NONLINEAR DYNAMICS IN HIGH-FREQUENCY INTRADAY FINANCIAL DATA: EVIDENCE FOR THE UK LONG GILT FUTURES MARKET

DAVID G. MCMILLAN
ALAN E. H. SPEIGHT*

Recent research investigating the properties of high-frequency financial data has suggested that the stochastic nonlinearity widely present in such data may be characterized by heterogeneous components in conditional volatility, and nonlinear dependence of threshold autoregressive form due to market frictions. This article tests for the presence of such effects in intraday long gilt futures returns on the UK LIFFE market. Tests against the null of linearity indicate the significance of smooth transition autoregressive nonlinearities in such returns at the 5-min frequency, which entails a first-order autoregressive process with switching intercept. This nonlinear structure is robust to the presence of asymmetric and component structures in conditional variance, and consistent with the existence

*Correspondence author, Department of Economics, University of Wales, Swansea, SA2 8PP, UK; e-mail: a.speight@swan.ac.uk

Received September 2000; Accepted March 2002

- *David G. McMillan is a Lecturer in the Department of Economics at the University of St. Andrews, United Kingdom.*
- *Alan E.H. Speight is a Professor of Economics in the Department of Economics at the University of Wales in Swansea, United Kingdom.*

The Journal of Futures Markets, Vol. 22, No. 11, 1037–1057 (2002)
Published online in Wiley InterScience (www.interscience.wiley.com). DOI: 10.1002/fut.10043

of heterogeneous traders facing different levels of transaction costs, noise trader risk, or capital constraints. Jrl Fut Mark 22:1037–1057, 2002

INTRODUCTION

The increasing availability of intraday financial data in recent years has presented new opportunities to examine financial market dynamics and market microstructure. While many initial investigations of such data focused on the search for generic nonlinearities of deterministic form, the emergent consensus from such studies is that there is little evidence to support the proposition that high-frequency returns are characterized by low-dimensional chaotic processes, but there is substantial evidence for the presence of stochastic nonlinear dependence, and these conclusions would seem to hold equally well for exchange rates, equity prices and commodity prices, and for both spot and futures markets (Goodhart & O'Hara, 1997).

The genre of models of autoregressive conditional heteroskedasticity (ARCH: Bollerslev, 1986; Engle, 1982) have typically provided the means for modeling stochastic nonlinear dependence in financial data at daily and lower frequencies, largely due to the empirical success of such models in capturing the time-varying conditional volatility that is characteristic of the returns distributions of many financial assets. A popular explanation for the presence of such ARCH effects in asset returns is provided by the mixture of distributions hypothesis (MDH), which holds that the distribution of returns follows a mixture of normals with changing variance, the rate of new information arrival providing the stochastic mixing variable (Clark, 1973; Diebold, 1986).[1] In extension of this approach, the examination of high-frequency intraday data has recently prompted several researchers to suggest that volatility may more accurately be characterized by heterogenous components, reflecting either heterogeneous information flows (Andersen & Bollerslev, 1997a) or the actions of heterogeneous market traders (Müller et al., 1997).

However, while time-varying conditional volatility offers a potential explanation for nonlinear dependence in high-frequency financial data, it remains equally possible that such dependence is, in fact, due to

[1]Other explanations for time-varying volatility based on considerations of market microstructure include that of Kyle (1985), whereby information held by an informed trader is transmitted into prices gradually through information diffusion. An alternative rationale is provided the theoretical model of Timmermann (1995), where the source of volatility clustering is incomplete learning and limited knowledge of the process generating fundamentals. For reviews, see Bollerslev, Chou and Kroner (1992), Bera and Higgins (1993), and Bollerslev, Engle, and Nelson (1994).

nonlinearity in conditional mean. Indeed, there is increasing recognition that financial markets may be characterized by nonlinear behavior resulting from a variety of sources of market frictions, including transaction costs and noise traders, which create a band of price movements around the equilibrium price such that arbitrageurs only actively trade when deviations from equilibrium become sufficiently large. Consequently, the dynamic behavior of returns will differ according to the size of the deviation in prices from equilibrium, giving rise to asymmetric dynamics for returns of differing size (inter alia, Dumas & Luciano, 1991; Schleifer, 2000; Sercu et al., 1995).

This article reports the results of empirical testing for the presence of the preceding forms of nonlinear dependence in the context of intraday UK LIFFE long gilt futures returns data. The nonlinear conditional mean model adopted is the smooth transition autoregressive (STAR) model (Chan & Tong, 1986; Granger & Teräsvirta, 1993; Teräsvirta, 1994, 1998; Teräsvirta & Anderson, 1992) that allows for differing market dynamics according to the magnitude of returns in keeping with the forms of market friction outlined above. Following confirmatory preliminary tests for the presence of threshold nonlinearities, STAR conditional mean estimates are reported. The robustness of that nonlinear mean structure to the presence of ARCH effects is examined through joint estimation under maximum likelihood using one of two extensions of the basic GARCH framework that permit conditional variance asymmetry or heterogeneity, respectively. The former is provided by the exponential-GARCH (EGARCH) model of Nelson (1991), which has close correspondence with the MDH described above. The latter is provided by the Engle and Lee (1999) component-GARCH (CGARCH) model, which permits the decomposition of conditional volatility into long-run and short-run elements, in keeping with recently advanced notions of heterogeneous volatility components in intraday financial data.

The remainder of the article is organized as follows. In the following section we provide a brief overview of the theoretical literature concerning the potential for threshold nonlinearities in financial market returns data. Then we outline the empirical models to be estimated and their properties. In the Data and Preliminary Diagnostics section we describe the data and the institutional setting from which it is drawn, provide nonparametric kernel density estimates of the data distributions, and report results of the preliminary tests for nonlinearity in conditional mean. The Empirical Results section discusses issues of model specification and evaluation, and reports conditional mean and variance estimates. Then we provide a summary of our findings and conclusions.

THEORETICAL BACKGROUND

As noted in the introduction, an issue that has received much attention in the empirical finance literature of late, and that offers an appealing explanation for asymmetries in market returns, is related to the interaction of informed traders and "noise traders."[2] The existence of noise trading means that profitable opportunities will inevitably arise for privately informed and arbitrage traders. In early recognition of the potential nonlinear consequences of such interaction, Cootner (1962) noted that the activities of noise traders will cause prices to hit upper or lower "reflecting barriers" around equilibrium, and thus trigger arbitrage activities by informed traders that push prices back to equilibrium. Cootner argued further that the position of such barriers is likely to depend on the size of market frictions such as transactions costs, giving rise to a band of price movements around the equilibrium price. Thus, informed traders only actively trade when deviations from equilibrium are sufficiently large to make arbitrage trade profitable. However, under this approach, such opportunities are unlikely to be long lived, existing only for as long as the reassessment of underlying fundamentals in the light of market "news" may warrant (He & Modest, 1995).

A related explanation for the possible existence of nonlinear adjustment, originally advocated in the context of foreign exchange markets, holds that small deviations from fundamental equilibrium may be considered relatively unimportant by both the market and policy makers. Consequently, only limited significance will be attached to market forces that are not governed by economic fundamentals, such as trading behavior based on technical analysis and chartism (Taylor & Allen, 1992). However, as the market price becomes increasingly misaligned, pressure from both the market and policy makers could be expected to return the exchange rate to its fundamental equilibrium value. Indeed, a series of recent articles examining both real and nominal exchange rates has

[2]The rationale generally offered for noise trading is that it allows privately informed traders to profitably exploit their informational advantage, without which market efficiency would not be assured (e.g., Kyle, 1985). That rationale does not, however, explain the reasons for noise trading, on which there are differing views. Noise trading may be regarded as either rational agents trading for liquidity and hedging purposes, consistent with a fully rational efficient-markets perspective (Ausubel, 1990a, 1990b; Biasis & Hillion, 1994; Diamond & Verrechia, 1981; Dow, 1995; Dow & Gorton, 1994, 1997), or irrational (or not fully rational) agents trading on beliefs and sentiments that are not justified by news concerning underlying fundamentals (Black, 1986; Schleifer & Summers, 1990; De Long et al., 1990). An interesting alternative interpretation recently offered by Dow and Gorton (1997) suggests that delegated portfolio managers may engage in noise trading to appease clients or managers who are unable to distinguish purposeful inaction from nonpurposeful inaction, as a result of which the amount of noise trading can be large compared to the amount of hedging volume, and Pareto improving.

reported affirmatory evidence of threshold effects (Coakley & Fuertes, 2000, 2001; Martens et al., 1998; Obstfeld & Taylor, 1997). More formal models of international goods and exchange market arbitrage examined by Dumas and Luciano (1991) and Sercu et al. (1995) have similarly suggested that the presence of transactions cost that result in small imbalances being left uncorrected may impart nonlinear adjustment to equilibrium due to nonlinear factors affecting the costs of arbitrage, such that the speed of reversion to equilibrium increases with the size of the deviation.

The above models rely on transactions costs to limit arbitrage around fundamental equilibrium and thus create bands of inactivity about the equilibrium price that must be breached before arbitrage takes place, such that small and large deviation from equilibrium are characterized by different dynamics. An alternative explanation put forward by Schleifer (2000) argues that, even in the absence of transaction costs, noise trading can produce limits to arbitrage such that large deviations from equilibrium are required before arbitrage occurs. This Schleifer ascribes to "mis-price deepening." That is, where noise traders beliefs become even more extreme before they are subject to correction, providing an additional source of risk to be faced by arbitrageurs, particularly if they are subject to a short horizon.[3] Consequently, the risk of a further movement of noise traders opinion away from the fundamental equilibrium must be borne in mind by arbitrageurs, and may limit their willingness to take a contrary position to noise traders. For example, if noise traders are pessimistic today about an asset and have thereby driven its price down, an arbitrageur buying this asset needs to recognize that noise traders might become even more pessimistic and drive the price down even further in the near future; if the arbitrageur has to liquidate their position before the price recovers, they will suffer a loss, and fear of this loss could limit the original arbitrage position. Conversely, an arbitrageur selling an asset short when bullish noise traders have driven its price up needs to be aware that noise traders might become even more bullish tomorrow, and so must take a position that accounts for the risk of a further price rise when they have to buy back the asset.

[3]Shleifer (2000) justifies the assumption that arbitrageurs may be faced by short horizons on the basis that arbitrageurs typically do not manage their own money but are agents for investors who evaluate their performance at regular intervals and reward them accordingly. Mispricings that take longer to correct than the evaluation horizon may, therefore, reduce arbitrageurs' remuneration. Further, many arbitrageurs borrow money and securities from intermediaries to put on their trades and, while they have to pay interest, they also face the risk of liquidation by lenders if prices move against them and the value of collateral falls.

EMPIRICAL MODELS

On the arguments of the preceding section, while the actions of individual arbitrage traders may be represented by a simple threshold model that imposes an abrupt switch in behavior, only if all traders act simultaneously will this also be the observed market outcome. For a market of many traders acting at slightly different times a smooth transition model is therefore arguably more appropriate than a "heaviside" model. To investigate the possibility of market nonlinearities due to the interaction of informed and noise traders as described above, we therefore consider the nonlinear smooth transition autoregressive, STAR(p), model (Granger & Teräsvirta, 1993; Teräsvirta & Anderson, 1992), which has the general form:

$$r_t = \pi_0 + \sum_{i=1}^{p} \pi_i r_{t-i} + F(r_{t-d})\left(\theta_0 + \sum_{i=1}^{p} \theta_i r_{t-i}\right) + \varepsilon_t \tag{1}$$

$$F(r_{t-d}) = 1 - \exp(-\gamma(r_{t-d} - c)^2); \quad \gamma > 0 \tag{2}$$

where r_t denotes the asset return calculated as the differenced logarithm of the asset price $r_t = \log(p_t/p_{t-1})$, and $F(.)$ denotes a transition function defined over a transition variable, here provided by the lagged return value, r_{t-d}, where d is a delay parameter. The interpretation of (1) is that r_t is described by the linear model of autoregressive order p in the second term on some occasions, and by that process with the addition of the potentially nonlinear component in the compound third term on other occasions. Alternatively, the components $F(r_{t-d})(\theta_0)$ and

$$F(r_{t-d})\left(\sum_{i=1}^{p} \theta_i r_{t-i}\right)$$

may be interpreted as rendering the intercepts and autoregressive parameters of the model time-varying, and (1) therefore as belonging to the class of state-dependent models (Priestley, 1988). The specific transition function $F(.)$ in (2) is the exponential form, where γ is a smoothing or transition parameter, and c a threshold parameter, the combination of (1) and (2) yielding the exponential-STAR (ESTAR) model, whereby the parameters in (4) change symmetrically about c with r_{t-d}, such that as $(r_{t-d} - c) \to 0$, $F(r_{t-d}) \to 0$, and as $(r_{t-d} - c) \to \infty$, $F(r_{t-d}) \to 1$,while as either $\gamma \to \infty$ or $\gamma \to 0$ the model reduces to the linear AR form. Thus, the ESTAR model implies that the dynamic process for moderate returns will differ from that for larger returns, irrespective of sign, in keeping with the arguments outlined in the preceding section.

The first model of conditional volatility examined is the exponential GARCH (EGARCH) model (Nelson, 1991). As noted above, the selection of the EGARCH model is motivated by its close relationship with the mixture of distributions hypothesis (Clark, 1973), which views the variability of security prices as arising from differences in information arrival rates.[4] Let the asset return r_t have conditional variance $h_t^2 = \text{var}(r_t \mid \psi_{t-1})$, where ψ_{t-1} defines the set of all information available at time $t - 1$, such that the first-order EGARCH model is given by:

$$\log(h_t^2) = \omega + \alpha(|\varepsilon_{t-1}|/|h_{t-1}|) + \zeta(\varepsilon_{t-1}/h_{t-1}) + \beta \log(h_{t-1}^2) \quad (3)$$

where the logarithmic form ensures conditional variance non-negativity without the necessity of constraining the coefficients of the model, the parameter α captures the volatility clustering effect that is characteristic of ARCH processes, a positive value indicating that large (small) shocks tend to follow large (small) shocks of random sign, the parameter β captures the degree of persistence in shocks to volatility with half-life decay given by $\log(0.5)/\log(\beta)$, and the potentially asymmetric effect of positive and negative shocks on conditional variance is captured by a nonzero value for the parameter ζ.

While the EGARCH model assumes homogeneity of the price discovery process, as noted in the Introduction, it has been suggested that intraday returns volatility may more realistically comprise heterogeneous components. Such components may reflect differing market reactions to differing sources and types of news (Andersen & Bollerslev, 1997a), or the differing reactions of market agents with heterogeneous positions and time horizons to the same items of news (Müller et al., 1997). On either view, returns volatility will consequently be dominated by transient or short-run volatility over higher data frequencies and by more persistent or long-run volatility over lower data frequencies. To examine the data for the possible presence of such components we implement the component-GARCH (CGARCH) model of Engle and Lee (1999), which permits the decomposition of volatility into a long-run component and a short-run component, the model being described by the joint process:

$$h_t^2 = q_t + \alpha(\varepsilon_{t-1}^2 - q_{t-1}) + \beta(h_{t-1}^2 - q_{t-1}) \quad (4a)$$

$$q_t = \omega + \rho q_{t-1} + \phi(\varepsilon_{t-1}^2 - h_{t-1}^2) \quad (4b)$$

where the forecasting error $(\varepsilon_t^2 - h_t^2)$ serves as the driving force for the time-dependent movement of the long-run component, q_t, and the difference between the conditional variance and long-run volatility $(h_t^2 - q_t)$ defines the short-run component. The initial impact of a shock to the transitory component is quantified by α, and β indicates the

[4]For further details of the MDH interpretation of the EGARCH model see, for example, Bera and Higgins (1993, pp. 324–327) and Bollerslev et al. (1994, pp. 2994–2994).

degree of memory in the transitory component, the sum of these parameters providing a measure of transitory shock persistence. The initial effect of a shock to the long-run component is given by ϕ, with persistence measured by the autoregressive root, ρ, and where $1 > \rho > (\alpha + \beta)$ the transitory component decays more quickly than the long-run component.[5]

DATA AND PRELIMINARY DIAGNOSTICS

The data analyzed consists of the prices of UK government bond (long gilt) futures contracts traded on the London International Financial Futures and Options Exchange (LIFFE). The long gilt futures contract is of interest as a heavily traded investment and hedging instrument, the main users of which LIFFE identifies as market makers, institutional investors, and issuers of long-term debt, for purposes of hedging, investment, asset allocation, portfolio insurance, and duration adjustment, such activities being primarily driven by consideration of long-run factors and underlying fundamentals. A further feature of the long gilt futures market is its low margin requirement, which encourages a degree of short-term speculation and provides circumstances conducive to noise-trading of the manner described above. The sample covers the period 24th January 1992 to 30th June 1995. The contract price data, p, is sampled at 5-min intervals and transformed to yield the returns series, $r_t = \log(p_t/p_{t-1})$, with the overnight return excluded so as to ensure consistent time-series.[6] With 846 trading days in the sample period, this yields 80,163 observations.

[5]The conditional variance is covariance stationary provided that both the long-run and short-run components are covariance stationary, as satisfied by $\rho < 1$ and $(\alpha + \beta) < 1$, respectively, while the additional restriction of non-negativity on the model parameters ensures that h_t^2 is non-negative as long as q_t is non-negative. It should also be noted that while the CGARCH model can be extended to asymmetric form, given the lack of empirical support for conditional variance asymmetry in the EGARCH model estimates reported below, we do not pursue that extension here.

[6]LIFFE futures contracts have four quarterly delivery months, in March, June, September, and December. Because several contracts may be traded simultaneously, and given the continuous series requirement, a decision must be made as to which contract price to take at any given point in time. In the case of Long Gilt futures, the contract can be delivered at the seller's discretion on any business day in the delivery month up to 2 days prior to the last business day of that month. In their examinations of bond futures, Becker, Finnerty, and Kopecky (1993, 1995, 1996) use the nearest delivery date contract until 2 days from the start of the delivery month, at which point they switch to using the next nearest delivery contract. We follow Abhyankar et al. (1995) and Buckle et al. (1998) in basing the choice of contract on traded volume. Thus, the switch of contract here occurs on the day on which volume in the second nearest contract exceeds traded volume in the nearest contract. This is approximately 1 month before expiry of the nearest contract, but is not a fixed distance from expiry. Having already excluded the overnight return there is no further requirement to adjust the series as a result of the change in contracts. Nevertheless, the effect of this splicing of contracts on the estimates reported in the text is tested for as a matter of empirical robustness, the results of which are noted where appropriate.

As has been noted elsewhere, intraday data is strongly characterized by high-frequency periodicity corresponding to proximity in time to market opening and close, macroeconomic and other systematic news releases, and other factors. The strength of these intraday effects is such that failing to adjust for them can result in misleading analysis of the dynamic dependencies in the data (Andersen & Bollerslev, 1997b; Goodhart et al., 1993; Goodhart & O'Hara, 1997; Guillaume et al., 1997). Prior to estimation, we therefore follow Andersen and Bollerslev (1997b) in standardizing returns by the mean absolute value for each intraday time 5-min interval.[7,8] Summary statistics for the data, both before and after adjustment by standardization, including measures of central tendency, skewness, kurtosis, tests of normality, and selective correlogram values for the levels and squares of the series, are reported in Table I. Self-evidently, adjustment increases the range and standard deviation of the underlying series, which has the indirect benefit of aiding parameter convergence in estimation. Otherwise the basic properties of the date are little affected. The distributional properties of the adjusted data are further illustrated in Figure 1, which depicts the results of nonparametric Epanechnikov kernel density estimation, where bandwidth selection is determined according to the data-based criteria of Silverman (1986). The "peakedness" and "fat-tails" due to leptokurtosis in the data, as indicated by the kurtosis statistics in Table I, is clearly obvious, and further motivates the consideration of GARCH processes below. Additionally evident are "shoulders" in the distributions (also present in the comparable distributions of the unadjusted data, and common in such high-frequency data), which suggests a concentration of data points a margin either side of the zero mean, and more so on the upper side of the distribution. This property further suggests to us the influence of significant market frictions, such that beyond small return values a range of price changes become more pronounced and numerous, and reinforces our consideration of threshold models able to accommodate this feature below. Before proceeding to the estimation of such

[7]Various alternative adjustments for systematic intraday effects have been proposed in the literature, including the use of interval dummies (Baillie & Bollerslev, 1990, 1991), time-scaling (Dacorogna et al., 1993), and artificial neural networks (Lo, 1994). So as not to compound the potential nonlinearities we are testing for, we forego the latter approaches in favor of the methodology described in the text. For a more detailed discussion of the intraday deterministic patterns in LIFFE futures returns and volume data, including that analyzed here, see Gwilym, McMillan, and Speight (1999).
[8]Given that the floor trading times for Long Gilt futures changed on August 1, 1994 from 8:30 a.m. to 4:15 p.m. to 8:00 a.m. to 4:15 p.m., this standardization exercise is conducted separately for both subperiods so as not to confound the intraday deterministic pattern. The effect of this change in trading times on the estimates reported is tested for as a matter of empirical robustness, the results of which are noted where appropriate.

TABLE I
Summary Statistics: Unadjusted and Adjusted Data

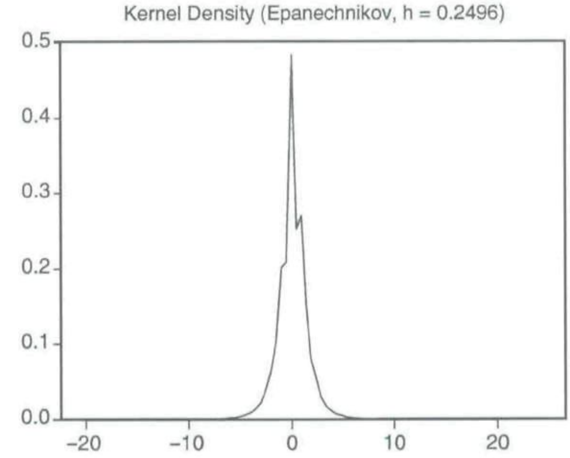
	Unadjusted	*Adjusted*
Mean	2.55×10^{-7}	0.000298
Median	0.000000	0.000000
Standard deviation	0.000562	1.5116
Minimum	0.0062	−22.0515
Maximum	0.0090	26.1938
Skewness	0.06	0.07
Kurtosis	11.58	11.11
Normality	246,163.7	219,835.0
Q_1	441.77	579.30
Q_{10}	468.53	618.51
Q_{20}	499.89	640.74
Q_1^2	2,028.6	1,962.1
Q_{10}^2	10,569.0	11,139.0
Q_{20}^2	14,947.0	15,026.0

Notes. For data description and details of the adjustment procedure used to accommodate intradaily "seasonal" patterns through scaling by time interval mean values, see text. Summary statistics are self-explanatory. Additionally, "normality" is the Jarque-Bera test of the null hypothesis of normality, distributed as χ_2^2, which is clearly rejected throughout. Q_i ($i = 1, 10, 20$) are selected values from the correlogram of the data, and Q_i^2 ($i = 1, 10, 20$) are selected values from the correlogram of the squares of the data.

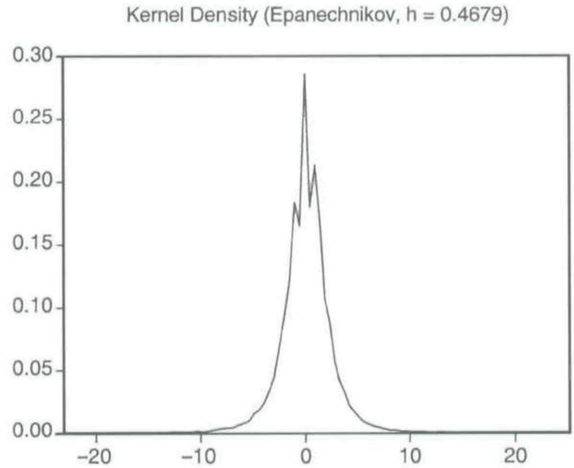
models, however, we first consider formal statistical tests for the presence of threshold nonlinearity.

The specification of preliminary linear AR(p) models is determined by reference to the autocorrelation and partial autocorrelation functions, the Schwarz criterion, the estimated log-likelihood, and residual tests for serial correlation. This identification procedure indicates that an AR(2) process is appropriate.[9] Given an appropriately specified AR model, we test for the presence of conditional mean nonlinearity following the procedure detailed in Teräsvirta and Anderson (1992), Granger and Teräsvirta (1993), and Teräsvirta (1994, 1998), which entails testing for threshold nonlinearities against the null of linearity over a range of suitable possible values for the delay parameter d. The resulting LM-type test of AR(p) linearity assuming known d is equivalent to the test of

[9]While not reported in full, autoregressive model parameter estimates are (with Bollerslev-Wooldridge robust standard errors): $\hat{\pi}_0 = 0.0003$ (0.0053); $\hat{\pi}_1 = -0.0869$ (0.0056); $\hat{\pi}_2 = -0.0228$ (0.0053). The absence of remaining serial correlation is confirmed using the heteroskedasticity-robust LM test of Wooldridge (1990), the results of which are available upon request. The presence of negative autocorrelation in high-frequency returns is not uncommon, various explanations for which have been proposed, including inventory concerns of market makers and bid-ask bounce. For further discussion, see Goodhart and O'Hara (1997, pp. 95–96).



(a) Five Minute Frequency



(b) Fifteen Minute Frequency

FIGURE 1
Nonparametric density plots of adjusted (interval-mean scaled) data.

the null hypothesis of linearity $H_0: \beta_{2j} = \beta_{3j} = \beta_{4j} = 0$ $(j = 1, \ldots, p)$ against the alternative in the following artificial regression:

$$y_t = \beta_0 + \sum_{j=1}^{p} \beta_{1j} y_{t-j} + \sum_{j=1}^{p} \beta_{2j} y_{t-j} y_{t-d} + \sum_{j=1}^{p} \beta_{3j} y_{t-j} y_{t-d}^2 + \sum_{j=1}^{p} \beta_{4j} y_{t-j} y_{t-d}^3 \quad (5)$$

The test statistic, computed as $LM = T(SSR_0 - SSR_1)/SSR_0$, where T denotes sample size, SSR_0 is the sum of squared residuals from the

TABLE II
Linearity and STAR Test Results

Linearity Test				*STAR Test*		
y_{t-d}				*Null*		
$d = 1$	$d = 2$	$d = 3$	$d = 4$	H_{04}	H_{03}	H_{02}
14.82 (0.19)	55.74 (5.67×10^{-8})	37.45 (9.68×10^{-5})	21.83 (0.03)	1.94 (0.93)	43.93 (7.63e-08)	9.88 (0.13)

Notes. For linearity and STAR model selection tests see Equation (5) and accompanying text. Reported values are test statistics with associated *p*-values in parentheses.

linear AR(p) model, and SSR_1 is the sum of squared residuals obtained from (5), is asymptotically distributed as $\chi^2_{(p/2)(p+1)+2p^2}$, where d is unknown. Where linearity is rejected for more than one value of the delay parameter, then d is determined such that $p(d) = arg\ \min_{1 \le d \le D} p(d)$, where $p(d)$ refers to the probability value at which the null of linearity is marginally rejected. The rationale for this selection method is as follows. When d is correctly specified, (5) is the appropriate auxiliary regression against the nonlinear alternative. If another d is selected, then (5) is mis-specified. The power of the corresponding test against the mis-specified nonlinear model will thus be weaker than the power of the test based upon the correctly specified auxiliary regression. As reported in full in Table II, application of this test procedure for all possible delay values $1 \le d \le 4$ confirm rejection of the null hypothesis of linearity in favor of STAR nonlinearity; the only exception to linearity rejection at the 5% significance level arises for $d = 1$, with application of the minimum $p(d)$ rule indicating $d = 2$.[10]

Alternatives specifications for the transition function include the logistic and heaviside functions, yielding LSTAR and TAR models, respectively. The former has the capacity to accommodate the latter, but more generally permits smooth transition between differing dynamics associated with positive and negative signs for $(r_{t-d} - c)$. Again following

[10]While it is in principle possible to compute test values using the heteroskedasticity-robust procedure outlined in Granger and Teräsvirta (1993, pp. 69–70), for example, the fact that heteroskedasticity under the null hypothesis can be the result of nonlinearity that is not taken into account in the conditional mean means that the test correction often removes most of the power of the test to detect nonlinearity (Van Dijk et al., 2001). Our approach, therefore, follows the suggestion of Wooldridge (1990), in conducting STAR tests under a robust covariance matrix in the presence of heteroskedasticity of unknown form using the method of White (1980). In view of the overwhelming rejection of linearity at the delay parameters identified in the article using this approach, we are confident that our conclusion that threshold nonlinearity is present in the data is robust.

Teräsvirta (1994, 1998), on the basis of a series of nested tests using (6) and given the limitation of assuming d is known, in which case the LM test statistic is asymptotically distributed as χ^2_{3p}, it is possible to discriminate between the ESTAR and LSTAR variant. The sequence of hypotheses tested are:

(i) $H_{04}: \beta_{4j} = 0$; (ii) $H_{03}: \beta_{3j} = 0 \mid \beta_{4j} = 0$; (iii) $H_{02}: \beta_{2j} = 0 \mid \beta_{4j} = 0$;

$(j = 1, \ldots, p)$. The rationale behind this sequence is based on interpreting the coefficients as functions of the parameters of the STAR model. Thus, if (i) is accepted and (ii) is rejected, the ESTAR variant is preferred. If either (i) is rejected, or (i) and (ii) are accepted and (iii) is rejected, the LSTAR variant is preferred. Adopting the $d = 2$ value indicated above, and as reported more fully in Table II, test statistics values are (i) 1.94, (ii) 43.93, and (iii) 9.88, relative to χ^2_6 critical values of 12.59 and 10.64 at the 5 and 10% significance levels, respectively, clearly confirming preference for an ESTAR specification. Given this diagnostic support for a nonlinear STAR model over the linear AR alternative as a description of conditional mean structure in long gilt futures returns, we proceed to full estimation of the ESTAR model in the following section.

EMPIRICAL RESULTS

Estimation of all models below is by iterative nonlinear least squares. The validity of estimated models is appraised on the basis of the significance of autoregressive terms and examination of coefficient estimates, in particular ensuring that the transition value, c, is within the range of $\{r_t\}$. The Akaike and Schwarz information criteria are also used to guide selection among competing models (Teräsvirta, 1994, 1998). We also examine the dynamic properties of the regimes corresponding to $F(r_{t-d}) = 0$ and $F(r_{t-d}) = 1$ by inspecting the roots of the relevant characteristic polynomials, as well as the dynamic properties of the full models.[11]

ESTAR-EGARCH results are reported in the first column of Table IV. Of immediate note is the reduction in model order relative to the linear case. For the resulting ESTAR(1)-EGARCH model, the central regime corresponding to $F(r_{t-d}) = 0$, which arises as $(r_{t-d} - c) \to 0$,

[11]In the absence of a general analytical solution, the latter procedure is performed numerically, using data generated from the estimated model after setting the error term to zero, with a sequence of observed values of the series acting as starting values, several of the latter being considered. For the class of models under investigation, this may result in a unique stable equilibrium, a limit cycle such that a set of values repeat themselves perpetually, chaotic realizations whereby a small change in initial values results in divergent but stable limit points, or explosive values.

TABLE III
Estimation Results

Model	Parameter										
	π_0	π_1	θ_0	γ	c	ω	ρ	ϕ	α	β	ζ
ESTAR-EGARCH	−0.0957	−0.1052*	0.2814*	0.1400**	1.7663*	−0.0927*	—	—	0.1591*	0.9668*	−0.0084
	(0.0120)	(0.0051)	(0.1053)	(0.0763)	(0.8961)	(0.0066)			(0.0076)	(0.0066)	(0.0052)
ESTAR-CGARCH	−0.1337*	−0.1091*	0.2529*	0.1286**	3.8721*	1.9950*	0.9973*	0.0158*	0.0741*	0.8307*	—
	(0.0365)	(0.0040)	(0.0564)	(0.0765)	(1.0110)	(0.1425)	(0.0005)	(0.0021)	(0.0048)	(0.0115)	

Notes. For model mnemonics and specifications, see text. Bollerslev–Wooldridge (1992) robust standard errors in parentheses. Asterisk(s) denote coefficient significance at the 5%(10%) level.

TABLE IV
Residual Diagnostics

Model	Log L	AIC	BIC	Skew	Kurt	JB	A_1	A_{10}	A_{20}
ESTAR-EGARCH	−137417.1	3.4288	3.4298	0.03	12.99	333435.4*	51.00*	67.21*	73.84*
ESTAR-CGARCH	−136584.3	3.4080	3.4092	0.14	10.61	193467.6*	9.18*	10.78	17.44

Notes. For model mnemonics and specifications, see text. Additionally, Log L denotes the maximized log likelihood value, AIC and BIC denote the Akaike and Schwarz (Bayesian) information criteria, Skew and Kurt are regular measures of skewness and kurtosis respectively, JB is the Jarque-Bera test of the null of normality, distributed as χ^2_2 and A is the regular ARCH LM test for lags $i = 1,10,20$ distributed as χ^2_i. Asterisk(s) denote test significance at the 5%(10%) level.

is described by an AR(1) process, while the outer regimes corresponding to $F(r_{t-d}) \neq 0$ as $(r_{t-d} - c) \rightarrow \infty$ as invokes an additional AR(0) process. Thus, there is significant negative autocorrelation in returns, irrespective of size, with the nonlinearity present being described by a shifting intercept dependent on the magnitude of returns relative to the interval norm. The latter implies significant negative drift in returns in the neighborhood of the threshold value, but, that tendency is counteracted by positive drift for larger returns as $F(r_{t-d}) \rightarrow 1$. Both regimes of the model are trivially stationary, and the full model is characterized by stable roots and a near-zero unique limit point (0.0044), with rapid adjustment to equilibrium within approximately eleven periods, or 55 min.

The estimated value of the threshold parameter, c, suggests that the central regime characterizes returns that are less than $1\frac{3}{4}$ times higher than the interval norm (standardized in relation to the intraday interval average). Transition between regimes is dictated by the estimated transition function, which is portrayed in Figure 2. The minimum of the function corresponds with the threshold parameter, its width in the neighborhood of c determines the range of the central regime, while its steepness (symmetric about c) determines the speed of transition between the center and outer regimes. The estimated transition parameter value of 0.14 (0.32 after reversing the scale transformation) is significant at the 10% level, and suggests only a moderate speed of transition between regimes, and therefore, a tendency for returns to sojourn in the center regime.[12] The midpoints between regimes occur for the data values (−1.5, 5.0) expressed as multiples of interval means. That is, the midway transition point between the center and outer regimes is passed when returns are either falling by $1\frac{1}{2}$ times their average for that intraday 5-min interval, or rising by five times their normal interval value. An approximate measure of the width of the center regime, corresponding to $F(r_{t-d}) < 0.2$, yields the pair of data points (0.0, 3.5), that is returns ranging from zero to $3\frac{1}{2}$ times the relevant 5-min interval norm. Conditional variance parameters for the ESTAR-EGARCH model indicate a high and significant measure of persistence in shocks to volatility

[12]A practical problem frequently encountered in the estimation of STAR models concerns convergence and precision in estimates of the smoothing or transition parameter, γ. In particular, a large γ value results in a steep slope for the transition function at c, and a large number of observations in the neighborhood of c are, in principle, required to estimate γ accurately. Consequently, with changes in γ having only a minor effect upon the transition function, the convergence of γ can prove problematic. A solution to this problem, suggested by Teräsvirta (1994, 1998) and adopted in estimation here, is to scale the smoothing parameter by the variance of the transition variable, $\sigma^2(r_{t-d})$, yielding the revised transition function, $F(r_{t-d}) = 1 - \exp[-\gamma(r_{t-d} - c)^2/\sigma^2(r_{t-d})]$, with appropriate adjustment required in interpretation of the resulting estimate of γ.

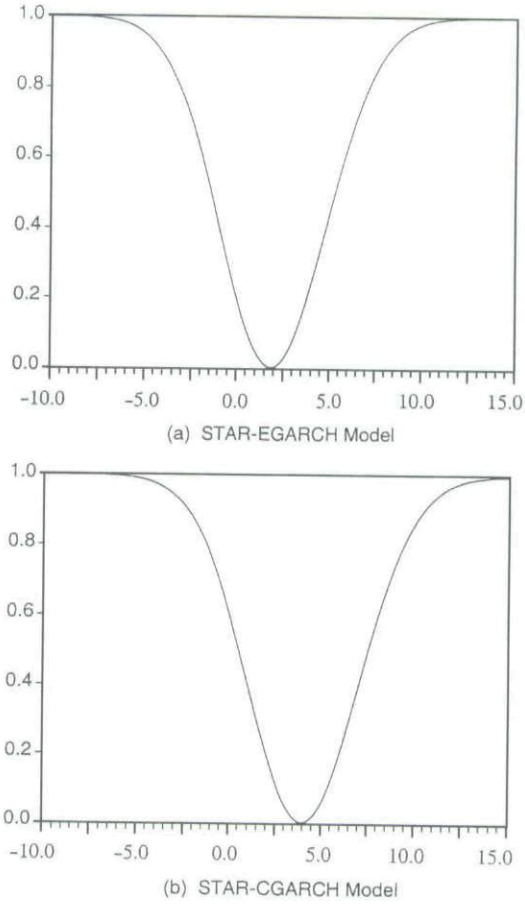


FIGURE 2
Estimated transition functions—5-min frequency.

of 0.97, implying half-life decay in just under 2 h, and significant volatility clustering, but no evidence of significant asymmetry in volatility with respect to shocks of differing sign.

ESTAR-CGARCH estimates confirm the magnitude and significance of the ESTAR parameters discussed above, but with some increase in the threshold parameter, decrease in the transition parameter, and the additional significance of the center regime intercept. The preceding discussion, therefore, continues to hold, but the midpoints between regimes now occurs for data values (0.5, 7.5), or returns of 1/2 and $7\frac{1}{2}$ times their interval average, while the central regime has a width corresponding to $F(r_{t-d}) < 0.2$ of (1.9, 5.8), or returns of approximately two to six times their average interval value. This estimated transition function is portrayed in Figure 2(b). Concerning the CGARCH parameter estimates, the initial effect of a shock to the permanent component of volatility, as quantified by the parameter ϕ, is fairly modest at under 0.2, while the autoregressive root, ρ, at over 0.99, suggests very strong persistence in the effect of such shocks, with a half-life decay of approximately 4 days. Both parameters of the transitory component are significant, and provide a joint persistence measure of over 0.9, implying half-life decay in transitory volatility shocks of approximately 35 min.

Residual diagnostics for the preceding models are reported in Table IV, and indicate residual non-normality, primarily due to excess kurtosis, reinforcing the use of Bollerslev-Wooldridge robust standard errors in the appraisal of parameter significance conducted above. However, LM tests indicate the presence of remaining ARCH effects, although only of first order form for the CGARCH case.

SUMMARY AND CONCLUSIONS

Motivated by considerations of market frictions and heterogeneities in information flows and market agents, the empirical evidence reported here has sought to identify the source of nonlinear dependence in futures returns, with particular regard to the potential for such dependence to arise in both conditional mean and conditional variance. Preliminary tests against the null of linearity confirm the presence of smooth transition autoregressive nonlinearity in the conditional mean of UK long gilt futures returns at the 5-min frequency. The estimated linear model is second-order autoregressive, while the nonlinear STAR model consists of a first-order autoregressive process with switching intercept. That structure is robust to the joint estimation of conditional variance processes of either exponential-GARCH or component-GARCH type. The former confirms the presence of significant clustering and persistence in conditional volatility, while the latter entails the successful decomposition of volatility into a long-lived permanent component and a more ephemeral transitory component. These results are consistent with the existence of heterogeneous traders (or, alternatively, heterogeneous information flows), such that different traders face different levels of transaction costs or noise trader risk, hence imparting a smooth transition threshold as opposed to the abrupt switch that would occur if all traders faced equivalent costs and risk.[13] Further, the existence of volatility components with differing degrees of persistence supports the contention that different traders operate at different time horizons, with

[13]On the formal modeling of noise trading, investor sentiment and limited arbitrage, see, for example, Schleifer (2000).

scalpers and arbitrage traders operating over small time intervals and reacting to small deviations of the asset price from its fundamental equilibrium, while fund managers and corporate treasurers take a longer view of the market. What remains is to determine whether the nonlinearities identified here have implications for the dynamic relationship between the price of futures contracts and the underlying cash market, and whether these nonlinearities are a feature of the open outcry pit trading system in use over the data period analyzed here or extend to the more recent period of electronic trading. These provide interesting avenues for further research.

BIBLIOGRAPHY

Abhyankar, A., Copeland, L. S., & Wong, W. (1995). Nonlinear dynamics in real-time equity market indices: Evidence from the UK. Economic Journal, 105, 864–880.

Andersen, T. G., & Bollerslev, T. (1997a). Heterogeneous information arrivals and return volatility dynamics: Uncovering the long-run in high frequency returns. Journal of Finance, 52, 975–1005.

Andersen, T. G., & Bollerslev, T. (1997b). Intraday periodicity and volatility persistence in financial markets. Journal of Empirical Finance, 4, 115–158.

Ausubel, L. M. (1990a). Insider trading in a rational expectations economy. American Economic Review, 80, 1022–1041.

Ausubel, L. M. (1990a). Partially-revealing rational expectations equilibrium in a competitive economy. Journal of Economic Theory, 50, 93–126.

Baillie, R. T., & Bollerslev, T. (1990). A multivariate generalised ARCH approach to modelling risk premia in forward foreign exchange rate markets. Journal of International Money and Finance, 9, 309–324.

Baillie, R. T., & Bollerslev, T. (1991). Intra-day and inter-market volatility in foreign exchange markets. Review of Economic Studies, 58, 565–585.

Becker, K. G., Finnerty, J. E., & Kopecky, K. J. (1993). Economic news and intraday volatility in international bond markets. Financial Analysts Journal, 49, 81–86.

Becker, K. G., Finnerty, J. E., & Kopecky, K. J. (1995). Domestic macroeconomic news and foreign interest rates. Journal of International Money and Finance, 14, 763–783.

Becker, K. G., Finnerty, J. E., & Kopecky, K. J. (1996). Macroeconomic news and the efficiency of international bond futures markets. Journal of Futures Markets, 16, 131–145.

Bera, A. K., & Higgins, M. L. (1997). ARCH and bilinearity as competing models for nonlinear dependence. Journal of Business and Economic Statistics, 15, 43–50.

Biasis, B., & Hillion, P. (1994). Insider and liquidity trading in stock and options markets. Review of Financial Studies, 7, 743–780.

Black, F. (1986). Noise. Journal of Finance, 41, 529–543.

Bollerslev, T. (1986). Generalised autoregressive heteroscedasticity. Journal of Econometrics, 31, 307–327.

Bollerslev, T., Chou, R. Y., & Kroner, K. F. (1992). ARCH modelling in finance. Journal of Econometrics, 52, 5–59.

Bollerslev, T., Engle, R. F., & Nelson, D. B. (1994). ARCH models. In R. F. Engle, & D. L. McFadden (Eds.), Handbook of econometrics (Vol. IV). Amsterdam: Elsevier Science Publisher B.V.

Buckle, M. J., ap Gwilym, O., Thomas, S. H., & Woodhams, M. S. (1995). Empirical regularities in interest rate and equity index futures markets and the effects of macroeconomic announcements. Journal of Business Finance and Accounting, 25, 921–944.

Chan, K., & Tong, H. (1986). On estimating thresholds in autoregressive models. Journal of Time Series Analysis, 7, 179–194.

Clark, P. K. (1973). A subordinate stochastic process model with finite variance for speculative prices. Econometrica, 41, 135–155.

Coakley, J., & Fuertes, A. M. (2000). Short run real exchange rate dynamics post-73. The Manchester School (Special Issue: Topics in Macroeconomics), 68, 461–475.

Coakley, J., & Fuertes, A. M. (2001). A nonlinear analysis of excess foreign exchange returns. The Manchester School, 69, 623–642.

Cootner, P. H. (1962). Stock prices: Random versus systematic changes. Industrial Management Review, 3, 24–45.

Dacorogna, M. M., Müller, U. A., Nagler, R. J., Olsen, R. B., & Pictet, O. V. (1993). A geographical model for the daily and weekly seasonal volatility in the FX market. Journal of International Money and Finance, 12, 413–438.

DeLong, J. B., Shleifer, A., Summers, L. H., & Waldmann, R. J. (1990). Noise trader risk in financial markets. Journal of Political Economy, 98, 703–738.

Diamond, D. W., & Verrecchia, R. E. (1981). Information aggregation in a noisy rational expectations economy. Journal of Financial Economics, 9, 221–235.

Diebold, F. X. (1986). Modelling the persistence of conditional variances: A comment. Econometric Reviews, 5, 51–56.

Dow, J. (1995). Arbitrage, hedging and the welfare economics of financial innovation: A model of cross-market liquidity effects (Working Paper). London: London Business School.

Dow, J., & Gorton, G. (1994). Arbitrage chains. Journal of Finance, 49, 819–849.

Dow, J., & Gorton, G. (1997). Noise trading, delegated portfolio management and economic welfare. Journal of Political Economy, 105, 1024–1050.

Dumas, B., & Luciano, E. (1991). An exact solution to a dynamic portfolio choice problem under transaction costs. Journal of Finance, 46, 577–595.

Engle, R. F. (1982). Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom Inflation. Econometrica, 50, 987–1007.

Engle, R. F., & Lee, G. G. J. (1999). A permanent and transitory component model of stock return volatility. In R. F. Engle & H. White (Eds.), Cointegration, causality, and forecasting: A festschrift in honour of Clive W. J. Granger. Oxford: Oxford University Press.

Goodhart, C. A. E., & O'Hara, M. (1997). High frequency data in financial markets: Issues and applications. Journal of Empirical Finance, 4, 73–114.

Goodhart, C. A. E., Hall, S. G., Henry, S. G. B., & Pesaran, B. (1993). News effects in a high frequency model of the sterling-dollar exchange rate. Journal of Applied Econometrics, 8, 1–13.

Granger, C. W. J., & Teräsvirta, T. (1993). Modelling nonlinear economic relationships. Oxford: Oxford University Press.

Guillaume, D. M., Dacorogna, M. M., Davé, R. R., Müller, U. A., Olsen, R. B., & Pictet, O. (1997). From the bird's eye to the microscope: A survey of new stylized facts of the intra-daily foreign exchange markets. Finance and Stochastics, 1, 61–76.

Gwilym, ap, O., McMillan, D. G., & Speight, A. E. H. (1999). The intra-day relationship between volume and volatility in LIFFE futures markets. Applied Financial Economics, 9, 593–604.

He, H., & Modest, D. (1995). Market frictions and consumption-based asset pricing. Journal of Political Economy, 103, 94–117.

Kyle, A. S. (1985). Continuous auctions and insider trading. Econometrica, 53, 1315–1335.

Lo, A. (1994). Neural networks and other nonparametric techniques in economics and finance. In Blending quantitative and traditional equity analysis (pp. 25–36). Association for Investment Management and Research.

Martens, M., Kofman, P., & Vorst, T. C. F. (1998). A threshold error correction model for intraday futures and index returns. Journal of Applied Econometrics, 13, 246–263.

Müller, U. A., Dacorogna, M. M., Davé, R. D., Olsen, R. B., Pictet, O. V., & von Weizsäcker, J. E. (1997). Volatilities of different time resolutions—Analysing the dynamics of market components. Journal of Empirical Finance, 4, 213–239.

Nelson, D. B. (1991). Conditional heteroscedasticity in asset returns: A new approach. Econometrica, 59, 347–370.

Obstfeld, M., & Taylor, A. M. (1997). Non-linear aspects of goods-market arbitrage and adjustment: Heckscher's commodity points revisited. Journal of the Japanese and International Economies, 11, 441–479.

Sercu, P., Uppal, R., & Van Hulle, C. (1995). The exchange rate in the presence of transaction costs: Implications for tests of purchasing power parity. Journal of Finance, 50, 1309–1319.

Schleifer, A. (2000). Inefficient markets: An introduction to behavioural finance. Oxford: Oxford University Press.

Schleifer, A., & Summers, L. H. (1990). The noise trader approach to finance. The Journal of Economic Perspectives, 4, 19–33.

Silverman, B. W. (1986). Density estimation for statistics and data analysis. London: Chapman and Hall.

Teräsvirta, T. (1994). Specification, estimation, and evaluation of smooth transition autoregressive models. Journal of the American Statistical Association, 89, 208–218.

Teräsvirta, T. (1998). Modelling economic relationships with smooth transition regressions. In A. Ullah & E. Dvid (Eds.), Handbook of applied economic statistics (pp. 507–552). New York: Marcel Dekker.

Teräsvirta, T., & Anderson, H. M. (1992). Characterising nonlinearities in business cycles using smooth transition autoregressive models. Journal of Applied Econometrics, 7, S119–S136.

Timmermann, A. (1995). Volatility clustering and mean reversion of stock returns in an asset pricing model with incomplete learning (Discussion Paper No: 95-23). La Jolla: UCSD, Department of Economics.

Van Dijk, D., Teräsvirta, T., & Franses P. H. (2002). Smooth transition autoregressive models—a survey of recent developments. Econometeric Reviews, 21, 1–47.

White, H. (1980). A heteroscedasticity-consistent covariance matrix estimator and a direct test for heteroscedasticity. Econometrica, 48, 817–838.

Wooldridge, J. M. (1990). A unified approach to robust, regression-based specification tests. Econometric Theory, 6, 17–43.