreted as a measure of the price skewness. To see this, set $K = \mathrm{E}(\tilde{P}_{A1}) = \bar{P}_{A1}$. If the price distribution is skewed to the right (left), the following two properties must hold (see Vercammen, 1995):

$$G(\bar{P}_{A1}) > (<)\ 1 - G(\bar{P}_{A1}) \tag{17}$$

$$\int_{\bar{P}_{A1}}^{P_{A1}^{\max}} (P_{A1} - \bar{P}_{A1})^2\, \mathrm{d}G(P_{A1}) > (<) \int_{P_{A1}^{\min}}^{\bar{P}_{A1}} (P_{A1} - \bar{P}_{A1})^2\, \mathrm{d}G(P_{A1}) \tag{18}$$

where $[P_{A1}^{\min}, P_{A1}^{\max}]$ is the support of G. Equation (17) states that the majority of the probability mass is to the left (right) of $\bar{P}_{A1}$, whereas eq. (18) states that the upper semivariance is larger (less) than the lower semivariance should the price distribution be skewed to the right (left). Adding $\int_{\bar{P}_{A1}}^{P_{A1}^{\max}} (P_{A1} - \bar{P}_{A1})^2\, \mathrm{d}G(P_{A1})$ to both sides of eq. (18) yields

$$2\,\mathrm{Cov}[\tilde{P}_{A1}, \max(\tilde{P}_{A1} - \bar{P}_{A1}, 0)] > (<) \mathrm{Var}(\tilde{P}_{A1}) \tag{19}$$

if the price distribution is skewed to the right (left). From eq. (17), $G(\bar{P}_{A1}) > (<)\ 1/2$, and thus, eq. (19) becomes

$$\mathrm{Cov}[\tilde{P}_{A1}, \max(\tilde{P}_{A1} - \bar{P}_{A1}, 0)] > (<)\ [1 - G(\bar{P}_{A1})]\mathrm{Var}(\tilde{P}_{A1})$$

if the price distribution is skewed to the right (left). Given an under hedge via the futures contracts, the distribution of the hedger's end-of-period wealth inherits the direction of the price skewness. The call options on futures play a role in reducing the skewness of the distribution of wealth. Succinctly, if the price distribution is skewed to the right (left), the hedger should adopt an option strategy that skews the distribution of wealth toward the left (right). Writing (Buying) the call options on futures achieves this goal. A short (long) position in the call options on futures builds up the probability mass more (less) quickly for relatively high (low) values of end-of-period wealth than for low (high) ones, thereby skewing the distribution of wealth back toward the left (right). This is referred to as the direct demand for the call options on futures.

If $\text{Cov}[\tilde{P}_{A1}, \max(\tilde{P}_{A1} - K, 0)] \geq [1 - G(K)]\text{Var}(\tilde{P}_{A1})$, the price distribution is not skewed to the left. The direct demand and the indirect demand reinforce each other, making a short position in the call options on futures optimal. However, if $\text{Cov}[\tilde{P}_{A1}, \max(\tilde{P}_{A1} - K, 0)] < [1 - G(K)]\text{Var}(\tilde{P}_{A1})$, the direct demand and the indirect demand work in the opposite directions. Because the indirect demand gets stronger when δ increases, a short (long) position in the call options on futures is optimal when δ is sufficiently large (small), such that the indirect (direct) demand dominates the direct (indirect) demand.

A NUMERICAL EXAMPLE

Suppose that the spot prices of the two grades are the same at $t = 0$. That is, $P_{A0} = P_{B0}$. Define $\tilde{u} = \tilde{P}_{A1} - P_{A0}$ and $\tilde{v} = \tilde{P}_{B1} - P_{B0}$ so that $\tilde{F}_1 = \min(\tilde{u}, \tilde{v}) + P_{A0}$. The two price changes, $\tilde{u}$ and $\tilde{v}$, are assumed to be jointly normally distributed with zero mean and unit variance.[12] Define the joint probability density function of $\tilde{u}$ and $\tilde{v}$ as

$$\phi(u, v) = \frac{1}{2\pi\sqrt{1 - \rho^2}} \exp\left[-\frac{u^2 - 2\rho uv + v^2}{2(1 - \rho^2)}\right]$$

where ρ is the correlation coefficient between $\tilde{u}$ and $\tilde{v}$. The hedger is assumed to have a negative exponential utility function, $U(W) = -\exp(-\lambda W)$, where λ is the hedger's constant coefficient of absolute risk aversion.

Consider first the case where the call options on futures are not available. Let $Q = 1$. In this case, given the unbiasedness of the futures price, the hedger's expected utility is given by

$$EU_0 = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} -\exp\{-\lambda\{u + [M - \min(u, v)]H\}\}\phi(u, v)\,\mathrm{d}u\,\mathrm{d}v$$

[12]See Kamara and Siegel (1987) for empirical justification of the assumption of equal variance in the red hard and red soft wheat, the two main delivery grades on the CBOT wheat futures market.

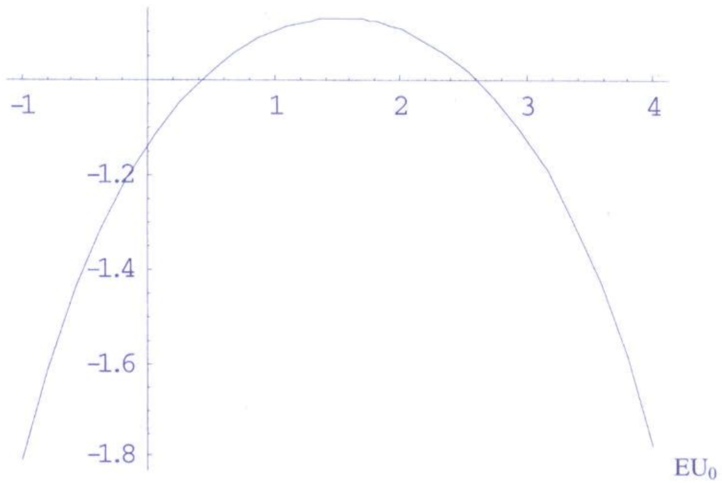

FIGURE 1
Relationship between expected utility and the futures position.

where $M = \mathrm{E}[\min(\tilde{u}, \tilde{v})] = -\sqrt{(1-\rho)/\pi}$. Setting $\rho = 0.8$ and $\lambda = 0.5$, Figure 1 depicts EU_0 as a function of H.[13]

The maximum is achieved when H is approximately equal to 1.5. When $H = 1.5$, $EU_0 = -0.868376$. Therefore, the certainty equivalent return per unit of output is equal to $\mathrm{CER}_0 = 0.282261$.[14]

Now, consider the case where the call options on futures are available. Let $Q = 1$ and $K = F_0$. In this case, given the joint unbiasedness of the prices of the futures contracts and the call options on futures, the hedger's expected utility is given by

$$EU_1 = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} -\exp\{-\lambda\{u + [M - \min(u,v)]H + [C - \max(\min(u,v) - M, 0)]Z\}\}\phi(u,v)\,\mathrm{d}u\,\mathrm{d}v$$

Setting $\rho = 0.8$ and $\lambda = 0.5$, Figure 2 depicts the surface for EU_1 as a function of H and Z.

[13]The choice of $\rho = 0.8$ is in the domain of the values chosen by Kamara and Siegel (1987) ($\rho = 0.9$, 0.95, 0.99, and 1). The choice of $\lambda = 0.5$ is taken to highlight the effects of delivery risk and risk aversion in the simulation exercise. Indeed, the numerical results are unaffected as long as $\lambda Q = 0.5$, where H and Z are interpreted as hedge ratios. For example, in the case that $\lambda = 0.0001$ and $Q = 5000$, the hedger is less risk averse, but his/her end-of-period wealth is measured in dollars per 5000 units of output, making his/her maximum expected utility remain at the same level.

[14]For any random wealth, $\tilde{W}$, the certainty equivalent wealth, W^C, is defined as $U(W^C) = \mathrm{E}[U(\tilde{W})]$. Because returns are considered in the simulation exercise rather than wealth, the certainty equivalent return with futures hedging, CER_0, is found by solving $U(\mathrm{CER}_0) = -\exp(-0.5 \times \mathrm{CER}_0) = -0.868376$ to yield $\mathrm{CER}_0 = 0.282261$.

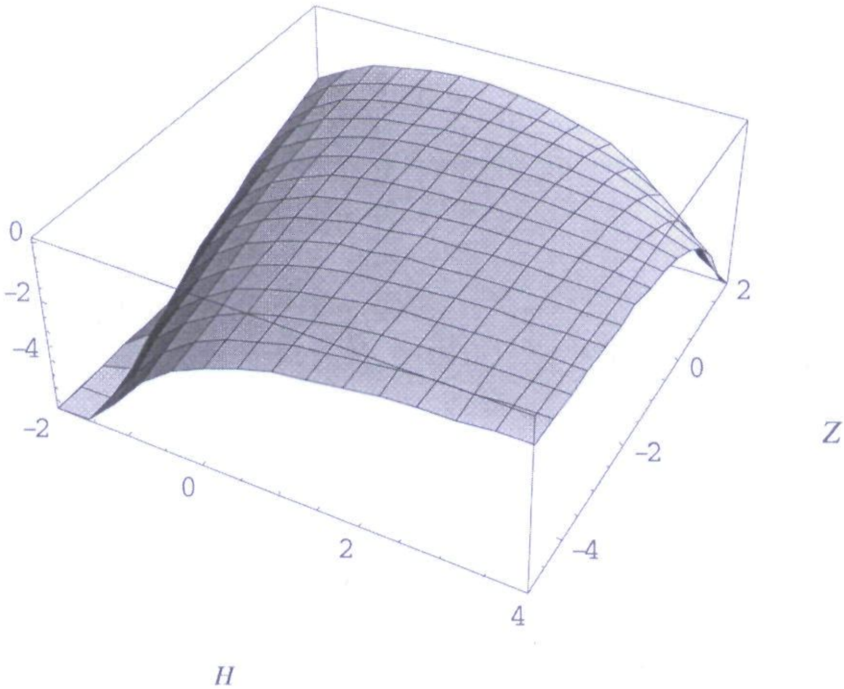

FIGURE 2
Expected utility as a function of futures and options positions.

The maximum is achieved when H and Z are approximately equal to 2.7 and -1.95, respectively. When $H = 2.7$ and $Z = -1.95$, $EU_1 = -0.785346$. Therefore, the certainty equivalent return per unit of output is equal to $\text{CER}_1 = 0.483262$, which is 71.21% higher than the one when only the futures contracts are used.[15]

CONCLUSIONS

Multiple delivery specifications exist on nearly all commodity futures contracts. Sellers typically are allowed to deliver any of several grades of the underlying commodity and at any of several locations. This flexibility drives the futures price on the delivery day to deviate from the spot price of the par-deliver grade at the par-delivery location. As such, hedgers face an additional delivery risk vis-à-vis the price risk. This study has examined the optimal hedging strategy for a risk-averse hedger in the presence of delivery risk. It is shown that the hedger deviates from a full hedge via unbiased futures contracts. This gives rise to some residual

[15]Solving $U(\text{CER}_1) = -\exp(-0.5 \times \text{CER}_1) = -0.785346$ yields $\text{CER}_1 = 0.483262$.

price risk that is hedgeable by trading options on futures. Because there are two distinct sources of uncertainty, the hedger requires at least two different hedging instruments to meet his/her hedging need. Thus, this study has provided a rationale for the hedging role of options when futures markets allow for multiple delivery specifications.

APPENDIX

Because the prices of the futures contracts and the call options on futures are jointly unbiased,

$$F_0 = \mathrm{E}(\tilde{P}_{A1}) - p\delta \tag{A.1}$$

$$C = p\mathrm{E}[\max(\tilde{P}_{A1} - \delta - K, 0)] + (1 - p)\mathrm{E}[\max(\tilde{P}_{A1} - K, 0)] \tag{A.2}$$

Differentiating eqs. (A.1) and (A.2) with respect to p yields,

$$\frac{\mathrm{d}F_0}{\mathrm{d}p} = -\delta \tag{A.3}$$

$$\frac{\mathrm{d}C}{\mathrm{d}p} = \mathrm{E}[\max(\tilde{P}_{A1} - \delta - K, 0)] - \mathrm{E}[\max(\tilde{P}_{A1} - K, 0)] \tag{A.4}$$

Totally differentiating eqs. (6) and (7) with respect to p and evaluating at $p = 0$ yields

$$\frac{\mathrm{d}H^*}{\mathrm{d}p} = \frac{EU_{Zp}EU_{HZ} - EU_{Hp}EU_{ZZ}}{EU_{HH}EU_{ZZ} - EU_{HZ}^2} \tag{A.5}$$

$$\frac{\mathrm{d}Z^*}{\mathrm{d}p} = \frac{EU_{Hp}EU_{HZ} - EU_{Zp}EU_{HH}}{EU_{HH}EU_{ZZ} - EU_{HZ}^2} \tag{A.6}$$

where

$$EU_{HH} = U''[(F_0 - P_{A0})Q]\mathrm{Var}(\tilde{P}_{A1}) \tag{A.7}$$

$$EU_{ZZ} = U''[(F_0 - P_{A0})Q]\mathrm{Var}[\max(\tilde{P}_{A1} - K, 0)] \tag{A.8}$$

$$EU_{HZ} = U''[(F_0 - P_{A0})Q]\mathrm{Cov}[\tilde{P}_{A1}, \max(\tilde{P}_{A1} - K, 0)] \tag{A.9}$$

$$EU_{Hp} = \{U'[(F_0 + \delta - P_{A0})Q] - U'[(F_0 - P_{A0})Q]\}\delta \tag{A.10}$$

$$EU_{Zp} = \{U'[(F_0 + \delta - P_{A0})Q] - U'[(F_0 - P_{A0})Q]\} \times \{C - \mathrm{E}[\max(\tilde{P}_{A1} - \delta - K, 0)]\} \tag{A.11}$$

because $H^* = Q$, $Z^* = 0$, $F_0 = \mathrm{E}(\tilde{P}_{A1})$, and $C = \mathrm{E}[\max(\tilde{P}_{A1} - K, 0)]$ when $p = 0$. Substituting eqs. (A.7) to (A.11) into eqs. (A.5) and (A.6) yields eqs. (10) and (11), respectively.

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