A NOTE ON LOSS AVERSION AND FUTURES HEDGING

DONALD LIEN

This note examines the effect of loss aversion on the futures trading behavior of a short hedger. Using a modified constant-absolute-risk-aversion utility function, I show that loss aversion has no effect in an unbiased futures market. It has different, predictable impacts when the futures market is in backwardation or contango. ©2001 John Wiley & Sons, Inc. Jrl Fut Mark 21: 681–692, 2001

INTRODUCTION

The traditional analysis of futures trading relies on the expected utility framework. The mean-variance approach is then a special case. The popular minimum-variance hedge ratio is derived under this approach for an extremely risk-averse hedger. Kahneman and Tversky (1979) and Tversky and Kahneman (1986, 1992), however, argued that individuals do not act to maximize expected utility; instead, they behave according to a set of rules collectively known as *prospect theory.* One of the rules suggested that individuals care for changes in wealth, instead of wealth itself. Loss aversion is another important rule.[1]

[1]Rabin (1998) provided and discussed in detail several psychological findings relevant to economic analysis.

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For correspondence, Donald Lien, Department of Economics, University of Kansas, Lawrence, Kansas 66045-2113; e-mail: dlien@ukans.edu

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■ *Donald Lien is a professor in the Department of Economics at the University of Kansas in Lawrence, Kansas.*

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There are two versions of loss aversion. Strong loss aversion states that individuals tend to be more sensitive to wealth losses than to wealth gains. In the literature, generally a risk-neutral utility function is assumed. Imposing strong loss aversion decomposes the utility function into two lines, one for gains and the other for losses, such that the slope for the latter is larger than that for the former. This approach assumes constant sensitivity to losses. However, weak loss aversion accounts for diminishing sensitivity to losses. That is, an individual is less sensitive to additional losses when the loss is large. As a result, the individual becomes risk-loving over losses, and the corresponding utility function is convex.

Thaler, Tversky, Kahneman, and Schwartz (1997) and Gneezy and Potters (1997) provided consistent support for loss aversion through experimental studies. Odean (1998) showed investors are reluctant to realize losses. Heisler (1996) and Locke and Mann (1999) established loss aversion for speculators and floor traders on futures markets, respectively. In particular, they established the so-called disposition effect, an excessive tendency to sell winners and hold losers, from loss aversion.

Benartzi and Thaler (1995) discussed the implications of loss aversion for equity premium puzzles. Langer and Weller (1997) considered the effects of loss aversion for collateralization. Bowman, Minehart, and Rabin (1999) found that loss aversion helps to explain some stylized facts in a consumption-saving model. Shumway (1997) derived an asset pricing formula under loss aversion and found loss aversion helps to explain asset returns.

To my knowledge, Albuquerque (1999) is the only one until now to consider futures hedging under loss aversion. He adopted a linear utility function. Consequently, loss aversion simply reduces to downside risk minimization.[2] This note attempts to explore the effects of loss aversion on futures hedging with constant-absolute-risk-aversion (CARA) utility functions. More specifically, I investigate the response of the optimal futures position once loss aversion is incorporated into the utility function.

[2]Two other articles by Gomes (2000) and Berkelaar and Kouwenberg (2000) are related to our problem. Gomes considered a risk-free asset along with a risky asset. He adopted a constant-relative-risk-aversion (CRRA) utility function. To derive an analytical solution within a one-period framework, he assumed there are only two possible states for the price of the risky asset. Berkelaar and Kouwenberg considered a general optimal portfolio choice with loss-averse investors. They assumed CRRA utility functions and a continuous-time framework. Asset prices are described by geometric Brownian motions.

UTILITY FUNCTIONS

To isolate loss aversion from risk aversion, many authors adopted a linear utility function such that $U(x) = x$ when $x \geq 0$ and $U(x) = kx$ when $x \leq 0$, $k > 1$. Thus, marginal utility is 1 when there are gains and greater than 1 when there are losses. The individual is, therefore, more sensitive to losses than to gains. Also, there is constant sensitivity to losses.

Tversky and Kahneman (1992) considered the following utility function: $V(x) = x^{\alpha}$; when $x \geq 0$ and $V(x) = -\lambda(-x)^{\beta}$ when $x \leq 0$, where $\lambda > 0$. On the basis of experimental evidence, they provided estimates for α, β, and λ as 0.88, 0.88, and 2.25, respectively. Shumway (1997) adopted this specification, with the condition $\alpha = \beta$, in his analysis. The utility function is concave when there are gains and convex when there are losses.

This specification begins with a CRRA utility function, which is only well-defined for positive arguments. In the finance literature, it is common to consider a CARA utility function. To satisfy loss aversion, consider the following utility function: $W(x) = 1 - \exp(-\theta x)$ when $x \geq 0$ and $W(x) = -\lambda[1 - \exp(\theta x)]$ when $x \leq 0$, where θ and λ are greater than 0. Similar to $V(x)$, $W(x)$ is concave in gains and convex in losses.[3] Although I retain the name for θ, risk aversion, from the conventional CARA utility function, it has a different meaning in this framework. In particular, an increase in θ does not imply the individual becomes more risk-averse. However, the utility function becomes more concave in gains but also becomes more convex in losses. The effects of θ on optimal decisions, therefore, cannot be inferred from the CARA framework.

To analyze the effects of loss aversion, I compare the optimal futures positions before and after loss aversion is incorporated into the utility function. Thus, $U(x)$ is compared to a linear utility function, $V(x)$ is compared to the CRRA utility function, and $W(x)$ is compared to the CARA utility function. The first case is of limited interest because linear utility leads to an unlimited position in futures. The CRRA utility function is not suitable for this purpose because it is not defined when x is nonpositive. Therefore, I consider only the CARA case.

[3]When $x < 0$, $W'(x) = \lambda\theta\exp(\theta x) > 0$ and $W''(x) = \lambda\theta^2\exp(\theta x) > 0$. Thus, $W(x)$ is convex when x is negative. Suppose that $x > 0$, $W'(x) = \theta\exp(-\theta x) > 0$ and $W''(x) = -\theta^2\exp(-\theta x) < 0$. That is, $W(x)$ is concave when x is positive. There is a kink point at the origin unless $\lambda = 1$. Note that $W'(-x) = \lambda W'(x)$ when $x > 0$, implying greater sensitivity to losses. Although the CRRA-generated utility function $V(x)$ in not suitable for our study here, $V(x)$ shares the same properties as $W(x)$.

BASIC FRAMEWORK

Consider a one-period model. An individual who has committed to a unit of spot position needs to choose an optimal futures position to maximize his expected utility. Let Δp and Δf denote changes in spot and futures prices, respectively. Given the initial wealth W_0, the end-of-period wealth is

$$W_1 = W_0 + \Delta p + Q\Delta f \tag{1}$$

where Q is the futures position. The individual is long in the futures market when $Q > 0$ and short when $Q < 0$. A larger Q implies that the individual buys more or sells fewer futures contracts. The utility function for the hedger is $u(W_1)$. Let $E(\cdot)$ denote the expectation operator with respect to Δp and Δf. The individual will choose the optimal futures position, Q^*, so that $E[u(W_1)]$ achieves a maximum.

To proceed further, it is assumed that Δp and Δf are jointly normally distributed. I first consider the CARA utility function:

$$u(x) = 1 - \exp(-\theta x), \quad \theta > 0 \tag{2}$$

so that $u(0) = 0$. Given the normality assumptions and the CARA utility specification, the solution to the expected utility maximization problem is the same as that derived from the mean-variance approach. That is, the optimal futures position is chosen to maximize the mean-variance function, $E(W_1) - (\theta/2)\mathrm{Var}(W_1)$. Equivalently, it is chosen to maximize $\mu - (\theta/2)\sigma^2$ where $\mu = E(\Delta p + Q\Delta f)$ and $\sigma^2 = \mathrm{Var}(\Delta p + Q\Delta f)$. Let Q^{mv} denote the optimal futures position under the CARA utility function.

EFFECTS OF LOSS AVERSION

When loss aversion is incorporated, the appropriate utility function is $W(x)$. The prospect theory suggests that x should be the change in wealth, that is, $W_1 - W_0$. Thus, under loss aversion the individual chooses the optimal futures position to maximize $EW(x) = EW(\Delta p + Q\Delta f)$, where $x = \Delta p + Q\Delta f$ is a normal random variable such that $E(x) = \mu$ and $\mathrm{Var}(x) = \sigma^2$. Denote the solution Q^{la}. We are interested in the relationship between Q^{mv} and Q^{la}, the optimal futures positions before and after loss aversion is taken into account.

Let $f(x)$ denote the probability density function of x. To evaluate $E[W(x)]$, note that

$$W^- = \int_{-\infty}^{0} [1 - e^{\theta x}] f(x)\, dx$$

$$= \Phi\left(\frac{-\mu}{\sigma}\right) - \Phi\left(\frac{-\mu - \theta\sigma^2}{\sigma}\right) \exp\left(\theta\mu + \frac{\theta^2\sigma^2}{2}\right) \tag{3a}$$

$$W^+ = \int_{0}^{\infty} [1 - e^{-\theta x}] f(x)\, dx$$

$$= \Phi\left(\frac{\mu}{\sigma}\right) - \Phi\left(\frac{\mu - \theta\sigma^2}{\sigma}\right) \exp\left(-\theta\mu + \frac{\theta^2\sigma^2}{2}\right) \tag{3b}$$

where Φ is the cumulative density function of a standard normal random variable. Note that $EW(x) = W^+ - \lambda W^-$.

Before proceeding with the general case, consider a scenario in which both spot and futures markets are unbiased. Thus, $E(\Delta p) = E(\Delta f) = \mu = 0$. Consequently,

$$EW(x) = \frac{1 - \lambda}{2} + (\lambda - 1)\Phi(-\theta\sigma)\exp\left(\frac{\theta^2\sigma^2}{2}\right) \tag{4}$$

when $\lambda = 1$ and $EW(x) = 0$ regardless of the futures position. From Equation 4, we have

$$\frac{\partial EW(x)}{\partial\sigma} = \theta(\lambda - 1)\exp\left(\frac{\theta^2\sigma^2}{2}\right)[-\phi(-\theta\sigma) + \theta\sigma\Phi(-\theta\sigma)] \tag{5}$$

The term within the bracket is always negative.[4] Hence, $EW(x)$ increases as σ decreases when $\lambda > 1$. The optimal futures position, Q^{la}, is then chosen to minimize the variance of x. Thus, $Q^{la} = Q^{mv}$. However, if $\lambda < 1$, $EW(x)$ decreases with decreasing σ. Consequently, the optimal futures position, Q^{la}, should maximize the variance of x. Proposition 1 summarizes the results:

Proposition 1: Suppose that both spot and futures markets are unbiased. Loss aversion has no impact on the optimal futures position when $\lambda > 1$. If $\lambda < 1$, the solution does not exist when loss aversion is taken into account.

[4]Let $z = -\theta\sigma$. Consider $H(z) = \phi(z) + z\Phi(z)$. It can be shown that $H'(z) = \Phi(z) > 0$ and $H(z)$ approaches zero when z approaches negative infinity. Thus, $H(z) > 0$. The term within the brackets of Equation 5 is simply $-H(z)$.

Note that λ measures the sensitivity to losses (relative to gains) at $x = 0$. Loss aversion requires $\lambda > 1$, implying that the concavity on the gain region dominates the convexity on the loss region. As a result, the optimal futures position remains unchanged. In fact, Proposition 1 can be easily extended to the case in which only the futures market is unbiased. Underlying this assumption, $\partial\mu/\partial Q = 0$ and, therefore, $\partial EW(x)/\partial Q = [\partial EW(x)/\partial\sigma](\partial\sigma/\partial Q)$.

CONTANGO AND BACKWARDATION

We return to the more general case. From Equations 3a and 3b, after algebraic manipulation, it can be shown that

$$\frac{\partial EW(x)}{\partial\mu} = \theta\Phi\left(-\theta\sigma + \frac{\mu}{\sigma}\right)\exp\left(-\theta\mu + \frac{\theta^2\sigma^2}{2}\right) + \lambda\theta\Phi\left(-\theta\sigma - \frac{\mu}{\sigma}\right)\exp\left(\theta\mu + \frac{\theta^2\sigma^2}{2}\right) \tag{6}$$

$$\frac{\partial EW(x)}{\partial\sigma} = k - \theta^2\Phi\left(-\theta\sigma + \frac{\mu}{\sigma}\right)\exp\left(-\theta\mu + \frac{\theta^2\sigma^2}{2}\right) + \lambda\theta^2\Phi\left(-\theta\sigma - \frac{\mu}{\sigma}\right)\exp\left(\theta\mu + \frac{\theta^2\sigma^2}{2}\right) \tag{7}$$

$$k = (1 - \lambda)\theta\phi\left(\frac{\mu}{\sigma}\right) \tag{8}$$

where $\phi(\cdot)$ is the probability density function of a standardized normal random variable.[5] Consequently, the optimal futures position (if it exists) must satisfy the following condition:

$$\frac{\partial EW(x)}{\partial Q} = (1 - \lambda)\theta\phi\left(\frac{\mu}{\sigma}\right)\frac{\partial\sigma}{\partial Q} + k_1\frac{\partial[\mu - (\theta/2)\sigma^2]}{\partial Q} + k_2\frac{\partial\mu}{\partial Q} = 0 \tag{9}$$

[5]In deriving Equation 6, we used the following property:

$$\phi\left(\frac{-\mu}{\sigma}\right) = \exp\left(\theta\mu + \frac{\theta^2\sigma^2}{2}\right)\phi\left(-\theta\sigma - \frac{\mu}{\sigma}\right)$$

at Q^{la}, where

$$k_1 = \theta\Phi\left(-\theta\sigma + \frac{\mu}{\sigma}\right)\exp\left(-\theta\mu + \frac{\theta^2\sigma^2}{2}\right) - \lambda\theta^2\Phi\left(-\theta\sigma - \frac{\mu}{\sigma}\right)\exp\left(\theta\mu + \frac{\theta^2\sigma^2}{2}\right) \tag{10}$$

$$k_2 = 2\lambda\theta\Phi\left(-\theta\sigma - \frac{\mu}{\sigma}\right)\exp\left(-\theta\mu + \frac{\theta^2\sigma^2}{2}\right) \tag{11}$$

Recall that from Proposition 1, $Q^{la} = Q^{mv}$ when $\mu = 0$ [i.e., $E(\Delta p) = E(\Delta f) = 0$]. If both $E(\Delta p)$ and $E(\Delta f)$ are small, the solution will exist in the presence of loss aversion. That is, $\partial^2 EW(x)/\partial Q^2$ is negative at Q^{la}, which is in a small neighborhood of Q^{mv}.

We can now evaluate $\partial EW(x)/\partial Q$ at Q^{mv}. Note that $\partial[\mu - (\theta/2)\sigma^2]/\partial Q = 0$ at Q^{mv}, the optimal futures position under the CARA utility, implying $\partial\sigma/\partial Q = (\partial\mu/\partial Q)/\theta\sigma = E(\Delta f)/\theta\sigma$ at Q^{mv}. Consequently,

$$\frac{\partial EW(x)}{\partial Q} = \left[(1 - \lambda)\phi\left(\frac{\mu}{\sigma}\right)\frac{1}{\sigma} + 2\lambda\theta\Phi\left(-\theta\sigma - \frac{\mu}{\sigma}\right) \times \exp\left(-\theta\mu + \frac{\theta^2\sigma^2}{2}\right)\right]E(\Delta f) \tag{12}$$

when $Q = Q^{mv}$. Because $\lambda > 1$, the first term within the bracket is negative. The second term is, however, positive. When λ and θ are both small, the first term dominates the second term. The sum becomes negative. Consequently, $\partial EW(x)/\partial Q$ is positive when $E(\Delta f)$ is positive and vice versa. Using the second-order condition and the condition that $\lambda > 1$, we establish Proposition 2:

Proposition 2: Suppose that both λ and θ are small. When the futures market is in backwardation, that is, $E(\Delta f) > 0$, loss aversion induces the hedger to buy fewer (or sell more) futures contracts, $Q^{la} < Q^{mv}$. However, when the futures market is in contango, that is, $E(\Delta f) < 0$, loss aversion induces the hedger to buy more (or sell fewer) futures contracts, $Q^{la} > Q^{mv}$.

Suppose that $E(\Delta f) > 0$. Although the individual will assume a short futures position to hedge the spot position, the speculative motives lead him to undertake some long futures positions as he expects to earn profits on these positions (buy low and sell high). Thus, the short hedger does not hedge as much as that dictated by the risk concerns. A loss-averse individual, however, is sensitive to losses (in particular, moderate and large losses). An expected profitable position may incur losses.

Consequently, the loss-averse individual will buy fewer (or sell more) futures contracts. However, when $E(\Delta f) < 0$, a short position is expected to generate profits. In response, the short hedger will sell more futures contracts than that prescribed by risk considerations. However, because of the sensitivity to losses, a loss-averse individual will sell fewer (or buy more) futures contracts.

Proposition 2 is consistent with the Mean-Variance (MV) framework with increased risk aversion. At Q^{mv}, $\partial[\mu - (\theta/2)\sigma^2]/\partial Q = 0$. By comparative static analysis,

$$\frac{\partial Q^{mv}}{\partial \theta} = \frac{(1/2)(\partial \sigma^2/\partial Q)}{\partial^2[\mu - (\theta/2)\sigma^2]/\partial Q^2} \tag{13}$$

From the second-order condition, the sign of $\partial Q^{mv}/\partial \theta$ is opposite to the sign of $\partial \mu/\partial Q$. Equivalently, the sign of $\partial Q^{mv}/\partial \theta$ is opposite to the sign of $E(\Delta f)$. When $E(\Delta f) > 0$, an increase in θ leads to a reduction in Q^{mv}. However, when $E(\Delta f) < 0$, Q^{mv} increases with increasing θ. This interpretation should be treated with caution, as in this framework an increase in θ does not necessarily imply the hedger becomes more risk-averse.

We now return to Equation 12. When λ and θ are sufficiently large, the second term within the bracket dominates the first term. As a result, $\partial EW(x)/\partial Q$ is negative when $E(\Delta f)$ is positive and vice versa. Proposition 3 follows:

Proposition 3: Suppose that λ and θ are sufficiently large. When the futures market is in backwardation, that is, $E(\Delta f) > 0$, loss aversion induces the hedger to buy more (or sell fewer) futures contracts, $Q^{la} > Q^{mv}$. However, when the futures market is in contango, that is, $E(\Delta f) < 0$, loss aversion induces the hedger to buy fewer (or sell more) futures contracts, $Q^{la} < Q^{mv}$.

When λ and θ are both sufficiently large, the individual's utility decreases drastically if a small loss is incurred. To reduce the possibility of small losses, the individual becomes more speculative. Thus, fewer futures contracts will be sold when $E(\Delta f) > 0$, as a long position is expected to be profitable. Similarly, when $E(\Delta f) < 0$, more futures contracts will be sold, as a short position is expected to be profitable.

The conclusions of Proposition 3 contrast with those derived from the MV analysis with increased risk aversion. As pointed out previously, with an increase in θ the utility function becomes more concave in gains and more convex in losses. When λ is large, the latter effect becomes more dominant and henceforth produces results contradictory to the MV analysis.

Propositions 2 and 3 were established for the case in which both $E(\Delta p)$ and $E(\Delta f)$ are small such that $EW(x)$ is locally concave in the neighborhood of Q^{mv}. However, the two propositions hold for the general case as long as the expected utility function $EW(x)$ is globally concave in Q.

NUMERICAL RESULTS

To gain more insights, let us conduct some numerical exercises. The parameter values are chosen as follows: $E(\Delta p) = 0.005$ or -0.005; $E(\Delta f) = E(\Delta p)$; $\text{Cov}(\Delta p, \Delta f) = 0.9$ or 0.95; $\text{Var}(\Delta p) = \text{Var}(\Delta f) = 1$; $\theta = 10$, 1, or 0.1; and $\lambda = 2$ or 3. Table I summarizes some numerical results.

The cases in which $\theta = 10$ or 1 and $\lambda = 2$ or 3 fall into the category described in Proposition 3. Hence, $Q^{la} > Q^{mv}$ when $E(\Delta f) > 0$ and $Q^{la} < Q^{mv}$ when $E(\Delta f) < 0$. For example, suppose that $\text{Cov}(\Delta p, \Delta f) = 0.90$. When $E(\Delta f) = 0.005$, $\theta = 10$, $\lambda = 2$, $Q^{la} = -0.87$, and $Q^{mv} = -0.90$; when $E(\Delta f) = -0.005$, $\theta = 10$, $\lambda = 3$, $Q^{la} = -0.92$,

TABLE I
Numerical Results

$E(\Delta p)$	$E(\Delta f)$	$\text{Cov}(\Delta p, \Delta f)$	θ	λ	Q^{la}	Q^{mv}
0.005	0.005	0.9	10	2	−0.87	−0.90
0.005	0.005	0.9	10	3	−0.88	−0.90
0.005	0.005	0.9	1	2	−0.89	−0.895
0.005	0.005	0.9	1	3	−0.894	−0.895
0.005	0.005	0.9	0.1	2	−0.892	−0.85
0.005	0.005	0.9	0.1	3	−0.894	−0.85
0.005	0.005	0.95	10	2	−0.934	−0.95
0.005	0.005	0.95	10	3	−0.938	−0.95
0.005	0.005	0.95	1	2	−0.944	−0.946
0.005	0.005	0.95	1	3	−0.946	−0.946
0.005	0.005	0.95	0.1	2	−0.944	−0.90
0.005	0.005	0.95	0.1	3	−0.946	−0.90
−0.005	−0.005	0.9	10	2	−0.932	−0.90
−0.005	−0.005	0.9	10	3	−0.92	−0.90
−0.005	−0.005	0.9	1	2	−0.91	−0.906
−0.005	−0.005	0.9	1	3	−0.906	−0.906
−0.005	−0.005	0.9	0.1	2	−0.908	−0.95
−0.005	−0.005	0.9	0.1	3	−0.906	−0.95
−0.005	−0.005	0.95	10	2	−0.968	−0.95
−0.005	−0.005	0.95	10	3	−0.962	−0.95
−0.005	−0.005	0.95	1	2	−0.956	−0.954
−0.005	−0.005	0.95	1	3	−0.954	−0.954
−0.005	−0.005	0.95	0.1	2	−0.956	−1.0
−0.005	−0.005	0.95	0.1	3	−0.954	−1.0

Note: $E(\Delta p)$ = expected change in spot prices; $E(\Delta f)$ = expected change in futures prices; $\text{Cov}(\Delta p, \Delta f)$ = covariance between change in spot prices and change in futures prices; θ = risk-aversion coefficient; λ = loss-aversion coefficient; Q^{la} = optimal futures position under loss aversion; Q^{mv} = optimal futures position without loss aversion.

and $Q^{mv} = -0.90$. However, the cases in which $\theta = 0.1$ satisfy Proposition 2. That is, $Q^{la} < Q^{mv}$ when $E(\Delta f) > 0$ and $Q^{la} > Q^{mv}$ when $E(\Delta f) < 0$. For example, when $E(\Delta f) = 0.005$, $\theta = 0.1$, $\lambda = 3$, $Q^{la} = -0.894$, and $Q^{mv} = -0.85$; when $E(\Delta f) = -0.005$, $\theta = 0.1$, $\lambda = 2$, $Q^{la} = -0.908$, and $Q^{mv} = -0.95$.

From Table I, we can observe the following: (a) The hedger trades more in the futures market (i.e., the absolute value of Q^{la} increases) as the covariance between change in spot prices and change in futures prices increases. This result is expected as a larger covariance (which is also the correlation in the numerical setup) improves the hedging effectiveness of the futures contract. (b) If $E(\Delta f) > 0$, the hedger trades more when θ decreases or λ increases. In this case, the hedger expects to make profits on long positions. If he is less risk-averse, he will buy more futures contracts. When he is more loss-averse, he also buys more contracts to avoid small losses. (c) However, if $E(\Delta f) < 0$, the hedger trades more when θ increases or λ decreases. Herein, the short positions are expected be profitable. The hedger will buy fewer contracts due to speculative motives. However, he also has to take into account the risk in the spot position. When θ increases, the risk concern dominates the speculative motives. As a result, the hedger buys more futures contracts. However, an increasing λ promotes the speculative motives, leading the hedger to buy fewer contracts.

For a direct hedge, the assumption that $E(\Delta p) = E(\Delta f)$ seems reasonable. For a cross hedge, the relationship between $E(\Delta p)$ and $E(\Delta f)$ may be arbitrary. Consider the following model: $\Delta p = -0.0095 + 0.9\Delta f + \varepsilon$, where ε is a disturbance term with $E(\varepsilon) = 0$, $\text{Var}(\varepsilon) = 0.19$, and $\text{Cov}(\varepsilon, \Delta f) = 0$. Suppose that $E(\Delta f) = 0.005$ and $\text{Var}(\Delta f) = 1$. This model implies $E(\Delta p) = -0.005$, $\text{Var}(\Delta p) = 1$, and $\text{Cov}(\Delta p, \Delta f) = 0.9$. I found in this case that $Q^{la} = -0.856$ and $Q^{mv} = -0.90$. However, Table I shows that $Q^{la} = -0.87$ and $Q^{mv} = -0.90$ when $E(\Delta p) = 0.005$ (and other parameter values remain unchanged). That is, as the spot return increases, the optimal futures position under loss aversion decreases when $E(\Delta f) > 0$. Consequently, the difference between Q^{mv} and Q^{la} shrinks when the spot return increases. Similar simulations show that the optimal futures position under loss aversion increases with increasing spot return when $E(\Delta f) < 0$. The gap between Q^{mv} and Q^{la} is then enlarged.

SOME FURTHER REMARKS

This framework can be easily extended to determine the optimal spread positions. Herein Δp and Δf are changes in the prices of two different futures contracts. The previous result provides the optimal spread ratio. A further analysis helps to determine the exact optimal spread positions.

Following the prospect theory, I define utility in term of change in wealth, $W_1 - W_0$. Traditionally, however, the utility is defined over the terminal wealth, W_1. In this case, the CARA loss-averse utility function is as follows: $W(x) = 1 - \exp[-\theta(W_0 + \Delta p + Q\Delta f - L)]$ if $W_1 \geq L$ and $W(x) = -\lambda\{1 - \exp[\theta(W_0 + \Delta p + Q\Delta f - L)]\}$ if $W_1 < L$, where L is the reference point. Note that $L = W_0$ corresponds to the prospect theory. In other words, prospect theory adopts the status quo as the reference point.

To consider the general case, let $\Delta q = W_0 - L + \Delta p$. Then, we have $W(x) = 1 - \exp[-\theta(\Delta q + Q\Delta f)]$ when $\Delta q + Q\Delta f \geq 0$ and $W(x) = -\lambda\{1 - \exp[-\theta(\Delta p + Q\Delta f])\}$ when $\Delta q + Q\Delta f < 0$. This utility function is similar to the one under prospect theory with Δq replacing Δp. Consequently, all three propositions derived under prospect theory extend to the case when the utility is defined over the terminal wealth. However, as the reference point (L) increases, $E(\Delta q)$ decreases. Table I indicates that the difference between Q^{mv} and Q^{la} will enlarge accordingly. That is, loss aversion has more impacts on futures hedging when the reference point improves. Intuitively, when the reference point improves, a larger portion of the CARA utility function has to be modified to account for loss aversion. The loss-averse utility function differs from the CARA utility function to a greater extent. Consequently, the two optimal hedge ratios are farther apart.

CONCLUSIONS

I examined the effect of loss aversion on the futures trading behavior of a short hedger. I began with a CARA utility function and modified the functional form to incorporate loss aversion. When comparing the optimal futures positions under each utility function, I found that in an unbiased futures market, loss aversion has no effect at all. Under backwardation, loss aversion leads to less active trading when the hedger is very risk-averse and loss-averse and vice versa. Opposite results were established for contango.

Finally, I characterized the effects of the risk-aversion coefficient and the loss aversion coefficient on futures trading. Under backwardation, a smaller risk-aversion coefficient or a larger loss-aversion coefficient increases trading volume. For contango, a larger risk-aversion coefficient and a smaller loss-aversion coefficient promotes futures trading.

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