RISK AVERSION, DISAPPOINTMENT AVERSION, AND FUTURES HEDGING

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This article examines the effect of disappointment aversion on futures hedging. We incorporated a constant-absolute-risk-aversion (CARA) utility function into the disappointment-aversion framework of Gul (1991). It is shown that a more disappointment-averse hedger will choose an optimal futures position closer to the minimum-variance hedge than will a less-disappointment-averse hedger. The effect of disappointment aversion is stronger when the hedger is less risk averse. A small disappointment aversion will cause a near-risk neutral hedger to take a drastically different position. In addition, a more-risk-averse or disappointment-averse hedger will have a lower reference point. Numerical results indicate that the reference point of a disappointment-averse hedger tends to be lower than that of a conventional loss-averse hedger. Consequently, the disappointment-averse hedger will act more conservatively, not exploiting profitable opportunities as much as the conventional loss averse hedger will. © 2002 John Wiley & Sons, Inc. Jrl Fut Mark 22:123–141, 2002

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INTRODUCTION

Friedman–Savage standard expected-utility (EU) framework is adopted widely in studying behaviors under uncertainty (Friedman & Savage, 1948, 1952). While alternative nonexpected-utility (NEU) approaches were available, most researchers continued to rely upon EU analysis. In recent years, several newly found market anomalies promoted research in NEU analysis. Among them, the prospect theory of Kahneman and Tversky (1979) was found to be useful in resolving the equity premium puzzle (see Benartzi & Thaler, 1995).

Central to the prospect theory is the concept of loss aversion. An individual is loss averse if the loss is weighted more than the gain. The reference point is either status quo or some other predetermined points. In several experiments, the subjects were found to exhibit loss aversion over endowments (see, for example, Thaler, Tversky, Kahneman, & Schwartz, 1997; Koonce, McAnally, & Mercer, 2000). Odean (1998) demonstrated that stock-market investors are reluctant to realize losses. Heisler (1996) and Locke and Mann (1999) found futures traders to be loss averse as well. Shumway (1997) derived implications of loss aversion for capital-asset pricing models and provided empirical support. Gomes (2001a) and Berkelaar and Kouwenberg (2000a, 2000b) analyzed the optimal portfolio choices under loss aversion. Albuquerque (1999) and Lien (2000) considered the effects of loss aversion on optimal futures hedging.

A predetermined reference point, however, is less than satisfactory. The disappointment-aversion framework proposed by Gul (1991) resolves this problem. Specifically, the disappointment-aversion utility function is a one-parameter extension of the conventional utility function. It maintains the loss-aversion property such that any outcome above a reference point is down-weighted. The reference point is determined endogenously to be the certainty equivalent of the expected outcome. Ang, Bekaert, and Liu (2000) compared the two approaches and discussed undesirable properties of the loss-aversion utility function relative to the disappointment-aversion utility function. Among them, the latter is axiomatic and normative, whereas the former is ad hoc.

Aizenman (1998) applied disappointment aversion to analyze the precautionary saving decision problem. Aizenman and Marion (1999) demonstrated that the disappointment-aversion preference could generate first-order negative effects of volatility on investment, which is consistent with the observed fact. On the other hand, the traditional expected utility theory predicts only a second-order effect of volatility. Ang et al. (2000) constructed a dynamic model within the disappointment-aversion framework to explain the large variations in the cross-section of equity holdings and the equity premium puzzle. Lien (2001b) examined optimal futures hedging under disappointment aversion, assuming that the spot price had only two possible realizations.

This article extends the above study by allowing spot and futures prices to be continuous. Specifically, it is assumed that the two prices are jointly normally distributed. For analytical purposes, we assume a constant-absolute-risk-aversion (CARA) utility function for the hedger. In the absence of disappointment aversion, the distribution assumption and the utility-function specification reduces the expected utility analysis to the mean–variance method. The effect of disappointment aversion is characterized by the divergence of the optimal futures position from that prescribed by the mean–variance consideration. We show that, under a backwardation or a contango equilibrium, both disappointment aversion and risk aversion act to reduce speculation intensity. Either aversion promotes the hedger to become more cautious and undertake a more conservative action. Risk aversion changes the curvature of the utility function. The more risk averse the hedger, the more concave is his utility function. Disappointment aversion changes how the hedger weighs the bad outcomes. The effect of disappointment aversion strengthens as the risk aversion decreases. Thus, disappointment aversion may reduce greatly the trading volume for a near-risk-neutral hedger.

Another interesting finding concerns the reference point. As the hedger becomes more risk averse or disappointment averse, he will choose a lower reference point. In addition, numerical results indicate that the reference point of a disappointment-averse hedger tends to be lower than that of a conventional loss-averse hedger. Thus, the disappointment-averse hedger will act more conservatively, not exploiting profitable opportunities as much as will the conventional loss-averse hedger.

UTILITY-FUNCTION SPECIFICATION

Consider a one-period model. An individual has a unit of long position in the spot market. He will assume a futures position to maximize his expected utility over the end-of-period wealth. Let Δp and Δf denote changes in spot and futures prices in this period, respectively. Given the initial wealth W_0, the end-of-period wealth is:

$$W_1 = W_0 + \Delta p + Q\Delta f, \tag{1}$$

where Q is the futures position. When $Q > 0$, the hedger buys futures contracts; when $Q < 0$, he sells futures contracts. In the following discussion, we find the optimal futures position to be negative (i.e., a short futures position). Thus, a smaller Q indicates a larger short position. The hedger has a constant-absolute-risk-aversion (CARA) utility function:

$$u(x) = -\exp(-\theta x), \tag{2}$$

where $\theta > 0$ is the risk-adverse coefficient. The utility function is strictly increasing and strictly concave.

Disappointment-aversion-utility function, denoted by $\Psi_{u,\beta}(.)$, is defined as follows:

$$\Psi_{u,\beta}(W_1) = u(W_1) \qquad \text{if } W_1 \leq V, \tag{3a}$$

$$\Psi_{u,\beta}(W_1) = \frac{u(W_1) + \beta u(V)}{1 + \beta} \qquad \text{if } W_1 > V; \tag{3b}$$

where β is called the disappointment-aversion coefficient and V is defined as follows: $u(V) = \mathrm{E}\lfloor \Psi_{u,\beta}(W_1) \rfloor$, the expectation of $\Psi_{u,\beta}(W_1)$. Note that V is the reference point such that any outcome better than V is discounted. Alternatively, let $g(W)$denote the probability density function of W, it can be shown that

$$u(V) = E[u(W)] - \beta \int_{-\infty}^{V} [u(V) - u(W)] g(W) dW. \tag{4}$$

When $\beta = 0$, $u(V) = \mathrm{E}[u(W)]$, the traditional expected utility function. If $\beta > 0$, the hedger is disappointed by losses such that there is a loss of utility $[u(V) - u(W)]$ whenever $W < V$. Thus, β measures the degree of disappointment aversion: the larger β, the more weight is applied to the loss of utility.

Suppose that Δp and Δf are jointly normally distributed with $\mathrm{E}(\Delta p) = \mu_p$, $E(\Delta f) = \mu_f$, $\mathrm{var}(\Delta p) = \sigma_p^2$, $\mathrm{var}(\Delta f) = \sigma_f^2$, and $\mathrm{cov}(\Delta p, \Delta f) = \sigma_{pf}$; where var(.) is the variance operator and cov(.,.) is the covariance operator. The total portfolio return $x = \Delta p + Q\Delta f$ also is distributed normally with mean $\mu = \mu_p + Q\mu_f$ and variance $\sigma^2 = \sigma_p^2 + 2Q\sigma_{pf} + Q^2\sigma_f^2$.

OPTIMAL-HEDGING STRATEGY

The hedger chooses the optimal futures position Q^* to maximize the expected DA utility, $\mathrm{E}(\Psi_{u,\beta}(W_1))$. By definition,

$$\mathrm{E}(\Psi_{u,\beta}(W_1)) = \mathrm{C} + \mathrm{D}, \text{ where} \tag{5a}$$

$$C = \int_{-\infty}^{V-W_0} u(W_0 + x)\frac{1}{\sqrt{2\pi}\sigma}\exp\left[\frac{(x-\mu)^2}{-2\sigma^2}\right]dx, \tag{5b}$$

$$D = \int_{V-W_0}^{\infty} \left\{\frac{u(W_0 + x) + \beta u(V)}{1+\beta}\right\}\frac{1}{\sqrt{2\pi}\sigma}\exp\left[\frac{(x-\mu)^2}{-2\sigma^2}\right]dx. \tag{5c}$$

Substituting the relationships $u(V) = \mathrm{E}(\Psi_{u,\beta}(W_1))$ and $u(x) = -\exp(-\theta x)$, equation (5) leads to the following:

$$\left(\frac{-\beta}{1+\beta}\right)\Phi\left(\frac{V - W_0 - \mu + \theta\sigma^2}{\sigma}\right)\exp\left(\frac{\theta^2\sigma^2}{2} - \mu\theta - W_0\theta\right)$$
$$-\frac{1}{1+\beta}\exp\left(\frac{\theta^2\sigma^2}{2} - \mu\theta - W_0\theta\right)$$
$$-u(V)\left[1 - \frac{\beta}{1+\beta}\Phi\left(\frac{-V + W_0 + \mu}{\sigma}\right)\right] = 0, \tag{6}$$

where $\Phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}}\exp(-\frac{z^2}{2})\,dz$, the cumulative density function of the standardized normal random variable.

Equation (6) implicitly defines V as a function of μ and σ: $V = g(\mu, \sigma)$. The latter, in turn, are functions of Q. Let $H(\mu, \sigma, V)$ denote the left-hand side of equation (6). We have $\partial V/\partial\mu = -(\partial H/\partial\mu)/(\partial H/\partial V)$ and $\partial V/\partial\sigma = -(\partial H/\partial\sigma)/(\partial H/\partial V)$. To maximize the expected DA utility, $u(V)$, the hedger chooses Q^* to maximize V. The first-order condition specifies $(\partial V/\partial\mu)(\partial\mu/\partial Q) + (\partial V/\partial\sigma)(\partial\sigma/\partial Q) = 0$. Equivalently, it is required

$$\frac{\partial H}{\partial\mu}\frac{\partial\mu}{\partial Q} + \frac{\partial H}{\partial\sigma}\frac{\partial\sigma}{\partial Q} = 0. \tag{7}$$

From the definition of $H(\mu, \sigma, V)$, we have:

$$\frac{\partial H}{\partial\mu} = \frac{\theta}{1+\beta}\left[1 + \beta\Phi\left(\frac{V - W_0 - \mu + \theta\sigma^2}{\sigma}\right)\right]\exp\left(\frac{\theta^2\sigma^2}{2} - \mu\theta - W_0\theta\right), \tag{8}$$

$$\frac{\partial H}{\partial\sigma} = \frac{\beta\theta u(V)}{1+\beta}\phi\left(\frac{-V + W_0 + \mu}{\sigma}\right) - \frac{\theta^2\sigma}{1+\beta}\left[1 + \beta\Phi\left(\frac{V - W_0 - \mu + \theta\sigma^2}{\sigma}\right)\right]$$
$$\times\exp\left(\frac{\theta^2\sigma^2}{2} - \mu\theta - W_0\theta\right), \tag{9}$$

where $\phi(x) = \frac{1}{\sqrt{2\pi}} \exp(-\frac{x^2}{2})$ is the probability density function of the standardized normal random variable. Substituting equations (8) and (9) into (7),

$$-\frac{\beta}{\sigma} \exp(-\theta V)\phi\left(\frac{-V + W_0 + \mu}{\sigma}\right)\left(\sigma_{pf} + Q\sigma_f^2\right)$$
$$+ \left[1 + \beta\Phi\left(\frac{V - W_0 - \mu + \theta\sigma^2}{\sigma}\right)\right] \exp\left(\frac{\theta^2\sigma^2}{2} - \mu\theta - W_0\theta\right)$$
$$\times \left(\mu_f - \theta\sigma_{pf} - \theta Q\sigma_f^2\right) = 0 \tag{10}$$

Equations (6) and (10) jointly determine the optimal futures position Q^* and the endogenous reference point V^*. From equation (10), we find

$$sign\left(\sigma_{pf} + Q\sigma_f^2\right) = sign\left(\mu_f - \theta\left(\sigma_{pf} + Q\sigma_f^2\right)\right). \tag{11}$$

Proposition 1: In an unbiased futures market (i.e., $\mu_f = 0$), the optimal hedging strategy for a disappointment-averse hedger is the same as that for a mean–variance hedger.

Proof: when $\mu_f = 0$, equation (11) implies that $\sigma_{pf} + Q^*\sigma_f^2$ and hence $Q^* = \sigma_{pf}/\sigma_f^2$. On the other hand, a mean–variance hedger chooses the optimal futures position, denoted by Q^m to maximize the following:

$$MV = (\mu_p + Q\mu_f) - \frac{\theta}{2}\left(\sigma_p^2 + 2Q\sigma_{pf} + Q^2\sigma_f^2\right) \tag{12}$$

The first-order condition leads to

$$\mu_f - \theta\left(\sigma_{pf} + Q^m\sigma_f^2\right) = 0. \tag{13}$$

When μ_f, equation (13) implies $Q^m = -\sigma_{pf}/\sigma_f^2 = Q^*$.

Note that the mean–variance hedger corresponds to the case when there is no disappointment (or loss) aversion. Therefore, Proposition 1 states that disappointment (or loss) aversion has no effect on futures hedging when the futures market is unbiased.

Lence (1995) and Rao (2000) established conditions under which the optimal hedge ratio is independent of risk preference within the expected utility framework. It may appear that Proposition 1 constructs a special case of their results. However, the endogenous determination of the reference point would suggest otherwise.

In the following, we will call a short hedge position of size $Q^F = \sigma_{pf}/\sigma_f^2$ the (adjusted) full hedge. Note that it is the minimum-variance position and equals one when futures and spot returns are perfectly correlated . Thus, σ_{pf}/σ_f^2 is the full hedge adjusted by the imperfect correlation between the two returns. A short position with a size that exceeds (or falls below) the full hedge is called an overhedge (or underhedge).

BACKWARDATION EQUILIBRIUM

A futures market is in backwardation (or contango) if the expected futures return is positive (or negative). Disappointment aversion affects the optimal hedging strategy in these situations. This section considers the backwardation equilibrium cases.

Proposition 2: When the futures market is in a backwardation equilibrium, the optimal futures hedging strategy is to underhedge. However, a disappointment-averse hedger will choose a position closer to the full hedge than will a nondisappointment-averse hedger.

Proof: We will prove by contradiction. Suppose that $\sigma_{pf} + Q^*\sigma_f^2 < 0$. Given $\mu_f > 0$, we have μ_f^2, which contradicts to equation (11). Thus, we must have $\sigma_{pf} + Q^*\sigma_f^2 > 0$, implying $Q^* > -\sigma_{pf}/\sigma_f^2$. In other words, the optimal strategy is an underhedge. Furthermore, equation (11) implies $\partial MV/\partial Q = \mu_f - \theta(\sigma_{pf} + Q\sigma_f^2) > 0$ when $Q = Q^*$. Because MV is a concave function in Q and $\partial MV/Q = 0$ when $Q = Q^m$, we have $Q^m > Q^*$. The mean–variance hedger has a smaller short position in the futures market when compared to a disappointment adverse hedger.

Proposition 2 compares the case $\beta = 0$ to the case $\beta > 0$. To analyze the effect of disappointment aversion on the optimal futures position for the general case, we need to evaluate $\partial Q^*/\partial\beta$. Let $G(Q, \beta)$ denote the left-hand-side of equation (10). By comparative static analysis,

$$\frac{\partial Q^*}{\partial \beta} = -\left(\frac{\partial G}{\partial Q}\right)^{-1}\left(\frac{\partial G}{\partial \beta}\right) \tag{14}$$

when evaluated at $Q = Q^*$. Recall that equation (10) is the first-order condition of the expected utility-maximization problem. Suppose that the second-order condition holds. Then $\partial G/\partial Q < 0$ when $Q = Q^*$. Moreover, at this point,

$$\frac{\partial G}{\partial \beta} = -\frac{1}{\beta}\exp\left(\frac{\theta^2\sigma^2}{2} - \mu\theta - W_0\theta\right)\left(\mu_f - \theta\sigma_{pf} - \theta Q\sigma_f^2\right), \tag{15}$$

which is negative (see the proof of Proposition 2). As a result, $\partial Q^*/\partial\beta < 0$. While backwardation creates an incentive for the hedger to buy more (or sell less) futures contracts for speculation purposes (due to a positive expected return), aversion to disappointment prevents the hedger from taking full advantage of the opportunity. As a hedger becomes more disappointment averse, a greater weight is imposed on the potential disappointment. Thus, while underhedging his position, the hedger will choose a smaller deviation from the full hedge.

Similarly, we evaluate the effects of θ (the risk-aversion coefficient) and σ_{pf} (the covariance between spot and futures returns) on the optimal futures position, $\partial Q^*/\partial\theta$ and $\partial Q^*/\partial\sigma_{pf}$. The signs of these two effects are determined by the signs of $\partial G/\partial\theta$ and $\partial G/\partial\sigma_{pf}$, respectively:

$$\frac{\partial G}{\partial \sigma_{pf}} = -\exp\left(\frac{\theta^2\sigma^2}{2} - \mu\theta - W_0\theta\right)\left[1 + \beta\Phi\left(\frac{V - W_0 - \mu - \theta\sigma^2}{\sigma}\right)\right] \times \left(\frac{\mu_f}{\sigma_{pf} + Q^*\sigma_f^2}\right), \tag{16}$$

$$\frac{\partial G}{\partial \theta} = K\left(\mu_f - \theta\sigma_{pf} - \theta Q^*\sigma_f^2\right)\exp\left(\frac{\theta^2\sigma^2}{2} - \mu\theta - W_0\theta\right), \text{ where} \tag{17}$$

$$K = V - \mu - W_0 + \frac{\mu_f\sigma^2}{\sigma_{pf} + Q^*\sigma_f^2} - \frac{\sigma_{pf} + Q^*\sigma_f^2}{\mu_f - \theta\left(\sigma_{pf} + Q^*\sigma_f^2\right)}. \tag{18}$$

From equation (16), we have $\partial G/\partial\sigma_{pf} < 0$ when $Q = Q^*$; henceforth, $\partial Q^*/\partial\sigma_{pf} < 0$ since both μ_f and $\sigma_{pf} + Q^*\sigma_f^2$ are positive. That is, as spot and futures returns become more positively correlated (i.e., the futures contract provides better hedging effectiveness under the conventional definition), the (short) hedger will sell more futures contracts. The result is consistent with that predicted by the mean–variance approach.

The sign of $\partial G/\partial\theta$ is the same as that of K, which is not easy to determine. Suppose that β is small, such that $\beta \leq 1$. Appendix B shows $K < 0$. Therefore, $\partial G/\partial\theta < 0$ and $\partial Q^*/\partial\theta < 0$. That is, a risk-averse hedger will sell more futures contracts. As a hedger becomes more risk averse, he is less willing to assume the risk in the spot position. Consequently, he will choose to hedge more. Proposition 3 summarizes the above findings.

Proposition 3: Suppose that the futures market establishes a backwardation equilibrium. The hedger will assume a larger (short) futures position if (i) he is more disappointment averse, (ii) he is more risk averse, or (iii) spot and futures returns are correlated more positively.

Appendix B demonstrates that the same conclusion applies when β is sufficiently large. The effect of the risk aversion is similar to that of the disappointment aversion. Either aversion promotes the hedger to become more cautious and undertake a more conservative action. Risk aversion changes the curvature of the utility function. The more risk averse the hedger, the more concave is his utility function. Disappointment aversion changes how the hedger weighs the bad outcomes. It affects the shape of utility function asymmetrically. Both aversions affect the endogenously determined reference point.

CONTANGO EQUILIBRIUM

We now turn to the case of a contango equilibrium, in which $\mu_f < 0$. The optimal futures position is characterized by the following proposition.

Proposition 4: When the futures market is in a contango equilibrium, the optimal futures hedging strategy is to overhedge. However, a disappointment-averse hedger will choose a position closer to full hedge than will a nondisappointment-averse hedger.

Proof: Given that $\mu_f < 0$, if $\sigma_{pf} + Q^*\sigma_f^2$, then $\mu_f - \theta(\sigma_{pf} + Q^*\sigma_f^2) < 0$. This contradicts to equation (11). Consequently, we must have $\sigma_{pf} + Q^*\sigma_f^2 < 0$ from which $Q^* < -\sigma_{pf}/\sigma_f^2$. That is, the optimal strategy is to overhedge. Because $\partial MV/\partial Q = \mu_f - \theta(\sigma_{pf} + Q^*\sigma_f^2) < 0$ when $Q = Q^*$, we have $Q^m < Q^*$.

Note that a contango promotes the hedger to sell more (or buy less) for speculation purposes. The aversion to disappointment, however, prevents the hedger from taking full advantage of the profitable opportunity.

For the more general case, equation (15) indicates that $\partial Q^*/\partial\beta > 0$ when a contango equilibrium prevails in the futures market. Thus, a more disappointment-averse hedger will have a smaller (short) futures position: the overhedge amount decreases as the hedger becomes more disappointment averse. Because $\mu_f < 0$, $\theta(\sigma_{pf} + Q^*\sigma_f^2) < 0$, and $\mu_f - \theta(\sigma_f + Q^*\sigma_f^2)$, the sign of equation (16) remains unchanged, whereas the sign of equation (17) is reversed. Proposition 5 follows.

Proposition 5: Suppose that the futures market establishes a contango equilibrium. The hedger will assume a larger (short) futures position if (i) he is less disappointment averse, (ii) he is less risk averse, or (iii) spot and futures returns are more positively correlated.

Once again, the effect of the risk aversion is similar to that of the disappointment aversion. Both aversions reduce the incentives for the hedger to take a large short position.

ENDOGENOUS REFERENCE POINT

In this section, we examine the response of the reference point to changes in the risk-aversion and disappointment-aversion coefficients. From equation (6), we have

$$\frac{\partial H}{\partial V}\frac{\partial V}{\partial \beta} + \frac{\partial H}{\partial Q}\frac{\partial Q}{\partial \beta} + \frac{\partial H}{\partial \beta} = 0. \tag{19}$$

At the optimal futures position, $\partial H/\partial Q = 0$. Consequently,

$$\frac{\partial V^*}{\partial \beta} = -\frac{\partial H/\partial \beta}{\partial H/\partial V}. \tag{20}$$

Straightforward calculations show that both $\partial H/\partial \beta$ and $\partial H/\partial V$ are negative (see Appendix A). Therefore, $\partial V^*/\partial \beta < 0$. As the hedger becomes more disappointment averse, the reference point depreciates. In other words, the hedger, knowing that he will be much more disappointed at the bad outcomes, adjusts his benchmark to a lower level to reduce possible disappointment.

Similarly, the effect of risk aversion can be analyzed as follows:

$$\frac{\partial V}{\partial \theta} = -\frac{\partial H/\partial \theta}{\partial H/\partial V}. \tag{21}$$

The sign of $\partial V^*/\partial \theta$ is the same as that of $\partial H/\partial \theta$. It can be shown that

$$\begin{aligned}\frac{\partial H}{\partial \theta} = {} & \frac{-e^{-\theta V^*}A}{1+\beta}\left[\beta\Phi\left(\frac{V^* - W_0 - \mu}{\sigma}\right) + 1\right] \\ & - \frac{\beta\sigma}{1+\beta}\phi\left(\frac{A}{\sigma}\right)\exp\left(\frac{\theta^2\sigma^2}{2} - \mu\theta - W_0\theta\right),\end{aligned} \tag{22}$$

where $A = \phi\sigma^2 - \mu - W_0 + V^*$.

The second term in the right-hand side of equation (22) is always negative, whereas the sign of the first term may be positive or negative depending upon the sign of A. When A is non-negative, $\partial V^*/\partial \theta < 0$. On the other hand, Appendix C shows that $A < 0$ when β is sufficiently large. If A is negative but close to zero, then the second term will dominate the first term, and once again $\partial V^*/\partial \theta < 0$. Similar to the case of disappointment aversion, an increase in the risk aversion depresses the benchmark in either scenario.

Suppose that β is small. The first term tends to dominate the second term. Consequently, the sign of $\partial V^*/\partial \theta$ is opposite to that of A. Consider the case where $\beta = 0$ (i.e., the hedger is not averse to disappointment). We have

$$V = \mu + W_0 - \frac{\theta\sigma^2}{2} \text{ and} \tag{23}$$

$$\frac{\partial V^*}{\partial \theta} = -\frac{\sigma^2}{2} < 0. \tag{24}$$

Alternatively, it is found that $A = \theta\sigma^2/2 > 0$. Thus, we expect $\partial V^*/\partial\theta$ to be negative when β is small. We now summarize the results in this section by the following proposition.

Proposition 6: As the hedger becomes more disappointment averse, the reference point will be adjusted accordingly downward. A similar effect is displayed in risk aversion when the disappointment-aversion coefficient is small.

NUMERICAL RESULTS

We conducted a numerical exercise to solve equation (6) and (10). The parameter values are as follows: $\mu_p = \pm 0.005$; $\mu_f = \pm 0.005$; $\sigma_{pf} = 0.9$ or 0.95; $\sigma_p^2 = \sigma_f^2 = 1$; $\theta = 0.01, 0.1, 1, 10$; $\beta = 0, 0.1, 0.2, 0.4, 0.8$. To save space, only parts of the results are presented (see Tables I–IV). Overall the results conform to the above theoretical analysis.

1. When the market is in backwardation (i.e., $\mu_f > 0$), the absolute value of Q^* is always less than σ_{pf}. That is, the optimal strategy is to underhedge. In addition, when β increases, Q^* decreases and gets closer to $-\sigma_{pf}$. The more disappointment averse the hedger is, the more futures contracts he sells. For example, assume $\mu_p = 0.005$, $\sigma_{pf} = 0.9$, and $\theta = 0.1$. A hedger with a disappointment-aversion coefficient of 0.2 will sell 0.8813 units of futures contracts, whereas he will sell 0.8921 units if the coefficient increases to 0.8 (see Table I). A mean–variance hedger (corresponding to the case $\beta = 0$) sells 0.85 units and a bona fide hedger (who attempts to minimize the variance) sells 0.90 units. Similar effects are observed for a change in the risk-aversion coefficient. Consider the case $\mu_p = -0.005$, $\sigma_{pf} = 0.9$, and $\beta = 0.2$. A hedger with a risk-aversion coefficient of 1 sells 0.8957 units. If the coefficient increases to 10, he will sell 0.8995 units (see Table II).
2. When the market is in contango (i.e., $\mu_f < 0$), the absolute value of Q^* is larger than σ_{pf}, indicating that the optimal strategy is to overhedge.

TABLE I

Optimal Futures Positions and Benchmarks

μ_p	μ_f	σ_{pf}	θ	β	V^*	Q^*
0.005	0.005	0.90	0.01	0	0.0008	−0.4000
0.005	0.005	0.90	0.1	0	−0.0089	−0.8500
0.005	0.005	0.90	1	0	−0.0945	−0.8950
0.005	0.005	0.90	10	0	−0.9495	−0.8995
0.005	0.005	0.90	0.01	0.2	−0.0321	−0.8717
0.005	0.005	0.90	0.1	0.2	−0.0407	−0.8813
0.005	0.005	0.90	1	0.2	−0.1259	−0.8957
0.005	0.005	0.90	10	0.2	−0.9672	−0.8995
0.005	0.005	0.90	0.01	0.8	−0.1025	−0.8909
0.005	0.005	0.90	0.1	0.8	−0.1110	−0.8921
0.005	0.005	0.90	1	0.8	−0.1958	−0.8967
0.005	0.005	0.90	10	0.8	−1.0065	−0.8995
0.005	0.005	0.95	0.01	0.2	−0.0229	−0.9294
0.005	0.005	0.95	0.1	0.2	−0.0273	−0.9350
0.005	0.005	0.95	1	0.2	−0.0711	−0.9459
0.005	0.005	0.95	10	0.2	−0.5033	−0.9495

TABLE II

Optimal Futures Positions and Benchmarks

μ_p	μ_f	σ_{pf}	θ	β	V^*	Q^*
−0.005	0.005	0.90	0.01	0	−0.0092	−0.4000
−0.005	0.005	0.90	0.1	0	−0.0189	−0.8500
−0.005	0.005	0.90	1	0	−0.1045	−0.8950
−0.005	0.005	0.90	10	0	−0.9595	−0.8995
−0.005	0.005	0.90	0.01	0.2	−0.0421	−0.8717
−0.005	0.005	0.90	0.1	0.2	−0.0507	−0.8813
−0.005	0.005	0.90	1	0.2	−0.1359	−0.8957
−0.005	0.005	0.90	10	0.2	−0.9772	−0.8995
−0.005	0.005	0.90	0.01	0.8	−0.1125	−0.8909
−0.005	0.005	0.90	0.1	0.8	−0.1211	−0.8921
−0.005	0.005	0.90	1	0.8	−0.2058	−0.8967
−0.005	0.005	0.90	10	0.8	−1.0165	−0.8995
−0.005	0.005	0.95	0.01	0.2	−0.0329	−0.9294
−0.005	0.005	0.95	0.1	0.2	−0.0373	−0.9350
−0.005	0.005	0.95	1	0.2	−0.0811	−0.9459
−0.005	0.005	0.95	10	0.2	−0.5133	−0.9495

Moreover, as β increases, Q^* increases as well, and approaches $-\sigma_{pf}$. To illustrate, let $\mu_p = 0.005$, $\sigma_{pf} = 0.9$, and $\theta = 0.1$. A hedger with a disappointment-aversion coefficient of 0.2 will sell 0.9188 units of futures contracts, whereas he will sell 0.9078 units if the coefficient increases to 0.8 (see Table III). For comparison, a mean–variance hedger (corresponding to the case $\beta = 0$) sells 0.95 units and a bona fide hedger (who attempts to minimize the variance) sells

TABLE III
Optimal Futures Positions and Benchmarks

μ_p	μ_f	σ_{pf}	θ	β	V^*	Q^*
−0.005	−0.005	0.90	0.01	0	−0.0002	−1.4000
−0.005	−0.005	0.90	0.1	0	−0.0099	−0.9500
−0.005	−0.005	0.90	1	0	−0.0955	−0.9050
−0.005	−0.005	0.90	10	0	−0.9505	−0.9005
−0.005	−0.005	0.90	0.01	0.2	−0.0331	−0.9283
−0.005	−0.005	0.90	0.1	0.2	−0.0417	−0.9188
−0.005	−0.005	0.90	1	0.2	−0.1269	−0.9043
−0.005	−0.005	0.90	10	0.2	−0.9682	−0.9005
−0.005	−0.005	0.90	0.01	0.8	−0.1035	−0.9091
−0.005	−0.005	0.90	0.1	0.8	−0.1121	−0.9078
−0.005	−0.005	0.90	1	0.8	−0.1968	−0.9033
−0.005	−0.005	0.90	10	0.8	−1.0075	−0.9005
−0.005	−0.005	0.95	0.01	0.2	−0.0234	−0.9706
−0.005	−0.005	0.95	0.1	0.2	−0.0278	−0.9650
−0.005	−0.005	0.95	1	0.2	−0.0716	−0.9541
−0.005	−0.005	0.95	10	0.2	−0.5038	−0.9505

TABLE IV
Optimal Futures Positions and Benchmarks

μ_p	μ_f	σ_{pf}	θ	β	V^*	Q^*
0.005	−0.005	0.90	0.01	0	0.0098	−1.4000
0.005	−0.005	0.90	0.1	0	0.0001	−0.9500
0.005	−0.005	0.90	1	0	−0.0855	−0.9050
0.005	−0.005	0.90	10	0	−0.9405	−0.9005
0.005	−0.005	0.90	0.01	0.2	−0.0231	−0.9283
0.005	−0.005	0.90	0.1	0.2	−0.0317	−0.9188
0.005	−0.005	0.90	1	0.2	−0.1169	−0.9043
0.005	−0.005	0.90	10	0.2	−0.9582	−0.9005
0.005	−0.005	0.90	0.01	0.8	−0.0935	−0.9091
0.005	−0.005	0.90	0.1	0.8	−0.1021	−0.9078
0.005	−0.005	0.90	1	0.8	−0.1868	−0.9033
0.005	−0.005	0.90	10	0.8	−0.9975	−0.9005
0.005	−0.005	0.95	0.01	0.2	−0.0134	−0.9706
0.005	−0.005	0.95	0.1	0.2	−0.0178	−0.9650
0.005	−0.005	0.95	1	0.2	−0.0616	−0.9541
0.005	−0.005	0.95	10	0.2	−0.4938	−0.9505

0.90 units. Once again, similar effects are observed for a change in the risk-aversion coefficient. Consider the case $\mu_p = 0.005$, $\sigma_{pf} = 0.9$, and $\beta = 0.8$. A hedger with a risk-aversion coefficient of 0.01 sells 0.9091 units. If the coefficient increases to 10, he will sell 0.9005 units (see Table IV).

3. There is a substitution effect between risk aversion and disappointment aversion such that the effect of either aversion on Q^* decreases

when the other aversion coefficient increases. The effect of disappointment aversion is stronger when the hedger is less risk averse. A small disappointment aversion will cause a near-risk-neutral hedger to take a drastically different position. For example, in Table I, when $\beta = 0$, the optimal futures position decreases from -0.4 to -0.85 if θ increases from 0.01 to 0.1. In case that $\beta = 0.8$, the same change in θ reduces Q^* from -0.8909 to -0.8921. Similarly, when $\theta = 0.1$, Q^* decreases from -0.8813 to -0.8921 as β increases from 0.2 to 0.8. On the other hand, if $\theta = 10$, β produces nearly no effect on Q^*.

4. As σ_{pf} increases, the absolute value of Q^* increases as well. When the covariance between Δp and Δf increases, the futures contract becomes a more effective hedge instrument. The hedger then sells more futures contracts. For example, suppose that $\mu_p = \mu_f = -0.005$, $\theta = 0.1$, $\beta = 0.2$, and $\sigma_{pf} = 0.9$. The optimal futures position is to short 0.9188 units of futures contracts. The number increases to 0.9650 when $\sigma_{pf} = 0.95$ (see Table III).
5. The reference point V^* is depressed if the risk aversion or the disappointment aversion increases. Consider the following case: $\mu_p = -0.005$, $\mu_f = -0.005$, $\sigma_{pf} = 0.9$, $\theta = 1$, and $\beta = 0.2$. Suppose that θ increases to 10, then V^* decreases from -0.1397 to -0.9772. Similarly, if β increases to 0.8, V^* decreases from -0.1397 to -0.2058.
6. For a loss-averse hedger, the reference point is fixed. There are two possible choices: $V = 0$ or $V = \mu_p$. The former method assumes no spot or futures position under the status quo. The latter approach assumes the hedger has a nontradable spot position and the benchmark is referred to the case of no futures trading. From Tables I–IV, it is found that V^* is always < 0 (and μ_p), except in a few cases where $\beta = 0$. That is, a disappointment-averse hedger is more loss averse than a *conventional* loss-averse hedger. Consequently, the optimal futures position for the disappointment-averse hedger is closer to the full hedge than that for the conventional loss-averse hedger.

CONCLUSIONS

Recent literature incorporates loss aversion and disappointment aversion into the studies of portfolio choice and futures hedging. In this study, we adopted Gul's disappointment-aversion framework with a CARA utility function and a normal distribution for wealth. We found that disappointment aversion has no effect on the hedging decision when the market is unbiased. In case of backwardation or contago, both disappointment aversion and risk aversion result in more cautious behavior and an optimal contract size closer to the minimum variance hedge. There is a substitution effect between the two aversions such that the effect of the disappointment aversion strengthens as the risk aversion decreases. In particular, disappointment aversion may reduce greatly the trading volume for a near-risk-neutral hedger.

Another interesting finding of this study concerns the reference point. A more risk- or disappointment-averse hedger will choose a lower reference point. In addition, numerical results indicate that the reference point of a disappointment-averse hedger tends to be lower than that of a conventional loss-averse hedger. Consequently, the disappointment-averse hedger will act more conservatively, not exploiting profitable opportunities as much as the conventional loss-averse hedger.

APPENDIX A

Recall that $H(.)$ is defined as the left-hand side of equation (6). Define B as follows: $B = \exp(\theta^2\sigma^2/2 - \mu\theta - W_0\theta)$. We have

$$\frac{\partial H}{\partial \beta} = \frac{-1}{(1+\beta)^2}\left[-B\Phi\left(\frac{-V+W_0+\mu}{\sigma} - \theta\sigma\right) + e^{-\theta V}\,\Phi\left(\frac{-V+W_0+\mu}{\sigma}\right)\right]. \tag{A1}$$

On the other hand, upon rearranging equation (6),

$$B\left[\beta\Phi\left(\frac{V-W_0-\mu}{\sigma} + \theta\sigma\right) + 1\right] = e^{-\theta V}\left[\beta\Phi\left(\frac{V-W_0-\mu}{\sigma}\right) + 1\right]. \tag{A2}$$

As a result, $B < e^{-\theta V}$. Upon substituting this relationship into (A1), we find $\partial H/\partial\beta < 0$.

It can also be shown that

$$\frac{\partial H}{\partial V} = -\theta e^{-\theta V}\left[1 - \frac{\beta}{1+\beta}\,\Phi\left(\frac{-V+W_0+\mu}{\sigma}\right)\right], \tag{A3}$$

which is always negative. Finally, equation (A2) is applied to derive equation (22) for $\partial H/\partial\theta$.

APPENDIX B

First, consider the case $\beta \leq 1$. We will prove that $K < 0$ where

$$K = V - \mu - W_0 + \frac{\mu_f \sigma^2}{\sigma_{pf} + Q^* \sigma_f^2} - \frac{\sigma_{pf} + Q^* \sigma_f^2}{\mu_f - \theta(\sigma_{pf} + Q^* \sigma_f^2)}. \quad (18)$$

Denote $R = \sigma_{pf} + Q^* \sigma_f^2$. Appendix A shows

$$B = \exp(\theta^2 \sigma^2/2 - \mu\theta - W_0\theta) < \exp(-\theta V), \text{ or equivalently,} \quad (B1)$$

$$\frac{V - \mu - W_0}{\sigma} < -\frac{\theta\sigma}{2}. \quad (B2)$$

As a result, we have the following inequality:

$$K < \left(\frac{\mu_f}{R} - \theta\right)\sigma^2 - \frac{1}{(\mu_f/R) - \theta} + \frac{\theta\sigma^2}{2}. \quad (B3)$$

From equation (10), after algebraic manipulations, we derive

$$\frac{\mu_f}{R} - \theta = \frac{\beta\phi(s)}{\sigma[\beta\Phi(s) + 1]}, \quad (B4)$$

where $s = (V - \mu - W_0)/\sigma < 0$. Upon substituting (B4) into (B3) and applying (B2),

$$K < \left[\frac{\beta\phi(s)}{\beta\Phi(s) + 1} - \frac{\beta\Phi(s) + 1}{\beta\phi(s)} - s\right]\sigma. \quad (B5)$$

Given that $\beta \leq 1$, the above inequality implies

$$K < \frac{\phi(s)}{\Phi(s)} - \frac{\Phi(s) + 1}{\phi(s)} - s, \text{ the sign of which is the same as } L(s): \quad (B6)$$

$$L(s) = \phi^2(s) - \Phi^2(s) - \Phi(s) - s\phi(s)\Phi(s). \quad (B7)$$

The remaining task is to show $L(s) < 0$ whenever s is negative. Note that $L(0) < 0$ and $L(s) < 0$ when $-s$ is sufficiently large. The first derivative of $L(s)$ is given as follows:

$$L'(s) = \phi(s)[-3s\phi(s) - 1 + (s^2 - 3)\Phi(s)]. \quad (B8)$$

Let s^* satisfy the condition $L'(s^*) = 0$. That is, s^* is either a local maximum point or a local minimum point. We have

$$L(s^*) = \phi^2(s^*) - (2/3)\Phi(s^*) - (s^{*2}/3)\Phi^2(s^*). \quad (B9)$$

Consider the following function: $M(z) = \phi^2(z) - (2/3)\Phi(z)$. It can be shown easily that $M(z)$ achieves a global maximum at $z = -1/3$. Moreover, $M(-1/3) < 0$ implying $M(z) < 0$ for every z. From this relationship, we find $L(s^*) < 0$ for every s^*. Consequently, $L(s) <$ whenever s is negative.

Now we turn to the case that β is sufficiently large. Letting β approach infinity, equation (B4) becomes

$$\frac{\mu_f}{R} - \theta = \frac{\phi(s)}{\sigma\Phi(s)}; \text{ therefore,} \tag{B10}$$

$$K = \left[s + \frac{\phi(s)}{\Phi(s)} - \frac{\Phi(s)}{\phi(s)}\right]\sigma. \tag{B11}$$

The sign of K is the same as that of $J(s) = s\phi(s)\Phi(s) + \phi^2(s) - \Phi^2(s)$. We have

$$J'(s) = -\phi(s)\Phi(s) - s\phi(s)(s\Phi(s) + \phi(s)). \tag{B12}$$

It can be shown that $s\Phi(s) + \phi(s)$ is always positive. Consequently, $J'(s) < 0$. Note that $J(0) = (1/2\pi) - (1/4) < 0$, we have $J(s) < 0$ (and $K < 0$) whenever $s < 0$.

APPENDIX C

Suppose that β is sufficiently large. By letting β approach infinity, equation (6) is reduced to the following:

$$\exp(-\theta V^*) = \left[\frac{\Phi(s + \theta\sigma)}{\Phi(s)}\right] \exp\left(\frac{\theta^2\sigma^2}{2} - \mu\theta - W_0\theta\right) \tag{C1}$$

where, as defined before, $s = (V - \mu - W_0)/\sigma < 0$. Let $M(s) = \Phi(s + \theta\sigma) - \exp(\theta^2\sigma^2/2)\Phi(s)$. Then

$$M'(s) = \frac{1}{\sqrt{2\pi}} \exp\left(\frac{s^2 + \theta^2\sigma^2}{-2}\right)[\exp(-s\theta\sigma) - \exp(\theta^2\sigma^2)]. \tag{C2}$$

As a result, $M'(s) > 0$ if $s < -\theta\sigma$ and $M'(s) < 0$ if $s > -\theta\sigma$. Because $M(-\infty) = 0$, we have $M(s) > 0$ when $s < -\theta\sigma$. Consider now

$$M\left(\frac{-\theta\sigma}{2}\right) = \Phi\left(\frac{\theta\sigma}{2}\right) - \exp\left(\frac{\theta^2\sigma^2}{2}\right)\Phi\left(\frac{-\theta\sigma}{2}\right). \tag{C3}$$

Let $z = \theta\sigma/2$ and let $T(z) = \Phi(z) - \exp(z^2/2)\Phi(-z)$. Upon taking the derivative, we have

$$T'(z) = \phi(z) + \exp(z^2/2)[\phi(z) - z\phi(-z)]. \tag{C4}$$

It can be shown that $\phi(z) - z\Phi(-z)$ is always positive, or equivalently, $T'(z) > 0$ for every z. Because $T(0) = 0$, $T(z) > 0$ when $z > 0$. In other words, $M(-\theta\sigma/2)$ is positive. Using the property of $M'(s)$, we conclude $M(s) > 0$ when $s < -\theta\sigma/2$.

If we substitute this relationship into equation (C1), then

$$\exp(-\theta V^*) > \exp\left(\theta^2\sigma^2 - \mu\theta - W_0\theta\right), \text{ implying} \tag{C5}$$

$$A = V^* + \theta\sigma^2 - \mu - W_0 < 0. \tag{C6}$$

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