MEAN REVERSION AND BASIS DYNAMICS

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An analytical relationship between basis change autocorrelations and thin trading effects together with partial adjustment factors is developed. Less than full price adjustments are demonstrated to lead to negative autocorrelations in basis innovation series in addition to those induced by thin trading effects. Numerical and empirical analyses explore the interrelationships between these effects and provide evidence for the presence of both effects in intradaily cash and futures data. © 2001 John Wiley & Sons, Inc. Jrl Fut Mark 21: 797–818, 2001

INTRODUCTION

The mean reversion in stock index futures basis changes has been empirically confirmed in a number of studies (e.g., MacKinlay & Ramaswamy, 1988, in the United States; Yadav & Pope, 1990, in the United Kingdom). A commonly accepted rationale for this mean reversion has been arbitrage activities in the stock index futures markets when the basis moves out of the no-arbitrage bounds. Miller, Muthuswamy, and

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Whaley (1994), however, argued that the mean reversion derives from statistical artifacts, in the sense that thin trading in the stock market index can lead to the documented negative autocorrelations.

Miller et al. (1994) analytically demonstrated how thin trading can induce negative autocorrelations in the basis change series and then empirically verified that the negative autocorrelations are reduced in degree when thin trading is controlled for. However, they did find that some negative autocorrelations remained in the basis change time series.

In this article, we introduce another phenomenon into the price process, namely, the dynamics of the stock index futures and cash markets. The interrelationships in a temporal sense between these markets have been analyzed in a number of articles (e.g., Kawaller, Koch, & Koch, 1987; Stoll & Whaley, 1990; Theobald & Yallup, 1998). The dynamics have generally been investigated within a lead-lag time series framework. We introduce the dynamics into the price processes via partial (or speed of) adjustment coefficients, as suggested and analyzed in Amihud and Mendelson (1987); the equivalence between partial adjustment coefficients and the lead-lag relationships mentioned previously is demonstrated in the next section. The impact of the introduction of partial adjustment coefficients is, *inter alia,* the induction of negative autocorrelations in basis changes. Numerical and empirical analyses are conducted that are indicative of the presence of partial adjustment effects. Because part of the negative autocorrelation is induced by a phenomenon that can be exploited by market participants (i.e., the differential speeds of adjustment across the cash and futures markets), arbitrage activities can still be associated with basis mean reversion. Arbitrage profits will be proportional to the differences in the partial adjustment coefficients in the cash and futures markets (see Theobald & Yallup, 1998).

BASIS MEAN-REVERSION ANALYTICS

The purpose of this section is to analytically demonstrate how the introduction of cash-futures dynamics, in the form of speeds of adjustment coefficients, impact on the mean reversion in the basis. The equivalence of the speeds of adjustment approach (Amihud & Mendelson, 1987; Theobald & Yallup, 1998) to the lead-lag approach to dynamics (Stoll & Whaley, 1990) is first demonstrated, and then the analytics of basis mean reversion in the presence of both thin trading and partial adjustments of prices are developed.

Articulation Between Partial Adjustments and Lead-Lag Specifications

The lead-lag relationship between cash and futures markets is generally investigated within a time series regression framework with leading and lagged regressor variables (e.g., Kawaller et al., 1987; Stoll & Whaley, 1990). Perhaps the simplest specification would be a regression of the form

$$R(s, t) = a + b\,R(f, t) + c\,R(f, t-1) + u(t) \tag{1}$$

where $R(s, t)$ $[R(f, t)]$ is the observed (i.e., potentially non-fully adjusting) return on the cash (futures) market and $u(t)$ is the stochastic disturbance term. Then, if the cash market prices only partially adjust to information,[1] with a partial adjustment of p (to be defined in the next section; essentially, $p = 1$ for full adjustment and $p < 1$ for less than full adjustment), the ordinary least-squares estimators of model coefficients can be shown to be given by

$$b = p\,\mathrm{cov}[R^1(s, t), R(f, t)]\{\mathrm{var}[R(f, t)]\}^{-1} \tag{2a}$$

$$c = p(1 - p)\,\mathrm{cov}[R^1(s, t), R(f, t)]\{\mathrm{var}[R(f, t)]\}^{-1} \tag{2b}$$

where cov and var are the covariance and variance operators, respectively, and $R^1(s, t)$ is the fully adjusting cash market return. Clearly, when $p = 1$, corresponding to full adjustment, $c = 0$. However, when $p < 1$, that is, when adjustment is less than full, $c \neq 0$, and the inference would be that the fully adjusting futures market leads the partially adjusting cash market.

Price Series Modeling

The stock index series is assumed to incorporate both partial adjustment and infrequent trading effects. The partial adjustment of the series is defined by the following formulation:

$$S(t) - S(t-1) = p\{V(t) - S(t-1)\} \tag{3}$$

where $V(t)$ is the full information index level in period t, $S(t)$ is the partially adjusting index level (without any infrequent trading effects), and p is the partial adjustment coefficient defined over the range[2] $0 < p < 2$

[1]It is assumed for analytical simplicity that the more liquid futures market fully adjusts in this exposition.

[2]To ensure that the stock index process does not become explosive.

(when $p = 1$, the index series fully adjusts, whereas when $p < 1$, the index series only partially adjusts). The formulation at Equation 3 is similar to the Amihud and Mendelson (1987) specification. However, a noise-spread variable is not included because the bid-ask effects are largely canceled out in what is, effectively, a large portfolio[3] (Miller et al., 1994, made a similar assumption in their work).

Defining $s(t) = S(t) - S(t-1)$, $v(t) = V(t) - V(t-1)$, and $s(o, t)$ as the observed innovation series, which are subject to both less than full adjustment and infrequent trading problems,[4] following Miller et al. (1994), we have

$$s(o, t) = \varnothing\, s(o, t-1) + (1 - \varnothing)\, s(t) \tag{4}$$

where $\varnothing$ is the thin trading parameter defined for the range $0 < \varnothing < 1$. As $\varnothing$ approaches 0, the trading approaches the continuous, whereas as $\varnothing$ approaches 1, the trading approaches the discontinuous. Combining Equations 3 and 4 yields

$$\begin{aligned} s(o, t) = {} & p(1 - \varnothing)\, v(t) + (1 - p + \varnothing)\, s(o, t-1) \\ & - \varnothing(1 - p)\, s(o, t-2) \end{aligned} \tag{5}$$

Equation 5 can be alternatively expressed in terms of lag operators (L) as

$$s(o, t) = (1 - \varnothing)(1 - \varnothing L)^{-1}\, [1 - (1 - p)L]^{-1}\, p\, v(t) \tag{6}$$

The partial adjustment process for the futures series can be similarly defined as

$$F(t) - F(t-1) = q[Z(t) - F(t-1)] \tag{7}$$

where $F(t)$ is the partially adjusting futures price with no noise–spread effects[5] (a spread variable is introduced in Equation 8), $Z(t)$ is the fully adjusting futures price, and q is the partial adjustment coefficient, again defined within the range $\{0, 2\}$. Defining $f(t) = F(t) - F(t-1)$ and $z(t) = Z(t) - Z(t-1)$ and introducing a spread variable θ (see Miller et al., 1994; Roll, 1984) defined within the range $\{-1, 0\}$, we have

$$f(o, t) = f(t) + \theta f(t-1) \tag{8}$$

[3]Furthermore, we define model 3 in terms of prices to correspond with Miller et al.'s (1994) analysis rather than in terms of logs of prices, as used in Amihud and Mendelson (1987).

[4]As in Amihud and Mendelson (1987), $V(t)$ is assumed to follow a random walk with drift.

[5]Although spread effects are averaged or diversified away in a portfolio or index, they will be present in a single security such as a futures contract. Thin trading will not be a problem in highly liquid futures markets. The modeling of the futures (and index) series specifically follows Miller et al.'s (1994) with the addition of partial adjustments in both markets.

where $f(o, t)$ is the observed futures series subject to spread and partial adjustment effects. Combining Equations 7 and 8 yields

$$f(o, t) = qz(t) + \theta\, qz(t-1) + (1-q)\, f(o, t-1) \qquad (9)$$

or, in terms of lag operators,

$$f(o, t) = (1 + \theta L)[1 - (1-q)L]^{-1}\, qz(t) \qquad (10)$$

when $p = 1$ and $q = 1$, the expressions at Equations 6 and 10 correspond to those used in Miller et al. (1994).

Futures-Variance/Index-Variance Ratio

In their Proposition 1, Miller et al. (1994) proved that the ratio of the variance of observed futures price changes to the variance of observed index changes, $R(o)$, is greater than 1 when innovations in the true futures and index series are equal. In Miller et al., $R(o)$ is calculated as $(1 + \theta^2)[(1 - \varnothing)/(1 + \varnothing)]^{-1}$

We can compute the variance of the observed index innovation series with both thin trading and partial adjustment effects as

$$\text{var}[s(o, t)] = \left\{\frac{p}{2-p} \cdot \frac{1-\varnothing}{1+\varnothing}\right\} \text{var}[v(t)] \qquad (11)$$

by applying the variance operator to Equation 6 and rearranging and summing the geometric progression. When $p < 1$ (ie., with less than full adjustment), the observed variance will be pushed further downward relative to the downward bias induced by infrequent trading.

Similarly, applying the variance operator to Equation 10, we can derive the variance of the observed futures price changes as

$$\text{var}[f(o, t)] = \frac{q}{2-q} \cdot (1 + \theta^2)\, \text{var}[z(t)] \qquad (12)$$

Again, the variance is biased downward when adjustments are less than full. The first proposition in Miller et al. (1994) can then be extended as Proposition 1.

Proposition 1

If the bid-ask spread in the futures market is economically significant ($\theta < 0$), if the stocks within the index portfolio trade infrequently

($\varnothing > 0$), if the index speed of adjustment is less than the futures speed of adjustment[6] ($q > p$), and if the true innovations of the futures and index have the same variance $\{\mathrm{var}[z(t)] = \mathrm{var}[v(t)]\}$, the variance ratio of observed futures price changes to observed index level changes is greater than 1.

Proof of Proposition 1: For $\varnothing > 0$ and $q > p$,

$$R(o)^2 = \frac{\mathrm{var}[f(o, t)]}{\mathrm{var}[s(o, t)]} = \frac{\dfrac{(q)}{(2 - q)}(1 + \theta^2)}{\dfrac{(p)}{(2 - q)}\dfrac{(1 - \varnothing)}{(1 + \varnothing)}} > 1 \tag{13}$$

When the futures market adjusts more fully than the index, the variance ratio $R(o)$, becomes larger than for infrequent trading-spread effects alone. When $p = q$ (i.e., there is no differential speed of adjustment between the cash and futures markets), $R(o)$ depends on $\{\varnothing, \theta\}$ only.

Autocorrelations in Basis Changes

Miller et al. (1994) demonstrated that the negative first-order autocorrelation that has been observed in basis change time series can be induced by infrequent trading and spread effects. In this section, less than full adjustments in futures and index innovation series are analytically demonstrated to further impact on first-order autocorrelations.

The first-order autocorrelation coefficient for observed basis changes is contained in Miller et al.'s (1994) Proposition 2. In the presence of less than full partial adjustments, this proposition becomes the following.

Proposition 2

The first-order autocorrelation of observed basis changes in terms of true underlying parameters is

$$\rho_1[b(o, t)]$$
$$= \frac{\alpha - \beta - \lambda + \delta}{\left\{\dfrac{p}{2 - p}\dfrac{1 - \varnothing}{1 + \varnothing}\right\}\mathrm{var}[v(t)] + \dfrac{q}{2 - q}(1 + \theta^2)\,\mathrm{var}[z(t)] - 2\varepsilon} \tag{14}$$

[6]This is likely to be the case for the more liquid futures contract; Theobald and Yallup (1998), using United Kingdom stock index futures data, provided some empirical support for this relationship.

where

$$\alpha = q^2(1 - q + \theta)\,\mathrm{var}[z(t)]\left\{1 + \frac{(1-q)(1-q+\theta)}{q(2-q)}\right\}$$

$$\beta = pq(1 - \varnothing)(1 - q + \theta)\,\mathrm{cov}[v(t), z(t)]$$

$$\times \left\{1 + \frac{(1-q)}{1 - \varnothing(1-q)}\left[1 - p + \varnothing + \frac{(1-p)^2(1-q)}{1-(1-p)(1-q)}\right]\right\}$$

$$\lambda = pq(1 - \varnothing)\,\mathrm{cov}[v(t), z(t)\left\{(1 - p + \varnothing)\right.$$

$$\left. + \frac{(1-q+\theta)}{1-(1-q)\varnothing}\left[\varnothing(1-p+\varnothing) + \frac{(1-p)^2}{1-(1-p)(1-q)}\right]\right\}$$

$$\delta = p^2(1-\varnothing)^2(1-p+\varnothing)\,\mathrm{var}[v(t)]\left\{1 + \frac{1}{1-\varnothing^2}\left[\varnothing(1-p+\varnothing)\right.\right.$$

$$\left.\left. + \frac{(1-p)^2(1+\varnothing^2)}{1-(1-p)\varnothing} + \frac{(1-p)^4}{[1-(1-p)\varnothing][1-(1-p^2)]}\right]\right\}$$

and

$$\varepsilon = pq(1-\varnothing)\left[1 + \frac{(1-q+\theta)(1-p+\varnothing)}{1-\varnothing(1-q)}\right.$$

$$\left. + \frac{(1-p)^2(1-q)(1-q+\theta)}{[1-\varnothing(1-q)][1-(1-p)(1-q)]}\right]\mathrm{cov}[z(t), v(t)]$$

The outline of the derivation of Equation 14 is contained in the appendix. As anticipated, Equation 14 reduces to the expression for the first-order autocorrelation in Miller et al. (1994) when $p = q = 1$. In the absence of spread effects ($\theta = 0$), infrequent trading effects ($\varnothing = 0$), and full adjustments in both futures and cash markets ($p = q = 1$), the first-order autocorrelation in basis changes will be 0.

Equation 14 is a somewhat complicated formulation; however, insights into its properties can be gleaned if we make strong assumptions regarding futures and cash markets. If, as in Miller et al. (1994), the variances of the true innovations in both series are assumed to be equal to $\{\mathrm{var}[z(t)] = \mathrm{var}[v(t)]\}$ and to be perfectly and positively correlated, insights into the nature of Equation 14 can be developed. In the presence of partial adjustments in the cash market only (i.e., $p \neq 1, q = 1$) and with an absence of spread effects assumed (i.e.,), the corresponding version of Miller et al.'s (1994) Proposition 3 can be developed as follows.

Proposition 3

If the bid-ask spread in the futures market is economically insignificant ($\theta = 0$), if the futures market fully adjusts ($q = 1$), if the stocks within the index portfolio trade infrequently ($\varnothing > 0$), if the index prices do not fully adjust ($p \neq 1$), and if the innovations of the true futures and the true index have the same variance $\{\mathrm{var}[z(t)] = \mathrm{var}[v(t)]\}$ and are perfectly positively correlated, the first-order correlation of observed basis changes is

$$\rho_1[b(o,t)] = \frac{-p(1-\varnothing)(1-p+\varnothing)\Bigl[1 - p(1-\varnothing) \times \Bigl\{1 + \frac{1}{1-\varnothing^2}\Bigl(\varnothing(1-p+\varnothing) + \frac{(1-p)^2(1+\varnothing^2)}{1-(1-p)\varnothing} + \frac{(1-p)^4}{(1-(1-p)\varnothing)(1-(1-p)^2)}\Bigr)\Bigr\}\Bigr]}{1 + p(1-\varnothing)\Bigl[\frac{1}{(2-p)(1+\varnothing)} - 2\Bigr]} \tag{15}$$

The relationship between the first-order correlation of the basis and the infrequent trading and speeds of adjustment parameters is graphically presented in Figure 1 and is discussed in the Numerical Analyses section.

The expression at Equation 15 can be further simplified if we assume that there is no infrequent trading in index stocks (i.e., $\varnothing = 0$). This then leads to a modified Proposition 3.

Proposition 3[1]

If the same conditions in Proposition 3 are assumed to hold and, additionally, the stocks in the index trade continuously ($\varnothing = 0$), the first-order autocorrelation of observed basis changes is

$$\begin{aligned}\rho_1[b(o,t)] &= \frac{\dfrac{-p^2(1-p)^2}{[1-(1-p)^2]}}{1 + p\left[\dfrac{1}{2-p} - 2\right]} = \frac{-p}{2} \quad \text{for } p \neq 1 \\ &= 0 \qquad \text{for } p = 1 \text{ and } p = 0\end{aligned} \tag{16}$$

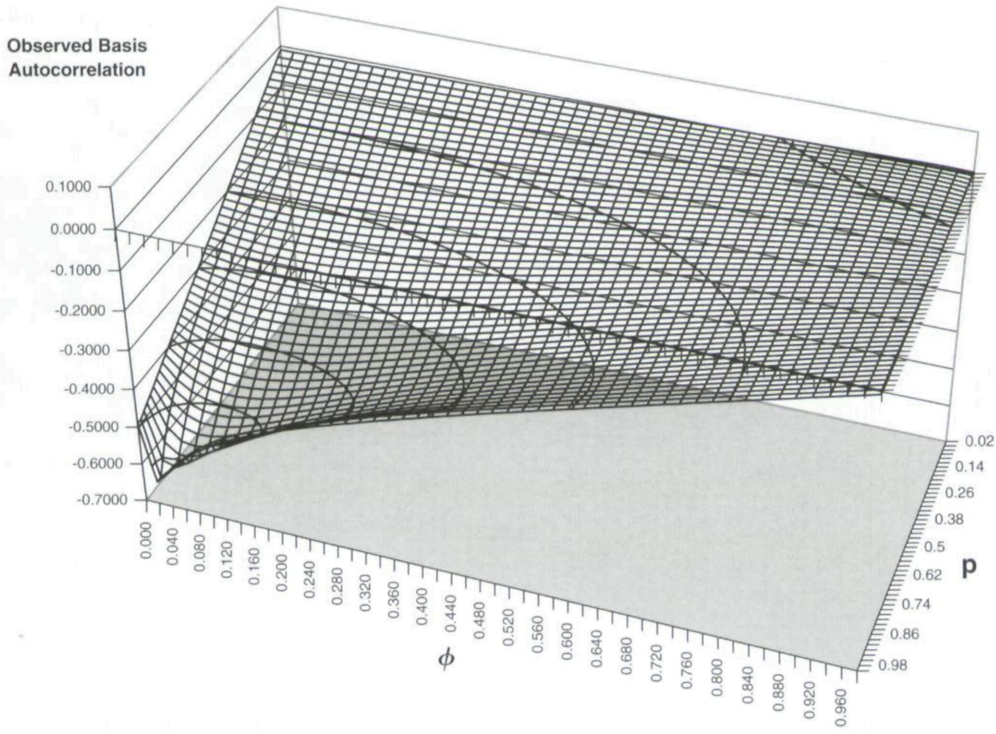

FIGURE 1

Equation 15: Observed basis autocorrelation when index stocks trade infrequently and index stock prices do not fully adjust to new information.

Equation 16 clearly demonstrates that when the stock index does not fully adjust, the observed basis will be negatively autocorrelated at lag 1 for speeds of adjustment less than 1. Figure 1 demonstrates that with $\varnothing = 0$, the degree of first-order basis autocorrelation becomes more negative as p increases; intuitively, when $p = 0$, there will be no adjustment and, hence, zero autocorrelation. Autocorrelation then becomes more negative until adjustment becomes full (i.e., $p = 1$). Although the partial adjustment factor, p, in our analysis plays a role similar to that of the infrequent trading variable, $\varnothing$, in Miller et al.'s (1994) analysis in this very simple scenario, in the more complex settings, at Equations 14, 15, and 17 (shown later), the two effects do not jointly enter into the analytics in a simple and symmetrical manner.

The analysis can be extended to the case where innovations in the index and the futures contract are not perfectly and positively correlated and do not have the same variance. This would be the case if, for example, the stock index pays uncertain amounts of dividends before the futures expire or the interest rate is stochastic. Proposition 4 can then be developed as follows.

Proposition 4

If the bid-ask spread in the futures market is economically insignificant ($\theta = 0$), if the futures market fully adjusts (i.e., $q = 1$), if the stocks within the index portfolio trade infrequently ($\varnothing > 0$), and if the index prices do not fully adjust (i.e., $p \neq 1$) the first-order autocorrelation of observed basis changes is

$$\rho_1[b(o,t)] =$$

$$\frac{-p(1-\varnothing)(1-p+\varnothing) \times \left[R\rho - p(1-\varnothing)\left\{1 + \frac{1}{1-\varnothing^2}\left(\varnothing(1-p+\varnothing) + \frac{(1-p)^2(1+\varnothing^2)}{1-(1-p)\varnothing} + \frac{(1-p)^4}{[1-(1-p)\varnothing][1-(1-p)^2]}\right)\right\}\right]}{R^2 + p(1-\varnothing)\left[\frac{1}{(2-p)(1+\varnothing)} - 2R\rho\right]} \quad (17)$$

where ρ is the contemporaneous correlation between innovations in the index and futures contract and R is the ratio of the standard deviations of the true innovations of the futures contract to those of the index.

The expression at Equation 17 can be further simplified if we assume that there is no infrequent trading in index stocks (i.e., $\varnothing = 0$). This leads to a modified Proposition 4.

Proposition 4[1]

If the same conditions in Proposition 4 are assumed to hold and, additionally, the stocks in the index trade continuously ($\varnothing = 0$), the first-order autocorrelation of observed basis changes is

$$\rho_1[b(o,t)] = \frac{p(1-p)[1-(2-p)\,R\rho]}{p + (2-p)(R^2 - 2p\,R\rho)} \quad (18)$$

The numerator of Equation 18 is negative for $0 < p < 1$ if $R\rho > (2-p)^{-1}$. The relationship between basis autocorrelations, the ratio of the standard deviations of the true innovations in the futures contract to those in the index and the contemporaneous correlation between index and futures innovations, is graphically presented in Figure 2 for a speed of adjustment of 0.9. This diagram is discussed in the Numerical Analyses section. With

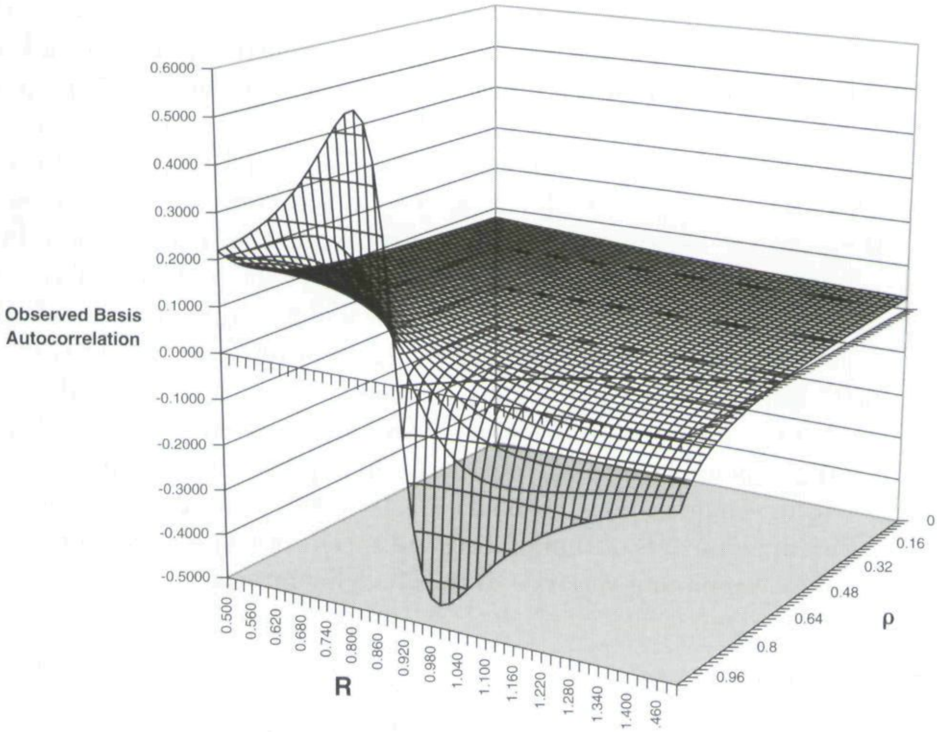

FIGURE 2

Equation 18: Observed basis autocorrelation when index prices do not fully adjust ($p = 0.9$).

the denominator, a variance that will be strictly positive, basis changes will again be negatively autocorrelated[7] when $R\rho > (2 - p)^{-1}$.

Effectively, within this framework negative first-order autocorrelations are introduced into the basis change series by at least two factors: (a) thin trading in the stock index and (b) partial adjustments in the stock index (where futures contracts are mispriced relative to the cost of carry model, the reversion toward the cost of carry price will be a further cause of negative autocorrelation; see Figlewski, 1984). Controlling for one of these factors only, as Miller et al. (1994), who controlled for infrequent trading only, will not completely remove negative autocorrelations.

Thin trading in the index derives from a measurement error or statistical problem, and as such, mean reversion deriving from thin trading

[7]With full adjustment (i.e., $p = 1$) and infrequent trading (i.e., $\varnothing > 0$), the corresponding condition is

$$R\rho > (1 - \varnothing)\left\{1 + \varnothing \frac{(1 + \varnothing)}{1 - \varnothing^2}\right\}$$

will not be exploitable as argued by Miller et al. (1994). However, the negative autocorrelations induced by lags in adjustment toward intrinsic values will be exploitable. That is, if the futures market speed of adjustment is greater than that of the cash market (i.e., $q > p$), when significant positive (negative) innovations are incorporated into the futures price, the delay in the cash market price reaction may be exploited with the positive (negative) futures price movement as a signal to go long (short) in the cash market. (Stoll & Whaley, 1990, made a similar argument). Theobald and Yallup (1998) indicated that the incremental arbitrage potential deriving from this effect is approximately equal to $qz(t) - pv(t)$. For example, when the futures market fully adjusts and the cash market completely fails to adjust toward intrinsic values (i.e., $q = 1$, $p = 0$), the exploitable arbitrage potential will approximate the innovation in the futures market, $z(t)$ (Theobald & Yallup, 1997, also demonstrated the differential impacts of thin trading and speeds of adjustment on the estimation of minimum-variance futures hedging ratios).

NUMERICAL ANALYSES

The analytics developed in the Basis Mean-Reversion Analytics section indicated that when the speeds of adjustment were complete in the cash and futures markets (i.e., $p = q = 1$), the autocorrelation results were identical to those of Miller et al. (1994). At the extremes of completely discontinuous trading (i.e., $\varnothing = 1$) and no adjustment toward intrinsic values (i.e., $p = 0$), basis change autocorrelations will be 0. The approaches toward these limits are clearly visible in Figure 1, where Equation 15 is plotted in three dimensions as $p_1[b(o, t)]$ against $\varnothing$ and ρ. The surface edge corresponding to $\varnothing = 0$ (i.e., continuous trading) demonstrates the results derived from Equation 16; that is, $\rho_1[b(o, t)] = -p/2$ for $p \neq \{1,0\}$. This result demonstrates an important feature of the impact of partial adjustments on basis change mean reversion. That is, sluggish speeds of adjustment have their greatest impacts when adjustments are close to, but not, complete.[8] This result is important because in close-to-efficient markets, the partial adjustment factor is not likely to be far from 1.[9]

Figure 2 presents a plot of $\rho_1[b(o, t)]$ against ρ and R for a partial adjustment factor of 0.9 in the stock index (this plot corresponds to

[8]Miller et al. (1994) similarly reported for the case of infrequent trading that "mean reversion becomes greater as the frequency of stock market trading increases" (p. 499).

[9]Theobald and Yallup (1998) found that partial adjustment factors for the U.K. FTSE 100 stock market index were typically greater than 0.9 at the daily differencing level (when corrected for thin trading).

TABLE I
Abstract of Simulated Values of $\{\rho, \varnothing, p, R\}$ Generating $\rho_1[b(o, t)] = -.369$

Partial Adjustment (p)	*Infrequent Trading* ($\varnothing$)	*Correlation* (ρ)	*Variance Ratio* (R)
1.000	0.26	1.000	1.000
0.900	0.00	0.998	1.020
0.800	0.10	1.000	1.179
0.800	0.20	0.986	0.973

Figure 3 in Miller et al., 1994), that is, the space corresponding to Equation 18. As in Miller et al., negative autocorrelations only occur in the regions where $\rho \approx 1$ and $R > 1$.

Further numerical analyses of the plausibility of the analytical results are provided by a simulation across values of $\{\rho, \varnothing, p, R\}$ in Equation 17 to generate the first-order autocorrelations of basic changes estimated in Miller et al. (1994) for the whole data set (see Miller et al., Table V, first row). An abstract of these results is contained in Table I. As can be seen, with complete adjustment ($p = 1$), perfect correlations ($\rho = 1$), and homoskedasticity ($R = 1$), the degree of infrequent trading should be 0.26 (i.e., 26% of the index should be infrequently traded). Plausible combinations of $\{\rho, \varnothing, p, R\}$ exist, as evidenced by Table I, which generate the observed autocorrelation. Miller et al. (1994) found that after correction for infrequent trading, $\rho_1[b(o, t)]$ declined to $-.252$. Simulating across $\{p, \rho_1, R\}$, with $\varnothing = 0$ again generates plausible values consistent with this autocorrelation. For example, when $p = 0.9$, $\rho = 0.991$, and $R = 1.069$, $\rho_1[b(o, t)]$ estimated from Equation 17 is $-.252$. In summary, the incorporation of partial adjustment factors with reasonable values for the stock market index generates basis change autocorrelations of the order empirically estimated by Miller et al. (1994).

EMPIRICAL ANALYSES

The speeds of adjustment and thin trading effects are further analyzed with intradaily, high-frequency data for the Financial Times Stock Exchange (FTSE)-100 stock index futures contracts and the FTSE-100 stock market index for January 28, 1999 to December 3, 1999. The data set comprised 215 days of data and provided 19,738 observations at 5-min intervals, 6,442 observations at 15-min intervals, 3,113 observations at 30-min intervals, and 1,556 observations at 60-min intervals. Summary statistics for the intradaily data are contained in Table II. As can be seen, the basis change exhibited mean reversion at each of these

TABLE II
Intradaily Data Summary Statistics

Statistic	*Differencing Interval* 5 min	15 min	30 min	60 min
Basis change autocorrelation	−0.3288	−0.3858	−0.315	−0.2229
Futures–cash variance ratio	1.1872	1.0806	1.0264	0.9924
Futures change autocorrelation	−0.0098	0.0126	−0.0142	−0.0452
Cash change autocorrelation	0.0841	0.0441	0.014	−0.04767
Futures–cash change autocorrelation	0.4906	0.7491	0.8704	0.9183

differencing intervals manifested by negative autocorrelations for each of the intradaily differencing intervals. The magnitude of the negative basis change autocorrelation declined with intervalling as anticipated because, on *a priori* grounds, thin trading and speed-of-adjustment effects will be reduced as the differencing interval is increased. The futures-cash variance ratio was greater than 1 for differencing intervals up to and including 30 min.

The presence of differential speeds of adjustment in relation to thin trading effects was effectively already investigated for U.S. stock index futures intradaily data by Stoll and Whaley (1990). That is, after the cash market return series was purged for thin trading effects with the residuals from an Autoregressive Moving Average (ARMA)(2,3) model, statistically significant lead-lag relationships between the cash and futures markets were still found to be present by Stoll and Whaley within the context of econometric specifications based on Equation 1.

The Stoll and Whaley (1990) type of analysis was extended to the sample of U.K. intradaily cash and futures data, and the estimated coefficients for Equation 1 are contained in Table III. The cash return series were purged of thin trading effects with the ARMA(2,3) specification used in Stoll and Whaley and with an ARMA(1,0) specification (which was generally found to be the best specification for the U.K. sample data with the Akaike Information Criterion). The results for the two ARMA specifications were very similar, and so that the table may be reported in a parsimonious fashion, only the results for the ARMA(2,3) specification are reported.

As can be seen in Table III, the coefficients on the lagged futures return variable (coefficient c) are positive and always statistically significantly different from 0 at the 1% level for the unadjusted cash data. However, the futures may lead the cash market because of thin trading effects in the cash variable and/or higher speeds of adjustment in the futures market. Purging the cash market of thin trading effects with the

TABLE III

Coefficient Estimates for Equation 1 $R(s, t) = a + bR(f, t) + cR(f, t-1) + u(t)$

		Coefficients			
Return Interval	*Cash Returns*	*a*	*b*	*c*	R^2
5 min	No adjustment	0.0000	0.4534*	0.3717*	40.4%
5 min	ARMA(2,3) residuals	0.0004	0.4522*	0.3384*	37.5%
15 min	No adjustment	−0.0009	0.7189*	0.2048*	60.7%
15 min	ARMA(2,3) residuals	0.0020	0.7180*	0.1728*	59.4%
30 min	No adjustment	−0.0032	0.8626*	0.0859*	76.7%
30 min	ARMA(2,3) residuals	0.0032	0.8606*	0.0739*	76.3%
60 min	No adjustment	−0.0061	0.9266*	0.0335*	84.6%
60 min	ARMA(2,3) residuals	0.0067	0.9218*	0.0782*	84.4%

*Statistically significantly different from 0 at the 1% level.

ARMA(2,3) residuals will control for the thin trading effect, and any statistically significant coefficients on the lagged futures variable may be ascribed to differential speeds of adjustments as between the cash and futures markets. The results reported in Table III indicate that purging the cash return variable of thin trading effects does not remove the statistical significance of the lagged futures return variable coefficient, and as such, the futures market leads the cash market in this situation because of phenomena other than thin trading, such as differential speeds of adjustment. As anticipated, the magnitude of the lagged futures coefficient is lower for the adjusted series (i.e., one source of the leading effect has been removed), and the coefficients decline with intervalling (i.e., differential speeds of adjustment decline with intervalling), apart from the ARMA-purged 60-min cash return case.

The articulation between thin trading and adjustment effects was investigated with daily data for the FTSE-100 futures and cash markets by Theobald and Yallup (1998), who found that the results were dependent on the model structures or estimators used. With the covariance ratio estimator adjusted for thin trading that was developed in that article, it was found at the daily level that although statistically significant thin trading effects were present (around 77% of the index return corresponded to the period end), speeds of adjustment were not statistically significantly different from 1.[10] However, purging the data of thin trading effects with the ARMA(2,3) model suggested by Stoll and Whaley (1990)

[10]The speeds of adjustment estimates were quite unstable, reflecting the fact that the covariance ratio estimator contained covariances very close to 0 in both the numerator and denominator when adjusted for thin trading.

and with the covariance ratio estimator unadjusted for thin trading effects led to speeds of adjustment that were statistically significantly different from 1 (except when the 1987 crash data were excluded).

The aforementioned analysis is extended in this article to the estimation of intradaily speeds of adjustment for the U.K. cash and futures markets with the estimators developed by Theobald and Yallup (1998). The estimator for the speed of adjustment in the cash market is given by

$$1 - p = \mathrm{cov}[R(f, t - 1), R(s, t)]\,\{\mathrm{cov}[R(f, t), R(s, t)]\}^{-1} \tag{19}$$

and the estimator for the futures market is

$$1 - q = \mathrm{cov}[R(f, t), R(s, t - 1)]\,\{\mathrm{cov}[R(f, t), R(s, t)]\}^{-1} \tag{20}$$

Because the estimators at Equations 19 and 20 may be viewed as instrumental variable estimators, the sampling distribution of the estimators is readily available (see Theobald & Yallup, 1998). The cash market speed of adjustment was estimated with both unadjusted cash return data and the ARMA(1,0) residuals of the cash return series.

The estimated speeds of adjustment are contained in Table IV. As anticipated, the estimates increase with intervalling (see the appendix of Theobald & Yallup, 1998, for an analytical demonstration of this property) for both cash return series and for the futures return series from the 15-min differencing interval onward.

The estimated speeds of adjustment are always very close to 1 for all differencing intervals in the case of the futures contract. Indeed, although the futures speeds of adjustment are statistically significantly different from 1 at the 15- and 60-min differencing intervals, the magnitude of the absolute departure from full adjustment is never economically significantly large, particularly when compared with the cash market adjustment speeds. The speeds of adjustment for the cash market are always lower

TABLE IV
Estimated Speeds of Adjustment

	Cash Market Speed of Adjustment		
Return Interval	*Unadjusted*	*ARMA(2,3) Residuals*	*Futures Market Speed of Adjustment*
5 min	0.1832*	0.2674*	0.9893
15 min	0.7048*	0.7488*	0.9625*
30 min	0.9158*	0.9303*	0.9919
60 min	1.0097	0.9613	1.0530

*Statistically significantly different from 1 at the 1% level.

than in the futures market, regardless of the return interval and return definition used. When the cash returns are purged of thin trading effects, the speed of adjustment increases, as anticipated, for the 5-, 15-, and 30-min differencing intervals, although the speeds of adjustment are always statistically significantly different from 1 and, for the 5-min and 15-min intervals, the departures from full adjustment are economically significant (indicating strong arbitrage potential at these differencing intervals). At the 60-min interval, the cash speed of adjustment is not statistically significantly different from 1 for both adjusted and unadjusted return series.

The impacts of the estimated speeds of adjustment were always to scale upward the futures-cash variance ratio developed in Equation 13 relative to the ratio developed in Miller et al. (1994) because the futures market adjusted more fully than the cash market. Because the estimated speed-of-adjustment coefficient of the cash market is less than 1 for all differencing intervals, this lack of full adjustment will contribute to the negative basis change autocorrelations as indicated by the analytics developed in the Basis Mean-Reversion Analytics section. For example, the condition for a negative basis change autocorrelation to occur in the absence of thin trading and spread effects as developed in Proposition 3[1] (and defined in Equation 16) requires that the cash speed-of-adjustment coefficient (p) be contained in the range $0 < p < 1$, a condition that is fulfilled for all the cash differencing intervals. However, because the cash speed of adjustment is not statistically significantly different from 1 at the 60-min differencing interval, the cash market adjustment speed will not make a statistically significant contribution to the negative basis change autocorrelation in this case. The speed of adjustment in the futures market is statistically significantly greater than 1 (i.e., the futures market overreacts) at this differencing interval, however. The condition for a negative basis change autocorrelation without spread and thin trading effects (i.e., $\theta = 0$ and $\varnothing = 0$) in the presence of full adjustment in the cash market (i.e., $p = 1$) is that the futures speed of adjustment, q, should be less than 1. Thus, at this differencing interval, the statistically significant overreaction in the futures market will lead to a reduction in the magnitude of the negative autocorrelation in the basis change variable, and as can be seen in Table II, the magnitude of the autocorrelation is at its minimum for this differencing interval.

The mean reversion that is present within the basis does derive from the actual characteristics of these markets rather than statistical artifacts, as argued by Miller et al. (1994). The predictability associated with basis mean reversion is then exploitable, and because the arbitrage potential is proportional to the differences between cash and futures market speeds of adjustments, these opportunities will be greatest at the shorter differencing intervals of 5 and 15 min.

The arbitrage potential was evaluated with a simple trading rule that employed significant movements in the futures market as lead indicators of cash market movements. The highest and lowest 5% of futures price movements over 5- and 15-min intervals were identified, and the subsequent returns to long cash and long-cash/short-futures strategies determined. The results contained in Table V indicate that arbitrage potential did exist but only when the trading rule was implemented immediately (i.e., with a 0 lag after significant futures movement). Returns were

TABLE V
Trading Rule Returns

		Cash Returns (%)		*Cash–Futures Returns (%)*	
Return Interval (min)	*Cash Purchase Lag (min)*	*5-min Futures Signal*	*15-min Futures Signal*	*5-min Futures Signal*	*15-min Futures Signal*
Panel A: Top 5% of Futures Returns over 5- and 15-min					
5	0	0.08967*	0.05626*	0.09707*	0.07037*
10	0	0.09874*	0.06080*	0.10642*	0.07308*
15	0	0.10194*	0.06771*	0.10896*	0.07512*
30	0	0.11544*	0.06109*	0.10864*	0.07346*
60	0	0.10577*	0.03915	0.10574*	0.08082*
5	5	0.00833	0.00434	0.00905	0.00264
10	5	0.01151	0.01094	0.01196	0.00447
15	5	0.01658	0.01824	0.00961*	0.00804
30	5	0.02301	0.00099	0.01306*	0.00647
60	5	0.01515	−0.01001	0.01425	0.00330
5	10	0.00305	0.00626	0.00220	0.00163
10	10	0.00759	0.01355	0.00013	0.00593
15	10	0.01340	0.01182	0.00099	0.00750
30	10	0.00938	−0.00691	0.00013	0.00372
60	10	0.00398	−0.00720	0.00139	0.00092
Panel B: Bottom 5% of Futures Returns over 5- and 15-min Intervals					
5	0	−0.09230*	−0.06142*	−0.09621*	−0.06931*
10	0	−0.09971*	−0.06112*	−0.09798*	−0.06658*
15	0	−0.10733*	−0.06437*	−0.10429*	−0.07000*
30	0	−0.11680*	−0.07676*	−0.11117*	−0.07059*
60	0	−0.11840*	−0.07953*	−0.11026*	−0.07685*
5	5	−0.00678	0.00037	0.00064	0.00411
10	5	−0.01396	−0.00280	−0.00504	0.00164
15	5	−0.01721	−0.00400	−0.00791	0.00242
30	5	−0.02636	−0.01567	−0.00556	0.00056
60	5	−0.02146	−0.01264	−0.01055	−0.00334
5	10	−0.00751	−0.00382	−0.00451	−0.00104
10	10	−0.01034	−0.00480	−0.00451	−0.00104
15	10	−0.00824	−0.01361	−0.00414	−0.00413
30	10	−0.01829	−0.01236	−0.00753	−0.00304
60	10	−0.01877	−0.01275	−0.00971	−0.00983

*Statistically significantly positive (Panel A) or negative (Panel B) returns at the 1% level.

larger when 5-min futures changes were used as the trading signal consistent with the more sluggish cash market speeds of adjustment at this differencing interval. A significant proportion of the strategy return occurred in the first 5 min of the strategy. Again, this is consistent with a speed-of-adjustment interpretation because the cash return that derives from a large futures price change (Δ) occurring i periods later will be $p(1 - p)^i\Delta$ and this return component will decline with increasing i when $0 < p < 1$. Of course, the subsequent return earned by this strategy could also derive from thin trading effects and speed-of-adjustment effects. With the analytical structure developed in Theobald and Yallup (1998), we found that at the 5- (15-) min differencing interval, 92% (95%) of the index return corresponds to the period end, proportions that are much more comparable than the cash market speeds of adjustment at these differencing intervals (0.2674 and 0.7488, respectively). Effectively, the relatively large changes in returns that occur when one moves from strategies that employ a 5-min futures movement to a 15-min movement as a trading signal cannot be wholly explained by the relatively small change in thin trading.

The results reported in this section provide empirical support for the underlying rationale of the analytics that were reported in the Basis Mean-Reversion Analytics section; that is, differential speeds of adjustment will occur across the futures and cash markets, with the former, more liquid market having the higher speed of adjustment, particularly at the shorter differencing intervals.

CONCLUSIONS

The incremental impacts of partial adjustments on the negative autocorrelations (mean reversion) in stock index futures basis changes were analytically demonstrated. Adjusting for infrequent trading effects alone will not remove the negative autocorrelations in basis changes. Numerical analyses with reasonable parameter values support the plausibility of this argument. Empirical evidence that provided support for the notion that differential speed-of-adjustment effects were present in addition to thin trading impacts was generated with a sample of U.K. intradaily data. The differentials were particularly apparent at the shorter differencing intervals (i.e., less than 15 min) where the negative autocorrelations were at their highest levels. Although the negative autocorrelations manifested by infrequent trading are not exploitable, the partial adjustment-induced autocorrelations can be exploitable by market participants.

APPENDIX

If the change in the observed basis is defined as $b(o, t)$, the first-order autocorrelation in the observed basis change series, $\rho_1[b(o, t)]$, is

$$\rho_1[b(o, t)] = \frac{\operatorname{cov}[b(o, t), b(o, t-1)]}{\operatorname{var}[b(o, t)]} \tag{A1}$$

with stationarity assumed.

The denominator at Equation A1 is

$$\operatorname{var}[b(o, t)] = \operatorname{var}[f(o, t)] + \operatorname{var}[s(o, t)] - 2\operatorname{cov}[f(o, t), s(o, t)] \tag{A2}$$

The first two terms at Equation A2 have been already derived from Equations 11 and 12. With Equations 6 and 10, we obtain

$$\begin{aligned}
&\operatorname{cov}[f(o, t), s(o, t)]\\
&\quad = \operatorname{cov}[q(1 + \theta L)(1 - (1 - q)L)^{-1} z(t),\\
&\qquad p(1 - \varnothing)(1 - \varnothing L)^{-1}(1 - (1 - p)L)^{-1} v(t)] &\text{(A3)}\\
&\quad = pq(1 - \varnothing)\operatorname{cov}[\{(1 + \theta L)(1 + (1 - q)L + (1 - q)^2 L^2 + \cdots)\} z(t),\\
&\qquad \{(1 + \varnothing L + \varnothing^2 L^2 + \cdots)(1 + (1 - p)L + \cdots)\, v(t)\}] &\text{(A4)}
\end{aligned}$$

With some rearrangement and summation of geometric progressions yields, when lagged crosscovariances between true futures and index changes are assumed to be 0,

$$\begin{aligned}
&\operatorname{cov}[f(o, t), s(o, t)]\\
&\quad = pq(1 - \varnothing)\left\{1 + \frac{(1 - q + \theta)(1 - p + \varnothing)}{1 - \varnothing(1 - q)}\right.\\
&\qquad \left. + \frac{(1 - p)^2(1 - q)(1 - q + \theta)}{(1 - \varnothing(1 - q))[1 - (1 - p)(1 - q)]}\right\} \operatorname{cov}[z(t), v(t)] &\text{(A5)}
\end{aligned}$$

The numerator at Equation A1 can be expressed, with Equations 6 and 10, as

$$\begin{aligned}
&\operatorname{cov}[f(o, t) - s(o, t), f(o, t - 1) - s(o, t - 1)]\\
&\quad = \operatorname{cov}[q(1 + \varnothing L)(1 - (1 - q)L)^{-1} z(t)\\
&\qquad - p(1 - \varnothing)(1 - \varnothing L)^{-1}(1 - (1 - p)L)^{-1} v(t),\\
&\qquad q(1 + \theta L)(1 - (1 - q)L)^{-1} z(t - 1)\\
&\qquad - p(1 - \varnothing)(1 - \varnothing L)^{-1}(1 - (1 - p)L)^{-1} v(t - 1)] &\text{(A6)}
\end{aligned}$$

Equation A6 can be expanded and decomposed into four terms (if non-contemporaneous crosscovariances between true futures and index price changes are assumed to be 0):

$$\begin{aligned}
&\operatorname{cov}[f(o,t)-s(o,t), f(o,t-1)-s(o,t-1)] \\
&\quad = q^2 \operatorname{var}[z(t)]\{(1-q+\theta)+(1-q)(1-q+\theta)^2 \\
&\qquad + (1-q)^3(1-q+\theta)^2+\cdots\} \\
&\qquad - pq(1-\varnothing)\operatorname{cov}[v(t),z(t)]\{(1-q+\theta) \\
&\qquad + (1-q)(1-q+\theta)(1-p+\varnothing)+\cdots\} \\
&\qquad - pq(1-\varnothing)\operatorname{cov}[v(t),z(t)]\{(1-p+\varnothing) \\
&\qquad + (1-q+\theta)[(1-p)^2+(1-p)\varnothing+\varnothing^2]+\cdots\} \\
&\qquad + p^2(1-\varnothing)^2\operatorname{var}[v(t)]\{(1-p+\varnothing)+(1-p+\varnothing)[(1-p)^2 \\
&\qquad + (1-p)\varnothing+\varnothing^2]+\cdots\} \qquad \text{(A7)}
\end{aligned}$$

Evaluating each of the four terms in turn yields

$$\begin{aligned}
&\operatorname{cov}[b(o,t), b(o,t-1)] \\
&\quad = q^2 \operatorname{var}[z(t)](1-q+\theta)\left\{1+\frac{(1-q)(1-q+\theta)}{q(2-q)}\right\} \\
&\quad - pq(1-\varnothing)(1-q+\theta)\operatorname{cov}[v(t),z(t)]\left\{1+\frac{(1-q)}{1-\varnothing(1-q)}\right. \\
&\quad \left.\left[1-p+\Re+\frac{(1-p)^2(1-q)}{1-(1-p)(1-q)}\right]\right\} \\
&\quad - pq(1-\varnothing)\operatorname{cov}[v(t),z(t)]\left\{(1-p+\varnothing)+\frac{(1-q+\theta)}{1-(1-q)\varnothing}\right. \\
&\quad \left.\left[\varnothing(1-p+\varnothing)+\frac{(1-p)^2}{1-(1-p)(1-q)}\right]\right\} \\
&\quad + p^2(1-\varnothing)^2(1-p+\varnothing)\operatorname{var}[v(t)]\left\{1+\frac{1}{1-\varnothing^2}\right.[\varnothing(1-p+\varnothing) \\
&\quad \left.+\frac{(1-p)^2(1+\varnothing^2)}{1-(1-p)\varnothing}+\frac{(1-p)^4}{[1-(1-p)\varnothing(1-(1-p)^2]}\right]\Bigg\} \qquad \text{(A8)}
\end{aligned}$$

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