HEDGING IN FUTURES AND OPTIONS MARKETS WITH BASIS RISK

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This paper analyzes the hedging decisions for firms facing price and basis risk. Two conditions assumed in most models on optimal hedging are relaxed. Hence, (i) the spot price is not necessarily linear in both the settlement price and the basis risk and (ii) futures contracts and options on futures at different strike prices are available. The design of the first-best hedging instrument is first derived and then it is used to examine the optimal hedging strategy in futures and options markets. The role of options as useful hedging tools is highlighted from the shape of the first-best solution. © 2002 John Wiley & Sons, Inc. Jrl Fut Mark 22:59–72, 2002

INTRODUCTION

Most models on optimal hedging account for the fact that the spot price at which output is sold typically differs from the price used to settle futures market positions. The firm thus faces basis risk in addition to price risk. However, such models usually assume that the spot price is linear in both the settlement price and the basis, and that either futures

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markets are only available (see, e.g., Benninga, Eldor, & Zilcha, 1983; Briys, Crouhy, & Schlesinger, 1993) or a single strike price for the option is at the firm's disposal (Moschini & Lapan, 1995). The purpose of this paper is to reexamine the problem of hedging price risk in the presence of basis risk under less restrictive assumptions: the spot price is not necessarily linear in the settlement price and the basis risk; futures contracts and options on futures at different strike prices contracts are available. This problem is analyzed in an original two-stage procedure. First, the optimal hedging instrument is designed in the presence of an unhedgeable and independent basis risk. Then, this first-best solution is used to derive important features of the optimal hedging strategy with futures contracts and options on futures when the basis risk is additive or multiplicative.

This paper shows that the role of options in the firm's hedging strategy is rationalized by the nonlinear payoff function of the first-best hedging instrument. This can be caused by a nonlinear relationship between the spot price and the settlement price, and/or by a nonadditive basis risk, and/or by biased financial markets. From the shape of the first-best hedging contract, the firm's hedging position in the futures market is characterized, and its options positions are derived under additive basis risk and under multiplicative small basis risk.

THE MODEL

Consider a firm that expects to sell a nonrandom quantity q of a given commodity at time $t = 1$, at the prevailing spot price $\tilde{p}$, where tilde denotes a random variable. The firm does not need to hold the inventory unhedged as financial markets for the good exist. The financial markets provide an imperfect hedging mechanism because of the presence of basis risk. In this context of incomplete markets, the relationship between the spot price and the settlement price is formalized by

$$\tilde{p} = g(\tilde{f}, \tilde{b}) \tag{1}$$

where g is a deterministic function, $\tilde{f}$ denotes the price used to settle hedging instruments where the support of its cumulative distribution function is contained in a compact interval $[0, f_{\max}]$ with $f_{\max} > 0$, and $\tilde{b}$ is the basis risk with $E_b\tilde{b} = 0$ where E_b denotes the expectation operator with respect to the random variable in subscript. The $\tilde{b}$ basis risk and the $\tilde{f}$ settlement price are assumed to be stochastically independent.[1] In addition, the spot price and the settlement price are assumed to be positively correlated or, equivalently, g is strictly increasing in the settlement price, i.e., $g_f \equiv \partial g/\partial f > 0$. Without loss of generality, a larger value of the $\tilde{b}$ risk leads to a larger spot price, i.e., $g_b \equiv \partial g/\partial b > 0$.

The hedging instrument is described by a couple $[I(.), P]$ where P is its premium and $I(f)$ is its payoff when the realized settlement price is f. The payoff function is assumed non-negative:

$$I(f) \geq 0 \qquad \text{for all } f \tag{2}$$

This condition is satisfied by existing hedging tools, e.g., option contracts. The price of the hedging contract is assumed to be proportional to the expected payoff:

$$P = (1 + \delta)E_f I(\tilde{f}) \tag{3}$$

with $\delta \geq 0$. The unbiased or "fair" hedging contract is defined such that, given the agent's expectations, its expected payoff net of the premium is zero, i.e., $P = E_f I(\tilde{f})$, or equivalently, $\delta = 0$. The contract is "overpriced" if $P > E_f I(\tilde{f})$, i.e., $\delta > 0$.[2]

The firm's preferences are represented by a von Neumann–Morgenstern utility function u which exhibits risk aversion, $u' > 0$, $u'' < 0$.[3] An optimal hedging contract on the settlement price in the presence of basis risk is obtained by finding the payoff function $I(.)$ and its premium P that maximize the firm's expected utility function of final wealth subject to the constraints above. Formally, this problem is stated

$$\max_{I(.),P} E_{f,p}\, u(\pi_0 + q\tilde{p} + I(\tilde{f}) - P) \tag{4}$$

subject to conditions (1), (2), and (3)

where π_0 is the firm's initial nonrandom wealth.

[1]This is a slightly stronger version of separability introduced by Benninga, Eldor, and Zilcha (1983). Separability will turn out to be a sufficient condition to derive results when the spot price is linear in both the settlement price and the basis risk, but it may not be not sufficient in a more general case, e.g., when the basis risk is multiplicative.

[2]It can be shown that the results will hold under the more general case where the price P depends only on the expected payoff: $P = c(E_f I(\tilde{f}))$ with $c(0) = 0$ and $c'(e) \geq 1$ for all e.

[3]The firm can also be assumed to be risk neutral but it is motivated to hedge by market imperfections (Greenwald & Stiglitz, 1993), direct and indirect costs of financial distress, and convex tax schedule (Froot, Scharfstein, & Stein, 1993; Smith & Stulz, 1985). Therefore, it behaves as if it was risk averse.

THE DESIGN OF THE FIRST-BEST HEDGING CONTRACT

The problem of optimal contract design is solved in two steps: First, the premium P is taken as given. Since the marginal payoff function $I'(f)$ appears neither in the objective function nor in the constraints, the problem can be solved by using Kuhn–Tucker condition for $I(f)$ for all $f \in [0, f_{\max}]$. Introducing condition (1) in problem (4), the first-order condition with respect to $I(f)$ is

$$E_b u'(\pi_0 + qg(f, \tilde{b}) + I(f) - P) + \lambda(f) - \mu\delta = 0 \qquad \text{for all } f \in [0, f_{\max}] \quad (5)$$

where μ and $\lambda(f)$ are the Lagrangian multipliers associated with conditions (3) and (2) respectively, with

$$\lambda(f)\begin{cases} = 0 & \text{if } I(f) > 0 \\ \geq 0 & \text{otherwise} \end{cases} \quad (6)$$

In the second step, program (4) is optimized with respect to P. This leads to the first-order condition

$$\mu = E_f E_b u'(\pi_0 + qg(\tilde{f}, \tilde{b}) + I(\tilde{f}) - P) \quad (7)$$

Since u is concave, it can be shown (see the Appendix) that a trigger price $\hat{f} \in [0, f_{\max}]$ exists such that the optimal hedging strategy satisfies

$$I^*(f)\begin{cases} > 0 & \text{if } f < \hat{f} \\ = 0 & \text{otherwise} \end{cases} \quad (8)$$

Differentiating Equation 5 with respect to leads to the marginal payoff function:

$$I^{*\prime}(f) = -q\,\frac{E_b\lfloor g_f(f, \tilde{b})u''(\tilde{\pi})\rfloor}{E_b u''(\tilde{\pi})} \qquad \text{for all } f < \hat{f} \quad (9)$$

where $\pi = \pi_0 + qg(f, b) + I^*(f) - P$. The optimal coverage function thus decreases with f. Introducing the first-order condition (7) into the first-order condition (5) and taking the expectation with respect to $\tilde{f}$ yields

$$E_f\lambda(\tilde{f}) = [\delta - 1]E_f E_b u'(\tilde{\pi}) \quad (10)$$

If the price of the hedging contract is unbiased, Equation 10 yields $E_f\lambda(\tilde{f}) = 0$. From condition (6), this implies that $\lambda(f) = 0$ for all $f \in [0, f_{\max}]$. Therefore, under unbiased prices, the optimal trigger level satisfies $\hat{f} = f_{\max}$. If the hedging contract is overpriced, then $E_f\lambda(\tilde{f}) > 0$, i.e., $\lambda(f) > 0$ for some $f \in [0, f_{\max}]$. This implies that $\hat{f} < f_{\max}$.

The above analysis on the design of the first-best hedging contract in the presence of basis risk can be summarized as follows. If the spot price is positively correlated with the settlement price, the optimal hedging contract displays a decreasing payoff function expressed by Equation 9 when the realized settlement price is lower than a trigger price. The optimal trigger price equals the maximum price if the hedging contract is unbiased. If the contract is overpriced, this optimal trigger price is lower than the maximum price.

OPTIMAL HEDGING STRATEGY WITH FUTURES AND OPTIONS CONTRACTS

The Setting

In real-world financial markets, standardized forms of hedging instruments are only available, such as futures contracts and options on futures. These tools, which are linear and piecewise linear with the settlement price respectively, usually preclude the firm from replicating the first-best hedging instrument. This thus creates a second source of incompleteness on financial markets, in addition to the first one caused by the unhedgeable basis risk.

Literature on optimal hedging with basis risk usually focused on the role of futures contracts (see, e.g., Benninga et al., 1983, 1984; Paroush & Wolf, 1989; Briys, Crouhy, & Schlesinger, 1993). An interesting exception is the paper by Moschini and Lapan (1995) that examines the hedging role of futures and options under multiple uncertainty. However, their model is based on several restrictive assumptions: the firm exhibits constant absolute risk aversion, random variables are normally distributed, and a single strike price for the option contract is considered.

In the current model, the firm can use not only futures contracts and options on futures, but also several strike prices are available among a set $K = \{k_1, k_2, \ldots, k_n\}$ with $0 < k_1 < k_2 < \cdots < k_n < f_{\max}$. If F denotes the price at which futures contracts can be purchased or sold, then $(F - f)$ denotes the realized profit per futures contract sold. Since a futures position can be duplicated with a put and a call, one of the three assets is redundant. The hedging strategy can thus be written as a combination of futures and straddles.[4] A short straddle at a strike price k_i has the net payoff $[P_i - |f - k_i|]$ where $P_i = (1 + \delta)E_f(|\tilde{f} - k_i|)$ is the price of this straddle. Then, assuming the firm sells x futures contracts and sells z_i straddles at a strike price k_i for $i = 1, \ldots, n$, the net payoff generated by this hedging strategy is

$$S(f) = x(F - f) + \sum_{i=1}^{n} z_i[P_i - |f - k_i|] \tag{11}$$

The hedging strategy S is piecewise linear with a kink at each strike price and its first derivative is

$$S'(f) = \begin{cases} -x + \sum_{i=1}^{n} z_i & \text{if } 0 \leq f < k_1 \\ -x - \sum_{i=1}^{j} z_i + \sum_{i=j+1}^{n} z_i & \text{if } f \in (k_j, k_{j+1}) \text{ for } j = 1, 2, \ldots, n-1 \\ -x - \sum_{i=1}^{n} z_i & \text{if } f > k_n \end{cases} \tag{12}$$

It is compared with the payoff function of the first-best hedging instrument expressed by Equations 8 and 9, net of its premium. The optimal hedging decisions are examined in two cases. First, the basis risk is assumed to interact in an additive manner with the settlement price. Then, it is assumed to interact in a multiplicative manner.

Additive Basis Risk

When the spot price is linear with respect to the basis risk, Equation 1 can be rewritten as

$$\tilde{p} = g(\tilde{f}, \tilde{b}) = h(\tilde{f}) + \tilde{b} \tag{13}$$

with $h' > 0$. Introducing (13) into (9) gives the optimal marginal payoff function

$$I^{*\prime}(f) = -qh'(f) \qquad \text{for all } f < \hat{f} \tag{14}$$

and its second derivative is

$$I^{*\prime\prime}(f) = -qh''(f) \qquad \text{for all } f < \hat{f} \tag{15}$$

[4]A short (long) straddle is equivalent to selling (buying) one put and one call at the same strike price.

Therefore, the form of the first-best payoff function depends neither on the firm's utility function nor on the distribution of the basis risk, but only on the relationship between the spot price and the settlement price. In order to derive the hedging strategy S from the first-best solution, the optimal hedging contract net of its premium is defined as

$$J^*(f) = I^*(f) - P = q \max\lfloor h(\hat{f}) - h(f), 0\rfloor - P; \quad \text{with } \hat{f} \in [0, f_{\max}] \tag{16}$$

First, assume that option prices are unbiased. This entails that $\hat{f} = f_{\max}$. The first-best solution (16) becomes

$$J^*(f) = q\lfloor E_f h(\tilde{f}) - h(f)\rfloor \tag{17}$$

for all $f \in [0, f_{\max}]$. If h is a linear function of the settlement price, i.e., $h(f) = \alpha + \beta f$, then the optimal net payoff function is

$$J^*(f) = q\beta\lfloor E_f\tilde{f} - f\rfloor \tag{18}$$

for all $f \in [0, f_{\max}]$. If the futures price is perceived unbiased, i.e., $F = E_f\tilde{f}$, the firm can perfectly replicate this first-best hedging contract by selling $q\beta$ futures contracts. Options contracts are thus not used as hedging instruments. This corresponds to a well-known result in the optimal hedging literature (see, e.g., Benninga et al., 1983, 1984).

If h is not a linear function, the first-best hedging contract cannot be perfectly replicated, and the optimal hedge ratios may depend on the firm's utility function. However, important features of the optimal hedging strategy that are independent of the firm's risk attitude can be derived from the first-best solution. Suppose that the spot price is increasing and concave with the settlement price, i.e., $h' > 0$ and $h'' < 0$. From Equations 14 and 15, the first-best payoff function is decreasing and convex for all $f \in [0, f_{\max}]$. Hence, the optimal hedging strategy S^* must be globally decreasing and convex in order to replicate as close as possible the first-best solution. From Equation 12, global convexity implies that

$$z_i^* < 0 \qquad \text{for } i = 1, 2, \ldots, n \tag{19}$$

and S^* is globally decreasing if

$$x > -\sum_{i=1}^{n} z_i \tag{20}$$

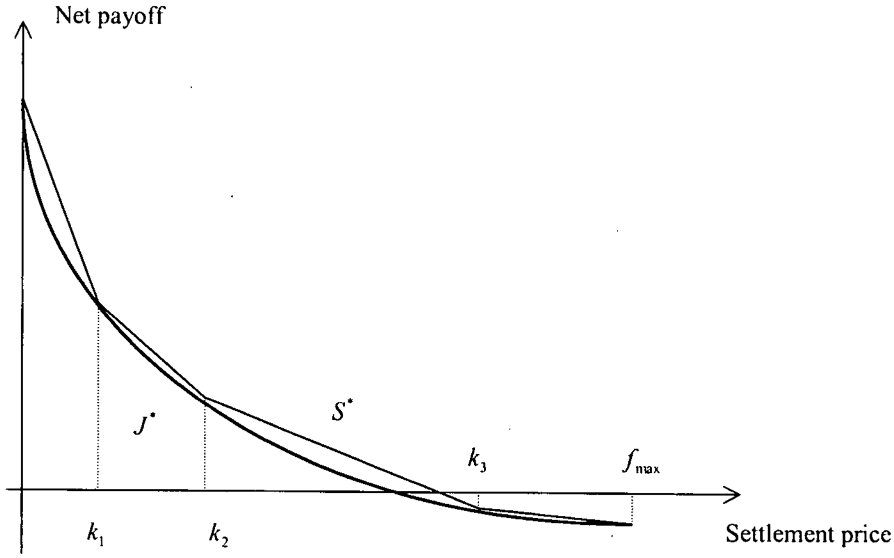

FIGURE 1
First-best hedging contract, J^*, and optimal hedging strategy with futures and options, S^*, when prices are unbiased.

Inequalities 19 and 20 mean that the optimal hedging strategy in the futures and options markets entails selling futures and buying straddles. In addition, the number of futures sold is higher than the number of straddles bought. This optimal strategy is illustrated in Figure 1 where three strike prices are available.

It can be shown from a similar analysis that the optimal hedge requires short futures positions and short straddle positions if the spot price is increasing and convex with the settlement price, i.e., $h' > 0$ and $h'' > 0$. The number of futures sold is higher than the number of straddles sold.

Hedging contracts are now assumed to be overpriced. The first-best solution (16) becomes

$$J^*(f) = q \max(h(\hat{f}) - h(f), 0) - P, \quad \text{with } \hat{f} < f_{\max} \tag{21}$$

When the spot price is linear with the settlement price, i.e., $h(f) = \alpha + \beta f$, then J^* can be rewritten as

$$J^*(f) = q\beta \max(\hat{f} - f, 0) - P, \quad \text{with } \hat{f} < f_{\max} \tag{22}$$

The form of the first-best solution is thus utility-free but the trigger price depends on the firm's preferences and on the premium. Hence, the first-best solutions can be replicated as follows. The firm buys $q\beta$ put options at a strike price $\hat{f}$. As a consequence, futures are not used as hedging instruments. It should be noticed that this result holds only if a strike price equal to the optimal trigger price is available in financial markets. Such an assumption seems to be realistic because real-world financial markets offer a large range of strike prices.

Multiplicative Basis Risk

The basis risk is now assumed to interact with the spot price in a multiplicative manner:

$$\tilde{p} = g(\tilde{f}, \tilde{b}) = h(f)(1 + \tilde{b}) \tag{23}$$

with $h' > 0$. Notice that such a specification allows to examine the optimal hedging decision for firms facing joint price and production risk (with no basis risk). In this context, the firm's revenue is $\tilde{f}Q(\tilde{f}, \tilde{b})$ where $\tilde{f}$ is the random price and the random output is defined by $Q(\tilde{f}, \tilde{b}) = qh(\tilde{f})(1 + \tilde{b})/\tilde{f}$, with $q > 0$.

Under condition 23, the marginal payoff function of the optimal hedging contract expressed in Equation 9 becomes

$$I^{*\prime}(f) = -qh'(f)\frac{E_b[\tilde{b}u''(\tilde{\pi})]}{E_b u''(\tilde{\pi})} \qquad \text{for all } f < \hat{f} \tag{24}$$

where $\pi = \pi_0 + qh(f)(1 + b) + I^*(f) - P$. It can be rewritten as

$$I^{*\prime}(f) = -qh'(f)\left[1 + \text{cov}_b\left(\tilde{b}, \frac{u''(\tilde{\pi})}{E_b u''(\tilde{\pi})}\right)\right] \qquad \text{for all } f < \hat{f} \tag{25}$$

where cov_b is the covariance operator with respect to $\tilde{b}$. The first-best marginal payoff function is based on two terms. The first right-hand side (RHS) term depends on the relationship between the spot price and the settlement price. This corresponds to the first-best marginal coverage in the case of an additive basis risk expressed in Equation 14. The second RHS term in brackets depends on the correlation between the $\tilde{b}$ basis risk and the second derivative of the firm's utility function. It is straightforward to show that the covariance term is negative, null or positive depending on whether the firms' marginal utility function is convex, linear or concave with wealth. The convexity of marginal utility function, $u''' > 0$, is justified by some fairly solid economic rationale. It is a necessary condition for decreasing absolute risk aversion. It is also a necessary and sufficient condition for an increase in future income to induce consumers to save more. This condition is called prudence by Kimball (1990). Under a prudent behavior, the optimal marginal payoff function thus satisfies:

$$-qh'(f) < I^{*\prime}(f) < 0 \qquad \text{for all } f < \hat{f} \tag{26}$$

If the firm exhibits prudence, the slope of its optimal payoff function under a multiplicative basis risk is, in absolute value, lower than the slope under an additive basis risk. This implies that, under unbiased markets, the number of futures contracts sold is lower under a multiplicative basis risk than under an additive one.

In the context of price and quantity uncertainty, Equation 26 means that the optimal hedge in the forward market is a short position if $h' > 0$ or, equivalently, if the elasticity of output with respect to price $\tilde{\eta} \equiv \partial \ln Q(\tilde{f}, \tilde{b})/\partial \ln \tilde{f}$ is higher than -1. When both risks are stochastically independent, i.e., $h' \equiv 1$, the optimal hedge of the prudent firm is less than the expected output q. Hence, hedging results derived by Losq (1982) turn out to be a specific case of the present analysis.

Contrary to the additive case, the sign of second derivative of the optimal payoff function turns out to be ambiguous. However, its nonlinearity under unbiased markets, which is generated by the nonlinear deterministic relationship between the spot price and the settlement price and/or by the correlation between the basis risk and the second derivative of the firm's utility function, allow option contracts to emerge as a useful hedging instrument in unbiased financial markets. This is illustrated in the following two cases.

First, suppose the firm's preferences are represented by a quadratic utility function.[5] The covariance term in Equation 25 equals zero and the first-best marginal payoff function becomes

$$I^{*\prime}(f) = -qh'(f) \qquad \text{for all } f < \hat{f} \tag{27}$$

This corresponds to the first-best solution under an additive basis risk. As a consequence, the features of the optimal hedging strategy derived for the risk-averse firms under an additive basis risk hold under a multiplicative basis risk for the firms with a quadratic utility function.[6] In particular, if h is a linear function and markets are unbiased, the payoff function

[5]This assumption is undesirable in the expected utility theory because this implies, among other things, that the firm's index of absolute risk aversion increases with wealth. Nevertheless, this utility function can also represent the preferences of a risk-neutral firm facing a quadratic and convex profit tax schedule.

[6]However, this does not mean that the optimal hedge ratios will be identical under additive and multiplicative basis risk.

of first-best hedging contract is linear in the settlement price and, therefore, it can be replicated only with futures.

Second, consider a "small" zero-mean basis risk $\tilde{b}$, i.e., the variance of $\tilde{b}$, denoted σ_b^2, is assumed to be "small." For the sake of simplicity, it is assumed $h(f) = f$. Expanding $u''(\pi)$ around $\overline{\pi} = E_b\tilde{\pi} = \pi_0 + qf + I(f) - P$ and ignoring the terms associated with the fourth and higher derivatives of the utility function produce:[7]

$$u''(\pi) \approx u''(\overline{\pi}) + (\pi - \overline{\pi})u'''(\overline{\pi}) \tag{28}$$

Taking the expectation with respect to $\tilde{b}$ yields:

$$E_b u''(\tilde{\pi}) \approx u''(\overline{\pi}) \tag{29}$$

This leads to the following approximation:

$$\frac{u''(\pi)}{E_b u''(\tilde{\pi})} \approx 1 - qfR(\overline{\pi})b \tag{30}$$

where $R(\pi) = -u'''(\pi)/u''(\pi)$ is the index of absolute prudence. Introducing (30) into (25) yields the closed-form solution:

$$I^{*\prime}(f) \approx -q\lfloor 1 - R(\overline{\pi})\sigma_b^2 f\rfloor \qquad \text{for all } f < \hat{f} \tag{31}$$

This can be rewritten as

$$I^{*\prime}(f) \approx -q[1 - 2fM(\overline{\pi})] \tag{32}$$

where $M(\pi) = \sigma_b^2 R(\pi)/2$ denotes the precautionary premium, by analogy with the risk premium derived by Pratt (1964). The optimal marginal coverage is thus negative for small basis risk. The lower the precautionary premium, the closer to $-q$ the approximate marginal coverage. Deriving the closed-form solution (31) with respect to f gives:

$$I^{*\prime\prime}(b) \approx qR(\overline{\pi})\sigma_b^2 + fR'(\overline{\pi})\sigma_b^2(q + I^{*\prime}(f)) \qquad \text{for all } f < \hat{f} \tag{33}$$

Replacing the marginal coverage $I^{*\prime}(f)$ by its closed-form solution in (31) and rearranging the terms yield

$$I^{*\prime\prime}(f) \approx qR(\overline{\pi})\sigma_b^2\lfloor 1 + f^2R'(\overline{\pi})\sigma_b^2\rfloor \qquad \text{for all } f < \hat{f} \tag{34}$$

The RHS term in brackets in the above equation is positive if the firm exhibits non-decreasing absolute prudence, i.e., $R' \geq 0$. Under the more

[7]This approximation leads to the mean-variance-skewness model proposed by Poitras (1993).

realistic assumption of decreasing absolute prudence (Kimball, 1993), it is positive for "small" zero-mean basis risk. Hence, the first-best coverage is increasing and convex with f and thus the optimal hedging strategy requires a short futures position and long straddles positions. Notice that if the index of partial prudence $fR(\pi)$ is constant and equal to $D > 0$, implying that $R(\pi)$ decreases with f, then from Equation 31 the marginal coverage is constant for all f. The first-best coverage is thus replicated by having a short futures position where the optimal hedge ratio is equal to $[1 - D\sigma_{\tilde{b}}^2]$.

Finally, the optimal hedging strategy is examined under the log-log model. This model is widely employed in empirical hedging studies. The relationship between the spot price and the settlement price is formalized by:

$$\log(\tilde{p}) = \alpha + \gamma \log(\tilde{f}) + \tilde{e} \tag{35}$$

where $\tilde{f}$ and $\tilde{e}$ are stochastically independent. Some authors using the log-log regression claim that $q\gamma$ is the optimal hedge ratio for all risk-averse hedgers as long as futures are unbiased (see, e.g., Baillie & Myers, 1991). Equation 35 can be rewritten as

$$\tilde{p} = A\tilde{f}^{\gamma}(1 + \tilde{b}) \tag{36}$$

where $\tilde{b} = 10^{\tilde{e}}/E_e 10^{\tilde{e}} - 1$ and $A = 10^{\alpha} E_e 10^{\tilde{e}}$. This is a particular case of the multiplicative relationship expressed in Equation 23 where $h(f) = Af^{\gamma}$. From Equation 24, the payoff function I^* of the first-best hedging contract under unbiased prices thus decreases or increases with the realized settlement price f, for all $f \in [0, f_{\max}]$, depending on whether the parameter γ is positive or negative. This entails that the optimal hedging strategy requires a short (long) futures position if γ is positive (negative). The optimal hedge ratio depends on the agent's utility function, as previously noticed by Lence (1995).

Suppose the firm exhibits a quadratic utility function. The first-best payoff function I^* is concave in f for all $f \in [0, f_{\max}]$ if $\gamma \in (0, 1)$ and it is convex otherwise. Therefore, the optimal hedging strategy also requires short straddle positions if $\gamma \in (0, 1)$ and long straddle positions if $\gamma < 0$ or $\gamma > 1$. When $\gamma = 1$, the first-best hedging contract is linear with the settlement price and thus it is perfectly replicated by selling qA futures at a price $F = E\tilde{f}$ under unbiased financial markets. Hence, the optimal futures hedge ratio under a log-log regression is utility-free if only if the firm's utility is quadratic and the absolute value of parameter γ equals one.

CONCLUSION

The hedging analysis is examined as a problem of optimal contract design. An original two-stage procedure is presented in which the first-best hedging instrument is derived and then it is used to analyze the optimal hedging strategy on financial markets. This procedure is applied to investigate the optimal hedging decisions against price risk in the presence of an unhedgeable and independent basis risk. The model differs from previous works in that neither the basis risk nor the settlement price are assumed to be linear with the spot price. In addition, options contracts with several strike prices are assumed to be at the firm's disposal. This contributes to enhancing the impact of the relationship between the spot price risk and the basis risk on the optimal hedging strategy. Under this first-best solution, payoffs are made when the realized settlement price is lower than a trigger price if the spot and the settlement price are positively correlated. The optimal trigger price is equal to the maximum settlement price under unbiased markets and it is lower than this maximum if the hedging contract is overpriced. The use of futures contracts and options on futures in unbiased financial markets is linked to the sign of the first and second derivatives of the first-best payoff function, respectively.

When the basis risk is additive, the form of the first-best payoff function is only based on the relationship between the spot price and the settlement price. The optimal hedging strategy requires a short futures position and short (resp. long) straddle positions if the spot price is a convex (resp. concave) function of the settlement price. The exact amounts of futures and options depend on the firm's attitude toward risk.

Under a multiplicative basis risk, the form of the first-best hedging instrument depends on the relationship between the spot price and the settlement price, and on the firm's prudent behavior. If the firm's preferences are represented by a quadratic utility function, this behavioral effect vanishes and, consequently, the features of the optimal hedging strategy derived under an additive basis risk hold. Under a small basis risk, the closed-form solution depends on the firm's precautionary premium. This multiplicative case corresponds to a model with no basis risk, but where price and production are subject to uncertainty.

This paper provides an original approach based on the design of the first-best hedging contract to examine the optimal hedging strategy on financial markets. Further research would be to reconsider this two-stage procedure in order to investigate the optimal hedging strategy under multiple uncertainty when two sources of risk can be hedged or insured with separate contracts.

APPENDIX

The existence of an unique trigger price is shown.

For all f such that $I(f) = 0$, define $K(f) = E_b u'(\pi_0 + qg(f, \tilde{b}) - P) - \mu\delta$. This function is negative from the first-order condition (5) and decreasing from $u' < 0$ and $g_f > 0$. If $K(0) > 0 \geq K(f_{max})$, then a unique value $\hat{f} \in [0, f_{max}]$ exists such that $K(\hat{f}) = 0$. The optimal form of the hedging contract in Equation 8 is thus derived. If $K(0) < 0$, then the non-negativity constraint (2) is always binding. This implies that $I(f) = 0$ for all $f \in [0, f_{max}]$ or, equivalently, $\hat{f} = 0$ in Equation 8. If $K(f_{max}) > 0$, then constraint (2) is never binding and thus $I(f) > 0$ for all $f \in [0, f_{max}]$. In Equation 8, this means that $\hat{f} = f_{max}$.

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