EXCESSIVE VARIATION IN RISK-FACTOR CORRELATIONS AND VOLATILITIES

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This article explores the time-series behavior of correlations of returns, volatilities of returns, volatilities of volatilities, and correlations of volatilities in domestic and international financial markets such as equity (indices), interest rates (bonds), and currency (exchange rates) using a Kalman filter approach to estimate the aforementioned parameters. The main findings include the following. First, the correlations of risk factors

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are highly unstable over time, both in terms of sign and absolute value. Second, the time variation of risk-factor volatilities is stochastically nonlinear. Long periods of deterministic volatilities are interrupted by sudden bursts of highly volatile periods. Third, daily volatilities of risk-factor volatilities fluctuate over time within a narrower band. Fourth, the correlations between volatilities are generally positive and relatively stable over time. These results have implications for financial risk management, dynamic asset allocation, and valuation of derivative securities. Jrl Fut Mark 22:1119–1146, 2002

INTRODUCTION

The significance of large unpredictable movements in implicit volatilities and covariances goes beyond the standard mean-variance portfolio analysis. A market practitioner uses estimates of volatility and correlation coefficients for two other important purposes. First, estimates of these parameters play a crucial role in *daily* risk management and form the basis of value at risk (VaR) calculations. Second, these estimates are used in calibrating dynamic models for the underlying assets during the valuation process of derivative securities. In a typical portfolio management problem where assets are linear in risk factors, *time variation* of volatilities and correlations can easily be considered by estimating conditionally time-varying variance–covariance matrices.[1] Yet, in risk management and valuation unstable daily or instantaneous correlations may lead to significant problems.

For example, many assets contain (several) implicit options. These options are sometimes compound, in the sense that they depend on the implicit volatilities as well as covariances of the underlying. If these covariances and volatilities exhibit strong unpredictable variation, then the pricing of portfolios of assets containing such options will indeed be more complicated.

The objective of this article is to investigate the linkages in terms of time-varying, stochastic volatilities in a sample of asset returns from three major markets (United States, Japan, and Europe) and for three types of instruments (interest rates, exchange rates, and stock returns). We focus on the time-series properties of volatilities of returns, correlations of returns, volatilities of volatilities, and correlations of volatilities. These parameters play a crucial role in the recommendations of the

[1] Within the asset allocation context, the problem reduces to one of time-varying estimation and can be handled in a straightforward fashion. The resulting portfolio weights are still optimal locally in a mean-square error sense.

Basel Committee concerning capital adequacy requirements against market risk.

The financial market turmoil of the 1990s is another reason that justifies this study. Market participants have realized that the estimated correlations and volatilities of implied volatilities can vary excessively during turbulent periods.

In risk management, the use of estimated volatilities and correlations is based on the assumption that the daily estimates of these parameters are "stable," so that yesterday's estimates can be used to determine today's VaR. High variation in the underlying correlations, both in terms of sign and in terms of size, is often ignored. However, the results provided in this article suggest that this may not be true. Using a Kalman filter approach, this article investigates the stability of the daily first- and second-order moments for important risk factors and concludes that they are *highly* unstable both in magnitude and sign.

Short-term volatility and correlation estimates are also used in calibrating equity prices and interest rate models. Here the instantaneous volatility and correlation parameters need to satisfy some important conditions. Depending on whether the volatility (correlation) coefficients are *constant, deterministically time-varying,* or *stochastic,* the pricing and hedging of interest rate derivatives will be different. In the case of equity and commodity derivatives depending on the magnitude of the variation, the same problem will again be present.

This article uses a robust set of estimators to provide evidence of *extreme* instability in the correlations of most risk factors and their volatilities. In particular, we show that the volatilities of the risk factors not only are unstable but also behave in a stochastically *nonlinear* fashion, where volatility itself becomes highly volatile for some brief periods and then reverts back to an essentially deterministic behavior. We also provide strong evidence on a relatively stable correlation between these stochastically varying volatilities. Although the risk factors appear to display *arbitrary* correlation when measured in terms of their *levels,* they display relatively stable correlation in their *volatilities*.

The results reported in this article also may be of interest in a broader sense. The financial market turmoil of 1997–99 has made clear how much international financial markets have become integrated. This is in part due to the gradual removal of controls on capital movements in many countries, in part to liberalization of rules regarding international diversification of portfolios of institutional investors, and in part to greater awareness on the part of investors of the benefits of international diversification. One would expect the increased integration to bring about closer connections between asset price movements in different markets and across different asset classes. The disturbance in any one location or market segment is rapidly transmitted elsewhere, as investors rebalance their portfolios in response to new information.

Implicit volatility and correlation estimates as well as asset market linkages have generated significant new research, especially after the contagion effect of the 1992 EMS crisis and the contagion of the Thai devaluation in 1997.

An excellent survey of the volatility dynamics can be found in Rebonato (2000). Andersen and Lund (1997); Gallant, Hsieh, and Tauchen (1997); Gallant and Tauchen (1999); and Durham and Gallant (2002) provide method of moments and maximum likelihood on the basis of estimation techniques for continuous time stochastic volatility models. Bates (1996) deals with jumps and stochastic volatility and discusses issues closely related to our empirical results. Neftci (2000) provides evidence that extreme movements in risk factors do not have a Gaussian behavior. Dumas, Fleming, and Whaley (1998) is a good source for implicit empirical volatility functions. Bertero and Mayer (1989) imply that interdependence between markets varies systematically with the degree of instability in the international system. They find evidence that correlations of returns increase when turbulence in the markets is higher. Similar results are reported in the study by Kupiec (1991) that covers not only national stock markets but also corporate bond yields and foreign exchange. Investigating not only equities but also foreign exchange and government bonds, McNelis (1993) finds significant correlations of volatilities across national markets.

This article is organized as follows. The next section lays out the definitions and notation for calculating volatilities and correlations of the daily financial time series. The empirical results and the data are presented in a recursive model. The conclusions discusses some implications of the findings for risk management, for short-term portfolio management and asset allocation decisions, and for asset pricing and the transmission of shocks across markets and asset classes.

DEFINITIONS AND NOTATION

As noted in the introduction, the main purpose of this article is to investigate the time-series behavior of correlations of returns, volatilities of returns, volatilities of volatilities, and correlations of volatilities in domestic and international financial markets such as equity (indices), interest rates (bonds), and currency (exchange rates) using a Kalman filter approach to estimate the aforementioned parameters. In particular we outline a simple statistical model that can be used to answer the following questions:

1. Are volatilities of returns stable over time? When they change, is the change smooth or violent?
2. Are correlation coefficients between returns stable over time, or do they fluctuate between +1 and −1?
3. Are there any systematic relationships between the risk-factor volatilities? For example, does the correlation between volatilities increase when the market volatility "jumps?"

We use a Kalman filter framework to provide answers to these questions. We report a narrow set of results from a very large pool, and we summarize our results only informally using charts rather than more formal measures. Nevertheless, we believe that our findings are suggestive of interesting patterns that have important implications in particular for risk management and valuation of some exotic options such as range structures. In this section we begin by laying out the definition of correlation within a continuous time-asset pricing framework.

We assume that $\{\mathbf{X}_t, \mathbf{Z}_t\}$ is a 2×1 vector of stochastic processes obeying the following stochastic differential equation:

$$\begin{bmatrix} d\mathbf{X}_t \\ d\mathbf{Z}_t \end{bmatrix} = \begin{bmatrix} a_t^x \mathbf{X}_t \\ a_t^z \mathbf{Z}_t \end{bmatrix} dt + \begin{bmatrix} \sigma_t^{11}\mathbf{X}_t & \sigma_t^{12}\mathbf{X}_t \\ \sigma_t^{21}\mathbf{Z}_t & \sigma_t^{22}\mathbf{Z}_t \end{bmatrix} \begin{bmatrix} d\mathbf{W}_{1t} \\ d\mathbf{W}_{2t} \end{bmatrix} \tag{1}$$

The $\{a_t^i, i = x, z\}$ and $\{\sigma_t^{ij}\}$ are the drift and diffusion parameters. The $\mathbf{W}_{1t}, \mathbf{W}_{2t}$ are two *independent* Wiener processes. Note carefully the way we have written the diffusion component. The diffusion component portrays the $\{\sigma_t^{ij}\}$ as the percentage of volatilities, although they may also potentially depend on $\mathbf{X}_t, \mathbf{Z}_t$. This is important because the markets quote percentage volatility, and we would like our tests tailored towards this convention.

In this bivariate framework, the (conditional) volatilities and the correlation between $\mathbf{X}_t$ and $\mathbf{Z}_t$ over a finite time interval Δ can be defined as follows. The volatility of $\mathbf{X}_t$ over an interval $[t - \Delta, t]$ is denoted by σ_t^x

$$\sigma_t^x = \left\{ \frac{1}{\Delta} E_t^P \left[\int_{t-\Delta}^{t} (d\mathbf{X}_s - a_s^x\, ds) \right]^2 \right\}^{1/2} \tag{2}$$

$$= \left\{ \frac{1}{\Delta} \int_{t-\Delta}^{t} [(\sigma_s^{11})^2 + (\sigma_s^{12})^2] ds \right\}^{1/2} \tag{3}$$

where the expectation $E[\cdot]$ is taken with respect to the information available at time t, denoted by H_t. P is the real-world probability. The fact that the Wiener processes increments are uncorrelated over time and uncorrelated with each other is used to derive the last equality.

Similarly for σ_t^z, the volatility of $\mathbf{Z}_t$, over $[t - \Delta, t]$, we have

$$\sigma_t^z = \left\{ \frac{1}{\Delta} E_t^P \left[\left[\int_{t-\Delta}^{t} (dZ_s - a_s^z \, ds) \right]^2 \right\} ^{1/2} \tag{4}$$

$$= \left\{ \frac{1}{\Delta} \int_{t-\Delta}^{t} [(\sigma_s^{21})^2 + (\sigma_s^{22})^2] \, ds \right\}^{1/2} \tag{5}$$

The (conditional) covariance, ρ_t^{xz}, between the increments of $\mathbf{X}_t$ and $\mathbf{Z}_t$ during Δ, is defined by

$$\rho_t^{xz} = E_t^P \left[\frac{1}{\Delta} \int_{t-\Delta}^{t} (d\mathbf{X}_s - a_s^x ds) \int_{t-\Delta}^{t} (d\mathbf{Z}_s - a_s^z ds) \right] \tag{6}$$

$$= \frac{1}{\Delta} \int_{t-\Delta}^{t} [\sigma_s^{11} \sigma_s^{21} + \sigma_s^{22} \sigma_s^{12}] \, ds \tag{7}$$

where the subscript s is a reminder of possible dependence of σ_s^{ij} on the $\mathbf{X}_t$, $\mathbf{Z}_t$.[2]

Finally, from these parameters, time-dependent conditional correlation coefficients can be calculated as

$$r_t^{xz} = \frac{\rho_t^{zx}}{\sigma_t^z \sigma_t^x} \tag{8}$$

The r_t^{xz} are considered as time-dependent "short-term" correlation coefficients between the risk factors $\mathbf{X}_t$ and $\mathbf{Z}_t$.

As can be seen from the expressions in 3, 5, and 7, the volatilities and covariances during a finite time interval Δ are (standard) integrals of expressions depending on $\{\sigma_s^{ij}, t \leq s \leq t + \Delta\}$. Further these nonanticipating random variables may depend on the $\mathbf{X}_t$ and $\mathbf{Z}_t$ in complicated ways and may make the time-dependent correlations r_t^{xz} fluctuate between $+1$ and -1.

The ensuing section describes a Kalman filter framework that can be used to obtain estimates for σ_t^x, σ_t^z, ρ_t^{xz} and especially of the r_t^{xz} for data that are *one day* apart.

[2] The integrals on the right-hand side of 3, 5, and 7 are defined pathwise.

A RECURSIVE MODEL

In the following, X_t represents an interest rate process, whereas Z_t denotes either an exchange rate or a stock price. Set $\Delta = 1$ so that $\{t = 0, 1, 2, \ldots\}$ is an index of *days* elapsed. Let H_t^x and H_t^z be the observed histories of the $\{X_t, Z_t\}$, respectively

$$H_t^x = \{X_t, X_{t-1}, \ldots, X_0\} \tag{9}$$

$$H_t^z = \{Z_t, Z_{t-1}, \ldots, Z_0\} \tag{10}$$

For interest rates, we let x_t be given by the first differences

$$x_t = X_t - X_{t-1} \tag{11}$$

We define z_t using logarithmic first differences

$$z_t = \log(Z_t) - \log(Z_{t-1}) \tag{12}$$

We assume that the x_t, z_t have the conditional means

$$E^P[x_t \mid H_{t-1}^x] = \bar{x}_t$$

$$E^P[z_t \mid H_{t-1}^z] = \bar{z}_t$$

Our objective is to estimate the parameters $\{\sigma_t^x, \sigma_t^z, r_t^{xz}\}$ defined in Equations 3, 5, and 8, respectively. In discrete time, these volatilities and correlations are calculated conditional on histories available at time $t - 1$, and they will in general be *time dependent*. To investigate the dynamics of these parameters, we exploit the Kalman filter framework.

There are two basic steps in the way we proceed.

We first identify the random variables $|x_t - \bar{x}_t|^2$, $|z_t - \bar{z}_t|^2$ and $[(x_t - \bar{x}_t)(z_t - \bar{z}_t)]$ as noisy observations for the unobserved time-varying parameters $(\sigma_t^x)^2$, $(\sigma_t^z)^2$, and ρ_t^{xz}, respectively. This is because we can always write, using orthogonal projections of the left-hand side variables on the spaces generated by H_{t-1}^x, H_{t-1}^z

$$|x_t - \bar{x}_t|^2 = E^P[x_t - \bar{x}_t \mid H_{t-1}^x]^2 + \epsilon_t^x \tag{13}$$

$$|z_t - \bar{z}_t|^2 = E^P[z_t - \bar{z}_t \mid H_{t-1}^z]^2 + \epsilon_t^z \tag{14}$$

$$(x_t - \bar{x}_t)(z_t - \bar{z}_t) = E^P[(x_t - \bar{x}_t)(z_t - \bar{z}_t) \mid H_{t-1}^x, H_{t-1}^z] + \epsilon_t^{xz} \tag{15}$$

where the $\{\epsilon_t^x, \epsilon_t^z, \epsilon_t^{xz}\}$ have zero mean and are serially uncorrelated as a result of orthogonality of projections.[3] We need to specify their variances explicitly. The conditional and the unconditional volatilities of these error terms are assumed to be the same and are given by

$$[E^P(\epsilon_t^x)^2]^{1/2} = \omega_\epsilon^x, \quad [E^P(\epsilon_t^z)^2]^{1/2} = \omega_\epsilon^z, \quad [E^P(\epsilon_t^{xz})^2]^{1/2} = \omega_\epsilon^{xz} \tag{16}$$

Rewriting Equations 13–15 in terms of the volatility and correlation parameters $\{\sigma_t^x, \sigma_t^x, \rho_t^{xz}\}$ introduced in the previous section, gives the *observation equations* of the Kalman filter

$$|x_t - \bar{x}_t|^2 = (\sigma_t^x)^2 + \epsilon_t^x \tag{17}$$

$$|z_t - \bar{z}_t|^2 = (\sigma_t^z)^2 + \epsilon_t^z \tag{18}$$

$$(x_t - \bar{z}_t)(z_t - \bar{z}_t) = \rho_t^{xz} + \epsilon_t^{xz} \tag{19}$$

The quantities on the left-hand side of these equations are observed. The terms on the right-hand side consist of two sets of time-dependent unobserved variables.

The $\{\epsilon_t^x, \epsilon_t^z, \epsilon_t^{xz}\}$ are unobserved disturbances to variances and covariances, respectively, whereas σ_t^x, σ_t^z and ρ_t^{xz} are unobserved time-varying H_{t-1}^x and H_{t-1}^z measurable variables. Clearly, these equations can be exploited in a Kalman filter framework if a second set of equations describing the (a prior) evolution of $\{\sigma_t^i, \rho_t^{ij}, i, j = x, z\}$ can be obtained.

To do this, note that thus far, we have not imposed any significant restrictions on the observed price series.[4] To complete the Kalman filter framework we now assume that the parameters σ_t^x, σ_t^z, and ρ_t^{xz} *smoothly* vary over time, and this evolution can be captured by the system of equations

$$\sigma_t^x = \alpha_x \sigma_{t-1}^x + v_t^x \tag{20}$$

$$\sigma_t^z = \alpha_z \sigma_{t-1}^z + v_t^z \tag{21}$$

$$\rho_t^{xz} = \alpha_{xz} \rho_{t-1}^{xz} + v_t^{xz} \tag{22}$$

where the $\{v_t^x, v_t^z, v_t^{xz}\}$ are zero-mean random variables whose conditional and unconditional second-order moments are denoted by

$$[E^P(v_t^x)^2]^{1/2} = \omega_v^x, \quad [E^P(v_t^z)^2]^{1/2} = \omega_v^z, \quad [E^P(v_t^{xz})^2]^{1/2} = \omega_v^{xz}$$

[3] They may very well be correlated among themselves, both over time and contemporaneously.

[4] This is apart from the implicit assumption that the respective conditional expectations exist.

The parameters $\{\omega_v^x, \omega_v^z, \omega_v^{xz}\}$ control the tightness of the priors in 20–22. In fact, smoothness of time-varying volatility and correlations implies that these parameters are *smaller* than the variances of $\{\epsilon_t^x, \epsilon_t^z, \epsilon_t^{xz}\}$ denoted by $\{\omega_\epsilon^i\}$.

The smoothness constraint also suggests that the $\alpha_x, \alpha_z, \alpha_{xz}$ are close to $+1$. The disturbance terms $\{v_t^x, v_t^z, v_t^{xz}\}$ are further assumed to be serially uncorrelated and uncorrelated among themselves.

Thus, we obtain a framework where the Kalman filter becomes the natural estimator. In fact, the general setup to estimate all the parameters can be described by a two-equation dynamic system

$$y_t = \beta\mu_t + \epsilon_t \tag{24}$$

$$\mu_t = \alpha\mu_{t-1} + v_t \tag{25}$$

where y_t is the noisy observation on the unobserved μ_t. The variances of ϵ_t and v_t are denoted, respectively, by ω_1 and ω_2. The covariance is denoted by ω_{12}.

The Kalman filter estimate of the parameter μ_t, denoted by $\hat{\mu}_t$

$$\hat{\mu}_t = E[\mu_t \mid H_{t-1}^y]$$

is then given by the following formula:

$$\hat{\mu}_t = \alpha\hat{\mu}_{t-1} + \left[\frac{\alpha\beta s_t + \omega_{12}}{\omega_2 + \beta^2 s_t}\right][y_t - \alpha\hat{\mu}_{t-1}] \tag{26}$$

where s_t is the conditional variance–covariance matrix of $\hat{\mu}_t$ and is given by the recursive equations

$$s_t = \alpha^2 s_{t-1} + \omega_1 \tag{27}$$

$$s_{t+1} = s_t - \frac{(\alpha\beta s_{t-1} + \omega_{12})^2}{(\omega_2 + \beta^2 s_{t-1})} \tag{28}$$

with s_0 specified a priori. Here ω_1, ω_2 are the volatilities of the errors in Equations 24 and 25, respectively. ω_{12} is the covariance between ϵ_t, v_t.

Selection of Parameters

The recursive formulae of 26–28 are applied to various interest rates, exchange rates, and stock prices. Once time-varying estimates for $\{\sigma_t^x, \sigma_t^z, \rho_t^{xz}\}$ are found, time-varying correlation coefficients r_t^{xz} are calculated and plotted. This means that for each pair of risk factors, Equations 26–28 have to be applied three times.

First, some parameters of the dynamic Equations 24 and 25 need to be specified a priori.

We assume that X_t, Z_t are reasonably close to being equilibrium prices so that their conditional means are zero

$$E^P[x_t \mid H^x_{t-1}] = 0$$

$$E^P[z_t \mid H^z_{t-1}] = 0$$

This is easier to justify for exchange rates than for the interest rates that are known to "follow" mean-reverting stochastic differential equations. However, even with mean-reverting interest rate processes, the *daily* drift will be negligible relative to the diffusion terms. In any case, the instability of volatilities and correlation coefficients is much larger to be explained by daily drifts.

Next, for the variance of the noisy observation, ω_1, we always use

$$\omega_1 = \frac{1}{T}\sum_{i=1}^{T}[y_t - \hat{y}]^2 \tag{29}$$

where T is arbitrarily selected as 150 days. We set the variance of the error in the state Equation 25 according to

$$\omega_2 = \lambda\omega_1 \tag{30}$$

Thus, λ is a proportionality constant that determines the relative weights put on the observations and on the priors at time t. We set λ equal to 0.2. Thus, we stack the cards *against* erratic movements in the time-varying parameters.

The correlation between ϵ_t and v_t in Equations 24 and 25 is set to zero

$$\omega_{12} = 0 \tag{31}$$

With respect to a priori dynamics of $\{\sigma^x_t, \sigma^z_t, \rho^{xz}_t\}$, we assume that

$$\alpha_x = \alpha_z = \alpha_{xz} = 0.9 \tag{32}$$

This assumption is consistent with the conventions used in risk-management literature.[5]

[5] For example, in RiskMetrics J.P. Morgan assumes that volatilities change by an exponential time-decay factor of 0.9.

Further, the parameter β is naturally selected as

$$\beta = 1 \tag{33}$$

Finally, the initial point for s_t was obtained by letting

$$s_1 = \omega_1 \tag{34}$$

To reduce the effects of initial point selection, the Kalman filter estimates for the first 15 days are subsequently dropped. It turns out that the results reported subsequently are not sensitive to *significant* variations in ω_i, α, β, and λ.

The Results

The results are based on the daily data on nine assets—the 10-year U.S., French, and Japanese government bond yields; the DEM/USD; Yen/USD and FF/USD exchange rates; and the U.S., Japanese, and French stock market indices (SP 500, Nikkei, and CAC40). The data are obtained from the Global Financial Data Inc. and cover the period from January 1, 1990 to December 31, 2000, yielding a total of 2700 daily observations after adjusting the data for correlation estimates.[6]

We report the results in two stages. First, we discuss the empirical findings that show an *extreme* variation in the volatilities and correlations of risk factors. Next, we discuss the volatilities and correlations of risk-factor volatilities. Both sets of empirical results provide strong evidence concerning practices of risk management and calibration of dynamic models for pricing purposes.

The first set of results concerns the *instability* of correlation coefficients between risk factors. We estimate time-varying correlation coefficients between risk factors and plot them. Conclusions are drawn from a visual examination of these charts.

Figure 1(a–c) displays the time-varying correlation coefficients between various risk factors. These figures were obtained by first applying the Kalman filter formulas in 26–28 to pairwise risk factors designed in the titles. The result is the estimate of the time-varying correlation, r_t^{xz}, that is plotted. We have the following observations concerning these time-varying correlations.

[6]In fact, many other series were tried. Interest rates on maturities shorter than 10 years and other risk factors such as monthly price indices were used. Data from smaller EC economies such as Spain were also investigated. The results were similar.

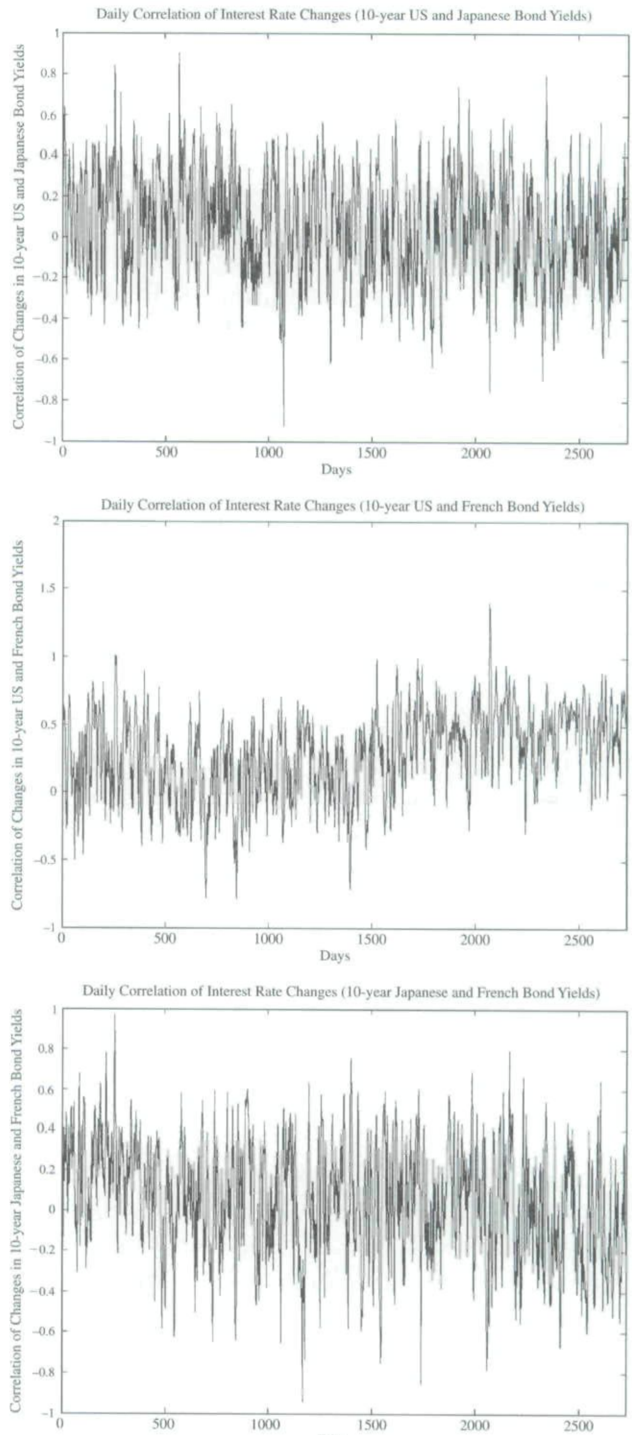

FIGURE 1(a)
Daily correlation of interest rate changes (Jan. 1, 1990–Dec. 31, 2000).

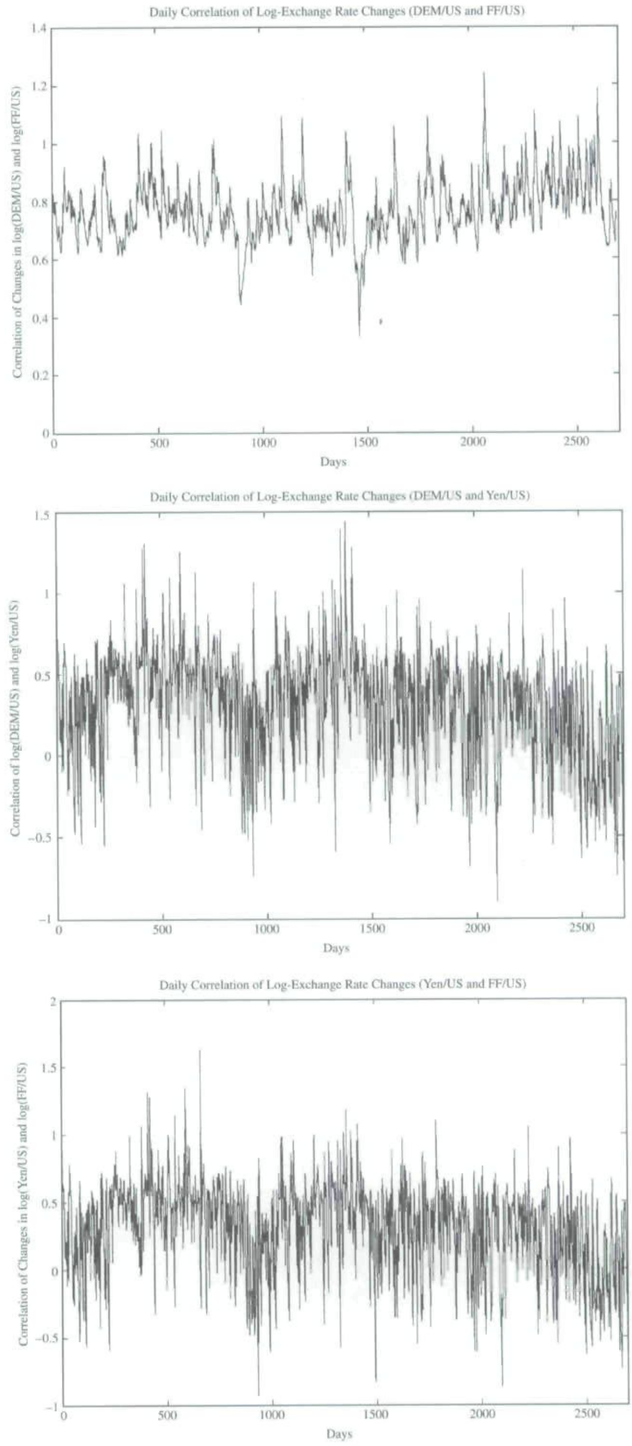

FIGURE 1(b)
Daily correlation of log-exchange rate changes (Jan. 1, 1990–Dec. 31, 2000).

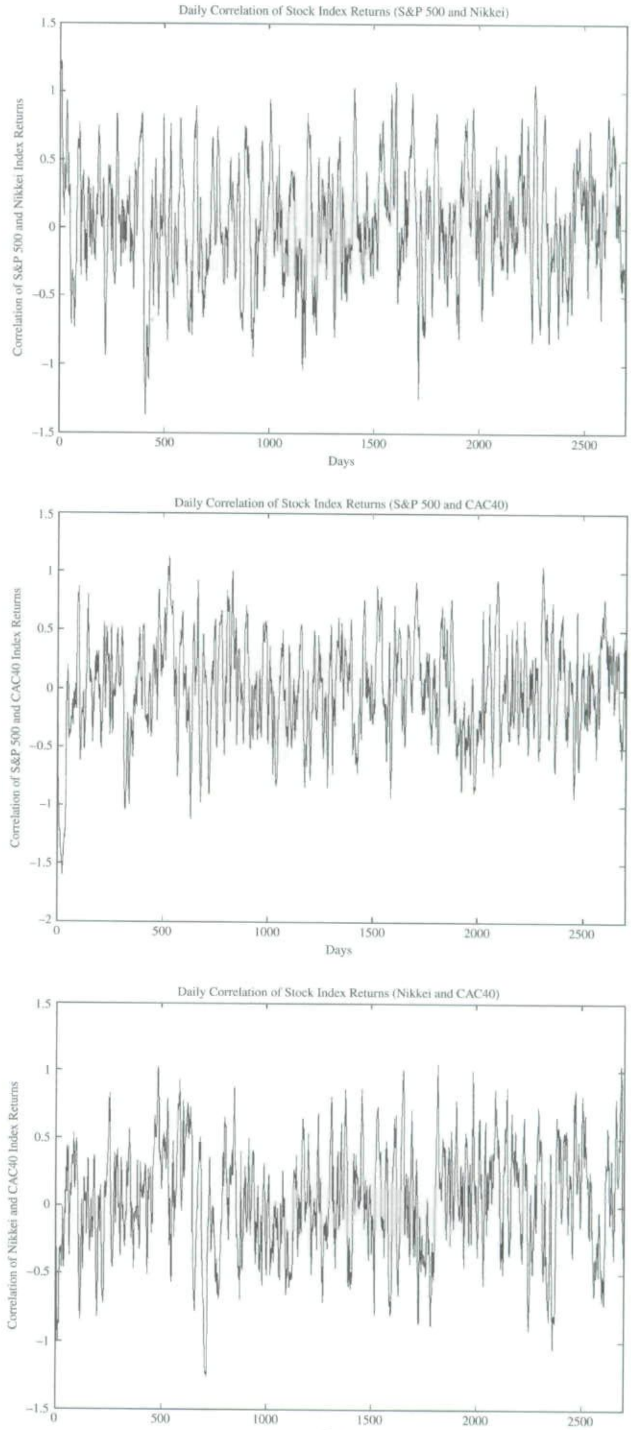

FIGURE 1(c)
Daily correlation of stock index returns (Jan. 1, 1990–Dec. 31, 2000).

With one exception, the *daily* correlations between risk factors are very *unstable*, varying from +1 to −1 throughout the sample period. This instability contrasts sharply with the very high positive correlation we find between US Dollar-French Frank and US Dollar-German Mark exchange rates as shown in Figure 1(b), which in fact provides some evidence that if there were meaningful correlation between risk factors, our procedures would have detected it. In fact, French Frank and German Mark are expected to be highly correlated because of their relation in EMU. Thus, when the Kalman filter is applied to estimate the corresponding r_t^{xz}, we obtain a correlation that is always positive and fluctuates around +0.8.[7]

Yet, when our procedure is applied to other risk factors, we see that the daily correlation coefficients are highly unstable. As expected, time-varying daily correlation coefficients for stock index returns are very unstable and fluctuate between +1 and −1. Time-varying daily correlation coefficients of 10-year government bond yields generally fluctuate within the band −0.6 and +0.6, but throughout the sample period it is difficult to make a statement concerning the *sign* of the daily correlations as well as their size.

As shown in Figure 1(a,b), daily correlations between exchange rates and interest rates belonging to different economies are *on average* positive but not throughout the sample period. In fact, we observe intermittent periods where this correlation shifts to negative territory, except for the correlation between DEM/USD and FF/USD. Finally, although not presented in this article, the daily correlations between changes in interest rates of two countries (e.g., United States and France) and their exchange rates (say FF/USD) are highly *unstable*, contrary to the expectation one may have from the uncovered interest rate parity. For these two variables we again have periods of high negative correlation as well as high positive ones.

Further, these switches between high negative and positive correlation seem to occur in a completely arbitrary fashion, to the extent that the use of daily estimated correlations for VaR calculations become quite suspect. Indeed, if short-term correlation coefficients display such extreme time variation, it becomes quite difficult to justify the use of "yesterday's" estimates in calculating "today's" VaR.

Figure 2(a–c) displays the volatility of interest rates, exchanges rates, and stock returns. The volatility of market variables are expressed in terms of percentages. For example, in Figure 2(c) daily volatility of

[7] There is one major instance of deviation, and that is the 1992 EMS foreign-exchange crisis.

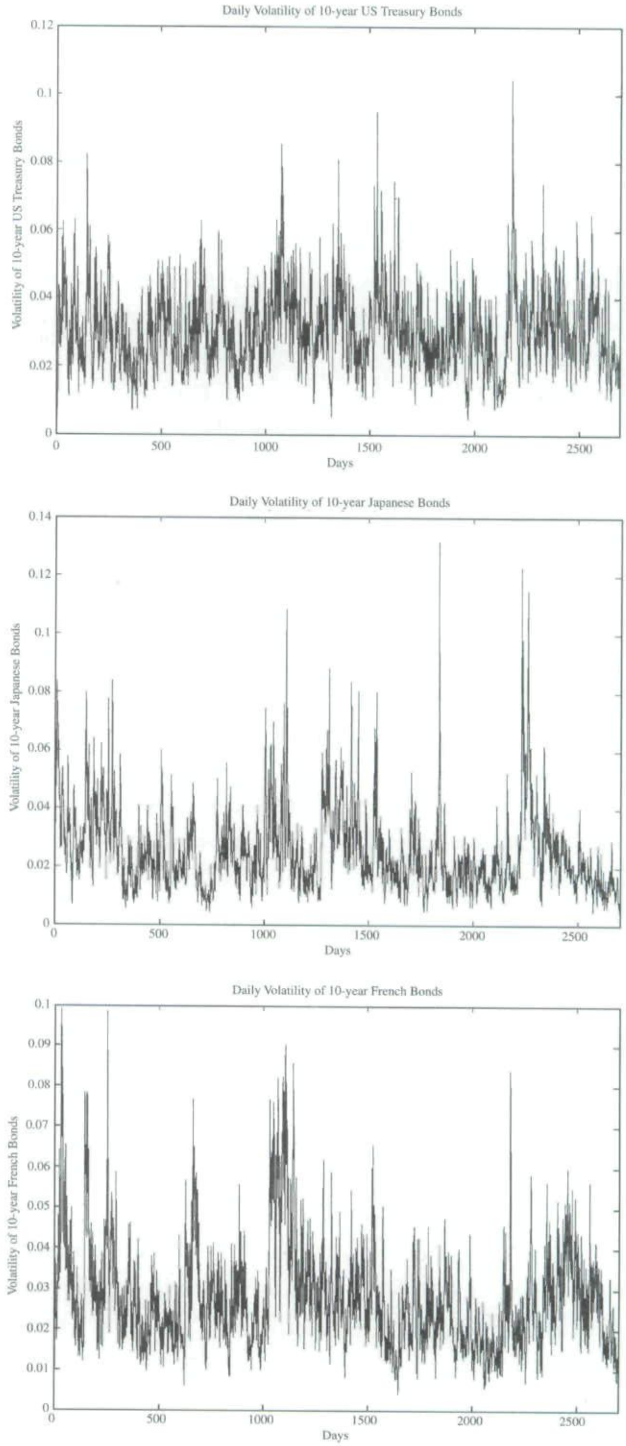

FIGURE 2(a)
Daily volatility of 10-year government bond yields (Jan. 1, 1990–Dec. 31, 2000).

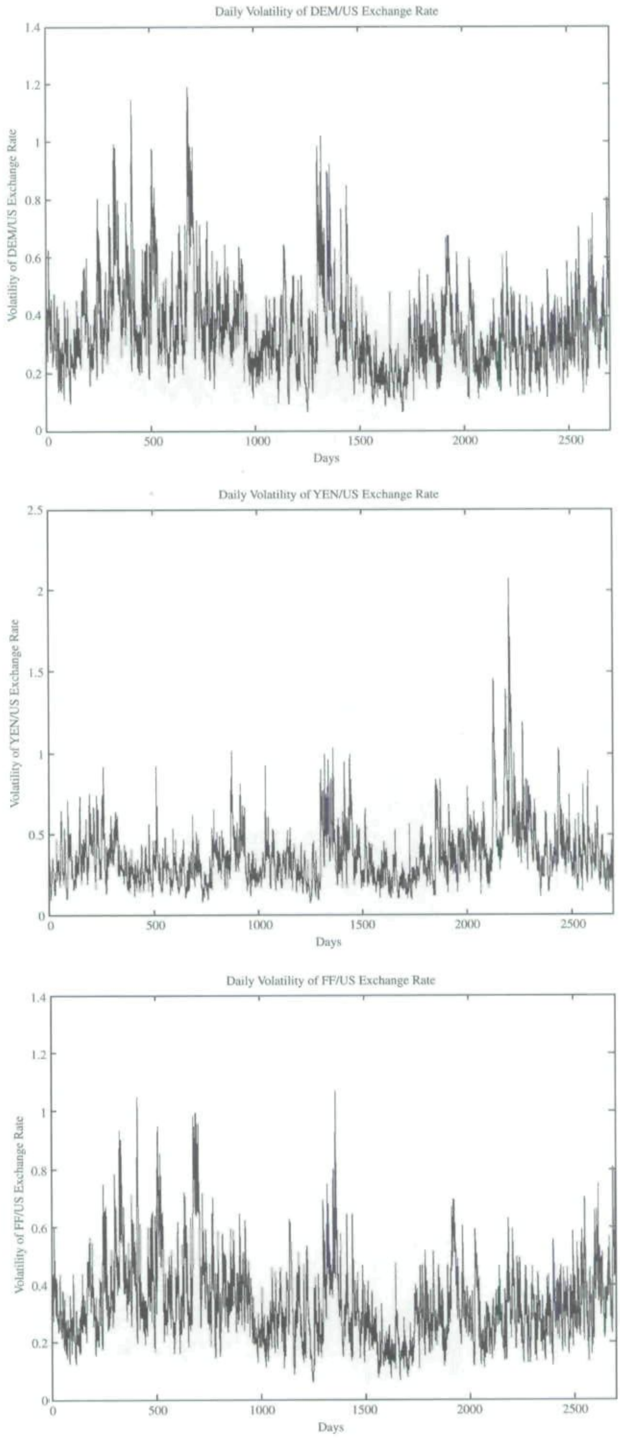

FIGURE 2(b)
Daily volatility of exchange rates
(Jan. 1, 1990–Dec. 31, 2000).

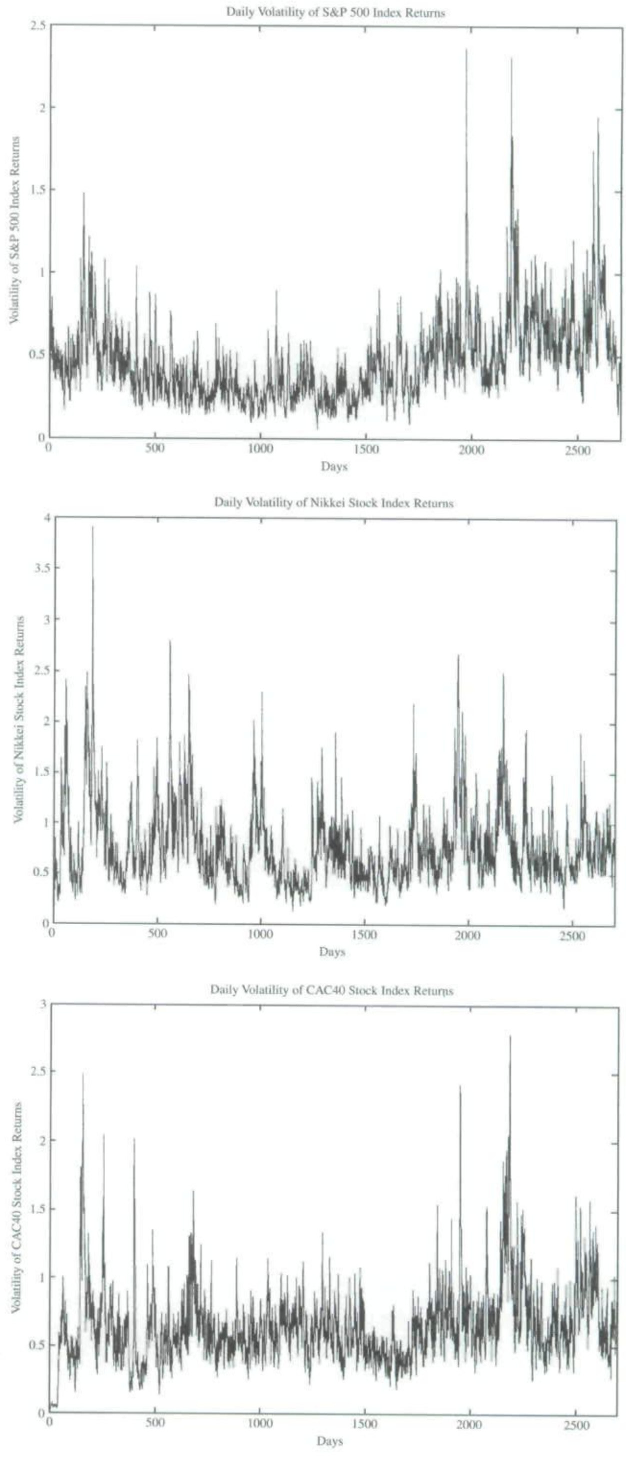

FIGURE 2(c)
Daily volatility of stock index returns
(Jan. 1, 1990–Dec. 31, 2000).

SP 500 index returns fluctuates between 0.1 and 2.4%. Daily volatility of Nikkei index returns fluctuates within the band of 0.4 and 3.9%, and averaged at around 1%. Daily volatility of CAC40 index returns fluctuates between 0.3 and 2.8%. As expected, the volatility of stock returns is higher than the volatility of exchange rates and interest rates. As depicted in Figure 2(a–c), the 10-year government bond yields are more stable than stock returns and exchange rates. In general, for all risk factors, deterministic periods interrupted by sudden bursts of volatility are clearly visible in Figure 2.

Thus, our first set of results seem to indicate that on a short-term basis, it is very difficult to use *stable* volatility or correlation coefficients for risk management and for pricing securities with historically estimated parameters. Instead, the results indicate that the implicit values for volatility and correlation parameters need to be extracted from liquid and up-to-date prices of properly chosen contingent claims. However, time variation induced by liquidity, for instance [as in Muthuswamy, Sarkar, Low, & Terry (2001)], would limit the effectiveness of this approach. It is interesting that econometric work based on monthly data may not show this weakness of historical second-moment estimation because the monthly second moments do appear to be quite stable. However, for pricing and risk-management purposes the use of monthly data will be less meaningful.

Our second set of results is given in Figures 3 and 4. The charts plotted in these figures were obtained from the same set of formulae as in the preceding case. In contrast with the results reported in the first preceding set, we now need to apply a *two-stage* process. At the first stage, time-varying volatilities for each risk factor are estimated with the Kalman filter, and this way a process of volatility estimates is generated. Then, time-varying volatilities and correlations were estimated using these estimated time-varying volatilities.

Figure 3(a–c) provides estimates of the daily time-varying correlations *between volatilities* of various risk factors. The daily correlations of volatilities are generally positive and fluctuate between 0 and +1. As shown in Figure 3, the correlations of interest rate volatilities fluctuate within a narrower band as compared with the correlations of stock return volatilities and the correlations of exchange rate volatilities.

Figure 4(a–c) presents the daily volatilities of interest rate volatility, exchange rate volatility, and stock return volatility. To calculate the data displayed here, we first applied the Kalman filter to obtain estimates of time-varying daily volatilities. Then, the Kalman filter was applied for a second time to obtain the volatilities of these first-round estimates.

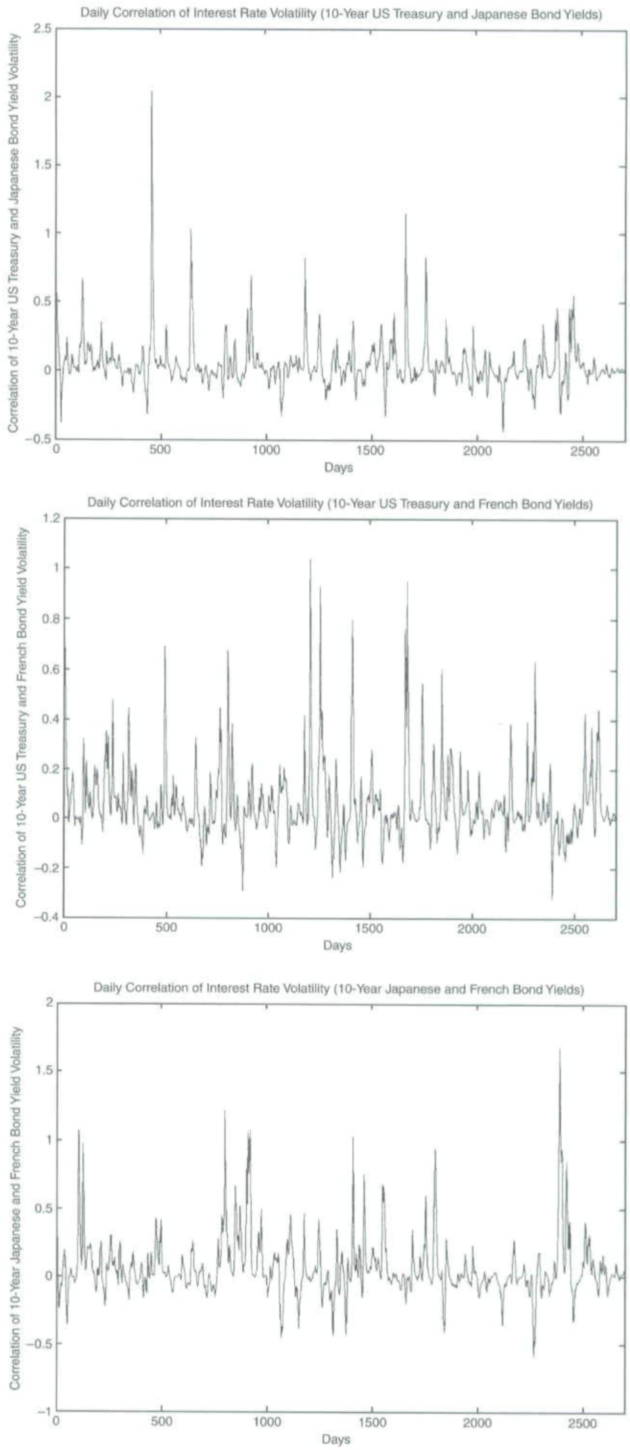

FIGURE 3(a)
Daily correlation of interest rate volatility (Jan. 1, 1990–Dec. 31, 2000).

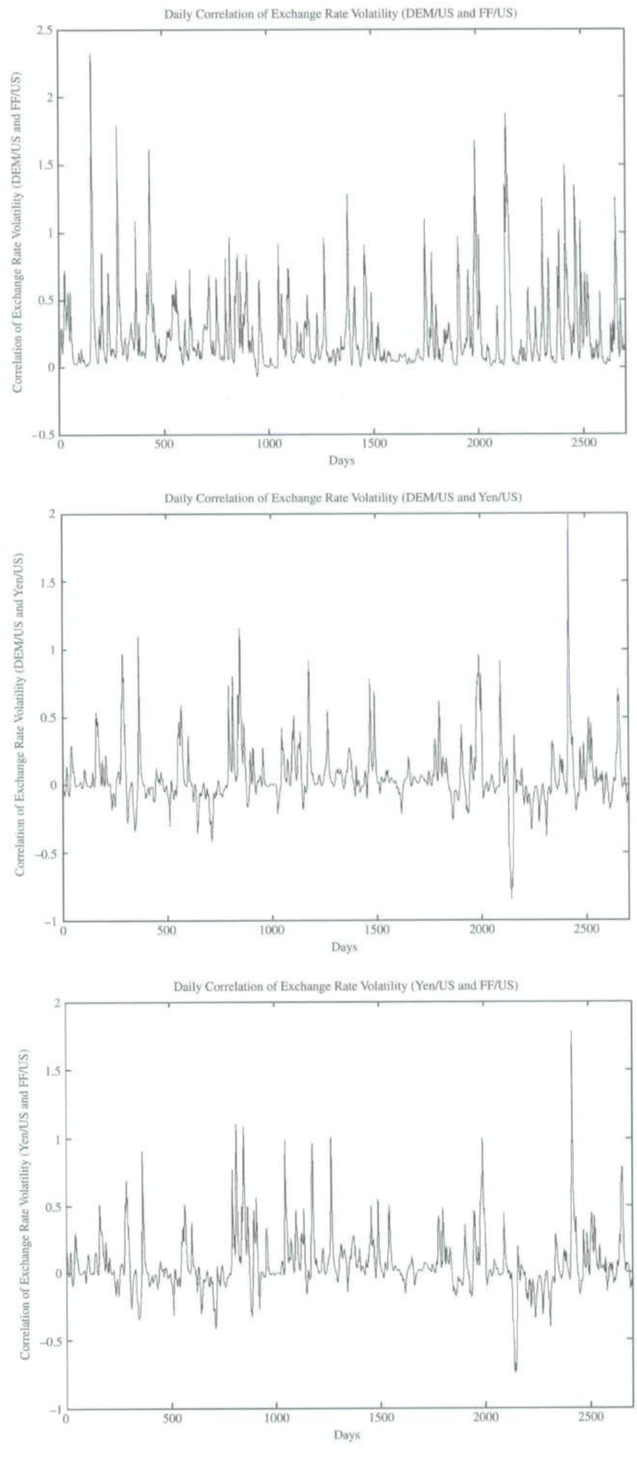

FIGURE 3(b)
Daily correlation of exchange rate volatility (Jan. 1, 1990–Dec. 31, 2000).

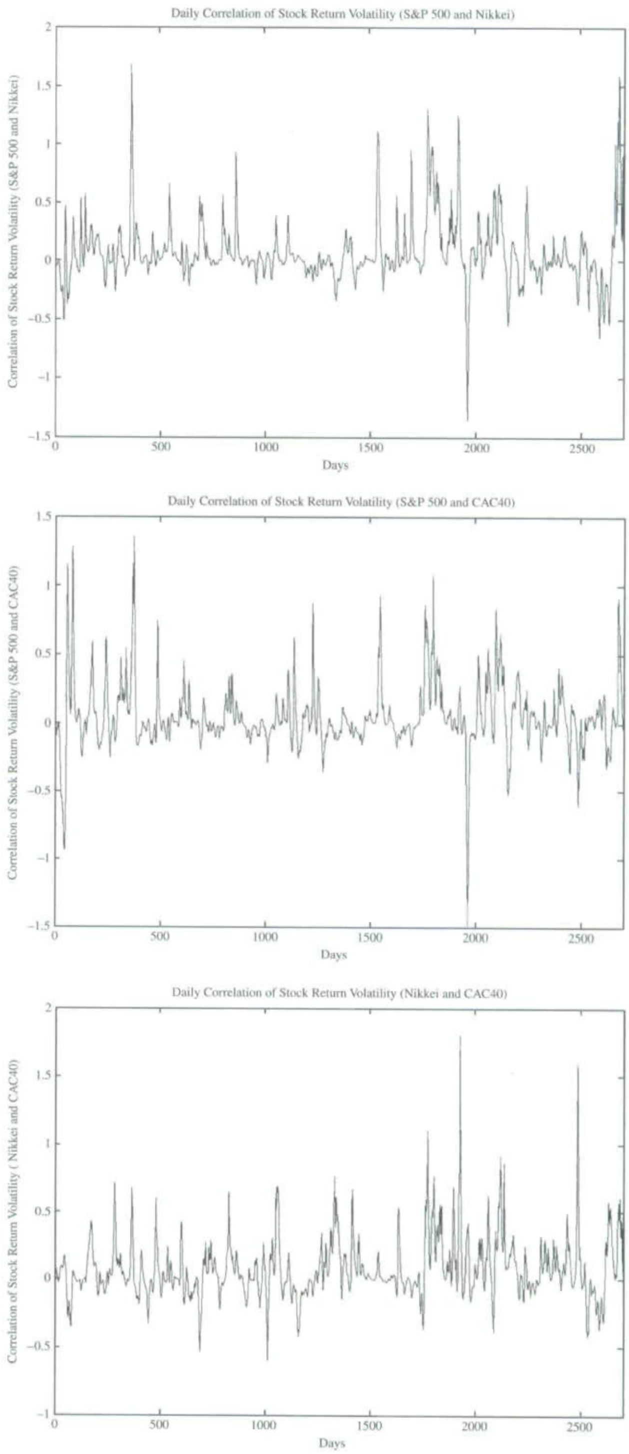

FIGURE 3(c)
Daily correlation of stock return volatility (Jan. 1, 1990–Dec. 31, 2000).

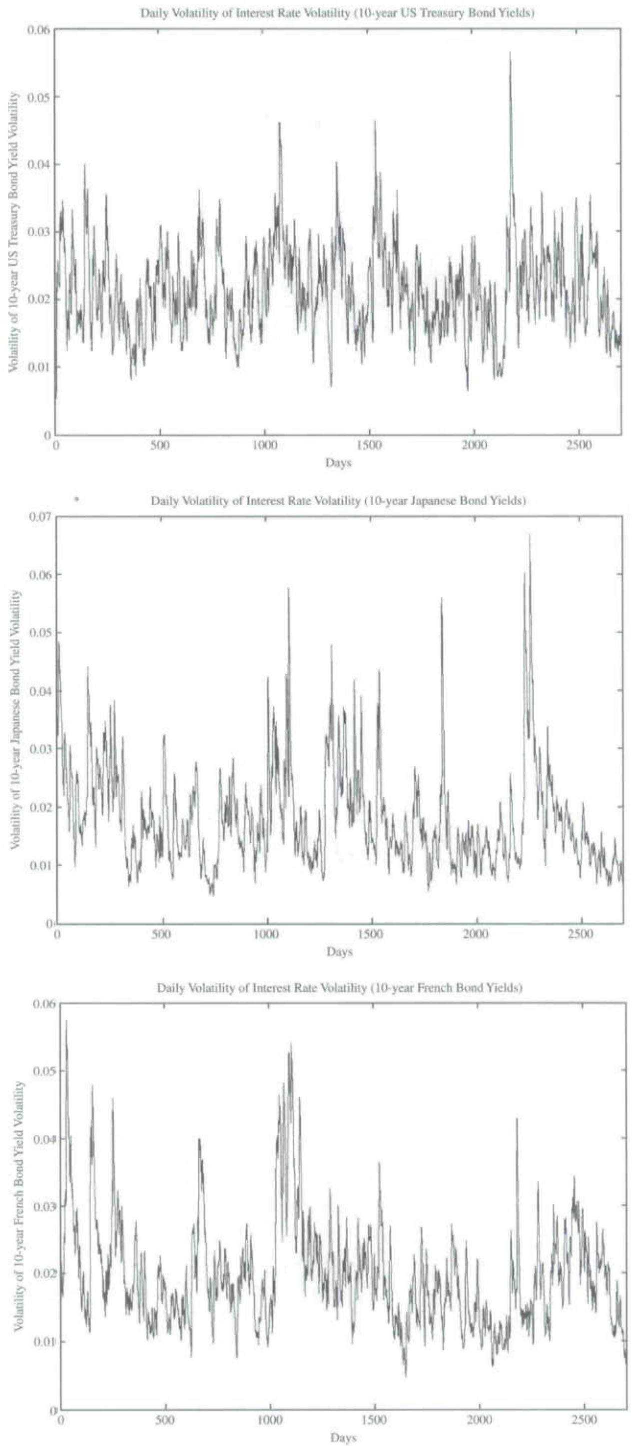

FIGURE 4(a)
Daily volatility of interest rate volatility
(Jan. 1, 1990–Dec. 31, 2000).

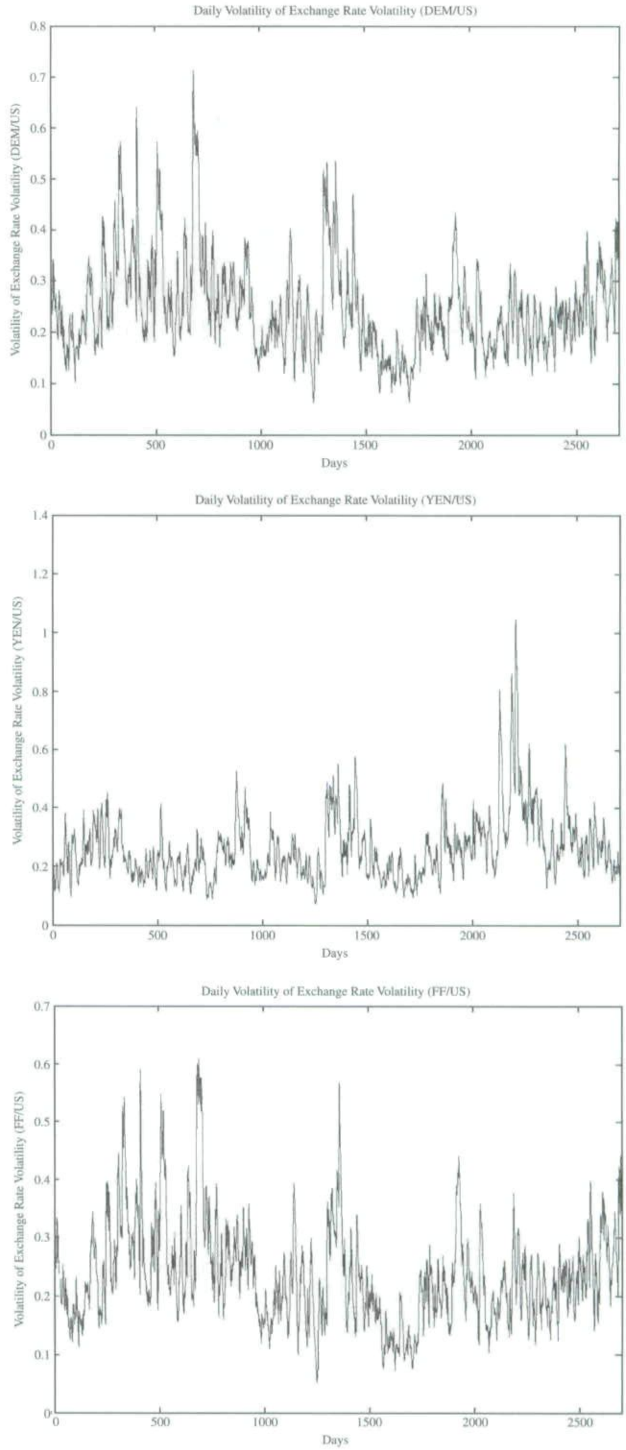

FIGURE 4(b)
Daily volatility of exchange rate volatility (Jan. 1, 1990–Dec. 31, 2000).

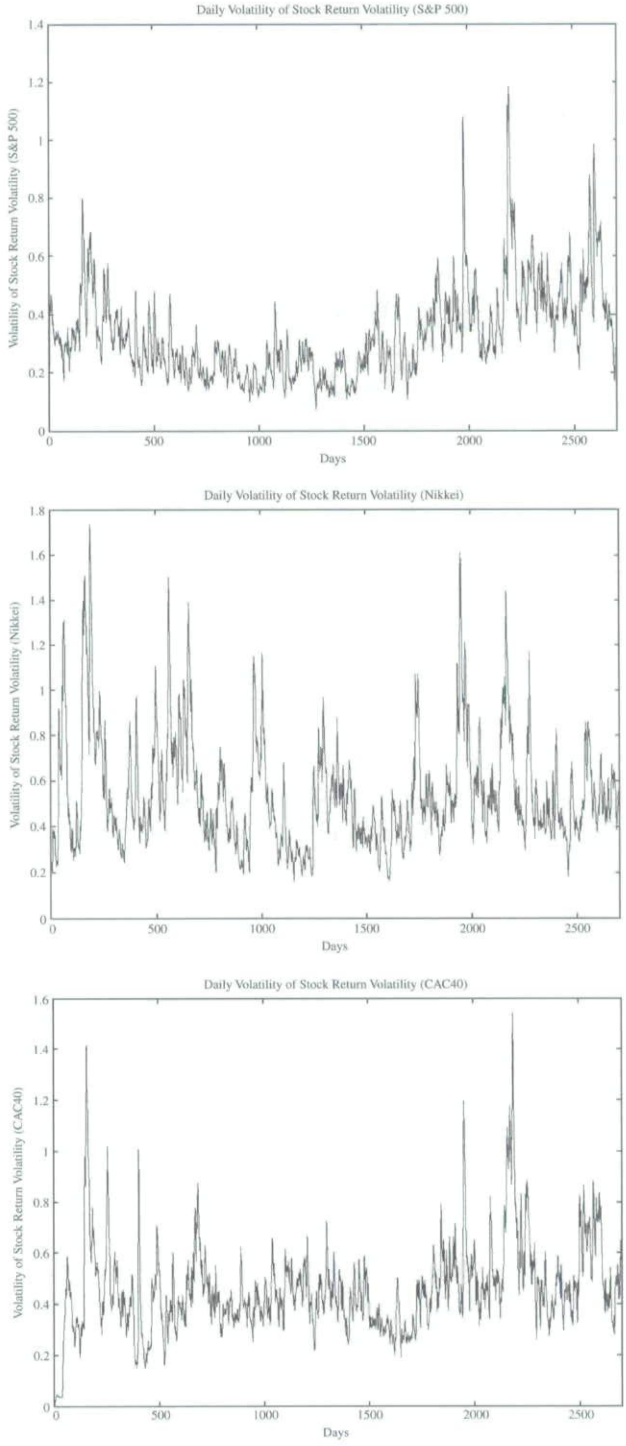

FIGURE 4(c)
Daily volatility of stock return volatility (Jan. 1, 1990–Dec. 31, 2000).

For example, the results of Figure 4(a) can be interpreted as displaying the volatility of the stochastic component in the 10-year government bond yield volatilities. The interesting result from Figure 4(a) is that most of the daily volatility of interest rate volatilities is close to zero. The volatility of volatility is expressed in terms of percentages. In all three cases, daily volatility of interest rate volatility fluctuates within the range of 0.01– 0.06%. Only at some occasional times do the volatilities become volatile. This occurs as a spike that then gradually goes back to zero within two to three weeks. We see that the size and the timing of these spikes are very highly correlated across the three interest rates.

As expected, the volatility of stock return volatility is higher than the volatility of interest rate and exchange rate volatilities. Specifically, as shown in Figure 4(c), daily volatility of stock return volatility fluctuates between 0.1 and 1.2% for SP 500, 0.2 and 1.7% for Nikkei, and 0.2 and 1.5% for CAC40. As expected, the daily volatility of interest rate volatility turns out to be much lower than than the volatility of stock return and exchange rate volatilities.

The results indicate that not only the volatilities seem to vary stochastically, but this variation has surprising characteristic. The variation in volatility processes is stochastically nonlinear, that is, long periods of stability are interrupted by sudden spurts of high volatility. This stochastically nonlinear behavior is present in all risk-factor volatilities. We also see that the correlations between the volatilities of risk factors are generally positive and vary high.

CONCLUSIONS

We can summarize our results in terms of three empirical propositions.

Proposition 1. *The correlations between risk factors are highly unstable even over short periods of time, both in terms of sign and absolute value. The exception is the correlations between DEM/USD and FF/USD exchange rates, which is close to* $+1$.

Proposition 2. *Although the correlations and volatilities of risk factors are themselves very unstable, the volatilities of risk factors are highly and positively correlated among themselves. The relatively more stable behavior of correlations between volatilities contrasts sharply with the instability of the correlation of the variables themselves.*

Proposition 3. *The time variation of volatilities is stochastically* nonlinear. *Long periods of "deterministic" volatilities are interrupted by sudden bursts of highly volatile periods. This pattern is repetitive.*

These results have the following implications.

Concerning risk management, according to the rules proposed by the Basel Committee, banks will not be able to use diversification arguments *across* asset classes when they calculate value at risk for the purpose of capital requirements. This has been criticized by the industry on the grounds that correlations of returns are typically less than 1 and that not allowing for the effects of diversification means that capital requirements will be unnecessarily high. Our results show that correlations can quite suddenly approach +1, although, on average, they are much smaller. However, because value at risk calculations is supposed to be valid in extreme situations, the assumption that the relevant correlations are equal to 1 is perhaps justified. Our results may therefore support the interpretations of the Basel Committee.

Concerning portfolio management and asset allocation, in view of the instability of correlations of returns in the short run, one cannot rely on standard optimal portfolio models for short-run portfolio management. The asset allocation problem needs to consider the unstable nature of these second-order moments.

This article indicates the presence of extremely large nonlinearities in the time series of correlations and volatilities of risk factors, whereas the correlation between volatilities is relatively stable, fluctuating within the band of 0 and +1. These results suggest that a valuation model with two underlying assets should not use constant volatilities for the asset returns. It would be more appropriate to use the two asset returns and the two volatilities as state variables as well as a constant positive correlation between the two volatilities. Therefore, the computation would be more complicated because there would be four state variables instead of two.

Concerning measurement of volatilities, the Kalman filter framework used in this article can be alternative approach to GARCH and stochastic volatility models. In addition, ARCH/GARCH models may not provide an accurate characterization of the actual volatility process because they do not incorporate the unobservable stochastic volatility factor into conditional variance or standard deviation. Stochastic volatility and regime-switching models may be a more appropriate way of capturing the stochastically nonlinear behavior of volatility fluctuations.

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