VALUATION AND HEDGING OF DIFFERENTIAL SWAPS

CHUANG-CHANG CHANG
SAN-LIN CHUNG
MIN-TEH YU*

This paper derives a general-form formula for pricing and hedging differential swaps with the principal denominated either in a domestic, foreign, or third-country currency. We first derive the formula for differential swaps with the principal in a domestic currency and identify an error in the formula of Wei (1994). We then show the pricing duality between differential swaps with the principal in a domestic currency and differential swaps with the principal in a foreign currency. Finally, we complete the pricing and hedging analysis on differential swaps by deriving a formula for differential swaps with the principal denominated in a third-country currency. Simulation results indicate that constant margin rates are generally smaller than interest rate differentials and decline with the tenor of swaps. Correlation parameters associated with the exchange rate play a more important role than correlation parameters among interest rates in pricing differential swaps. © 2002 John Wiley & Sons, Inc. Jrl Fut Mark 22:73–94, 2002

We are grateful to Andrew Chen, Son-Nan Chen, Alberto Moel, Jason Wei, and seminar participants at the 1997 Chinese Finance Association Annual Meeting, 1998 Multinational Finance Society Conference, and 1998 Financial Management Association Meeting for helpful comments and discussions.
*Correspondence author, Min-Teh Yu, Department of Finance, Yuan Ze University, Jung-Li 32003, Taiwan; e-mail: mtyu@saturn. yzu.edu.tw

Received March 2000; Accepted May 2001

■ *Chuang-Chang Chang is in the Department of Finance at National Central University in Taiwan.*

■ *San-Lin Chung is in the Department of Finance at National Central University in Taiwan.*

■ *Min-Teh Yu is with the Department of Finance at Yuan Ze University in Taiwan.*

The Journal of Futures Markets, Vol. 22, No. 1, 73–94 (2002)

INTRODUCTION

Recent years have witnessed tremendous growth in the volume and scope of both exchange-traded derivatives and over-the-counter customized products which involve multiple assets and currencies. The growth has given rise to "quanto" contracts with a property that enables investors to participate in foreign equity or bond markets without bearing the exchange rate risk. One popular quanto fixed-income product is a differential swap (diff swap, hereafter). A diff swap is a variation of an interest rate swap, distinguished by the fact that at least one of the payment rates refers to a currency different from the notional principal.

The first reported diff swap was negotiated in early 1991 between Credit Suisse First Boston and a Japanese insurance company. Since then, diff swaps have grown rapidly, because of the large differential in short-term interest rates across major countries. A recent estimate places the outstanding notional principal amount of diff swaps at $40 to $50 billion.[1] A simple example would be a U.S. dealer who enters into a five-year swap and agrees to pay the six-month dollar LIBOR rate plus a spread semi-annually. In return, the swap's counterpart pays the dealer the Japanese yen LIBOR rate. To the U.S. dealer, the notional principal of the traded diff swaps could be denominated in a domestic currency (U.S. dollar), foreign currency (say Japanese Yen), or in a third-country currency (say British pound).

The main rationale for entering into a diff swap is to exploit the interest rate differential without directly incurring exchange rate risk. Applications of diff swaps are quite broad. For example, money market investors may use diff swaps to take advantage of a high-yield currency; corporate borrowers with debt in a discount currency can use diff swaps to lower their effective borrowing costs from the expected persistence of a low nominal interest rate in the premium currency; portfolio managers can use diff swaps to enhance current yield if they expect high yields to persist in a discount currency. These positions, once contracted, are sensitive to interest rate changes associated with currency movements, but carry no direct currency risks.

There are only a few papers that discuss how to price and hedge diff swaps in the literature and most focus on the case where the notional principal is denominated in a domestic currency. Litzenberger (1992)

[1]For the early development of diff swap markets, see Cookson (1992), Das (1992a, 1992b), and Smith, Smithson, and Wakeman (1986); for pricing and the evolving market of interest rate swaps, see Bicksler and Chen (1986) and Wall and Pringle (1989).

first discusses diff swaps in his presidential address[2] and suggests some possible analytical valuation procedures. Jamshidian (1993) uses a replication approach to value diff swaps, but provides merely a general expression of the solution which cannot be used without further analysis. Amin and Jarrow (1991) develop a model for valuing currency options using the Heath, Jarrow, and Morton (1992, hereafter HJM) model of term structure to describe domestic and foreign interest rates.[3] Their method can be applied to valuing diff swaps, but they did not work on this direction.

Following Amin and Jarrow (1991), Turnbull (1993) adopts a multi-factor HJM model to describe the evolution of domestic and foreign forward interest rates and shows how to price and hedge the two common forms of diff swaps with the principal in a domestic currency and foreign currency. His model can be viewed as being complementary to the models of Amin and Jarrow (1991) and Jamshidian (1993), but differing in degree of explicitness. Using a mean-reverting Ornstein-Uhlenbeck process for both domestic and foreign spot interest rates and a lognormal process for the exchange rate, Wei (1994) develops a model of diff swaps with the principal in the domestic currency. His model works with a spot interest rate process, while Amin and Jarrow (1991) and Turnbull (1993) do with a forward rate process. The spot rate framework is not different from the forward rate framework. In fact, the spot rate model considered in Wei (1994) corresponds to the HJM model with an exponentially decaying variance of forward rates. We choose the spot rate framework, because we can directly compare our closed-form solutions with those of Wei (1994).

Our contributions to the literature are threefold. First, in the second section we derive the pricing formula for diff swaps with the principal in a domestic currency and point out the error in Wei's (1994) formula. Second, in the third section demonstrates that there exists a dual pricing relationship between a diff swap with the principal in a domestic currency and a diff swap with the principal in a foreign currency. Third, in the fourth section we derive a general pricing and hedging model for diff swaps with the principal in a third-country currency. We then carry out simulations and analyze their sensitivities to changes in parameters in the fifth section and draw conclusions in the final section.

[2]It was named "switch LIBOR swaps" in Litzenberger (1992), but is more popularly known as "diff swaps" today.

[3]Amin and Jarrow (1991) derive closed-form solutions for European options under the assumption that both domestic and foreign interest rates are normally distributed and the exchange rate is lognormally distributed.

DIFF SWAPS WITH THE PRINCIPAL IN A DOMESTIC CURRENCY

We follow Wei's (1994) framework and assume that both domestic and foreign interest rates follow a mean-reverting Ornstein–Uhlenbeck process and the exchange rate follows a lognormal process. The assumption about the dynamics of interest rates is drawn from Vasicek (1977). As pointed out by Longstaff (1995), there are several reasons why this assumption is suitable in the context of this model. First, as long as the value of interest rate is positive, these dynamics always imply a positive expected future value of interest rate. This merit is especially important since the primary effect of interest rate on the price of the interest rate derivatives is through its expected future values. Second, the probability of negative interest rates occurring is small for most realistic parameter values. Our framework can be expressed as

$$dr_d = k_d(\mu_d - r_d)\,dt + \sigma_d\, dw_d(t) \tag{1}$$

$$dr_f = k_f(\mu_f - r_f)\,dt + \sigma_f\, dw_f(t) \tag{2}$$

$$\frac{dX}{X} = \mu_X\, dt + \sigma_X\, dw_X(t) \tag{3}$$

where $r_d(r_f)$ is the domestic (foreign) risk-free interest rate, X is the spot exchange rate expressed as the domestic currency value of one unit of foreign currency, $\mu_d(\mu_f)$ is the expected long-run value of the domestic (foreign) interest rate, μ_X is the drift rate of the exchange rate (X), $k_d(k_f)$ is the reversion coefficient of the domestic (foreign) interest rate, $\sigma_d\,(\sigma_f)$ and σ_X are the standard deviation of the domestic (foreign) interest rate and exchange rate, w_d, w_f, and w_X are the standard Wiener process with instantaneous correlation ρ_{df}, ρ_{dX}, ρ_{fX} and among them, and t is the current time.

The domestic (foreign) risk-neutral process[4] of the domestic (foreign) interest rate and the exchange rate can also be identified as follows. Letting $\lambda_d(\lambda_f)$ be the market price of risk for the domestic (foreign) interest rate, we can express the domestic (foreign) risk-neutral process for the domestic (foreign) interest rate as

$$dr_d = [k_d\,(\mu_d - r_d) + \lambda_d\sigma_d]dt + \sigma_d\, dw_d(t)$$

$$dr_f = [k_f\,(\mu_f - r_f) + \lambda_f\sigma_f]dt + \sigma_f\, dw_f(t) \tag{4}$$

[4]It should be noted that there exist multiple risk neutral measures in the cross-country economy. Under the domestic (foreign) risk neutral measure, all asset prices normalized by the domestic (foreign) money market account follow martingales if the economy is arbitrage free.

Define $B_d(t, T)$ as the price at time t of a domestic zero-coupon bond that pays off one unit of domestic currency at maturity date T. Since $B_d(t, T) = E^d[\exp(-\int_t^T r_d(s)\,ds)]$, where E_d denotes the expected value in a domestic risk-neutral world, Vasicek (1977) shows that the bond price can be expressed as

$$B_d(t, T) = D \times \exp(-r_d \times F) \tag{5}$$

and

$$F = \frac{1 - \exp[-k_d(T - t)]}{k_d}$$

$$D = \exp\left[\left(\mu_d + \frac{\lambda_d \sigma_d}{k_d} - \frac{\sigma_d^2}{2k_d^2}\right) \times (F - (T - t)) - \frac{\sigma_d^2 F^2}{4k_d}\right]$$

Furthermore, using an arbitrage argument, we can obtain the domestic risk-neutral process for the exchange rate as given by Equation 6:

$$\frac{dX}{X} = (r_d(t) - r_f'(t))\,dt + \sigma_X\,dw_X(t) \tag{6}$$

where r_f' is the foreign interest rate process in the domestic risk-neutral world.[5]

Suppose that a domestic investor enters into a diff swap with the notional principal denominated in the domestic currency and will receive/pay interest rate payments based on the foreign/domestic interest rate. The swap's value to the domestic investor is then the difference between the present values of future cash inflows of foreign interest payments and cash outflows of domestic interest payments, i.e.,

$$V = V_f - V_d - \frac{c(t_{i+1} - t_i)}{360} P_d \sum_{i=1}^{n} B_d(t, t_i) \tag{7}$$

where $V_d(V_f)$ is the present value at time t of all domestic (foreign) interest rate payments during the remaining life of the swap, P_d is the notional principal denominated in the domestic currency, n is the number of interest rate resets left in the remaining life of the swap, t_i is the reset time that the payments predetermined at t_{i-1} are exchanged and payments for t_{i+1}

[5]Please see Equation 11 or footnote 6 for the relationship between r_f' (domestic risk-neutral process) and r_f (foreign risk-neutral process). The foreign interest rate process in the domestic risk-neutral world is identical to the true process (i.e., Equation 2) except that the drift term is adjusted such that the expected return of a foreign bond equals the domestic risk-free rate.

are determined, and c is the annualized constant margin rate or spread that is specified to compensate for the interest rate differential with the swap value set to zero at its inception date.

The size of the domestic interest payment determined (paid) at time $t_i(t_{i+1})$ is

$$CF_i^d = P_d\left[\frac{1}{E_{t_i}^d(\exp(-\int_{t_i}^{t_i+1} r_d(s)\,ds))} - 1\right]$$

To get the present value of the cash flow CF_i^d, we need to discount it back to time t using the domestic interest rate. Let V_i^d denote this present value; then,

$$\begin{aligned} V_i^d &= E^d\left[\exp\left(-\int_t^{t_i} r_d(s)\,ds\right) \times E_{t_i}^d\left(\exp\left(-\int_{t_i}^{t_i+1} r_d(s)\,ds\right)\right) \times CF_i^d\right] \\ &= P_dB_d(t, t_i) - P_dB_d(t, t_{i+1}) \end{aligned}$$

Let $r_d^0(r_f^0)$ denote the domestic (foreign) interest rate, which is known at time t and used to determine the payment at time t_1. The value of the domestic floater is then

$$\begin{aligned} V_d &= P_d \times r_d^0 \times B_d(t, t_1) + \sum_{i=1}^{n-1} V_i^d \\ &= P_d[(1 + r_d^0) \times B_d(t, t_1) - B_d(t, T)] \end{aligned} \tag{8}$$

The present value (V_i^f) of the foreign interest payment (CF_i^f) determined (paid) at time $t_i(t_{i+1})$ is similarly

$$V_i^f = E^d\left[\exp\left(-\int_t^{t_i} r_d(s)\,ds\right) \times E_{t_i}^d\left(\exp\left(-\int_{t_i}^{t_i+1} r_d(s)\,ds\right)\right) \times CF_i^f\right] \tag{9}$$

where

$$CF_i^f = P_d\left[\frac{1}{E_{t_i}^f(\exp(-\int_{t_i}^{t_i+1} r_f(s)\,ds))} - 1\right]$$

and $E_{t_i}^f$ is an expectation operator based on information available at time t_i in the foreign risk-neutral world.[6] Therefore, the present value at time

[6]Wei (1994) mistakenly uses the domestic risk-neutral process of the foreign risk-free rate (i.e., r_f') to determine the amount of foreign interest payment. In other words, the following formula in Wei is not correct:

$$CF_i^f = P_d\left[\frac{1}{E_{t_i}^d(\exp(-\int_{t_i}^{t_i+1} r_f'(s)\,ds))} - 1\right]$$

t of all foreign interest rate payments during the remaining life of the swap, V_f, can be written as

$$V_f = P_d \times r_f^0 \times B_d(t, t_i) + \sum_{i=1}^{n-1} V_i^f \tag{10}$$

To evaluate the expectation in (9), we follow Wei's proposition to relate the foreign risk-neutral world and the domestic risk-neutral world as follows:[7]

$$E^d\left[\exp\left(-\int_t^T r_f'(s)\,ds\right)\right]$$
$$= E^f\left[\exp\left(-\int_t^T r_f(s)\,ds\right)\right] \times \exp\left[\operatorname{cov}\left(\int_t^T r_f(s)\,ds, \int_t^T \sigma_X\,dw_X(s)\right)\right] \tag{11}$$

Finally, using the result of Equation 11 and the property of lognormal variable, we can evaluate and obtain

$$V_i^f = P_d \times \frac{B_d(t, t_{i+1})B_f(t, t_i)}{B_f(t, t_{i+1})} \exp(-b_1 + b_2 - b_3) - P_d B_d(t, t_{i+1}) \tag{12}$$

Since Wei misspecified the amount of foreign interest payment when evaluating V_i^f; hence the term of b_3 reported in his pricing formula is not correct. We derive the correct ones and display them as follows:

$$\begin{aligned} b_1 = \frac{\sigma_d \sigma_f \rho_{df}}{k_d k_f}\Bigg[& \frac{1}{k_f}\left(1+e^{-k_f(t_{i+1}-t)} - e^{-k_f(t_i-t)} - e^{-k_f(t_{i+1}-t_i)}\right) \\ & + \frac{1}{k_d + k_f}\Big(-e^{-(k_d+k_f)(t_{i+1}-t)} - e^{-k_d(t_{i+1}-t_i)} + e^{-k_d(t_{i+1}-t)-k_f(t_i-t)} \\ & + e^{-(k_d+k_f)(t_{i+1}-t_i)}\Big)\Bigg] \end{aligned}$$

$$\begin{aligned} b_2 = \frac{\sigma_f^2}{k_f^3}\Bigg[& 1 - \frac{3}{2}e^{-k_f(t_{i+1}-t_i)} + e^{-k_f(t_{i+1}-t)} - e^{-k_f(t_i-t)} - \frac{1}{2}e^{-2k_f(t_{i+1}-t)} \\ & + \frac{1}{2}e^{-2k_f(t_{i+1}-t_i)} + \frac{1}{2}e^{-k_f(t_{i+1}+t_i-2t)}\Bigg] \end{aligned}$$

$$b_3 = \frac{\sigma_f \sigma_X \rho_{fX}}{k_f^2} \times \left[1 - e^{-k_f(t_{i+1}-t_i)} + e^{-k_f(t_{i+1}-t)} - e^{-k_f(t_i-t)}\right]$$

[7]Alternatively, we can express the relationship between r_f' and r_f as

$$dr_f' = dr_f - \rho_{fX}\sigma_f\sigma_X\,dt$$

The terms b_1, b_2, and b_3 are meant to adjust for the effects of the correlation between domestic and foreign interest rates, the volatility of the foreign interest rate, and the correlation between the foreign interest rate and exchange rate.

All the terms in Equation 7 are now fully specified and the value of the diff swap to the investor who is paying the domestic interest rate and receiving the foreign interest rate can be evaluated by using Equations 7 through 12. Since the constant margin rate is specified to let the value of the diff swap be zero at the initialized date, the constant margin rate at the initialized date must be:

$$c^* = \frac{V_f - V_d}{\cdot\frac{(t_{i+1} - t_i)}{360} P_d \sum_{i=1}^{n} B_d(t, t_i)} \tag{13}$$

Hedging a diff swap is simple since it has linear payoffs. We will not derive the hedging formula for the cases of diff swaps with principal in domestic currency and foreign currency; instead we will do that for the case of diff swaps with principal in the third-country currency in the section entitled Pricing Diff Swaps—A General Form.

PRICING DUALITY OF DIFF SWAPS

This section will demonstrate that there exists a dual pricing relationship between the diff swap with principal denominated in a domestic currency and the diff swap with principal denominated in a foreign currency. In other words, we can translate the pricing formula of the diff swap with principal denominated in a domestic currency into that of the diff swap with principal denominated in a foreign currency and vice versa.

Consider a foreign investor who enters into an identical diff swap to that described in Section 2 except the notional principal is denominated in a foreign currency. To this investor, it is a diff swap with principal denominated in his currency, and its value in foreign currency is given in Equation 7, i.e.,

$$V = V_f - V_d - \frac{c(t_{i+1} - t_i)}{360} P_f \sum_{i=1}^{n} B_f(t, t_i) \tag{14}$$

where P_f is the notional principal denominated in the foreign currency, and other definitions are the same as in the previous section. Since the payment is in the foreign currency, the value of foreign interest payment in the foreign currency has exactly the same formula as that of V_d in Equation 8, that is,

$$V_f = P_f \times [(1 + r_f^0) \times B_f(t, t_i) - B_f(t, T)] \tag{15}$$

On the other hand, the value of domestic interest payment in foreign currency should follow the formula of V_f in Equation 9,

$$V_d = P_f \times r_d^0 \times B_f(t, t_i) + \sum_{i=1}^{n-1} V_i^d \tag{16}$$

where

$$V_i^d = P_f \times \frac{B_f(t, t_{i+1})B_d(t, t_i)}{B_d(t, t_{i+1})} \exp(-b_1^* + b_2^* - b_3^*) - P_f B_f(t, t_{i+1})$$

Note that the parameters in b_1^* and b_2^* are identical to those in b_1 and b_2 except that the subscripts "d" and "f"are interchanged. Moreover, term b_3^* is meant to adjust for the effect of the correlation between the domestic interest rate and the exchange rate ($\frac{1}{X}$, the foreign currency value of one unit of domestic currency). In summary, we have

$$\begin{aligned} b_1^* = \frac{\sigma_d \sigma_f \rho_{df}}{k_d k_f} \Bigg[& \frac{1}{k_d}(1 + e^{-k_d(t_{i+1}-t)} - e^{-k_d(t_i - t)} - e^{-k_d(t_{i+1}-t_i)}) \\ & + \frac{1}{k_d + k_f}(-e^{-(k_d+k_f)(t_{i+1}-t)} - e^{-k_f(t_{i+1}-t_i)} + e^{-k_f(t_{i+1}-t)-k_d(t_i-t)} \\ & + e^{-(k_d+k_f)(t_{i+1}-t_i)}) \Bigg] \end{aligned}$$

$$\begin{aligned} b_2^* = \frac{\sigma_d^2}{k_d^3} \Bigg[& 1 - \frac{3}{2} e^{-k_d(t_{i+1}-t_i)} + e^{-k_d(t_{i+1}-t)} - e^{-k_d(t_i-t)} - \frac{1}{2} e^{-2k_d(t_{i+1}-t)} \\ & + \frac{1}{2} e^{-2k_d(t_{i+1}-t_i)} + \frac{1}{2} e^{-k_d(t_{i+1}+t_i-2t)} \Bigg] \end{aligned}$$

$$b_3^* = \frac{-\sigma_d \sigma_X \sigma_{dX}}{k_d^2} \times [1 - e^{-k_d(t_{i+1}-t_i)} + e^{-k_d(t_{i+1}-t)} - e^{-k_d(t_i-t)}]$$

By invoking the law of one price, we can obtain the value in domestic currency of a diff swap denominated in a foreign currency by multiplying its value in the foreign currency with the spot exchange rate. In fact, one can also check the pricing formulae in Turnbull (1993) and identify a dual pricing relationship for diff swaps. We therefore conclude that the following proposition is true (without proof).

Proposition: There exists a dual pricing relationship for a diff swap with domestic currency denominated principal and a diff swap with foreign currency denominated principal.

PRICING DIFF SWAPS—A GENERAL FORM

It is quite common for a dealer to trade diff swaps with the notional principal denominated in a third-country currency; however, there is no pricing formula for this type. In this section we fill in the gap of the existing literature and derive pricing and hedging formulas for diff swaps with notional principal denominated in a third-country currency.

Suppose that a domestic (U.S.) investor enters into a diff swap with the notional principal denominated in a third-country currency (British Pound) and will receive/pay periodic interest payments based on a foreign (Japanese Yen LIBOR)/domestic (U.S. LIBOR) interest rate. The swap's value to the investor is then the difference between the present values of future cash outflows of domestic interest payments and cash inflows of foreign interest payments. In the same framework of the second section, the value of the diff swap (V) to the domestic investor who is paying a domestic interest rate and receiving a foreign interest rate is

$$V = V_f - V_d - \frac{c(t_{i+1} - t_i)}{360} P_{tc} X^* \sum_{i=1}^{n} B_{tc}(t, t_i) \tag{17}$$

where P_{tc} is the notional principal denominated in the third-country currency, X^* is the spot exchange rate expressed as the domestic currency value of one unit of the third-country currency, and other notations are defined as in the previous sections.

As shown in the Appendix, can be expressed as[8]

$$V_d = P_{tc} \times r_d^0 \times B_{tc}(t, t_i) X^* + \sum_{i=1}^{n-1} V_i^d \tag{18}$$

where

$$V_i^d = X^* \times P_{tc} \times \frac{B_{tc}(t, t_{i+1}) B_d(t, t_i)}{B_d(t, t_{i+1})} \exp(-c_1 + c_2 - c_3) - P_{tc} X^* B_{tc}(t, t_{i+1})$$

where $B_{tc}(t, t_i)$ is the time-t price of the third country's zero-coupon bond that pays one unit of the third-country currency at maturity date t_i, and

$$\begin{aligned} c_1 = \frac{\sigma_d \sigma_{tc} \rho_{dtc}}{k_d k_{tc}} \Bigg[& \frac{1}{k_d}\left(1 + e^{-k_d(t_{i+1}-t)} - e^{-k_d(t_i - t)} - e^{-k_d(t_{i+1}-t_i)}\right) \\ & + \frac{1}{k_d + k_{tc}}\left(-e^{-(k_d+k_{tc})(t_{i+1}-t)} - e^{-k_{tc}(t_{i+1}-t_i)} + e^{-k_{tc}(t_{i+1}-t) - k_d(t_i - t)} \right. \\ & \left. + e^{-(k_d+k_{tc})(t_{i+1}-t_i)}\right) \Bigg] \end{aligned}$$

[8]Since the domestic floater is paid in a third-country currency, it actually corresponds to a diff swap with the principal in a foreign currency. Thus, its pricing formula is similar to that presented in Section 3.

$$c_2 = \frac{\sigma_d^2}{k_d^3}\left[1 - \frac{3}{2}e^{-k_d(t_{i+1}-t_i)} + e^{-k_d(t_{i+1}-t)} - e^{-k_d(t_i-t)} - \frac{1}{2}e^{-2k_d(t_{i+1}-t)}\right.$$
$$\left. + \frac{1}{2}e^{-2k_d(t_{i+1}-t_i)} + \frac{1}{2}e^{-k_d(t_{i+1}+t_i-2t)}\right]$$
$$c_3 = \frac{-\sigma_d\sigma_{X^*}\rho_{dX^*}}{k_d^2} \times [1 - e^{-k_d(t_{i+1}-t_i)} + e^{-k_d(t_{i+1}-t)} - e^{-k_d(t_i-t)}]$$

where $\sigma_{tc}(\sigma_{X^*})$ is the standard deviation of the third-country interest rate (exchange rate, X^*), k_{tc} is the reversion coefficient of the third-country interest rate, and $\rho_{dtc}(\rho_{dX^*})$ is the correlation coefficient between the domestic interest rate and third-country interest rate (exchange rate, (X^*).

The present value at time t of all the interest rate payments during the remaining life of the swap, V_f, can be expressed as[9]

$$V_f = P_{tc} \times r_f^0 \times B_{tc}(t, t_i)X^* + \sum_{i=1}^{n-1} V_i^f \tag{19}$$

where

$$V_i^f = P_{tc}X^* \times \frac{B_{tc}(t, t_{i+1})B_f(t, t_i)}{B_f(t, t_{i+1})}\exp(b_2 + e_1 - e_2 - e_3)$$
$$- P_{tc}X^*B_{tc}(t, t_{i+1}) \tag{20}$$

and where

$$e_1 = \frac{\sigma_f\sigma_{X^*}\rho_{fX^*}}{k_f^2}[1 - e^{-k_f(t_{i+1}-t_i)} + e^{-k_f(t_{i+1}-t)} - e^{-k_f(t_i-t)}]$$

$$e_2 = \frac{\sigma_f\sigma_{tc}\rho_{ftc}}{k_f + k_{tc}}\left[\frac{1}{k_f}(1 + e^{-k_f(t_{i+1}-t)} - e^{-k_f(t_i-t)} - e^{-k_f(t_{i+1}-t_i)})\right.$$
$$+ \frac{1}{k_f + k_{tc}}(-e^{-(k_f+k_{tc})(t_{i+1}-t)} - e^{-k_{tc}(t_{i+1}-t_i)} + e^{-k_{tc}(t_{i+1}-t)-k_f(t_i-t)}$$
$$\left. + e^{-(k_f+k_{tc})(t_{i+1}-t_i)})\right]$$

$$e_3 = \frac{\sigma_f\sigma_X\rho_{fX}}{k_f^2} \times [1 - e^{-k_f(t_{i+1}-t_i)} + e^{-k_f(t_{i+1}-t)} - e^{-k_f(t_i-t)}]$$

[9]Since the foreign floater is paid in a third-country currency, its pricing formula is also similar to that presented in Section 3. However, it should be noted that the term used to adjust for the effect of the correlation between the foreign interest rate and exchange rate X*/X the foreign currency value of one unit of third-country currency) is decomposed into $(e_1 - e_3)$.

where ρ_{fX^*} is the correlation coefficient between the foreign interest rate and exchange rate (X^*).

To let the value of the diff swap equal zero at the initialized date, the constant margin rate must be as follows:

$$c^* = \frac{V_f - V_d}{\frac{(t_{i+1} - t_i)}{360} P_{tc} X^* \sum_{i=1}^{n} B_{tc}(t, t_i)} \tag{21}$$

We should point out that there is a simple way to check our general-form formula. If the third country is identical to the domestic country (i.e., $X^* = 1$, $\sigma_{X^*} = \rho_{dX^*} = 0$, $\sigma_d = \sigma_{tc}$, $k_d = k_{tc}$, and $\rho_{dtc} = 1$), our general-form formula will reduce to the pricing formula reported in Section 2. On the other hand, if the third country is identical to the foreign country (i.e., $X^* = X$, $\sigma_{X^*} = \sigma_X$, $\sigma_f = \sigma_{tc}$, $k_f = k_{tc}$, $\rho_{fX^*} = \rho_{fX}$, $\rho_{dtc} = \rho_{df}$, and $\rho_{ftc} = 1$), one can find that our general-form formula will reduce to the pricing formula, reported in Section 3, for diff swaps with the principal denominated in a foreign currency.

There are several interesting points worth discussing about the general-form pricing formula for diff swaps. First, the values of the diff swaps in our pricing model do not directly depend on the market prices of interest rate risks. The interest rate effects are embedded in the domestic, foreign, and third country's zero-coupon bond prices. Second, the structure of diff swaps seems to be unrelated to exchange rate behavior, but the value of the diff swap depends on the exchange rate volatility (σ_X, σ_{X^*}) and the correlation coefficients of ρ_{fX}, ρ_{dX^*} and ρ_{fX^*}. Third, the correlation between domestic and third-country interest rates and the correlation between foreign and third-country interest rates also play a role in the pricing formula. Hence, the key factors determining the values of diff swaps with the principal in a third-country currency are much more complicated than those of the diff swaps with the principal in a domestic or foreign currency.

As mentioned earlier, hedging a diff swap is relatively simple since it has linear payoffs. For a particular payment on the domestic interest rate leg, that is the value in Equation 18, we need three instruments to construct the hedging portfolio. This is because there are three factors in the term V_i^d, namely domestic interest rate, third-country interest rate, and exchange rate (X^*). The hedging portfolio can be shown as

$$V_i^d(hedge) = n_{d1} B_d(t, t_i) + n_{d2} B_d(t, t_{i+1}) + n_{y2} X^* B_{tc}(t, t_{i+1}) \tag{22}$$

where[10]

$$n_{d1} = \frac{-P_{tc}X^*B_{tc}(t, t_{i+1})}{B_d(t, t_{i+1})}\exp(-c_1 + c_2 - c_3)$$

$$n_{d2} = \frac{P_{tc}X^*B_{tc}(t, t_{i+1})B_d(t, t_i)}{B_d^2(t, t_{i+1})}\exp(-c_1 + c_2 - c_3)$$

$$n_{y2} = \frac{-V_i^d}{B_{tc}(t, t_{i+1})X^*}$$

and where V_i^d is given in Equation 19.

Since there are three factors in the term V_i^f, namely foreign and third-country interest rates and exchange rate, X^*, we also need three instruments to construct the hedging portfolio. The hedging portfolio is constructed as

$$V_i^f(hedge) = n_{f1}B_f(t, t_i) + n_{f2}B_f(t, t_{i+1}) + n_{y2}^* X^*B_{tc}(t, t_{i+1}) \quad (23)$$

where

$$n_{f1} = \frac{-P_{tc}X^*B_{tc}(t, t_{i+1})}{XB_f(t, t_{i+1})}\exp(b_2 + e_1 - e_2 - e_3)$$

$$n_{f2} = \frac{P_{tc}X^*B_{tc}(t, t_{i+1})B_f(t, t_i)}{XB_f^2(t, t_{i+1})}\exp(b_2 + e_1 - e_2 - e_3)$$

$$n_{y2}^* = \frac{-V_i^d}{B_{tc}(t, t_{i+1})X^*}$$

and where V_i^f is given in Equation 21. We can reduce the hedging formulas in our general-form solution to the special case when the third country is identical to the domestic country or the foreign country.

SIMULATION RESULTS

Diff Swaps in a Domestic Currency

This section carries out simulations to answer several questions that are interesting to traders in diff swap markets. These questions are: What is the relationship between the constant margin rate and the tenor? What is the magnitude of the constant margin rate relative to the current rate differential? Are the correlation parameters important for pricing diff swaps?

[10]The derivations of hedging ratios are available upon request from the authors.

We first carry out simulations using the pricing formula for diff swaps with the principal in a domestic currency as derived in Section 2. The simulation results are reported in Tables I and II. Several observations are found from Table I. First, the longer the tenor is for the swaps,

TABLE I
Effects of Interest Rate Differential on Constant Margin Rate
(Diff Swaps with Principal in Domestic Currency)

r_d	r_f	$r_f - r_d$	V_f	V_d	c^*
		$T - t = 1$ *year*			
0.05	0.10	0.05	9.79	5.12	0.0485
0.06	0.10	0.04	9.72	6.00	0.0390
0.07	0.10	0.03	9.65	6.87	0.0294
0.08	0.10	0.02	9.58	7.73	0.0197
0.09	0.10	0.01	9.52	8.58	0.0100
0.10	0.10	0.00	9.45	9.42	0.0003
0.11	0.10	−0.01	9.38	10.26	−0.0095
0.12	0.10	−0.02	9.32	11.09	−0.0194
0.13	0.10	−0.03	9.25	11.91	−0.0293
0.14	0.10	−0.04	9.19	12.73	−0.0392
0.15	0.10	−0.05	9.12	13.53	−0.0492
		$T - t = 3$ *year*			
0.05	0.10	0.05	27.49	15.64	0.0434
0.06	0.10	0.04	27.09	17.66	0.0351
0.07	0.10	0.03	26.70	19.62	0.0267
0.08	0.10	0.02	26.32	21.54	0.0183
0.09	0.10	0.01	25.94	23.41	0.0098
0.10	0.10	0.00	25.57	25.24	0.0013
0.11	0.10	−0.01	25.21	27.03	−0.0073
0.12	0.10	−0.02	24.85	28.77	−0.0159
0.13	0.10	−0.03	24.50	30.47	−0.0246
0.14	0.10	−0.04	24.15	32.13	−0.0333
0.15	0.10	−0.05	23.81	33.75	−0.0421
		$T - t = 5$ *year*			
0.05	0.10	0.05	42.83	25.88	0.0396
0.06	0.10	0.04	41.96	28.44	0.0322
0.07	0.10	0.03	41.11	30.91	0.0248
0.08	0.10	0.02	40.29	33.30	0.0173
0.09	0.10	0.01	39.48	35.60	0.0098
0.10	0.10	0.00	38.70	37.83	0.0022
0.11	0.10	−0.01	37.93	39.98	−0.0054
0.12	0.10	−0.02	37.18	42.05	−0.0131
0.13	0.10	−0.03	36.45	44.06	−0.0209
0.14	0.10	−0.04	35.74	45.99	−0.0287
0.15	0.10	−0.05	35.05	47.86	−0.0366

Note. This table computes the values of the two floaters (V_f and V_d) and the constant margin rate (c^*) under alternative interest differentials. The calculations are performed for three tenors. In each case, we assume that the reset period is six months and the other parameters are: $\sigma_f = \sigma_d = 0.02$, $\sigma_x = 0.3$, $\rho_{df} = 0.3$, $\rho_{dx} = -0.3$, $\mu_d = \mu_f = 0.1$, $k_d = k_f = 0.15$, $\lambda_d = \lambda_f = -0.1$, and $P_d = \$100$.

TABLE II
Effects of Correlations on Swap Values
(Diff Swaps with Principal in Domestic Currency)

	V_f	V_d	V	*Error (%)*	c^e *(bp)*
	Panel A: Varying ρ_{df} While Keeping ρ_{fX} Constant (at −0.3)				
ρ_{df}					
0.00	26.74	19.62	0.76	0.00	0.00
0.10	26.73	19.62	0.75	1.33	0.51
0.20	26.71	19.62	0.73	4.11	1.01
0.30	26.70	19.62	0.72	5.56	1.52
0.40	26.69	19.62	0.71	7.04	2.03
0.50	26.67	19.62	0.69	10.14	2.54
	Panel B: Varying ρ_{fx} While Keeping ρ_{df} Constant (at 0.3)				
ρ_{fx}					
0.00	26.45	19.62	0.47	0.00	0.00
−0.10	26.53	19.62	0.55	−14.55	−3.18
−0.20	26.62	19.62	0.64	−26.56	−6.37
−0.30	26.70	19.62	0.72	−34.72	−9.56
−0.40	26.79	19.62	0.80	−41.25	−12.74
−0.50	26.87	19.62	0.89	−47.19	−15.93

Note. This table demonstrates the effects of correlations on values of diff swaps. The swap is assumed to have three years of remaining life. The constant margin is set at 0.0024. The reset period is six months. In the calculations, we set $\sigma_f = \sigma_d = 0.02$, $\sigma_X = 0.15$, $r_d = 0.07$, $r_f = 0.1$, $\mu_d = \mu_f = 0.1$, $k_d = k_f = 0.15$, $\lambda_d = \lambda_f = -0.1$, and $P_d = \$100$. The second to last column reports the pricing errors when assuming zero correlation. For example, in Panel A, 10.14% is calculated as 100(0.76 − 0.69)/0.69. The last column (c^e) shows the equivalent margin rate that would have the same effect on the swap value as that caused by the correlation. For example, in Panel A, the swap value decreases by \$0.07(\$0.76 − \$0.69), when ρ_{df} changes from 0 to 0.50. Had we kept ρ_{df} at zero, but increased the constant margin by 2.54 basis points, the swap value would also have increased by \$0.07.

the lower is the constant margin rate. However, this characteristic is not universal. In some cases, the constant margin rate is high when the tenor is long. Second, the magnitude of the constant margin rate is generally smaller than the interest rate differential. For example, with a five-year swap, when the current domestic and foreign interest rates are 15% and 10% respectively, the investor, who pays a domestic interest rate, will incur a current loss of 5%. However, the counterpart only compensates the investor 366 basis points in the form of constant margin. The interest compensation comes from the downward-sloping domestic yield curve since $r_d > \mu_d$. This demonstrates that when one enters into a diff swap deal, it is more important to focus on the yield curve differential than on the current rate differential.

We additionally investigate how the correlation parameters affect the values of diff swaps, assuming that the swap has a remaining life of three years and the constant margin rate, set at the inception of the swap, is 2.4%. We then compute two sets of swap values by varying ρ_{df}

and ρ_{fX} and also compute their equivalent margin rates to show the change in margin rate that would have the same effect on the swap value as that caused by the change in correlation.

As shown in panel A of Table II, we find that the correlation between the domestic interest rate and foreign interest rate plays a relatively minor role in pricing diff swaps. For example, as ρ_{df} changes from 0 to 0.5, the change in the constant margin rate is only 2.54 basis points. However, from panel B of Table II, we find that the correlation between the foreign interest rate and exchange rate plays a more important role in pricing diff swaps. For example, as ρ_{fX} changes from 0 to −0.5, the change in the constant margin rate is 15.93 basis points.[11] The reasons behind these results can be explained by the fact that we set parameter values much higher for exchange rate volatility (0.15) than for interest rate volatility (0.02). The parameter values are chosen to reflect market realities. Hence the changes in the correlation between the exchange rate and interest rate will affect the constant margin rates of diff swaps more significantly than the changes in the correlation between interest rates. Due to the pricing duality between diff swaps with principal in domestic and foreign currency, it is not necessary to simulate for the case of diff swaps with the principal in a foreign currency.

Diff Swaps with Principal in Third-Country Currency

In the case of diff swaps with the principal denominated in a third-country currency, we first carry out simulations to answer the question on the relationship between the constant margin rate and the tenor. As reported in Table III, we find that the longer the tenor is for the swaps, the lower is the constant margin rate. Again, this characteristic is not universal. In some cases, the constant margin rate is high when the tenor is long. Second, as in the case of diff swaps with a domestic currency, the magnitude of the constant margin rate is generally smaller than the interest rate differential. This again supports the view that one should focus more on the yield curve differential than on the current rate differential when entering into a diff swap deal.

The last question that we try to answer is how the correlation parameters affect the values of diff swaps in the case of a third-country

[11]The simulation results are qualitatively similar to those reported in Wei (1994), but different in magnitude.

TABLE III

Effects of Interest Rate Differential on Constant Margin Rate
(The General-Form Diff Swaps)

r_d	r_f	$r_f - r_d$	V_f	V_d	c^*
		$T - t = 1$ *year*			
0.05	0.10	0.05	9.43	4.93	0.0485
0.06	0.10	0.04	9.43	5.82	0.0389
0.07	0.10	0.03	9.43	6.71	0.0293
0.08	0.10	0.02	9.43	7.60	0.0197
0.09	0.10	0.01	9.43	8.50	0.0100
0.10	0.10	0.00	9.43	9.41	0.0002
0.11	0.10	−0.01	9.43	10.32	−0.0096
0.12	0.10	−0.02	9.43	11.23	−0.0194
0.13	0.10	−0.03	9.43	12.15	−0.0293
0.14	0.10	−0.04	9.43	13.07	−0.0392
0.15	0.10	−0.05	9.43	14.00	−0.0492
		$T - t = 3$ *year*			
0.05	0.10	0.05	25.33	14.37	0.0432
0.06	0.10	0.04	25.33	16.50	0.0348
0.07	0.10	0.03	25.33	18.63	0.0264
0.08	0.10	0.02	25.33	20.78	0.0180
0.09	0.10	0.01	25.33	22.93	0.0095
0.10	0.10	0.00	25.33	25.09	0.0009
0.11	0.10	−0.01	25.33	27.26	−0.0076
0.12	0.10	−0.02	25.33	29.44	−0.0162
0.13	0.10	−0.03	25.33	31.63	−0.0248
0.14	0.10	−0.04	25.33	33.83	−0.0335
0.15	0.10	−0.05	25.33	36.04	−0.0422
		$T - t = 5$ *year*			
0.05	0.10	0.05	38.11	22.97	0.0391
0.06	0.10	0.04	38.11	25.86	0.0317
0.07	0.10	0.03	38.11	28.76	0.0242
0.08	0.10	0.02	38.11	31.68	0.0166
0.09	0.10	0.01	38.11	34.60	0.0091
0.10	0.10	0.00	38.11	37.53	0.0015
0.11	0.10	−0.01	38.11	40.47	−0.0061
0.12	0.10	−0.02	38.11	43.43	−0.0137
0.13	0.10	−0.03	38.11	46.40	−0.0214
0.14	0.10	−0.04	38.11	49.37	−0.0291
0.15	0.10	−0.05	38.11	52.36	−0.0368

Note. This table computes the values of the two floaters (V_f and V_d) and the constant margin rate (c^*) under alternative interest differentials. The calculations are performed for three tenors. In each case, we assume that the reset period is six months and the other parameters are: $X^* = 1$, $r_{tc} = 0.1$, $\sigma_f = \sigma_d = \sigma_{tc} = 0.02$, $\sigma_X = 0.3$, $\rho_{df} = \rho_{dtc} = \rho_{ftc} = 0.3$, $\rho_{fX} = \rho_{fX^*} = \rho_{dX^*} = -0.3$, $\mu_d = \mu_f = 0.1$, $k_d = k_f = k_{tc} = 0.15$, $\lambda_d = \lambda_f = \lambda_{tc} = -0.1$, and $P_{tc} = 100$.

TABLE IV
Effects of Correlations on Swap Values
(The General-Form Diff Swaps)

	V_f	V_d	V	*Error (%)*	c^e *(bp)*
Panel A: Varying ρ_{dtc} While Keeping Other Correlation Terms Constant					
ρ_{dtc}					
−0.30	25.33	18.71	0.54	5.56	1.48
−0.20	25.33	18.69	0.55	3.64	0.99
−0.10	25.33	18.68	0.56	1.79	0.49
0.00	25.33	18.67	0.57	0.00	0.00
0.10	25.33	18.66	0.59	−3.39	−0.49
0.20	25.33	18.64	0.60	−5.00	−0.99
0.30	25.33	18.63	0.61	−6.56	−1.48
Panel B: Varying ρ_{ftc} While Keeping Other Correlation Terms Constant					
ρ_{ftc}					
−0.30	25.41	18.63	0.69	−5.80	−1.50
−0.20	25.39	18.63	0.67	−2.99	−1.00
−0.10	25.38	18.63	0.66	−1.52	−0.50
0.00	25.37	18.63	0.65	0.00	0.00
0.10	25.36	18.63	0.64	1.56	0.50
0.20	25.34	18.63	0.62	4.84	1.00
0.30	25.33	18.63	0.61	6.56	1.50

Note. This table demonstrates the effects of correlations on values of diff swaps. The swap is assumed to have three years of remaining life. The constant margin is set at 0.024. The reset period is six months. In the calculations, we set $X^* = 1$, $r_0 = 0.07$, $r_f = r_{tc} = 0.1$, $\sigma_f = \sigma_d = \sigma_{tc} = 0.02$, $\sigma_X = 0.03$, $\rho_{df} = \rho_{dtc} = \rho_{ftc} = 0.3$, $\rho_{fX} = \rho_{fX^*} = \rho_{dX^*} = -0.3$, $\mu_d = \mu_f = 0.1$, $k_d = k_f = k_{tc} = 0.15$, $\lambda_d = \lambda_f = \lambda_{tc} = -0.1$, and $P_{tc} = 100$. The equivalent margin rate (c^e) and the pricing errors (*Error*) are defined as in Table II.

currency. From Equations 18 and 20, we know that there are five correlation parameters, ρ_{dtc}, ρ_{ftc}, ρ_{fX}, ρ_{fX^*}, and ρ_{dX^*}, that affect the swap's value. Results in panels A and B of Table IV indicate that the correlation between the domestic interest rate and third-country interest rate, and the correlation between the foreign interest rate and third-country interest rate play a minor role in diff swap pricing. However, results in panels A, B, and C of Table V indicate that the correlation between the foreign interest rate and exchange rate (X), the correlation between the foreign interest rate and exchange rate (X^*), and the correlation between the domestic interest rate and exchange rate (X^*) play a more important role in pricing diff swaps. As we discussed in the previous section, the reasons that correlations of interest rates associated with exchange rates play a more important role than correlations among interest rates themselves can be explained by the fact that exchange rate volatility is much higher than interest rate volatility.

TABLE V

Effect of Correlations on Swap Values (The General-Form Diff Swaps)

	V_f	V_d	V	*Error* (%)	c^e (*bp*)
Panel A: Varying ρ_{fX} While Keeping Other Correlation Terms Constant					
ρ_{fX}					
−0.30	25.33	18.63	0.61	−39.34	−9.42
−0.20	25.25	18.63	0.53	−30.19	−6.28
−0.10	25.17	18.63	0.45	−17.78	−3.14
0.00	25.09	18.63	0.37	0.00	0.00
0.10	25.01	18.63	0.29	27.59	3.14
0.20	24.93	18.63	0.21	76.19	6.28
0.30	24.85	18.63	0.13	184.62	9.41
Panel B: Varying ρ_{fX^*} While Keeping Other Correlation Terms Constant					
ρ_{fX^*}					
−0.30	25.33	18.63	0.61	39.34	9.43
−0.20	25.41	18.63	0.69	23.19	6.29
−0.10	25.49	18.63	0.77	10.39	3.14
0.00	25.57	18.63	0.85	0.00	0.00
0.10	25.65	18.63	0.93	−8.60	−3.14
0.20	25.73	18.63	1.01	−15.84	−6.29
0.30	25.81	18.63	1.09	−22.02	−9.43
Panel C: Varying ρ_{dX^*} While Keeping Other Correlation Terms Constant					
ρ_{dX^*}					
−0.30	25.33	18.63	0.61	−39.34	−9.32
−0.20	25.33	18.71	0.53	−30.19	−6.21
−0.10	25.33	18.79	0.45	−17.78	−3.11
0.00	25.33	18.87	0.37	0.00	0.00
0.10	25.33	18.95	0.30	23.33	3.11
0.20	25.33	19.03	0.22	68.18	6.21
0.30	25.33	19.10	0.14	164.29	9.32

Note. This table demonstrates the effects of correlations on values of diff swaps. The swap is assumed to have three years of remaining life. The constant margin is set at 0.024. The reset period is six months. In the calculations, we set $X^* = 1$, $r_d = 0.07$, $r_f = r_{tc} = 0.1$, $\sigma_f = \sigma_d = \sigma_{tc} = 0.02$, $\sigma_X = .03$, $\rho_{df} = \rho_{dtc} = \rho_{ftc} = 0.3$, $\rho_{fX} = \rho_{fX^*} = \rho_{dX^*} -0.3$, $\mu_d = \mu_f = 0.1$, $k_d = k_f = k_{tc} = 0.15$, $\lambda_d = \lambda_f = \lambda_{tc} = -0.1$, and $P_{tc} = 100$. The equivalent margin rate (c^e) and the pricing errors (*Error*) are

CONCLUSIONS

Under the assumptions that both domestic and foreign interest rates follow the mean-reverting Ornstein–Uhlenbeck process and the exchange rate follows the lognormal process, we derive a general-form formula for pricing and hedging diff swaps with the principal denominated either in a domestic, foreign, or third-country currency. We start our derivation from the most common type of diff swaps, the principal in a domestic currency, and identify an error in the formula of Wei (1994). We then demonstrate the pricing duality between diff swaps with the principal in a domestic currency and diff swaps with the principal in a foreign currency. Finally, we complete the pricing and hedging analysis on diff swaps by deriving formulae for diff swaps with the principal denominated in a third-country currency.

Simulation results show that the constant margin rate on average declines with the tenor of the swaps and the magnitude of the constant margin rate is generally smaller than the interest rate differential. Among domestic interest rate, foreign interest rate, third-country interest rate, and exchange rate, we found that correlations associated with the exchange rate play a more important role in pricing diff swaps than correlations among interest rates themselves.

APPENDIX

This appendix shows the derivation of V_i^d and V_i^f given in Equations 19 and 21. For a domestic investor, the domestic interest rate payment at time t_{i+1} denominated *in third-country currency* is

$$CF_i^d = P_{tc}E_{t_i}^d\left[\left(\frac{1}{E_{t_i}^d[\exp(-\int_{t_i}^{t_{i+1}} r_d(s)\,ds)]} - 1\right)\right] \tag{24}$$

Since the domestic price at time t_i of receiving one unit of the third-country currency at time t_{i+1} is $X_{t_i}^* \times B_{tc}(t_i, t_{i+1})$, we can discount CF_i^d backward (from time t_i to t) using the domestic interest rate to get the present value *in domestic currency* (at time t) as follows:

$$V_i^d = P_{tc}E^d\left[\exp\left(-\int_t^{t_i} r_d(s)\,ds\right)\left(\frac{1}{E_{t_i}^d[\exp(-\int_{t_i}^{t_{i+1}} r_d(s)\,ds)]} - 1\right)X_{t_i}^* B_{tc}(t_i, t_{i+1})\right] \tag{25}$$

According to Arnold (1974), we can rewrite $X_{t_i}^*$ as

$$X_{t_i}^* = X^* \exp\left[\int_t^{t_i} r_d(s)\,ds - \int_t^{t_i} r_{tc}'(s)\,ds - \int_t^{t_i} \frac{\sigma_{X^*}^2}{2}\,ds + \int_t^{t_i} \sigma_{X^*}\,dW_{X^*}(s)\right] \tag{26}$$

From the definition of bond price, we have

$$B_{tc}(t_i, t_{i+1}) = E_{t_i}^{tc}\left[\exp -\left(\int_{t_i}^{t_{i+1}} r_{tc}(s)\,ds\right)\right] \tag{27}$$

where E^{tc} is an expectation operator in the third country's risk-neutral world.

Substituting $X^*_{t_i}$ and $B_{tc}(t_i, t_{i+1})$, as shown in Equations 26 and 27, into Equation 25, we obtain

$$V_i^d = P_{tc}X^*E^d\left[\exp\left(-\int_t^{t_i} r'_{tc}(s)\,ds - \int_t^{t_i}\frac{\sigma_{X^*}^2}{2}\,ds + \int_t^{t_i}\sigma_{X^*}\,dW_{X^*}(s)\right)\right.$$
$$\left.\times\left(\frac{E_{t_i}^{tc}[\exp(-\int_{t_i}^{t_{i+1}} r_{tc}(s)\,ds)]}{E_{t_i}^{d}[\exp(-\int_{t_i}^{t_{i+1}} r_{d}(s)\,ds)]} - E_{t_i}^{tc}\left[\exp\left(-\int_{t_i}^{t_{i+1}} r_{tc}(s)\,ds\right)\right]\right)\right] \quad (28)$$

The transformation of two different risk-neutral worlds is given in Wei (1994) as the following

$$E^d\left[\exp\left(-\int_t^{T} r'_{tc}(s)\,ds\right]\right. = E^{tc}\left[\exp\left(-\int_t^{T} r_{tc}(s)\,ds\right)\right.$$
$$\left.\times\exp\left(\mathrm{cov}\left(\int_t^T r_{tc}(s)\,ds, \int_t^T \sigma_{X^*}\,dW_{X^*}(s)\right)\right)\right] \quad (29)$$

Finally, we use Equation 29 and the property of a lognormal variable to evaluate Equation 28 and obtain

$$V_i^d = X^* \times P_{tc} \times \frac{B_{tc}(t, t_{i+1})B_d(t, t_i)}{B_d(t, t_{i+1})}\exp(-c_1 + c_2 - c_3)$$
$$- P_{tc}X^*B_{tc}(t, t_{i+1}) \quad (30)$$

Similarly, V_i^f can be evaluated in the same way and we have

$$V_i^f = P_{tc}E^d\left[\exp\left(-\int_t^{t_i} r_d(s)\,ds\right)\left(\frac{1}{E_{t_i}^f[\exp(-\int_{t_i}^{t_{i+1}} r_f(s)\,ds)]} - 1\right)X^*_{t_i}B_{tc}(t_i, t_{i+1})\right] \quad (31)$$

Using Equations 26, 27, and 29 and the property of a lognormal variable, we obtain

$$V_i^f = P_{tc}X^* \times \frac{B_{tc}(t, t_{i+1})B_f(t, t_i)}{B_f(t, t_{i+1})}\exp(e_1 - e_2 - e_3) - P_{tc}X^*B_{tc}(t, t_{i+1}) \quad (32)$$

BIBLIOGRAPHY

Amin, K., & Jarrow, R. A. (1991). Pricing foreign currency options under stochastic interest rates. Journal of International Money and Finance, October, 301–329.

Amin, K., & Jarrow, R. A. (1994). Quanto swaps challenge: The result. Euromoney, October, 30.

Arnold, L. (1974). Stochastic differential equations: Theory and applications. New York: John Wiley.

Bicksler, J., & Chen, A. H. (1986). An economic analysis of interest rate swaps. Journal of Finance, 41, 645–655.

Cookson, R. (1992). Stock-still. Risk, November, 27–33.

Das, S. (1992a). Differential strip down. Risk, June, 65–72.

Das, S. (1992b). Differential operators. Risk, July, 51–53.

Fraser, K. M. (1992). Forex derivatives take a wild ride. Global Finance, November, 50–54.

Gastineau, G. (1994). An introduction to special-purpose derivatives: Rate differential swaps and deferred strike options. Journal of Derivatives, Spring, 59–62.

Heath, D., Jarrow, R. A., & Morton, A. (1992). Bond pricing and the term structure of interest rates: A new methodology for contingent claim valuation. Econometrica, 60, 77–105.

Jamshidian, F. (1994). Hedging quantos, differential swaps and ratios. Applied Mathematical Finance, 1–20.

Litzenberger, R. H. (1992). Swaps: Plain and fanciful. Journal of Finance, July, 831–850.

Mahoney, J. M. (1995). Correlation products and risk management issues. Economic Policy Review, Federal Reserve Bank of New York, October, 7–20.

Robinson, D. (1991). Diff swaps tempt the way. Euromoney, October, 10–11.

Smith, C. W., Smithson, C. W., & Wakeman, L. M. (1986). The evolving market for swaps. Midland Corporate Finance Journal, 20–32.

Turnbull, S. M. (1993). Pricing and hedging diff swaps. Journal of Financial Engineering, December, 297–333.

Vasicek, O. (1977). An equilibrium characterization of the term structure. Journal of Financial Economics, November, 177–188.

Wall, L. D., & Pringle, J. J. (1989). Alternative explanations of interest rate swaps: A theoretical and empirical analysis. Financial Management, 18, 59–73.

Wei, J. Z. (1994). Valuing differential swaps. Journal of Derivatives, Spring, 64–76.