STEP-RESET OPTIONS: DESIGN AND VALUATION

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This study proposes a new design of reset options in which the option's exercise price adjusts gradually, based on the amount of time the underlying spent beyond prespecified reset levels. Relative to standard reset options, a step-reset design offers several desirable properties. First of all, it demands a lower option premium but preserves the same desirable reset attribute that appeals to market investors. Second, it overcomes the disturbing problem of delta jump as exhibited in standard reset option, and thus greatly reduces the difficulties in risk management for reset option sellers who hedge dynamically. Moreover, the step-reset feature makes the option more robust against short-term price movements of the underlying and removes the pressure of price manipulation often associated with standard reset options.

To value this innovative option product, we develop a tree-based valuation algorithm in this study. Specifically, we parameterize the trinomial tree model to correctly account for the discrete nature of reset monitoring. The use of lattice model gives us the flexibility to price step-reset options with American exercise right. Finally, to accommodate the path-dependent exercise price, we introduce a state-to-state recursive pricing procedure to properly capture the path-dependent step-reset effect and enhance computational efficiency. © 2002 John Wiley & Sons, Inc. Jrl Fut Mark 22:155–171, 2002

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INTRODUCTION

The strike price of a reset option can be adjusted in favor of its holders if the price movement of the underlying breaches a prescribed level.[1] The reset design appeals to investors because it commands an edge in maintaining option value when the underlying is heading toward the wrong direction. Gray and Whaley (1997) look into the S&P 500 index put warrant, which has a periodic reset feature. Hsueh and Gou (1998), on the other hand, analyze the multiple reset feature that is included in most covered warrants traded in Taiwan. More recently, Cheng and Zhang (2000) discuss the pricing and hedging of reset options and propose a closed-form pricing formula for this increasingly popular derivative instrument.

Previous studies of reset options, however, also point out several common weaknesses associated with the reset design. Specifically, sellers of reset option face a challenging task of risk management. This is because when the reset condition is triggered, a reset option immediately turns into a vanilla option and thus produces a discontinuous delta for option sellers who hedge dynamically. Moreover, price manipulations usually intensify when the underlying is near the reset barrier, resulting in heightened market volatility and increased hedging costs.

It is worth noting that reset options are often likened to barrier options that are also popular in derivative markets. The values of both barrier and reset options are closely tied to whether the barrier condition is triggered. The consequence of barrier triggering, however, is quite different for these two types of options. Specifically, when the underlying crosses the pre-specified price level, a reset option takes on a new strike price while the barrier option begins or ceases to exist. Nevertheless, sellers of both options face similar hedging problems due to the existence of barriers in the option contract.

To alleviate the discontinuity of delta associated with barrier-type options, a family of Parisian-type options has been introduced into the market. Parisian options are a modified version of standard barrier options, where the knock in or knock out condition is triggered only after the underlying has spent a certain amount of time beyond the barrier. This novel specification has the effect of smoothing the option value and its delta (and gamma). Furthermore, the delayed triggering arrangement makes price manipulation of the underlying around barriers much more difficult.

[1]Many variants of reset design appear in the market, including multiple resets, reset with a floor, and reset based on the minimum of past prices (i.e., lookback reset), etc.

Motivated by the contract design of Parisian options, we propose in this study a step-reset option where the exercise price resets *gradually* as the underlying crosses the reset barrier. This novel design offers at least two important advantages to both issuers and investors. First, the step-reset specification rules out jumps in delta and thus makes reset options more amenable to dynamic hedging. Consequently, it greatly reduces the tension of price manipulation around the reset level. Furthermore, step-reset options command a much lower premium than their standard reset counterpart.[2] This is because investors pay only for the protection they really need (i.e., the underlying breaches the reset level and remains there), not for the possible windfall gains due to a sudden excursion of the underlying that touches off the reset condition.

STEP-RESET DESIGN

Consider a single reset call option with original strike price X and time to expiration T.[3] Let S be the underlying stock price and B the prescribed reset (barrier) level, the option will take on a lower strike price, X_N, anytime the underlying drops below B before the reset period τ runs out.[4] For a step-reset design, we define τ_B as the excursion time, which is the cumulative length of time during the reset period that the underlying touches or stays below the reset level, and the actual (realized) strike price (X^*) of a step-reset option is determined by the following formula:

$$X^* = X_N(\tau_B/\tau) + X(\tau - \tau_B)/\tau \qquad (1)$$

It is easy to see that if the underlying never reaches the reset level, τ_B is equal to 0 and the step-reset option reduces to a vanilla call option with the original strike price X. To the other extreme, if the excursion time equals the whole reset period, i.e., $\tau_B = \tau$, the step-reset option will be no different from a vanilla option with a lower strike price X_N.

[2]The main disadvantage of a reset option from a marketing standpoint is its cost. On average, the premium for reset options is about 6 to 10% higher than that of similar options without reset feature.

[3]Step-reset put options can be structured in a similar fashion. For brevity, we focus our discussion on call options in this paper.

[4]Most reset warrants traded in the market have reset period shorter than the life of option. If the reset period covers the whole length of option maturity, or $\tau = T$, then a reset call can be viewed as a combination of a knock-out option with strike X and a knock-in option with strike X_N, with both having the same barrier level.

Many other variations of a step-reset design are possible. For instance, one can specify additional reset levels to provide investors with added protection if so desired. For a two-level step-reset design, we can let X_{N1} and X_{N2} be the new exercise prices if the underlying falls below reset barriers $B1$ and $B2$, respectively. Further define excursion times τ_{B1} and τ_{B2} as the cumulative time that the underlying stays below $B1$ (but above $B2$) or $B2$, respectively, then the actual exercise price of this two-level step-reset option can be determined as

$$X^* = X_{N1}(\tau_{B1}/\tau) + X_{N2}(\tau_{B2}/\tau) + X(\tau - \tau_{B1} - \tau_{B2})/\tau \tag{2}$$

With multiple reset level specification, investors receive more protection from an adverse movement of the underlying price because the reset will be triggered sooner. In other words, the underlying does not have to drop as much to kick off the reset protection as in the one-level reset call design.

VALUATION OF STEP-RESET OPTIONS

While no prior study on the pricing of step-reset options is available, previous research on Parisian options may offer some insights. Chesney, Jeanblanc-Picque, and Yor (1997) examine the excursion of the underlying price process beyond a given barrier level, and define the knockout condition of a Parisian option as the underlying that stays *continuously* below the barrier for a prescribed time interval. They derive an analytical expression for the Laplace transform of a relevant probability density and price such Parisian options by a numerical inversion of the Laplace transform.

The requirement of a continuous price excursion may not be practical for our purpose of having a smooth exercise price reset process, since the reset condition will be easily interrupted due to short-term price movements. Linetsky (1998) considers a Brownian motion with killing at a finite rate ρ below the barrier, and defines the option payout by the formula: $e^{-\rho\tau*}\max(S_T - X, 0)$, where $\tau*$ is the *total* amount of time during the option life that the underlying price gets below the barrier. This specification does not require the underlying to stay continuously below the barrier as the original Parisian specification and is more in line with the spirit of our step-reset design. Since the knock-in/knock-out feature is activated only if the cumulative time spent beyond barriers exceeds some prescribed quantity, this specification resembles the averaging feature of an Asian option and is referred by some as the *Parasian* option.[5] Linetsky (1998) expresses the value of this variation of Parisan options as the discounted conditional expectation under the risk-neutral measure: $e^{-rT}E[e^{-\rho\tau*}\max(S_T - X, 0)]$. Then, using the Feynman-Kac approach, he solves the corresponding parabolic partial differential equation and obtains closed-form pricing solutions for a family of Parasian version of barrier options.

Closed-form solutions are available, however, only for European-type Parasian options. For options with early exercise rights or other contract modifications, numerical methods may be the only alternative. Recently, Haber, Schonbucher, and Wilmott (1999) advocate an explicit finite difference approach to solve the relevant partial differential equations and obtain the price and hedging variables for both Parisian and Parasian options. Like Parasian options, our proposed step-reset options are strongly path-dependent. The option payoff depends not only on the underlying price movements at maturity, but also on the path taken to get there because the option's exercise price is contingent upon the cumulative time spent below the reset barrier. Consequently, a closed-form solution for the resulting partial differential equation is difficult to obtain, if not impossible.[6] To value step-reset options, we employ a tree-based approach which offers enough flexibility to accommodate many contract details that are specific to the step-reset design.

A Trinomial Lattice Model for Step-Reset Options

Boyle and Lau (1994) point out that when pricing barrier-type options with tree-based models, it is critical to place a layer of price nodes exactly on the barrier, otherwise, erratic and wavy convergence pattern can occur and significant pricing error can persist even with a large number of time steps. Ritchken (1995) takes advantage of the flexibility offered by the trinomial model and proposes an ingenious way to "stretch" the tree so that a layer of price nodes can coincide exactly with the barrier. Formally, assume that the underlying price follows a geometric Wiener process with a drift rate $\mu = r - \sigma^2/2$, where r is the risk free interest rate and σ is the instantaneous volatility. Let the asset price at time t as $S(t)$, then $\ln S(t + \Delta t) = \ln S(t) + \Delta X(t)$, where $\Delta X(t)$ follows a normal distribution

[5] See, e.g., Harber, Schonbucher, and Wilmott (1999).

[6] When the reset period (τ) is specified to be shorter than the life of option (T), as most of the reset warrants we observe in the market are, the pricing problem is further complicated.

with mean $\mu\Delta t$ and variance $\sigma^2\Delta t$. To approximate $\Delta X(t)$ with a three-jump discrete variable, we can let

$$\Delta X(t) = \begin{cases} \lambda\sigma\sqrt{\Delta t} & \text{with probability } Pu \\ 0 & \text{with probability } Pm \\ -\lambda\sigma\sqrt{\Delta t} & \text{with probability } Pd \end{cases} \tag{3}$$

where λ is called a stretch parameter. To ensure that the discrete distribution of asset price converges to the continuous-time limit of a lognormal distribution, we match the first two moments of the trinomial distribution to their continuous-time counterparts, that is

$$E[\Delta X(t)] = \mu\Delta t = Pu(\lambda\sigma\sqrt{\Delta t}) + Pm(0) + Pd(-\lambda\sigma\sqrt{\Delta t}) \tag{4}$$

$$\begin{aligned} E[(\Delta X(t))^2] &= \sigma^2\Delta t + \mu^2\Delta t^2 \\ &= Pu(\lambda\sigma\sqrt{\Delta t})^2 + Pm(0)^2 + Pd(-\lambda\sigma\sqrt{\Delta t})^2 \end{aligned} \tag{5}$$

together with the constraint that $Pu + Pm + Pd = 1$, we have three equations and three unknowns. As dt approaches zero, the term containing Δt^2 can be ignored without losing generality, hence the system can be solved to give the probabilities as:

$$\begin{aligned} Pu &= \frac{1}{2\lambda^2} + \frac{\mu\sqrt{\Delta t}}{2\lambda\sigma} \\ Pm &= 1 - \frac{1}{\lambda^2} \\ Pd &= \frac{1}{2\lambda^2} - \frac{\mu\sqrt{\Delta t}}{2\lambda\sigma} \end{aligned} \tag{6}$$

The stretch parameter λ is used to control the distance between layers of price node in the lattice. Note that when $\lambda = 1$, Pm equals 0 and the trinomial lattice collapses to the standard binomial lattice. Also, to prevent the probabilities from turning negative, we restrict λ to be greater than 1.

To proceed with the pricing of step-reset call options, we can initially set the stretch parameter to one, thus the magnitude of one up or down movement is equal to $\sigma\sqrt{\Delta t}$. Assume that the reset level is B, and the spot price S is above the barrier, the number of down movements required to reach the reset barrier can be calculated as

$$N = \frac{\ln(S/B)}{\sigma\sqrt{\Delta t}} \tag{7}$$

or if $S < B$, the number of up movements to advance beyond the barrier level is[7]

$$N = \frac{\ln(B/S)}{\sigma\sqrt{\Delta t}} \tag{8}$$

Finally, to ensure that the reset barrier coincides with a layer of price nodes, we can calculate the appropriate stretch parameter as

$$\lambda = \frac{N}{\text{int}(N)} \tag{9}$$

where $\text{int}(N)$ represents the largest integer smaller than N.

Discretely Monitored Step-Reset Barrier

The monitoring of whether the reset barrier is breached is certainly of great importance to both option issuers and investors. Although a continuous barrier design may facilitate pricing, the practical implementation of continuous monitoring is not possible in the actual marketplace. Specifically, to trade reset options, it is critical that investors know the prevailing exercise price in order to value these options correctly, but it may not be feasible to keep all investors informed concurrently with any triggering of reset condition during intraday trading. To rule out any ambiguity, the usual practice, as adopted in Taiwan's warrant market, is to use the underlying's daily closing price as the benchmark and intraday transgressions of the reset barrier, if any, do not constitute reset incidence. Had the option's exercise price been changed due to reset triggering, it is publicly announced before the market reopens the next trading day. To make practical sense and follow market convention, our proposed step-reset option adopts the discrete (daily) monitoring of reset conditions so that the adjustment of exercise price, if any, is made after the market closes.

Treating discrete barrier as if it is continuous will overstate the probability of barrier triggering and lead to erroneous pricing.[8] To accommodate discrete monitoring within a tree-based framework, one can set the time step equal to the monitoring frequency and insert intraday time steps to improve the convergence of the lattice model. This

[7]This situation applies only to step-reset options. For a standard one-reset call, it turns into a vanilla option once the reset condition is triggered (i.e., $S < B$) and no more matching the barrier with a layer of price node is necessary.

[8]Previous research, including Chance (1994) and Kat and Verdonk (1995), has shown that substantial price differences exist between discrete and continuous barrier options.

brute-force method, however, can be very time-consuming and computationally inefficient as it requires a very large number of intraday time steps to obtain a reasonable accuracy level.

Cheuk and Vorst (1996) suggest a way to correct for the discontinuity problem. In short, they show that accurate results can be obtained by positioning the barrier in the middle of two layers of price nodes and this choice "does justice to the discontinuity of the value function and produces fast convergence."[9] This adjustment can be easily incorporated into our model by redefining the stretch parameter λ in Equation 9 as

$$\lambda = \frac{N}{\text{int}(N) - 0.5} \tag{10}$$

so that the reset barrier will be positioned exactly halfway between price nodes with $\text{int}(N) - 1$ and $\text{int}(N)$ net down movements, respectively.

State-to-State Recursive Pricing Algorithm

A major obstacle of pricing step-reset options is the uncertainty of its exercise price. From Equation 1, it is clear that the option's exercise price will depend upon the length of time the underlying stays below the reset level during the reset period. For discretely-monitored reset barrier, however, this problem turns out to be quite manageable since the number of possible exercise prices (hereafter, the number of states) does not grow exponentially.[10]

Consider a daily-monitored step-reset option with a reset period of τ days, we set the time step of the lattice model to be one day and define $S(i, j)$ as the underlying price at the jth node in time i. Depending on the price path taken to reach that node, the maximum number of possible states for the node (i, j) is simply $i + 1$, for $i \leq \tau$, and $\tau + 1$ for $\tau < i \leq T$. It is quite obvious that with multiple exercise prices for each price node, the traditional node-to-node backward induction pricing procedure no longer works for step-reset option valuation. To accommodate multiple states within each node, we modify the valuation procedure by

[9] Ahn, Figlewski, and Gao (1999) propose an adaptive mesh (trinomial) model to price discrete barrier option and Tian (1999) develops a flexible binomial model to account for the discreteness in barrier monitoring. Broadie, Glasserman, and Kou (1997), on the other hand, suggest a barrier level adjustment to enable the use of closed-form solutions for discrete barrier options. None of the models, however, is readily extendable to discrete step-reset options.

[10] With continuous monitoring, on the other hand, the number of possible exercise prices for step-reset options goes to infinity.

tracking instead the state-to-state recursive relationship between adjacent daily nodes across time.

To explore the recursive relationship between alternative states across time within a trinomial tree framework, we have to distinguish between different exercise prices at node (i, j). To this end, we designate k as the state variable which tracks the excursion time in days; i.e., the total number of days the underlying falls below the reset level. Define $C_{i,j,k}$ as the value of a step-reset option for the kth state at node (i, j). For each node (T, j) at the option maturity, there are $\tau + 1$ possible states representing an equal number of possible exercise prices resulting from the step-reset design. For each of the $\tau + 1$ states at node (T, j), backward induction is performed to calculate the option value for the same state of the corresponding node one step back. Figure 1 illustrates this state-to-state backward induction procedure within a trinomial lattice framework.

The state-to-state pricing procedure is carried out for every state of each node recursively from time period $T - 1$ to $\tau + 1$, and we can express the recursive pricing equation as follows:

$$C_{i,j,k} = [p_d C_{i+1,j,k} + p_m C_{i+1,j+1,k} + p_u C_{i+1,j+2,k}]$$

$$\text{for } i = T - 1 \text{ to } \tau + 1$$

Starting from time τ (i.e., the end of reset period) back to the current time 0, the recursive relationship needs to be modified to account for the relative position of the underlying at node (i, j) against the reset

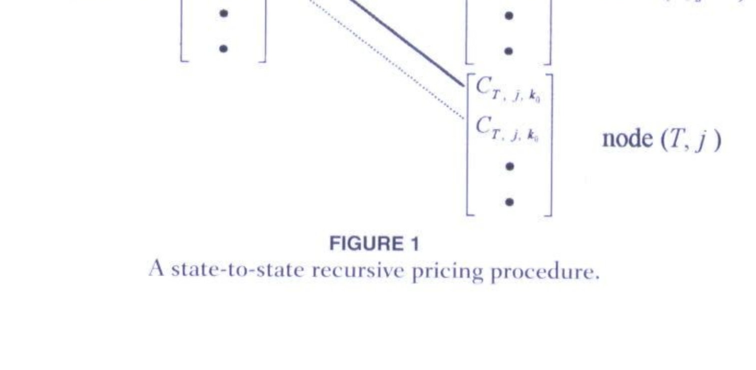


FIGURE 1
A state-to-state recursive pricing procedure.

level. Specifically, we know that $S(i, j)$ will move to $S(i + 1, j)$, $S(i + 1, j + 1)$, and $S(i + 1, j + 2)$ following a down-, level-, and up-movement in the underlying, respectively. When $S(i, j)$ is below the reset level, a down- or level-movement will keep $S(I + 1, j)$ and $S(i + 1, j + 1)$ below the reset; hence the state variable k associated with the node will increase by 1, which means one more day the underlying is below the reset. As for the up-movement, if $S(i + 1, j + 2)$ is still below the reset, the associated state variable will also become $k + 1$. However, if it moves above the reset level, the value of state variable k remains the same since there will be no increase in the time spent below barriers.[11]

We use $C_{i,j,k}$ to represent the option value where the underlying has spent $k - 1$ days below the reset barrier, given that $S(i, j)$ is above the reset level. On the other hand, if $S(i, j)$ is below the reset, $C_{i,j,k}$ is the option value reflecting that the underlying has spent k days below the reset level. We can then express the recursive pricing equation for step-reset option from time τ to 0 as

$$C_{i,j,k} = [p_d C_{i+1,j,k+1} + p_m C_{i+1,j+1,k+1} + p_u C_{i+1,j+2,k+1}] \qquad \text{for } S(i, j) \leq \text{reset level}$$

$$C_{i,j,k} = [p_d C_{i+1,j,k} + p_m C_{i+1,j+1,k} + p_u C_{i+1,j+2,k}] \qquad \text{for } S(i, j) > \text{reset level}$$

If the step-reset option has American exercise right, our tree-based model can handle it easily by comparing the early exercise value at node (i, j, k) with $C_{i,j,k}$ derived from the backward induction, and set $C_{i,j,k}$ equal to the greater of the two before the backward induction continues.

RESULTS AND DISCUSSIONS

Since no closed-form solution exists for step-reset options, we calculate Monte Carlo estimates to serve as our benchmark for testing the accuracy of our trinomial model. We assume a European step-reset option with one-year maturity, and reset periods of both 1 month and 2 months are examined separately. The Monte Carlo simulations are based on 264 time steps (representing as many trading days in a year) and 30,000 simulation runs. For each of the first 22 or 44 days during the reset period, the simulated underlying price is checked for the possible triggering of reset conditions.

[11]The situation of $S(i, j)$ being above the reset level can be analyzed in the similar way and is not repeated here for brevity.

TABLE I

Accuracy Tests of the Trinomial Lattice Model for Step-Reset Options

Spot Price		*Reset Period = 1 month*			*Reset Period = 2 months*		
		$\sigma = 0.2$	$\sigma = 0.5$	$\sigma = 0.8$	$\sigma = 0.2$	$\sigma = 0.5$	$\sigma = 0.8$
100	Our model	10.481	22.253	33.520	10.558	22.437	33.649
	MC	10.481	22.251	33.518	10.558	22.434	33.646
		(0.001)	(0.004)	(0.005)	(0.002)	(0.005)	(0.005)
95	Our model	7.793	19.594	30.499	7.960	19.702	30.546
	MC	7.793	19.593	30.499	7.959	19.697	30.545
		(0.003)	(0.006)	(0.005)	(0.004)	(0.005)	(0.005)
90	Our model	6.782	17.423	27.686	6.637	17.320	27.607
	MC	6.782	17.420	27.690	6.633	17.321	27.602
		(0.007)	(0.006)	(0.005)	(0.006)	(0.006)	(0.005)
85	Our model	6.042	15.407	24.984	5.651	15.108	24.784
	MC	6.043	15.406	24.983	5.650	15.105	24.776
		(0.004)	(0.005)	(0.005)	(0.005)	(0.005)	(0.005)
80	Our model	4.131	13.195	22.274	3.998	12.875	22.012
	MC	4.134	13.193	22.270	3.995	12.871	22.017
		(0.005)	(0.004)	(0.004)	(0.006)	(0.005)	(0.005)

Note. Option parameters are $X = 100$, $X_N = 90$, $T = 1$ year, $r = 0.05$, B (the reset level) $= 90$, and 264 trading days per year (22 days per month). Our trinomial tree model uses 3960 time steps (i.e., 264 daily time steps and 15 intraday time steps). The MC (Monte Carlo) simulations are based on 264 time steps and 30,000 trials, using the estimates from standard reset options of same parameters as the control variate for variance reduction purpose. The standard errors of the MC estimates are shown in parentheses.

If the underlying is equal to or below the reset level, one day is added to the excursion time. The total excursion time at the end of reset period is then used to determine the option's exercise price so that the value of step-reset option can be calculated for that run of simulation. To improve simulation efficiency, we use the estimate of the standard reset option as the control variate for variance reduction purpose. Our trinomial tree model also uses 264 daily time steps, but between daily time steps, we insert 15 intraday steps (m) to improve convergence of our model.[12] Hence, the total number of time steps for the tree is actually $n \times m = 3960$.

Table I presents the test results of step-reset options with daily monitoring, for five different levels of underlying stock prices and three volatility scenarios. As can be seen, our model yields nearly identical results to those of Monte Carlo simulations. The mean of the absolute price differences between the two models is only \$0.002 and none of the result rejects the hypothesis that our tree estimates are the true option values.

[12]In fact, very fast convergence of our model is achieved with less than 5 intraday time steps. A larger number of intraday time steps are needed only for situations where the underlying is very close to the reset barrier.

It is worth noting that, had the adjustment for discrete monitoring not been made, that is, by not placing the barrier between two layers of price nodes but exactly over a layer of nodes (as if the barrier is continuously monitored), the estimated values can be significantly biased. Table II compares the option values using the same trinomial model without correcting for the discrete step-reset barrier, with that from Monte Carlo simulations. As shown in the table, the tree estimates are all statistically different from the Monte Carlo values at the 1% significance level. These findings demonstrate the importance of correctly positioning the reset barrier in a tree-based model and lend support to the discrete barrier adjustment method suggested by Cheuk and Vorst (1996).

When the spot price happens to equal the reset barrier, it will be no longer possible to place the reset barrier between two layers of price

TABLE II

Pricing Errors When the Discreteness of Step-Reset Barrier Is Ignored

Spot	$\sigma = 0.2$			$\sigma = 0.5$			$\sigma = 0.8$		
Price	*CT*	*MC*	*Diff.*	*CT*	*MC*	*Diff.*	*CT*	*MC*	*Diff.*
				Panel A: Step-Reset Period = 1 month					
100	10.491	10.481 (0.001)	0.011	22.400	22.251 (0.004)	0.148	33.472	33.518 (0.005)	−0.046
95	7.723	7.793 (0.003)	−0.070	19.978	19.593 (0.006)	0.385	30.763	30.499 (0.005)	0.264
90	6.542	6.782 (0.007)	−0.240	17.334	17.420 (0.006)	−0.086	27.417	27.690 (0.005)	−0.274
85	6.141	6.043 (0.004)	0.098	15.740	15.406 (0.005)	0.334	25.195	24.983 (0.005)	0.211
80	4.123	4.134 (0.005)	−0.011	13.309	13.193 (0.004)	0.116	22.420	22.270 (0.004)	0.150
				Panel B: Step-Reset Period = 2 month					
100	10.587	10.558 (0.002)	0.028	22.578	22.434 (0.005)	0.144	33.569	33.646 (0.005)	−0.076
95	7.877	7.959 (0.004)	−0.082	20.010	19.697 (0.005)	0.313	30.743	30.545 (0.005)	0.198
90	6.429	6.633 (0.006)	−0.204	17.196	17.321 (0.006)	−0.125	27.403	27.602 (0.005)	−0.200
85	5.765	5.650 (0.005)	0.115	15.391	15.105 (0.005)	0.286	24.951	24.776 (0.005)	0.175
80	3.969	3.995 (0.006)	−0.026	12.997	12.871 (0.005)	0.126	22.144	22.017 (0.005)	0.127

Note. Option parameters are $X = 100$, $X_N = 90$, $T = 1$ year, $r = 0.05$, $B = 90$, and 264 trading days per year (22 days per month). The CT (continuous-barrier) trinomial model matches the barrier exactly with a layer of price nodes and monitor reset condition on daily nodes. The MC (Monte Carlo) simulations have 30,000 runs and use a control variate method for variance reduction. The standard errors of the MC estimates are shown in parentheses.

nodes. To overcome this problem, we perturb the spot price and calculate the value of step-reset option for the underlying prices slightly above and below the spot price. The correct option value at the spot price level is then obtained through interpolation. Option values at $S = 90$ in Table I are obtained using this procedure. For reset period of 1-month and volatility level of 0.5, for example, we calculate option values for both $S = 90.5$ and 89.5, and they are 17.621 and 17.225, respectively. The step-reset option value at spot price of 90 is then calculated as $(17.621 + 17.225)/2 = 17.423$, which is very close to the Monte Carlo estimate of 17.430, by a difference of only \$0.003.

Characteristics of Step-Reset Option Values

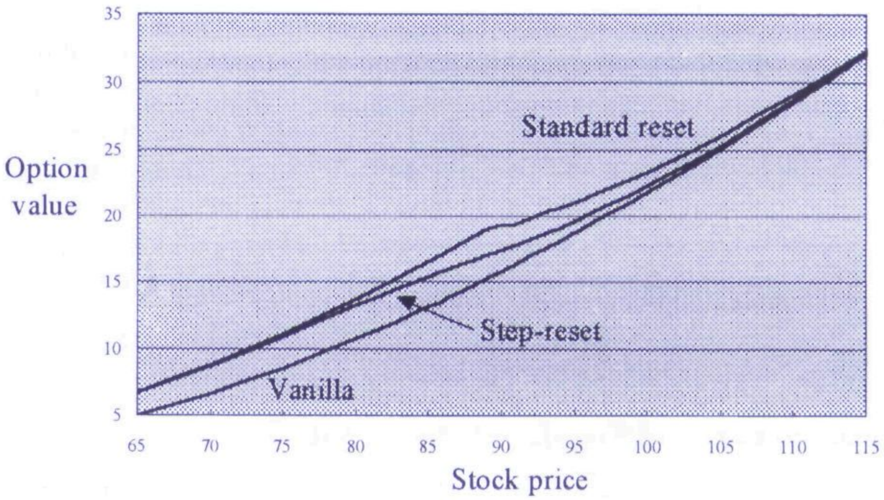
Table III shows the values of step-reset options across different levels of underlying price and volatility. For comparison purpose, we also calculate the values for both the standard reset option (trigger if touched) and the vanilla option (no reset). To facilitate discussion, we give a graphical display for the case of $\sigma = 0.5$ in Figure 2.

TABLE III

Value of Step-Reset, Standard Reset, and Vanilla Options Across Underlying Stock Prices

S	*Vanilla Option*	*Step Reset*	*Premium*	*Standard Reset*	*Premium*
60	3.576	4.936	38.04%	4.938	38.09%
65	4.964	6.687	34.71%	6.699	34.95%
70	6.621	8.700	31.40%	8.752	32.20%
75	8.544	10.907	27.66%	11.088	29.77%
80	10.726	13.195	23.01%	13.688	27.62%
85	13.156	15.407	17.11%	16.537	25.69%
87	14.195	16.235	14.37%	17.741	24.98%
89	15.270	17.033	11.55%	18.981	24.30%
90	15.821	17.488	10.54%	19.268	21.79%
91	16.381	17.819	8.78%	19.413	18.51%
93	17.526	18.669	6.52%	20.224	15.39%
95	18.705	19.594	4.75%	21.022	12.39%
100	21.793	22.253	2.11%	23.296	6.90%
105	25.068	25.296	0.91%	25.977	3.63%
110	28.515	28.624	0.38%	29.032	1.81%
115	32.119	32.169	0.16%	32.396	0.86%
120	35.865	35.888	0.06%	36.007	0.39%

Note. Option parameters are $X = 100$, $X_N = 90$, $T = 1$ year, $r = 0.05$, $\sigma = 0.5$, and $B = 90$. Vanilla option values are derived from B/S closed-form formula. Standard reset values are calculated using the same trinomial tree model but with "reset if touched" specification and the step-reset values are based on "gradual reset" specification. Premium figures are option premium over that of vanilla option in percentage term.

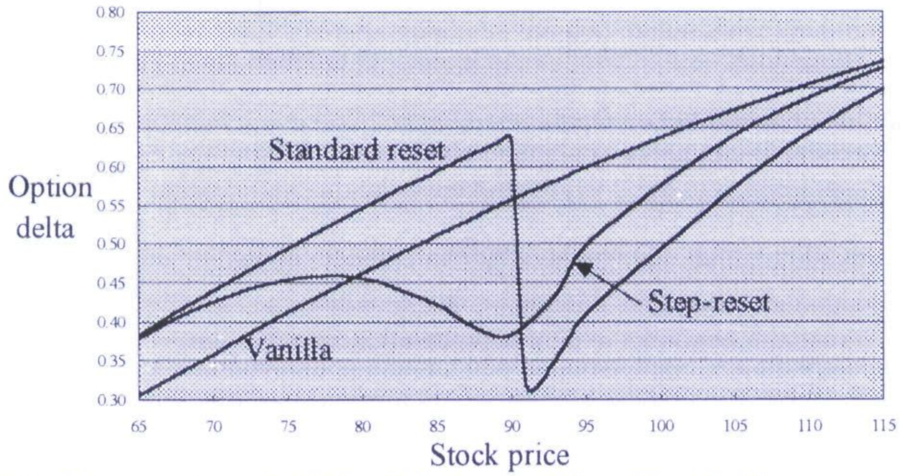


Note. Option parameters are $X = 100$, $X_N = 90$, $T = 1$ year, $r = 0.05$, $\sigma = 0.5$, and B (the reset level) equals 90.

FIGURE 2
Comparison of prices among step-reset, standard reset, and vanilla call options.

As can be seen clearly from the graph, the value of step-reset option falls in the middle of standard reset and vanilla options. Due to the reset feature, the values of both standard reset and step-reset options are always higher than that of the vanilla option. The step-reset feature, however, does not generate as much value when the underlying is above the reset level ($B = 90$ in our example), and that makes step-reset options behave more like a vanilla option. In fact, as the underlying moves higher, the reset feature loses its advantage and even standard reset option converges to vanilla option.

On the other hand, when the underlying price heads lower, the chance of its triggering and staying below the reset level increases, and the step-reset option becomes more and more like a standard reset option. This pattern of gradual increase in the option value indicates that investors are protected when the underling is heading toward the wrong direction. More importantly, its value preservation function is tied to the probability of the underlying staying below a prescribed level, that is, the higher the probability, the stronger the protection. This is in sharp contrast to the standard reset design where the protection is granted even if the underlying just skirts the reset level and rebounds afterwards. In terms of option premium, therefore, the step-reset design allows investors to pay for the protection that better fits their need. Table III shows that the premium for an at-the-money step-reset option ($S = 100$)



Note. Option parameters are $X = 100$, $X_N = 90$, $T = 1$ year, $r = 0.05$, $\sigma = 0.5$, and $B = 90$.

FIGURE 3
Delta of step-reset, standard reset, and vanilla call options.

TABLE IV
Delta of Step-Reset, Standard Reset, and Vanilla Options Across Underlying Stock Prices

Underlying Price	*Standard Reset*	*Step Reset*	*Vanilla*
60	0.322	0.322	0.251
65	0.382	0.378	0.304
70	0.439	0.425	0.358
75	0.494	0.454	0.411
80	0.546	0.455	0.462
85	0.593	0.422	0.510
87	0.611	0.397	0.528
89	0.628	0.380	0.547
90	0.637	0.387	0.555
91	0.317	0.395	0.564
93	0.354	0.436	0.581
95	0.409	0.499	0.598
100	0.492	0.574	0.637
105	0.572	0.640	0.673
110	0.643	0.689	0.706
115	0.699	0.728	0.735
120	0.743	0.759	0.763

Note. Option parameters are $X = 100$, $X_N = 90$, $T = 1$ year, $r = 0.05$, $\sigma = 0.5$, and $B = 90$. Delta for vanilla option are obtained from B/S closed-form formula, delta for standard reset option are calculated based on "reset if touched" specification, and figures for step-reset option are based on "gradual reset" specification; both are calculated using the trinomial tree model developed in this study.

is only 2% higher than that of the equivalent vanilla option, but it is nearly 7% higher for a standard reset option.

Delta of Step-Reset Options

Figure 3 illustrates the delta of step-reset options at various underlying stock prices and contrasts that with the standard reset and vanilla options, respectively. Numerical values for Figure 3 are given in Table IV. The familiar downward sloping delta curve of vanilla option poses a sharp contrast to the sudden delta jump near the reset barrier ($B = 90$) for the standard reset option. This highlights the risk management problem associated with standard reset options around the prescribed barrier. With the step-reset design, however, the spike in the delta is replaced by a much smoother U-shaped curve. This added continuity of delta around reset level greatly facilitates option sellers to hedge step-reset options dynamically by trading the underlying stock.

CONCLUSIONS

Derivative products with the exercise price reset feature appeared in the market a few years ago and have since gained wide acceptance and popularity. Similar to barrier options, the reset specification creates hedging problems for option sellers and induces potential manipulations of the underlying price around reset levels. In this study, we propose an improved and innovative reset design called step-reset options that allow us to retain the key attribute of a reset option but do without the disturbing risk management problems associated with it.

From a marketing standpoint, step-reset options command a much lower premium but preserve the same attractive merit of exercise price reset, and thus should generate wide acceptance in the market. In terms of risk management, step-reset options exhibit the desirable attribute of continuous delta and thus facilitate option sellers who hedge dynamically. Furthermore, the step-reset feature makes the option more robust against short-term movements of the underlying, and this can lessen the possibility of price manipulation around reset barrier and improve overall market stability.

To price step-reset options, we develop a tree-based valuation algorithm in this study. Our trinomial tree model is parameterized to specifically account for the discrete nature of reset monitoring. The use of lattice model also gives us the flexibility to handle American-style step-reset options with ease. To accommodate the path-dependent exercise price that step-reset options possess, we modify the traditional backward induction method and conduct instead a state-to-state recursive pricing procedure to properly capture the step-reset effect. Using Monte Carlo estimates as the benchmark, our trinomial tree model is shown to perform with great accuracy and efficiency.

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