THE DEMAND FOR HEDGING WITH FUTURES AND OPTIONS

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The optimal hedging portfolio is shown to include both futures and options under a variety of circumstances when the marginal cost of hedging is nonzero. Futures and options are treated as substitute goods, and the properties of the resulting hedging demand system are explained. The overall optimal hedge ratio is shown to increase when the marginal cost of trading options is reduced. The overall optimal hedge ratio is shown to decrease when the marginal cost of trading futures is decreased. One implication is that hedging demand can be stimulated by a reduction in the perceived cost of trading options through the education of hedgers about options and the initiation of programs such as the Dairy Options Pilot Program. The demand approach is applied to estimate optimal hedge ratios for dairy producers hedging corn inputs in five regions of Pennsylvania. © 2001 John Wiley & Sons, Inc. Jrl Fut Mark 21:693–712, 2001

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INTRODUCTION

The correspondence between optimal hedging behavior and the demand for hedging has not been transparent in the hedging literature. Authors such as Castelino (1989, 1992), Paroush and Wolf (1989), Moser and Helms (1990), Park and Antonovitz (1990), Castelino, Francis, and Wolf (1991), Briys, Crouhy, and Schlesinger (1993), Lapan and Moschini (1994), and Netz (1996) examined the role of basis risk within a price risk-management strategy. Hirshleifer (1988) showed that transaction costs drive hedgers from the market, Simaan (1993) and Lence (1995, 1996) found that opportunity costs reduce optimal hedge ratios, and Frechette (2000) showed how hedging costs affect the demand for futures, but none considered options. Wolf (1987), Lapan, Moschini, and Hanson (1991; hereafter called LMH), Moschini and Lapan (1992, 1995), and Sakong, Hayes, and Hallam (1993) looked at the trade-off between futures and options but not their costs.

A synthesis of these approaches leads to a view of futures and options as substitute goods. Risk managers are demanders of risk-management goods and are consumers of hedging products. Other market participants are suppliers of risk-management goods and producers of hedging products. Each good has a price that is determined by the equilibrium between consumers and producers.

The demander of a risk-management good chooses a hedge ratio or quantity for each good to maximize expected utility. Futures and options are substitute goods from the demander's perspective, and the price of each will affect the demand for both types of goods. The situation is analogous to the consumer choice problem with multiple goods and a budget constraint, except that a budget constraint is not necessarily binding for the hedger.

The marginal cost of hedging is unlikely to be zero. Brokerage fees, transaction costs, opportunity costs, learning costs, and other marginal costs associated with futures and options hedging must be included. For example, initiating a futures position in corn may involve a \$25 per contract brokerage fee and \$1,000 set aside in a margin account. If the hedger's annual discount rate is 12% in excess of the margin account's rate of return, the margin account costs \$120 per year per contract or \$10 per month per contract. Over a 6-month horizon, the total cost from both these sources will be \$85 per contract. Additional costs of transacting and administering the hedge could easily increase the marginal hedging cost to more than \$100 per contract or 2 cents per bushel.

Within a demand systems framework, it is easy to see that, with everything else fixed, fewer futures contracts will be acquired by the hedger if futures cost 2 cents per bushel than would be purchased if they were free. What is not clear is the effect on options.

The trade-off between futures and options was studied by LMH, but they did not consider costs. In this context, the results of LMH are reinvestigated and reinterpreted. They started with a base case of unbiased pricing and then considered the effect of alternative expectations (on the part of the hedger). The same framework is used here; I start with a base case of no marginal hedging costs and then consider the effect of positive marginal costs.

The main contribution of this article is a reinterpretation of the LMH model in terms of hedging costs, which leads to a reinterpretation of their primary result. Their primary result was that options add nothing to the optimal hedging strategy in unbiased markets. When futures and options are costless, their result is still correct, but when futures and options are not costless, the typical optimal hedging strategy will mix futures and options in some combination. This result is driven entirely by the costs faced by the hedger, including brokerage fees, transaction costs, opportunity costs, learning costs, and every other cost associated with futures and options hedging.

There have been other extensions to the LMH model that have also yielded nonzero optimal options hedge ratios. For example, Moschini and Lapan (1992) showed that options hedging may be valuable when some production decisions can be postponed. Moschini and Lapan (1995) and Sakong et al. (1993) demonstrated the same result under production uncertainty. All three assumed that hedging was costless but production was not predetermined. Options were shown to be useful for hedging because profits were nonlinear with respect to the random price. Options hedging was shown to have value in each case, but hedging costs were not considered. Here, the result is driven by hedging costs, not by nonlinearities or nonorthogonalities in the basis function, production uncertainty, or any other changes to the LMH model.

The importance of this result goes beyond academic circles. Agricultural hedging has grown in importance in the United States with the implementation of the FAIR Act, and agricultural futures and options have moved from relative obscurity to become part of the national consciousness. This research implies that hedgers ought to consider carefully the costs and benefits of hedging and to compare the costs of different risk-management products before choosing a hedging strategy. In most cases, a role exists for both futures and options, depending on the relative price of each and the marginal expected utility gained from hedging. The hedging demand framework is useful for understanding equilibrium hedging when hedging demand and supply conditions change because of government policy or actions by hedgers, brokerage houses, or exchanges.

MODEL

The model used here is based on the LMH model. The LMH notation has been adopted to emphasize the differences and similarities of the two models. The purpose is to make it clear how the LMH results can be reinterpreted in a demand systems framework. The variant of LMH to be used here is the one where hedging decisions are conditional on the output decision. No new insights are gained by the inclusion of output as a choice variable beyond what LMH, Moschini and Lapan (1992, 1995), and Sakong et al. (1993) already demonstrated. Profits are considered per unit profits, which is different from but not inconsistent with LMH. The choice variables must be interpreted then as hedge ratios rather than quantities hedged.

LMH selected an objective function with von Neumann and Morgenstern's (1944) expected utility paradigm. Kahl (1983), Berck (1981), Bond and Thompson (1985), Bond, Thompson, and Geldard (1985), Hirshleifer (1988), and Viaene and de Vries (1992) used Markowitz's (1952, 1959) mean-variance utility, but evidence indicates that neither Markowitz's (1952, 1959) mean-variance utility nor von Neumann and Morgenstern's (1944) expected utility can explain all the nuances of choice under uncertainty. Both remain in common use and differ little in prescriptive value for a wide range of models, as shown by Meyer (1987), Tew, Reid, and Witt (1991), Hanson and Ladd (1991), and Simaan (1993). Expected utility is used by Lapan and Moschini (1994), Moschini and Lapan (1992, 1995), Lence (1995, 1996), and LMH. In particular, LMH explained why mean-variance utility is inappropriate for choices involving options.

The hedger is assumed to maximize expected utility of profits by choosing a pair of hedge ratios, x and z, where x is the portion of output hedged in the futures market and z is the portion of output hedged in the options market. LMH normalized this by choosing x to be accomplished with short futures, so an output hedger will typically have a positive x, but they chose z to be accomplished with short puts, so an output hedger will typically have a negative z. As in LMH, only one option strike price, k, is considered. There are two periods in the LMH model, with the current futures price denoted f and the terminal futures price denoted p when realized or $\tilde{p}$ when treated as a random variable. The options premium is denoted r, and the terminal option value is denoted v or $\tilde{v}$: $v = 0$ if $p > k$, and $v = k - p$ if $p < k$. The utility function $u(.)$ is assumed to be continuous, monotonically increasing, and strictly concave: $u' > 0$ and $u'' < 0$. Utility is a function of profits, π, treated as a random variable:

$$\tilde{\pi} = \tilde{b} + (f - \tilde{p})x + (r - \tilde{v})z - c \tag{1}$$

where $\tilde{b}$ represents the local commodity price with spatial and temporal bases included and c represents costs per unit. All money values are adjusted by appropriate discount rates suitably defined.

The hedger's optimization problem is

$$\max_{\{x,\, z\}} E[u(\tilde{\pi})] \tag{2}$$

with E representing the expectations operator over all sources of uncertainty. The first-order conditions are

$$E[u'(\tilde{\pi})(f - \tilde{p})] = 0 \tag{3a}$$

$$E[u'(\tilde{\pi})(r - \tilde{v})] = 0 \tag{3b}$$

The second-order conditions are satisfied because of the conditions on $u(.)$.

LMH took the model further by specifying the nature of the basis risk:

$$\tilde{b} = \alpha + \beta\tilde{p} + \tilde{\theta} \tag{4}$$

where α and β are fixed parameters and $\tilde{\theta}$ is a random variable with mean zero and orthogonal to $\tilde{p}$ and, therefore, $\tilde{v}$. Profits can then be rewritten as

$$\tilde{\pi} = \pi_0 + \tilde{\theta} + \tilde{p}(\beta - x) - \tilde{v}z \tag{5}$$

with

$$\pi_0 = \alpha - c + fx + rz \tag{6}$$

as the deterministic part. Their insight is that setting $x = \beta$ and $z = 0$ satisfies the first-order conditions Equations 3a and 3b because the only remaining stochastic part of $\tilde{\pi}$ is $\tilde{\theta}$, which is orthogonal to $(f - \tilde{p})$ and $(r - \tilde{v})$, so that Equations 3a and 3b are satisfied if $E(f - \tilde{p}) = 0$ and $E(r - \tilde{v}) = 0$. These last two conditions are simply the market unbiasedness conditions for efficient futures and options pricing. They concluded that unbiased markets with a linear basis rule such as Equation 4 yield no role for options in an optimal hedging portfolio.

Interpreting the LMH model in a demand systems context means treating marginal hedging costs as prices faced by the hedger. The prices include such things as brokerage fees, opportunity costs, and learning costs associated with futures and options hedging. They may also include perceived costs associated with uncertainty but not modeled by the expected utility framework, such as distributional ambiguity and nonlinear responses to distributional properties. There are many costs faced by an atomistic hedger that make the net hedged price less than $E\tilde{p}$ for futures and the net option premium greater than $E\tilde{v}$ for options, even if the markets themselves are unbiased. The major innovation here is the treatment of hedging demand separately from the market pricing mechanism (hedging supply).

To proceed, consider a hedger who faces additional costs beyond f and r, the unbiased expectations of $\tilde{p}$ and $\tilde{v}$. Call these extra costs t_x per unit hedged with futures and t_z per unit hedged with options. This situation is analogous to LMH's results with biased markets, but the difference in interpretation is important. Profits now become

$$\tilde{\pi} = \tilde{b} + (f - \tilde{p})x + (r - \tilde{v})z - t_x|x| - t_z|z| - c \tag{7}$$

where $|.|$ is the absolute value operator. The absolute value operator is necessary because z is less than 0 in most cases for output hedgers, as discussed previously. In a parallel manner to Equations 3a and 3b, the first-order conditions can be derived with the sign operator, defined by $\text{sgn}(x) = |x|/x$ for nonzero x and $\text{sgn}(0) = 0$:

$$E[u'(\tilde{\pi})(f - \tilde{p} - t_x \,\text{sgn}(x))] = 0 \tag{8a}$$

$$E[u'(\tilde{\pi})(r - \tilde{v} - t_z \,\text{sgn}(z))] = 0 \tag{8b}$$

Profits can be expressed as in Equations 5 and 6:

$$\tilde{\pi} = \pi_0 + \tilde{\theta} + \tilde{p}(\beta - x) - \tilde{v}z \tag{9}$$

$$\pi_0 = \alpha - c + [f - t_x \,\text{sgn}(x)]x + [r - t_z \,\text{sgn}(z)]z \tag{10}$$

Now, the solutions to Equations 8a and 8b are different from the solutions to Equations 3a and 3b because

$$E[u'(\tilde{\pi})(f - \tilde{p})] = t_x Eu'(\tilde{\pi}) > 0 \text{ for positive } x \tag{11a}$$

$$E[u'(\tilde{\pi})(r - \tilde{v})] = t_z \,\text{sgn}(z) Eu'(\tilde{\pi}) < 0 \text{ for negative } z \tag{11b}$$

If we follow the logic through, the range of possible solutions becomes $0 \leq x < \beta$ and $z \leq 0$ when t_x and t_z are greater than 0. The markets themselves may be unbiased, yet the hedger may choose to mix futures and options simply because hedging is not free.

The implication is that futures and options are substitute goods within most typical hedging portfolios. Totally differentiating the first-order conditions Equations 8a and 8b, as in LMH's Equations 11 and 12 under a negative exponential utility, yields the signs of the demand slopes and cross-price slopes (partial derivatives):

$$-d|z|/dt_z > dx/dt_z = d|z|/dt_x > -dx/dt_x > 0 \tag{12}$$

The direct effects are negative, meaning the amount hedged with futures will decline as the marginal cost of hedging with futures increases, and the amount hedged with options will decline as the marginal cost of hedging with options increases. The cross-effects are positive, meaning the amount hedged with one product will increase when the price of the other product rises. The cross-effects are also equal, proving the demand system symmetry condition. These results were derived by LMH but not interpreted in this way.

A useful result from the previous arguments and Equation 12 is that $d|z|/dt_z < dx/dt_x < 0$, so options demand is more sensitive to price changes than futures demand. This result is consistent with the idea of options as a luxury good and futures as a necessary good.[1] The implication is that policy makers and market professionals wishing to encourage hedging ought to focus on options, not futures. Reducing the marginal cost of options (including opportunity costs, fees, and learning costs) could stimulate hedging demand and result in a larger aggregate hedge ratio.

To explore this issue further, consider the effect of t_x and t_z on the aggregate hedge ratio, $h = x + |z|$. From Equation 12, $dh/dt_x = dx/dt_x + d|z|/dt_x > 0$ and $dh/dt_z = dx/dt_z + d|z|/dt_z < 0$, which means that an increase in the marginal cost of hedging in the futures market (an upward shift in the hedging supply curve for futures) will lead to an increase in the overall hedge ratio. Similarly, an increase in the marginal cost of hedging in the options market (an upward shift in the hedging

[1]To understand this, interpret an increase in t_x as an equivalent decrease in t_z and a decrease in hedging expenditures denoted y. Similarly, an increase in t_z is equivalent to a decrease in t_x and a decrease in hedging expenditures. Consider that $dx/dt_z = d|z|/dt_x$, and dx/dt_x is smaller in magnitude than $d|z|/dt_z$, so dx/dy must be smaller in magnitude than $d|z|/dy$. Thus, the synthetic expenditure elasticity for futures is smaller than the expenditure elasticity for options, meaning futures are a necessary good and options are a luxury good.

supply curve for options) will lead in a decrease to the overall hedge ratio. Conversely, a decrease in the marginal cost of hedging with options will stimulate hedging demand, whereas a decrease in the marginal cost of hedging with futures will reduce total hedging demand.

This result seems counterintuitive at first until h is understood to be a composite good made up of futures and options. An increase in the price of one or the other component good will always lead to a reduction in the purchase of that good and an increase in the purchase of its substitute; however, the demand for the composite good is not constrained to decrease. Here h is behaving as a Giffen good when t_x increases and as a normal good when t_z increases.

The confusion is allayed when the aggregate price is homogenized: restrict both to move together one for one, so that $dt_x = dt_z = dt$. Thus, $dh/dt = dh/dt_x + dh/dt_z = dx/dt_x + d|z|/dt_x + dx/dt_z + d|z|dt_z$, the sign of which can be shown to be negative. Hedging shifts from options to futures as t increases because $dx/dt > 0$ and $d|z|/dt < 0$, but the total effect is to reduce the options position by more than the futures position as marginal costs increase. In aggregate, then, $dh/dt < 0$, which means the aggregate hedging demand curve slopes downward.

Beyond this, it is possible to characterize the share of hedging demand composed of futures and the share composed of options. The share comprised by futures is x/h, and $d(x/h)/dt_x = (dx/dt_x)/h - (x/h^2)(dx/dt_x + d|z|/dt_x) = [(h - x)dx/dt_x - xd|z|/dt_x]/h^2 = (|z|dx/dt_x - xd|z|/dt_x)/h^2 < 0$. The share composed of options is $|z|/h$, and $d(|z|/h)/dt_z = (xd|z|/dt_z - |z|dx/dt_z)/h^2 < 0$. Each demand share curve (or, equivalently, each demand curve conditional on h) slopes downward.

A marginal rate of substitution is inadequate to characterize demand properties. The usual marginal rate of substitution measures the variation in one good's quantity that keeps utility fixed as another good's quantity is changed exogenously. Here, however, expected utility has already been maximized, and no change in quantities can keep it fixed without a corresponding change in prices. Indifference curves can be constructed to illustrate iso-expected utility contours. They are ellipses with a downward sloping major axis in $x - |z|$ space. The center of the ellipse is the optimum point, which depends on t_x, t_z, and the other parameters.

Another way to characterize the demand for hedging is plotting expansion paths in $x - |z|$ space as t_x and t_z change. One such expansion path is illustrated in Figure 1. When $t_x = t_z = 0$, the optimum point is $(\beta, 0)$, as discussed previously. No options are purchased unless t_x is positive. As t_x increases, the expansion path moves northwest in the

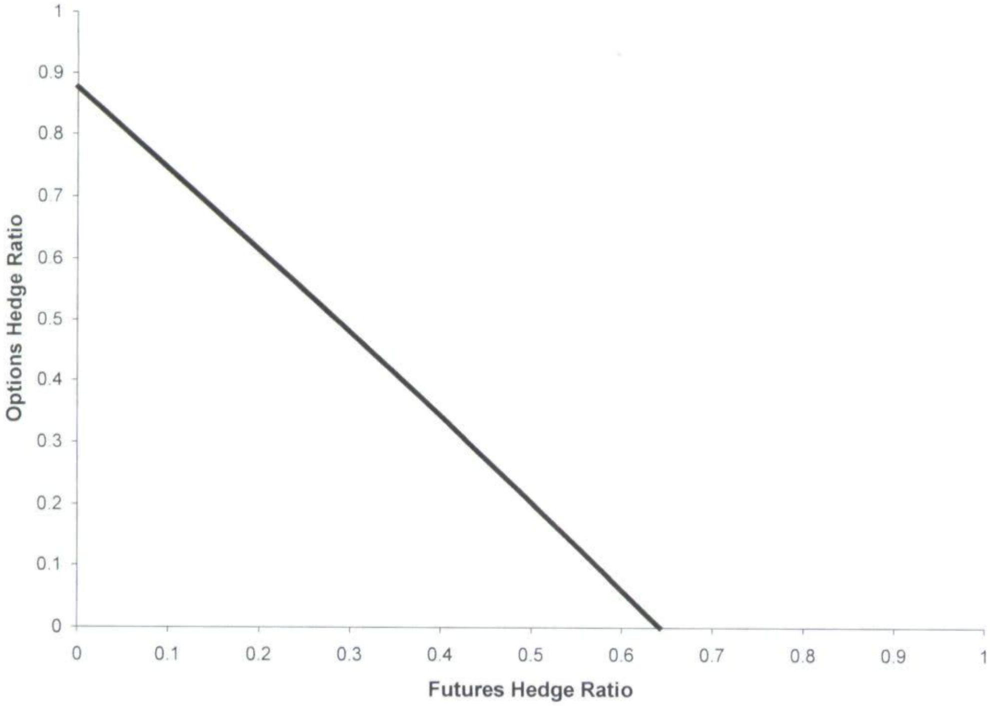

FIGURE 1

Hedge ratio expansion path: moderate risk aversion in Southeastern Pennsylvania ($t_z = 0.0$).

coordinate space until it meets the $|z|$ axis. Similarly, if $t_z = 0$ and t_x is large and positive, the expansion path moves from the $|z|$ axis southeast to the x axis as t_z rises. The slope of the expansion path depends on t_x, t_z, and the other parameters.

For most realistic values, the optimum is on neither the x axis nor the $|z|$ axis but somewhere between. In realistic situations, futures and options ought to be mixed. Futures cannot be said to be a better hedging product than options without reference to hedging costs.

This approach also yields a useful way to compute the value of a market's existence. In consumer theory, the consumer's surplus is the total benefit accruing to the consumer because the opportunity exists to purchase the good. Here, the hedger's surplus may be defined analogously, as in Frechette (2000). If $u^* = \max_{\{x,z\}} Eu(\tilde{\pi})$, the hedger's surplus in the options market is e_z, where $u^* = \max_{\{x|z=0\}} Eu(\tilde{\pi} + e_z)$. Similarly, the hedger's surplus in the futures market is e_x, where $u^* = \max_{\{z|x=0\}} Eu(\tilde{\pi} + e_x)$. The aggregate hedger's surplus for both markets is e_{xz}, where $u^* = Eu(\tilde{\pi} + e_{xz})$, holding $x = z = 0$. The value of e_{xz} will necessarily be greater than or equal to $e_x + e_z$ because futures and options are substitutes. The values depend on t_x, t_z, and the other parameters.

EMPIRICAL APPLICATION

This section applies the concepts from the previous section to the hedging decision for dairy producers in five local regions of Pennsylvania. The dairy producers hedge their purchases of corn with long futures and long call options, exactly the reverse of the LMH case. The analysis is essentially identical in every other way. The negative exponential (constant absolute risk aversion) utility function is assumed, which results in a convenient way to compare results for different levels of risk aversion. Basis is specified as local price minus Chicago price. Expected utility is then computed as a numerical double integral over price risk and basis risk and maximized (again numerically) with respect to x and z. The integrals were calculated by the trapezoidal method, and optimization was achieved by the simplex method.

Data

The data set is the same one used by Frechette (2000) and consists of (a) weekly corn cash prices collected by the Pennsylvania Department of Agriculture (PDA) and (b) the nearby corn futures price in Chicago. Local cash prices were collected through surveys and phone calls for five regions: Southeastern, Central, South Central, Western, and Lehigh Valley. The prices were collected and reported by the PDA on Monday mornings before the market opened, and the futures price that corresponds most closely is the previous Friday's settlement price for the nearby futures contract. If the Chicago Board of Trade was closed due to a holiday, the closest day was used, with the information sets matched as closely as possible in each case. All prices are reported in cents per bushel for the years 1997 and 1998.

Table I displays summary statistics and the conditional covariance structure used in this analysis. The table shows that the covariances are negative and relatively large in magnitude between the Chicago price and each regional basis, indicating that the hedge ratios may be quite low in these regions. These statistics represent actual results for the sample period, and the results represent optimal ex post behavior in the sense that hedgers are assumed to have known the covariance matrix before the sample period began. Individual hedgers' expectations will depend on the sample period and available information.

TABLE I
Conditional Covariance Structure for Regional Prices in Nominal Cents per Bushel (1997–1998)

Region	N[a]	*Price Mean*	*Price Variance*	*Basis Variance*	*Basis–Futures Covariance*	*Price–Futures Covariance*
Southeastern Pennsylvania	104	291.40	31.31	14.26	−21.59	38.66
Central Pennsylvania	104	287.40	8.91	37.33	−44.22	15.93
South Central Pennsylvania	104	284.80	15.14	41.69	−43.29	16.86
Western Pennsylvania	104	279.50	28.90	63.41	−47.27	12.88
Lehigh Valley	85	274.70	30.61	19.16	−25.30	35.86
Chicago Board of Trade (CBOT) Futures	104	255.22	60.27			

[a]Data collection for the Lehigh Valley began May 1997. All other data are for January 1997 to December 1998.

Unbiased markets are assumed. Thus, f is the futures price and r is the expected value of $\max\{\tilde{p} - k, 0\}$.[2] The estimates of basis risk and expected basis depend on the structural forecasting model chosen by the hedger. There are many such models in use, such as naive expectations, adaptive expectations, and rational expectations. The results depend on the model chosen, and yet there is no clear consensus in the literature to guide this choice. Fortunately, the results often are robust to any reasonable choice of forecasting method. Moschini and Hennessy (2000) considered this issue and concluded that a constant covariance matrix may not be a bad approximation and that conditional variance does not do much better than unconditional variance for use in estimating producers' responses to price risk. Each hedger has a unique perception of market structure, and no single model has come to dominate the literature.

To proceed, an adaptive expectations model is selected, as in Frechette (2000). To illustrate, note that adaptive expectations models can be written in autoregressive form as

$$E_t p_{t+1} = \alpha_0 + \alpha_1 p_t + \alpha_2 p_{t-1} + \dots . \tag{13}$$

In practice, Equation 13 is truncated at a lag length sufficient to balance accuracy against degrees of freedom, and an error term is appended. If the error term satisfies standard assumptions, then ordinary least squares can be used to get estimates of the α_i, which generates corresponding estimates of $E_t p_{t+1}$. The lag length is chosen by maximization of the adjusted R^2 statistic and testing of the standard ordinary-least-squares assumptions. The conditional covariance matrix is estimated by substitution of

[2]The base case for the strike price is at-the-money, but other strike prices are considered.

the expected local price minus the expected futures price for the expected basis. The conditional covariance matrix is assumed to be constant and to represent a bivariate normal distribution.

The coefficient of absolute risk aversion was set to span a range of possible risk preferences. Lapan and Moschini (1994) and Lence (1995) were used as a guide to select values for the coefficient of absolute risk aversion after the units were converted as in Raskin and Cochran (1986). The relative risk-aversion parameter in these studies ranged from 0 to 20 per year for a soybean farm. Adjusting to a weekly value (multiply by 52) in cents (divide by 100) and adjusting from an output-based quantity to a much smaller input quantity (divide by roughly 5) require a final scaling factor of roughly 0.1. The range from 0 to 20 corresponds to an approximate range for the coefficient of absolute risk aversion of 0 to 2 per \$0.01. Reasonable values to span this range were chosen as 2.00 for high risk aversion, 0.20 for moderate risk aversion, and 0.02 for low risk aversion.

Hedging Demands

The computational hedging demand curves cannot be expressed analytically. Table II displays the quantities demanded at regular grid points in (t_x, t_z) space for the Southeastern Pennsylvania region, with a coefficient of absolute risk aversion of 0.2 per \$0.01 (cents). Most hedgers face

TABLE II

Hedging Demand in Southeastern Pennsylvania for Moderate Risk Aversion

	t_z					
t_x	*0.0*	*0.5*	*1.0*	*1.5*	*2.0*	*2.5*
0.0	0.6422 0	0.6422 0	0.6422 0	0.6422 0	0.6422 0	0.6422 0
0.5	0.4629 0.2547	0.6003 0	0.6003 0	0.6003 0	0.6003 0	0.6003 0
1.0	0.2324 0.5704	0.5397 0.0339	0.5590 0	0.5590 0	0.5590 0	0.5590 0
1.5	0 0.8763	0.3193 0.3325	0.5173 0	0.5173 0	0.5173 0	0.5173 0
2.0	0 0.8763	0.0145 0.7274	0.3995 0.1239	0.4759 0	0.4759 0	0.4759 0
2.5	0 0.8763	0 0.7460	0.1084 0.4982	0.4344 0	0.4344 0	0.4344 0

Note. Futures hedge ratios are listed above options hedge ratios. t_x = marginal cost of trading futures (cents per bushel); t_z = marginal cost of trading options (cents per bushel).

price combinations in the lower left corner of Table II because of margin accounts. Margin accounts account for a large portion of the cost of hedging with futures but are not required for options hedging. For example, if an options position costs \$25 plus the premium, t_z is 0.5 cents per bushel, which corresponds to Column 2 in the table. If a futures position also costs \$25 but requires a margin account, t_x could be as high as 2.5 cents per bushel, depending on the hedger's discount rate. If $t_x = 1.5$, corresponding to the fourth row of Table II, the optimal hedge ratios are $x = 0.3193$ and $z = 0.3325$. Thirty-two percent of a hedger's corn purchases will be hedged with futures, and 33% will be hedged with options.

The four other regions of Pennsylvania display similar hedging demands. Figure 2 shows the futures hedging demand curves for each region, assuming the marginal cost of trading options is 0.5 cents per bushel. Figure 3 shows the corresponding options hedging demand curves, assuming the marginal cost of trading futures is 1.5 cents per bushel. The Southeastern region displays the highest demand for futures hedging, with a hedge ratio of 0.64 when the price is zero and a cutoff price of 2.05 cents per bushel, at which the demand curve intersects the price axis. The Western region displays the lowest demand for futures

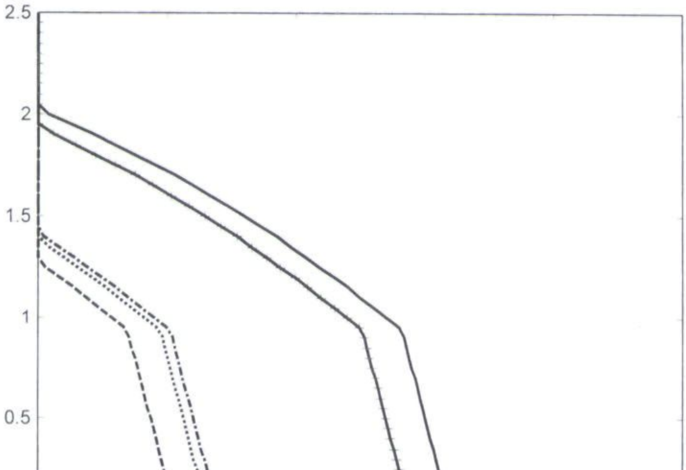

FIGURE 2

Futures hedging demands: moderate risk aversion ($t_z = 0.5$). The vertical axis represents the futures cost in cents per bushel; the horizontal axis represents the hedge ratio (futures). The Southeastern, Central, South Central, Western, and Lehigh Valley regions are represented by solid, dotted, dotted–dashed, dashed, crosshaired lines, respectively.

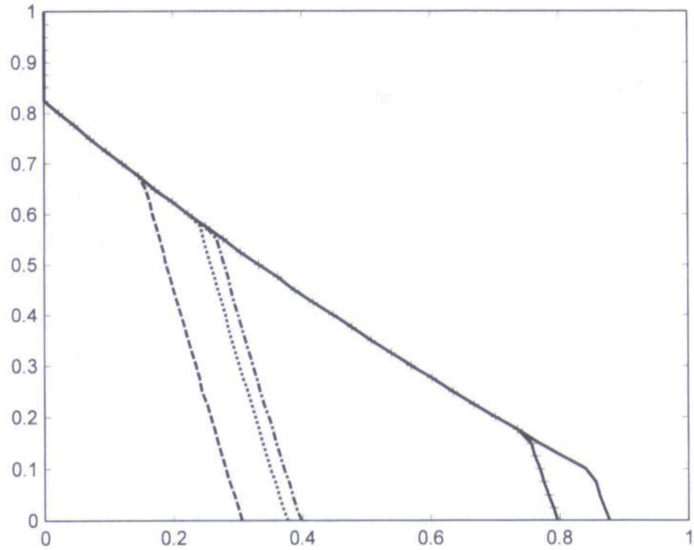

FIGURE 3

Options hedging demands: moderate risk aversion (t_x = 1.5). The vertical axis represents the option cost (beyond the premium) in cents per bushel; the horizontal axis represents the hedge ratio (options). The Southeastern, Central, South Central, Western, and Lehigh Valley regions are represented by solid, dotted, dotted–dashed, dashed, crosshaired lines, respectively.

hedging, with a hedge ratio of 0.21 when the price is zero and a cutoff price of 1.26. The Southeastern region displays the highest demand for options, with a hedge ratio of 0.88 when the price is zero and a cutoff price of 0.825. The Western region displays the lowest demand for options, with a hedge ratio of 0.30 when the price is zero and a cutoff price of 0.825. The options cutoff prices are the same for every region. The kinks in the demand curves correspond to these cutoff points where hedging no longer occurs in the second market.

Little difference is observed when the coefficient of absolute risk aversion is changed. Tables III and IV display the demand for futures and options for the Southeastern Pennsylvania region when the coefficient of absolute risk aversion is 2.0 and 0.02. Interestingly, for some combinations of t_x and t_z, the demand for hedging overall decreases as the level of risk aversion increases. This phenomenon occurs when options comprise a large part of the hedging portfolio and the optimal decision is to overhedge, compared to no-cost strategy. As the level of risk aversion rises, the way hedgers reduce risk is to reduce the hedge ratio, rather than increase it, because they are already overhedged. In summary, higher levels of risk aversion cause the overall hedge ratio to move toward the optimal no-cost hedge ratio, sometimes resulting in an increase and sometimes a decrease.

TABLE III

Hedging Demand in Southeastern Pennsylvania for High Risk Aversion

	t_z					
t_x	*0.0*	*0.5*	*1.0*	*1.5*	*2.0*	*2.5*
0.0	0.6418 0	0.6418 0	0.6418 0	0.6418 0	0.6418 0	0.6418 0
0.5	0.6239 0.0255	0.6377 0	0.6377 0	0.6377 0	0.6377 0	0.6377 0
1.0	0.6008 0.0571	0.6316 0.0034	0.6335 0	0.6335 0	0.6335 0	0.6335 0
1.5	0.5688 0.0990	0.6095 0.0332	0.6293 0	0.6293 0	0.6293 0	0.6293 0
2.0	0.5176 0.1623	0.5791 0.0727	0.6175 0.0124	0.6252 0	0.6252 0	0.6252 0
2.5	0.4067 0.2904	0.5308 0.1320	0.5884 0.0498	0.6211 0	0.6211 0	0.6211 0

Note. Futures hedge ratios are listed above options hedge ratios. t_x = marginal cost of trading futures (cents per bushel); t_z = marginal cost of trading options (cents per bushel).

TABLE IV

Hedging Demand in Southeastern Pennsylvania for Low Risk Aversion

	t_z					
t_x	*0.0*	*0.5*	*1.0*	*1.5*	*2.0*	*2.5*
0.0	0.6453 0	0.6453 0	0.6453 0	0.6453 0	0.6453 0	0.6453 0
0.5	0 0.9233	0.2270 0	0.2270 0	0.2270 0	0.2270 0	0.2270 0
1.0	0 0.9233	0 0	0 0	0 0	0 0	0 0
1.5	0 0.9233	0 0	0 0	0 0	0 0	0 0
2.0	0 0.9233	0 0	0 0	0 0	0 0	0 0
2.5	0 0.9233	0 0	0 0	0 0	0 0	0 0

Note. Futures hedge ratios are listed above options hedge ratios. t_x = marginal cost of trading futures (cents per bushel); t_z = marginal cost of trading options (cents per bushel).

These analyses are based on the availability of an at-the-money option contract. When other strike prices are considered, low strike prices (in-the-money) are always preferred to high ones. The mean Chicago price was 255.22 cents per bushel, and Table II shows the results for an at-the-money strike price of 255 cents per bushel for the

TABLE V
Hedging Demand in Southeastern Pennsylvania for In-The-Money Options

	t_z					
t_x	*0.0*	*0.5*	*1.0*	*1.5*	*2.0*	*2.5*
0.0	0.6418 0	0.6418 0	0.6418 0	0.6418 0	0.6418 0	0.6418 0
0.5	0.3779 0.3428	0.6003 0	0.6003 0	0.6003 0	0.6003 0	0.6003 0
1.0	0 0.8102	0.4600 0.1482	0.5587 0	0.5587 0	0.5587 0	0.5587 0
1.5	0 0.8102	0.0804 0.6150	0.5173 0	0.5173 0	0.5173 0	0.5173 0
2.0	0 0.8102	0 0.7109	0.1739 0.4161	0.4759 0	0.4759 0	0.4759 0
2.5	0 0.8102	0 0.7109	0 0.6217	0.2616 0.2336	0.4343 0	0.4343 0

Note. Futures hedge ratios are listed above options hedge ratios. t_x = marginal cost of trading futures (cents per bushel); t_z = marginal cost of trading options (cents per bushel).

base case with moderate risk aversion. Table V displays the hedging demands for Southeastern Pennsylvania with an in-the-money strike price of 253 cents per bushel. Options demand is higher for all combinations of t_x and t_z below the diagonal. In-the-money options with unbiased premiums are preferred because they offer more risk reduction than out-of-the-money options with no loss in expected value.

This result depends on how fast the cost of options changes as the strike price deviates from the current futures price. The marginal cost of supplying options away from the money may be substantial because of low liquidity. Deep in-the-money options will have a high marginal cost, and shallow in-the-money options will be preferred by input hedgers. The supply curve for options at different strike prices is left for future research.

The hedging demands discussed in the last section generate hedger's surplus estimates that depend on t_x, t_z, and the other parameters of the model. Table VI displays the hedger's surplus estimates under moderate and high levels of risk aversion for each of the five Pennsylvania regions. Hedger's surplus is displayed for futures (e_x), options (e_z), and total (e_{xz}). All surplus estimates are zero under low risk aversion, indicating that the opportunity to hedge is valueless or nearly valueless for hedgers with low risk aversion. Under moderate risk aversion, the Southeastern region had the highest hedger's surplus at 1.7 cents per bushel, and the Western, Central, and South Central regions had a hedger's surplus of zero for

TABLE VI
Hedger's Surplus

Location	*Moderate Risk Aversion*			*High Risk Aversion*		
	e_x	e_z	e_{zx}	e_x	e_z	e_{zx}
Southeastern Pennsylvania	0.088	0.052	1.685	0.937	0.005	23.002
Central Pennsylvania	0	0.049	0.190	0.260	0.005	3.909
South Central Pennsylvania	0	0.051	0.220	0.285	0.005	4.396
Western Pennsylvania	0	0.041	0.111	0.181	0.005	2.515
Lehigh Valley	0.059	0.052	1.324	0.821	0.005	19.104

Note. t_x = 1.5 cents per bushel; t_z = 0.5 cents per bushel; e_x = futures surplus (cents per bushel); e_z = futures surplus (cents per bushel); and e_{xz} = total surplus (cents per bushel).

futures. Under high risk aversion, the total surplus ranged from 2.5 cents per bushel to 23.0 cents per bushel.

The covariance structure between the Chicago price and each regional basis is the main determinant of the level of hedger's surplus estimated. Lower strike prices generated higher estimates of hedger's surplus, but changing the strike price did not affect the character of the hedger's surplus estimates. Quantitative comparisons are difficult to make because all surplus estimates are conditioned on the same fixed values for $t_x = 1.5$ cents per bushel and $t_z = 0.5$ cents per bushel, which may vary with the strike price as noted previously.

An important qualitative comparison can be made between e_x and e_z. The value of e_x is the value of the opportunity to trade futures if options are available to trade. The value of e_z is the value of the opportunity to trade options if futures are available to trade. In most cases, e_x and e_z are very small. For example, e_x is 0.088 cents per bushel and e_z is 0.052 cents per bushel for the Southeastern region at a moderate level of risk aversion. The surplus values are very small, indicating that futures and options are close substitutes.

Another qualitative comparison can be drawn between e_x and e_z on the one hand and e_{xz} on the other. The value of e_{xz} is the value of the opportunity to trade futures or options compared with the alternative of neither. At the moderate level of risk aversion, the Southeastern region had an e_{xz} value of 1.685 cents per bushel. Compared with 0.088 for e_x and 0.052 for e_z, 1.685 seems very large. This result indicates that hedgers place a relatively high value on the opportunity to hedge, but they are nearly indifferent between futures and options.

At a high level of risk aversion, hedging becomes more valuable to hedgers. The value of e_{xz} rises dramatically to 23.002 cents per bushel for the Southeastern region. The value of e_x rises to 0.937, and the value of e_z falls to 0.005, indicating that the opportunity to hedge with futures is more valuable than the opportunity to hedge with options for the given parameter values. The difference between e_x and e_z may be due to the hedger's assumed preference against upside and downside variance. This result indicates that the expected utility model may need to be reconsidered for studies of this sort in the future. It might be replaced by a model that more adequately represents hedgers' preference against downside risk without penalizing upside risk.

IMPLICATIONS AND CONCLUSIONS

There are several practical implications of these results. First, LMH's conclusion that options play no role in the optimal hedging portfolio needs to be further conditioned and clarified. LMH made it clear that they assumed markets were unbiased, and they explained how biased expectations may lead to different results. However, they did not consider that hedging is costly. In most real situations faced by hedgers, the optimal mix of futures and options will not be one-sided.

Second, the overall hedge ratio falls when the marginal cost of trading options increases, but in many cases it actually rises when the marginal cost of trading futures increases. The optimal position in those cases is an overhedged position that declines as the level of risk aversion increases. These counterintuitive conclusions indicate that overhedging may not always be speculative.

Third, options hedging is more sensitive to price than futures hedging. Hedging demand can be stimulated most easily by a reduction in the marginal cost of options trading. Such a reduction can be accomplished by a reduction in learning costs and management costs through the education of hedgers about options. It can also be stimulated directly by options being subsidized and hedgers being allowed to learn by experience, as with the Dairy Options Pilot Program. Targeting options is likely to have more effect on the overall hedge ratio than targeting futures.

Fourth, futures and options are highly substitutable, although not one for one. The hedger's surplus forgone by ignoring one market or the other is small in most cases, although the hedger's surplus forgone by ignoring both markets is much higher. Low levels of risk aversion make hedging unimportant and reduce the hedger's surplus to zero. For high and moderate levels of risk aversion, however, hedgers are nearly indifferent between the optimal futures-only strategy and the optimal options-only strategy. The optimal futures-only strategy involved a lower hedge ratio than the optimal options-only hedge ratio.

Overall, the relationship between futures and options still needs to be explored. The supply curve for hedging products needs to be developed in more detail. The relationship among the marginal costs of hedging options with different strike prices must be understood more fully. Alternative decision rules to expected utility should be considered to account properly for asymmetric attitudes toward upside and downside risk. Future research along these lines may contribute to a better understanding of hedging behavior and the development of more effective risk-management strategies for hedgers.

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