HEDGING IN INCOMPLETE MARKETS: AN APPROXIMATION PROCEDURE FOR PRACTICAL APPLICATION

WOLFGANG BREUER*
MARC GÜRTLER

Derivative financial instruments are frequently used as a tool for influencing the risk of entrepreneurial uncertain payoff. To this end, an approximation procedure is developed capable of calculating the optimal quantity of derivatives to be used. It is assumed that the entrepreneurial cash flow is governed by several stochastic factors and that derivatives are only available as a hedging tool for one of these factors. In general, it is easy to determine optimal hedging payment structures with respect to this factor, but real-life hedging opportunities will typically not allow to perfectly reproduce such a fictitious payment structure, thus leading to complex numerical optimization problems. Instead of directly approximating the entrepreneurial expected utility maximum, we suggest using the

The authors thank the anonymous referee for constructive comments. Of course, all remaining errors are the authors own.
*Correspondence author, Department of Finance, Aachen University of Technology, Templergraben 64, 52056 Aachen, Germany; e-mail: wolfgang.breuer@rwth-aachen.de

Received May 2000; Accepted September 2000

- *Wolfgang Breuer is a Professor in the Department of Finance at Aachen University of Technology in Aachen, Germany.*
- *Marc Gürtler is an Assistant Professor in the Department of Finance at Aachen University of Technology in Aachen, Germany.*

The Journal of Futures Markets, Vol. 21, No. 7, 599–631 (2001)

fictitious optimal hedging payment structure as a starting point and to minimize the quadratic deviation between payment structures realizable by financial derivatives actually available and the resulting entrepreneurial payoff achieving the fictitious optimal hedging payment structure. This approach proves to be rather easy. Indeed, under certain conditions an explicit solution can be reached. After analyzing the qualitative properties of our approximation solution, we examine its efficiency for two practical hedging problems. In the first example, we get nearly the same solutions with our approximation procedure as with a grid programming approach presented by some other authors. Among other things, our second example may explain why some special kinds of financial derivatives, known as *shared currency option under tenders,* are not used in international invitations for tenders even though they offer hedging opportunities that are otherwise not available. Jrl Fut Mark 21: 599–631, 2001

INTRODUCTION

Many economic activities lead to uncertain future payment structures that are governed by a number of stochastic factors. In such a situation, hedging is generally understood as the optimization of cash flows by the use of derivative financial instruments without expected payoffs being affected.[1] However, in incomplete markets, derivatives will not be available for all uncertain variables so that a perfect hedge leading to riskless future overall earnings will in general not be possible.

In a model context, this situation can be interpreted as a case in which an investor expects as a result of his business activities a payoff amounting to $f(\tilde{\gamma}, \tilde{r})$ at a future point in time $t = 1$. $\tilde{\gamma}$ and $\tilde{r}$ describe two uncertain variables influencing cash flows, whereby derivative instruments only are available on the basis of $\tilde{r}$ so that hedging opportunities are incomplete.[2] In this situation, we denote with $h(\tilde{r})$ the payoff structure of such a fictitious financial instrument that maximizes the investor's expected utility among all payoff structures $h^+(\tilde{r})$ with zero expectation:

$$E[U(f(\tilde{\gamma}, \tilde{r}) + h^+(\tilde{r}))] \rightarrow \max_{h+(.)}! \text{ s.t. } E[h^+(\tilde{r})] = 0. \qquad (1)$$

[1]Because in this context the use of derivative financial instruments does not change expectation values, hedging can be considered a recommendation for minimizing risk. For such an understanding of hedging, see Benninga, Eldor, and Zilcha (1983), and for an adequate interpretation of risk, see Rothschild and Stiglitz (1970).

[2]The model can easily be extended to N different nonhedgable random variables, $\tilde{\gamma}_1 \ldots \tilde{\gamma}_N$. For reasons of simplicity, only one random variable $\tilde{\gamma}$ will be assumed here.

Typically, it will not be possible to combine existing derivatives i with uncertain cash flows $f^{(i)}(\tilde{r})(i = 1 \ldots n)$ in such a way as to generate the payoff structure $h(\tilde{r})$. Thus, hedging opportunities will be incomplete not only with respect to $\tilde{y}$ but also with respect to $\tilde{r}$.

Consider, for example, the context of a single-index model where an investor has at his disposal a particular portfolio of assets with returns depending on the return on the market portfolio $\tilde{r}_M$. Moreover, business activities could generate further changes in payment structures summarized by $\tilde{y}$ for which, however, derivatives are not traded in the market. The investor would supplement the uncertain cash flow $f(\tilde{y}, \tilde{r}_M)$ with derivatives according to his individual preferences. As he may only have available to him forwards and options based on the market index, he would have to calculate the optimal number $x^{(1)}$ and $x^{(2)}$ of each of the two derivatives, thereby failing to achieve the fictitious optimal payment structure $h(\tilde{r}_M)$.

Furthermore, such a scenario characterizes particularly well situations where entrepreneurial earnings are exposed to quantity and price risks, but only price risk can be hedged. A concrete case would be one in which an entrepreneur would want to produce a certain quantity of a good to offer in the market. The uncertain sales volume $\tilde{y}$ would of course depend on the price $\tilde{p}$. If production costs $C(\tilde{y})$ at time $t = 1$ are taken into account, an uncertain net payment $f(\tilde{y}, \tilde{p}) = \tilde{p} \cdot \tilde{y} - C(\tilde{y})$ is generated. The entrepreneur could then use, if available, forwards and put options with underlying uncertain price $\tilde{p}$ in the market to safeguard against possible fluctuations in price,[3] whereas the uncertain sales volume $\tilde{y}$ cannot be hedged directly.

Finally, consider an American entrepreneur who is participating in an international tender, for example, a major construction project,[4] where bids have to be denominated in Deutschmark (DM). At time $t = 0$, the entrepreneur enters an offer B in foreign currency for which he is prepared to supply a certain (investment) good at a future point at time $t = 2$. At $t = 1$, the entrepreneur finds out whether he is awarded the contract and consequently given the order. At $t = 2$, on receipt of the order, the good is delivered and the amount in foreign currency B is handed over in return. The production costs calculated in domestic currency will also be incurred at $t = 2$ and be denoted C.

In such a situation at $t = 0$, the entrepreneur is confronted by two different risks. First, it is not clear whether he will be awarded the

[3]In such a situation, the entrepreneurial engagement in derivatives is called an *anticipatory hedge*. See Working (1962, p. 441).

[4]For scenarios of this type, see, for example, Lagerstam (1990).

contract. Second, the spot rate $\tilde{e}_2$ governing the exchange of the amount in foreign currency B he may receive at $t = 2$ is uncertain. With $\tilde{\gamma}$ as a Bernoulli-distributed random variable having at time $t = 1$ the value 1 on receipt of the order and otherwise the value 0, overall entrepreneurial payoffs in domestic currency will amount to $f(\tilde{\gamma}, \tilde{e}_2) = \tilde{\gamma} \cdot (B \cdot \tilde{e}_2 - C)$.[5] Although there are only limited possibilities for using derivatives to safeguard against an unfavorable tender outcome, that is, the case $\tilde{\gamma} = 0$, derivatives based on exchange rates are not difficult to obtain.

The last two scenarios are called the *production example* and the *tender example* and are used as a basis for our numerical examples later in this article.

In the situations outlined, the entrepreneur in his role as a rational decision maker is faced with the following optimization problem that aims at determining optimal engagements $x^{(1)} \ldots x^{(n)}$ in derivatives $i = 1 \ldots n$:

$$E\left[U\left(f(\tilde{\gamma}, \tilde{r}) + \sum_{i=1}^{n} x^{(i)} \cdot f^{(i)}(\tilde{r})\right)\right] \to \max_{x^{(1)} \ldots x^{(n)}}! \tag{2}$$

This problem can be solved without major difficulties as far as quadratic utility functions are concerned. For expected overall payments not depending on the values of $x^{(1)} \ldots x^{(n)}$, there are no opportunities to speculate, and the assumption of quadratic utility leads to simple variance minimization. Though easily resolvable, it is well known that quadratic utility functions are not very plausible because they imply entrepreneurial risk aversion to be increasing in wealth. Much more plausible are utility functions that result in constant or decreasing absolute risk aversion.[6] Another way to justify a simple variance-minimization approach in the absence of speculation possibilities would be to assume that the entrepreneurial payoffs are governed by a normal distribution. Even if the primitives ($\tilde{\gamma}$ and $\tilde{r}$) are normally distributed, the entrepreneurial total payoff will be so as well only in a very few cases. For example, $\tilde{\gamma}$ and $\tilde{r}$ must not be combined as a product $\tilde{\gamma} \cdot \tilde{r}$, and there should be no use of options because of their asymmetric payment structure, which immediately implies a nonzero skewness[7] of total payments. To summarize, although a variance-minimization approach is very easy to handle, in

[5]For simplicity, C is assumed to be independent of e_2.
[6]See, for example, Arrow (1971, p. 96).
[7]The skewness of a random variable can be defined as its third central moment. It is zero for all symmetric distributions, such as the normal distribution.

general it can be justified analytically only under very special additional assumptions.[8]

Therefore, we think there is a need to solve Problem 2 for other than quadratic utilities. Unfortunately, in such cases explicit solutions cannot as a rule be given, so that one is forced to rely on numerical optimization procedures. Moreover, such procedures are only capable of calculating an approximation of the true expected utility maximum. In what follows, we want to present an approximation procedure that is based on the determination of $h(r)$ according to Problem 1.

There is no doubt that the application of variance minimization or, somewhat more generally, mean-variance approaches (in case the mean of total payoffs can be influenced by engagements in financial instruments) is quite straightforward. However, because it is hard to give a convincing analytical foundation for such approaches, there are some attempts to justify them as an alternative approximation for the maximization of other more plausible utility functions. The most prominent of these contributions stems from Kroll, Levy, and Markowitz (1984), who showed that the use of quadratic utilities approximates the true expected utility optimum in several cases rather well. This result may or may not hold for scenarios other than the ones examined by Kroll et al. (1984). Because of such problems connected with mean-variance considerations, it does not seem to be a very big surprise that many authors explicitly refrain from assuming mean-variance preferences.[9] Despite this, anyone who is presenting an alternative approximation procedure must concede the relevance of the benchmark approximation described by simple mean-variance optimization. Any newly proposed approximation approach should do at least as well as straightforward mean-variance optimization. This objection is referred to in detail in connection with the discussion of the tender example. It is shown that simple variance minimization will do a good job only in cases with low risk aversion but may fail significantly otherwise.

[8]Besides quadratic utility or normally distributed payoffs, there are a few other specific conditions under which variance minimization can be justified. See, for example, Adler and Detemple (1988) for a hedging problem with engagements in financial futures in continuous time on the basis of Brownian motions. Moreover, Benninga, Eldor, and Zilcha (1985) showed that variance minimization by a perfect hedge, that is, a hedge that effectively eliminates all uncertainty, is optimal for any risk-averse decision maker in the absence of speculation possibilities. This last result is used in the SCOUTs and International Invitations for Tenders section to derive Equation 23.

[9]For example, in the last 11 volumes (1990, Issue 1, through 2000, Issue 4) of the Journal of Futures Markets, we counted seven articles that considered expected utility maximization problems without relying on mean-variance preferences: Lien (2000); Broll and Wong (1999); Lei, Liu, and Hallam (1995); Briys and Schlesinger (1993); Poitras (1993); Broll and Wahl (1992); and Park and Antonovitz (1990).

Procedures designed to solve expected utility maximization problems are often characterized by the undue length of time required for a calculation and, in the extreme case, by the ultimate failure to solve the optimization problem. The speed of calculation mainly becomes important if a great number of different parameter constellations are analyzed to examine the sensitivity of hedging solutions. Such a question is important for researchers as well as practitioners. Part of our tender example is a closer look at the question why firms actually do not use a certain kind of derivative named *shared currency option under tender* (SCOUT) even though at first sight it seems to contribute significantly to the completion of capital markets. To examine such a problem numerically, it is not sufficient to calculate just one optimal solution for one parameter constellation. In fact, calculations have to be done for a great number of different situations. Thereby, the number of single optimization problems under consideration increases progressively with the number of parameters to be varied. For example, with only six different parameters and five possible realizations for each of them, there are $5^6 = 15{,}625$ different decision problems that have to be solved numerically. Obviously, it does matter if each of these problems can be handled in as little as 90 s or if it takes 3,600 s for each single problem. In the first case, all optimizations can be accomplished in about 16.28 days; in the last case, it takes approximately 1.78 years. When we talk about practical applications of our approximation procedure, we primarily have such time-consuming sensitivity analysis in mind.

It does not seem controversial that expected utility maximization is one[10] adequate approach to analyze optimal entrepreneurial engagements in derivatives.[11] This alone creates a need for efficient numerical algorithms as we have just sketched. Indeed, expected utility maximization can also be applied to concrete practical decision problems. The specification of a utility function instead of simple variance minimization may seem rather impractical. Nevertheless, it should be noted that for the problems considered in this article, variance minimization is equivalent to implicitly assuming a quadratic utility function. Therefore, variance minimization is by no means a preference-free assumption, as it may seem at first glance. Moreover, this assumption is ad hoc in the same way as presupposing, for example, an exponential utility function. The only difference between these two specifications of entrepreneurial preferences refers to the fact that the second approach offers one more degree of freedom because of the necessity (or, to look at it in a more positive

[10]Value maximization is the other relevant approach.

[11]For examples, the reader should refer to the literature cited in Footnote 9.

manner, possibility) of specifying a risk-aversion parameter. In the tender example, we show that the specification of risk-aversion parameters can be done through the definition of adequate entrepreneurial relative risk discounts (RDs), which implicitly define the corresponding entrepreneurial risk-aversion parameter. The same holds true for other types of utility functions, such as those with constant relative risk aversion.

Moreover, it is not difficult to specify an adequate entrepreneurial utility function even in a more general setting, if we restrict ourselves to the consideration of so-called hyperbolic absolute risk-aversion (HARA-) utility functions. Utility functions of the HARA type are able to describe a wide range of entrepreneurial risk preferences (e.g., quadratic utility) and are completely characterized by just two parameters.[12] Therefore, besides the specification of a relative RD by the entrepreneur as outlined previously and presented in more detail in the SCOUTs and International Invitations for Tenders section, the entrepreneur has to answer only one additional question. For example, in general it would be sufficient if the decision maker states which relative portion of a certain initial wealth level he would *ceteris paribus* invest in some risky assets with clearly defined return distributions and which part he would invest securely. On the basis of this information, his (HARA-) utility function could be calculated. Even from this standpoint of practical determination, pure variance minimization thus seems to be inferior to the application of more plausible utility functions.

One may wonder if firms are able to use such sophisticated approaches to risk management. In the first place, if not, it is one task of academic research to explain to them this opportunity. Second, at least banks may be endowed with sufficient human capital to apply advanced methods of deriving optimal engagements in derivatives. Therefore, they could use expected utility maximization approaches to assess the attractiveness of newly designed financial instruments for their customers. In this way, unsuccessful financial innovations such as SCOUTs could be avoided or ideas could be developed to improve on them. We think that the approach presented in this article may support developments to build up a utility-oriented risk management.

After all, the creation of efficient approximation procedures is of sufficient academic and practical importance that it pays to take a closer look at the problem of approximation.[13]

[12] For a formal definition of the class of HARA-utility functions, see the Optimal Solution Analysis section, and for its interesting properties, see especially Cass and Stiglitz (1970).

[13] For another recent article that deals with the problem of approximation procedures for expected utility maximization, see Lei et al. (1995).

In the next section, our approximation procedure is derived. Some interesting properties of the function $h(r)$ are described in the section Optimal Solution Analysis, and the Cost Limit Analysis section presents an index to assess the quality of approximate hedging solutions. In the Hedging Uncertain Cash Flows From Corn Production in Iowa and SCOUTs and International Invitations for Tenders sections, the proposed procedure is tested with the production example and the tender example. The article finishes with a conclusion.

APPROXIMATION PROCEDURE

The problem of variation[14] as described in Problem 1 leads to the following Euler–Lagrange conditions with λ describing the Lagrange parameter:

$$E_\gamma[U'(f(\tilde{\gamma}, r) + h(r))|r] - \lambda = 0 \quad \text{for all } r \tag{3}$$

$$E[h(\tilde{r})] = 0. \tag{4}$$

Let f_H be the overall entrepreneurial net cash flow after hedging. Then, in the case of a quadratic utility function, $U(f_H) = a_1 \cdot f_H - a_2 \cdot f_H^2$, the optimal payment structure, $h(r)$, is[15]

$$h(r) = E[f(\tilde{\gamma}, \tilde{r})] - E_\gamma[f(\tilde{\gamma}, r)|r] \tag{5}$$

and in the case of an exponential utility function, $U(f_H) = -\exp(-a \cdot f_H)$, the analogue result is

$$\begin{aligned} h(r) = {} & \frac{1}{a} \cdot \ln(E_\gamma[\exp(-a \cdot f(\tilde{\gamma}, r))|r]) \\ & - \frac{1}{a} \cdot E_r[\ln(E_\gamma[\exp(-a \cdot f(\tilde{\gamma}, \tilde{r}))|\tilde{r}])] \end{aligned} \tag{6}$$

Moreover, in the case of other utility functions, the system of Equations

[14]The idea to determine the fictitious optimal cash flow can be traced back to Steil (1993), whose calculations refer to a special case of international tender participation similar to the one outlined in the introduction of this article. Therefore, his cash flow $f(\tilde{\gamma}, \tilde{r})$ is structured in a special way, and his two stochastic values are assumed to be stochastically independent. Moreover, the fictitious optimal solution is only used by Steil to provide a means of comparison and not for a solution approximation.

[15]The derivations for the following optimal cash flows are given in the Appendix. In addition, in the absence of a further stochastic variable $\tilde{\gamma}$, that is, $f(\tilde{\gamma}, \tilde{r}) = f(\tilde{r})$, the solution is always explicitly calculable as $h(r) = E[f(\tilde{r})] - f(r)$. Of course, this was to be expected because the cash flow $h(r)$ can only be used for the purpose of risk reduction, with the resulting total position describing a certain cash flow amounting to $E[f(\tilde{r})]$.

3 and 4 is numerically rather easy to solve.[16] We, therefore, assume that the optimal payment structure $h(r)$ is obtainable free of calculation costs. Let C_A denote calculation costs at $t = 1$ for any approximation solution X_A for the original decision problem (Problem 2). In addition, $h_X(\tilde{r}) := \sum_{i=1}^{n} x^{(i)} \cdot f^{(i)}(\tilde{r})$ describes the uncertain cash flow of a course of action $X = (x^{(1)} \ldots x^{(n)})$ resulting at $t = 1$. Furthermore, $\Pi_A > 0$ represents the maximum premium an investor would be prepared to pay for the (fictitious) optimal cash flow $h(\tilde{r})$ to avoid participating in the additional noise term of $\varepsilon_A(\tilde{r}) := h_A(\tilde{r}) - h(\tilde{r})$ with $h_A(\tilde{r}) := h_{X_A}(\tilde{r})$ and incurring calculation costs C_A. Thus, Π_A is calculated from the identity

$$E[U(f(\tilde{\gamma}, \tilde{r}) + h(\tilde{r}) - \Pi_A)] = E[U(f(\tilde{\gamma}, \tilde{r}) + h_A(\tilde{r}) - C_A)] \quad (7)$$

Π_A can be calculated for any possible approximation solution X_A, and the maximization of the entrepreneur's expected utility is equivalent to the minimization of Π_A. In the Appendix, the following simple estimation of Π_A is derived:

$$\Pi_A \leq \frac{1}{\lambda} \cdot \mathrm{stddev}[U'(f(\tilde{\gamma}, \tilde{r}) - C_A + h_A(\tilde{r}))] \cdot \mathrm{stddev}[\varepsilon_A(\tilde{r})] + C_A \quad (8)$$

To keep the premium as small as possible, it could be restricted by minimizing the right-hand side of Inequation 8. This method, however, does not necessarily guarantee an easier calculation than direct expected utility maximization. Therefore, we restrict ourselves to the minimization of the factor $\mathrm{stddev}[\varepsilon_A(\tilde{r})]$. This leads to the following optimization problem in which $\tilde{\varepsilon}_X := h_X(\tilde{r}) - h(\tilde{r})$ describes the noise term of an arbitrary course of action $X = (x^{(1)} \ldots x^{(n)})$:

$$\mathrm{var}[\tilde{\varepsilon}_X] = E\left[\left(\sum_{i=1}^{n} x^{(i)} \cdot f^{(i)}(\tilde{r}) - h(\tilde{r})\right)^2\right] \rightarrow \min_{x^{(1)} \ldots x^{(n)}}! \quad (9)$$

Obviously, the approximation procedure (Inequation 9) determines the solution $x^{(1)} \ldots x^{(n)}$ with the smallest possible distance to h, whereby the space of continuous functionals is normed by the standard deviation. If we abstract from calculation costs, the fictitious optimal solution h is immediately obtained with the existence of a noise term standard deviation of zero. The minimization of only one of the two standard

[16]Equations 3 and 4 are reduced to two interlocking root equations. For a given λ, the solution of Equation 3 results in $h(r, \lambda)$. According to Equation 4, λ is chosen in such a way as to fulfil the requirement of $m(\lambda) := E[h(\tilde{r}, \lambda)] = 0$. Therefore, an algorithm is first needed to calculate a root λ^* of $m(\lambda)$. By the use of this algorithm in turn, $h(r, \lambda^*)$ is always determined as a zero of Equation 3.

deviations mentioned thus seems particularly adequate if stddev$[\varepsilon_A(\tilde{r})]$ can be lowered to values near zero. In particular, in the Appendix we prove the following theorem.

Theorem 1

Consider a sequence $(D_n)_{n\in \mathrm{IN}}$ of decision problems according to Problem 9 with entrepreneurial access to n put options with different strike prices, where for $n \to \infty$, the range of their strike prices goes to infinity and the differences between them shrink to zero. If we abstract from calculation costs, for sufficiently high n the difference between the entrepreneur's expected utility maximum and its approximation according to the solution of Problem 9 falls below any arbitrary (positive) level.

From Problem 9, we can derive the following necessary and sufficient[17] conditions for the optimal strategy X:

$$E\left[\left(h(\tilde{r}) - \sum_{j=1}^{n} x^{(i)} \cdot f^{(j)}(\tilde{r})\right)\cdot f^{(i)}(\tilde{r})\right] = 0 \quad \forall i = 1\ldots n$$

$$\Leftrightarrow \sum_{j=1}^{n} x^{(j)} \cdot \underbrace{\mathrm{cov}[f^{(i)}(\tilde{r}), f^{(j)}(\tilde{r})]}_{=:c_{ij}} = \underbrace{\mathrm{cov}[f^{(i)}(\tilde{r}), h(\tilde{r})]}_{=:h_i} \quad \forall i = 1\ldots n$$

$$\Leftrightarrow C \cdot X = H \Leftrightarrow X = X_A = C^{-1} \cdot H \tag{10}$$

The Hessian $[\partial^2 (h - h_X)^2/(\partial x^{(i)} \partial x^{(j)})]_{i,j} = 2 \cdot (\mathrm{cov}[f^{(i)}(\tilde{r}), f^{(j)}(\tilde{r})])_{i,j}$ as twice a variance–covariance matrix is positive definite. Furthermore, the existence of the matrix inverse C^{-1} may be assumed. This implies that none of the available derivatives is redundant as a potential hedging tool. In this context, X denotes the value vector of all $x^{(i)}$, C is the variance–covariance matrix of $f^{(i)}$ and $f^{(j)}$, and H is the transpose of the vector of all h_i.

This leads to a clearly definable entrepreneurial hedging strategy, especially in the case of an exponential utility function. Our approach also applies to other utility functions because it is not difficult to determine the fictitious cash flow h numerically, and the solution of Equation 10 can be derived for any concrete functional form of h.[18] For

[17] For this proposition in a general portfolio theoretical framework, see, for example, Merton (1972, p. 1851).

[18] This illustrates the (numerical) advantage of the approximation procedure (Equation 10) over the direct optimization according to Problem 2. Problem 2 requires a numerical optimization procedure and the determination of a two-dimensional integral that varies in each optimization step. However, the solution X_A can be determined directly with Equation 10. For this procedure, only one numerical integration is needed as no change in the integrand is required.

this reason, the approximation procedure based on Problem 9 can be applied without any significant calculation costs being incurred, $C_A = 0$. Finally, the summation for all $i = 1 \ldots n$ of all necessary conditions weighted with the quantities $x^{(i)}$ results in the identity $\text{var}[h_A(\tilde{r})] = \text{cov}[h_A(\tilde{r}), h(\tilde{r})]$, so that the total risk resulting from the approximation hedge h_A only just matches its share of the total risk $\text{var}[h(\tilde{r})] = \text{cov}[h_A(\tilde{r}), h(\tilde{r})] + \text{cov}[h(\tilde{r}) - h_A(\tilde{r}), h(\tilde{r})]$ of the optimal cash flow h.

Of course, the procedure is also supposed to ensure that the exact solution is obtained on the basis of a quadratic utility function. To check this, an examination is carried out of the expected utility of a reference solution X of Problem 2:[19]

$$\begin{aligned} &E[U(f(\tilde{\gamma}, \tilde{r}) + h_X(\tilde{r}))] \\ &\quad = E[U(f(\tilde{\gamma}, \tilde{r}) + h(\tilde{r}))] \\ &\qquad + E[U'(f(\tilde{\gamma}, \tilde{r}) + h(\tilde{r})) \cdot (h_X(\tilde{r}) - h(\tilde{r}))] \\ &\qquad + \frac{1}{2} \cdot E[U''(f(\tilde{\gamma}, \tilde{r}) + h(\tilde{r})) \cdot (h_X(\tilde{r}) - h(\tilde{r}))^2] \end{aligned} \tag{11}$$

According to Equation 3, $E_\gamma[U'(f(\tilde{\gamma}, r) + h(r))|r]$ is constant for all r, and $E[h_X(\tilde{r}) - h(\tilde{r})]$ has the value zero because of assumed nonbias. Consequently, the second summand of the Taylor development vanishes. Because of the constant negative derivative $U''(f_H) = -2 \cdot a_2$, the maximization of the expected utility corresponds to the minimization of the noise term variance $E[(h_X(\tilde{r}) - h(\tilde{r}))^2]$.

Because the fictitious optimal solution $h(r)$ forms the basis for the approximation procedure, a short overview will be presented in the following section showing the qualitative progression of h with respect to specific scenarios of cash flow $f(\tilde{\gamma}, \tilde{r})$.

OPTIMAL SOLUTION ANALYSIS

For the approximation of the optimal solution h by appropriate derivatives, it is useful to obtain information on the qualitative progression of $h(r)$ so that its curvature can at least be visually approximated when derivative instruments are selected. For example, an entrepreneur might want to limit the number of derivatives used for reasons of calculation cost. His knowledge of the qualitative progression of $h(r)$ would enable him to (pre)select derivatives with suitable payment structures.

[19]Consideration ought to be given to the fact that in the case of a quadratic utility function, all derivatives of an order higher than 2 will disappear.

For a qualitative characterization of the optimal cash flow, it is, however, necessary to make assumptions about the original cash flow $f(\gamma, r)$. To this end, a theorem is formulated that is proven in the Appendix of this article and is presented next.

Theorem 2

Assume a utility function with positive decreasing marginal utility.

1. Under these assumptions, the following applies:

$$\frac{df}{dr}(\gamma, r) \underset{(\leq)}{\geq} 0 \text{ for all } \gamma, r \Rightarrow h'(r) \underset{(\geq)}{\leq} 0 \text{ for all } r.$$

$$\text{If, in addition, } P_\gamma\left(\frac{df}{dr}(\tilde{\gamma}, r_0) \underset{(<)}{>} 0 \,\middle|\, r_0\right) > 0,$$

$$\text{the result will be } h'(r_0) \underset{(>)}{<} 0.$$

2. Moreover, assuming the marginal utility function strictly convex,[20] the following holds:

$$\frac{d^2f}{dr^2}(\gamma, r) \leq 0 \text{ for all } \gamma, r \Rightarrow h''(r) \geq 0 \text{ for all } r.$$

$$\text{If, in addition, } P_\gamma\left(\frac{d^2f}{dr^2}(\tilde{\gamma}, r_0) \underset{(<)}{>} 0 \,\middle|\, r_0\right) > 0 \text{ or}$$

$$P_\gamma\left(\frac{d^2f}{d\gamma dr}(\tilde{\gamma}, r_0) \neq 0 \,\middle|\, r_0\right) > 0, \text{ the result will be } h''(r_0) > 0.$$

The monotone behavior denoted in 1. is to be expected as a risk-averse investor will have a preference for cash flows with only moderate fluctuations. With an optimal cash flow, a negative correlation to the original payment structure occurs with a diversification effect that balances fluctuations with counterfluctuations. In the case of a convex marginal utility function as described in 2., the entrepreneur will have positive preferences for a higher skewness of his overall position,[21] which indicates his interest in cash flow distributions with positive skewness. Consequently, one of his aims is avoiding unfavorable (negative) extreme values of his overall cash flow by a reduction in cash flow concavity. This

[20]For example, this requirement is met in the case of a decision maker with a constant or decreasing absolute risk aversion.

[21]See, for example, Rubinstein (1973) or Kraus and Litzenberger (1976).

corresponds with the result of 2. Analogous to this, in the case of strictly concave marginal utility and thus skewness aversion, even if this supposition appears implausible, a concave progression of $h(r)$ is obtained for a cash flow $f(\gamma, r)$ with convexity in r. In the case of a limited use of available derivatives, consideration ought to be given to the fact that combinations of derivatives can cause the required monotone and curvature behavior.

COST LIMIT ANALYSIS

The aforementioned approximation procedure (Equation 10) ought to be used only if the alternative option, that is, the calculation of an improved solution X of Problem 2, leads to higher discounts $\Pi_X \geq \Pi_A$. In this context, Π_A is determined without significant numerical difficulty with Equation 7. Nevertheless, more straightforward and explicit methods allowing additional interpretation can be applied to two special cases. The first one refers to an exponential utility function with constant absolute risk aversion. To this end, let CE_A be the entrepreneurial certainty equivalent in the case of an approximation solution A with $C_A = 0$. Then, with exponential utility the entrepreneur's overall welfare position for $C_A > 0$ amounts to $CE_A - C_A$. Finally, denote with CE_h the maximum attainable certainty equivalent by realizing the fictitious cash flow h. Therefore, we get from Equation 7 the following relationship:

$$CE_A - C_A = CE_h - \Pi_A \tag{12}$$

An alternative strategy X with $CE_X - C_X = CE_h - \Pi_X$ is not better than A if we have $CE_A - C_A \geq CE_X - C_X$. Because $C_A = 0$ for our approximation and $CE_X < CE_h$, it follows that X is not better than A for

$$\begin{aligned} & CE_A \geq CE_h - C_X \\ \Leftrightarrow & \frac{C_X}{CE_h} \geq 1 - \frac{CE_A}{CE_h} = \frac{\Pi_A}{CE_h} \end{aligned} \tag{13}$$

The right-hand side of Inequation 13 thus may serve as an adequate measure for assessing the quality of our approximation solution, which tells us the maximum possible gain in relative terms from implementing an engagement in hedging instruments other than our approximation solution A. In particular, if the relative costs C_X/CE_h for solving Problem 1 exactly lie above the measure of misconduct of the approximation solution (on the right hand of Inequation 13), strategy A is optimal. The same is true for any alternative solution with calculation costs C_X regardless of the true value for CE_X. Moreover, the index on the right-hand side of Inequation 13 is invariant to positive monotone transformations of the entrepreneurial utility function.[22] Certainly, this index can be applied to assess the quality of approximation procedures other than ours. This emphasizes the central role of $h(r)$ in determining the adequacy of any kind of approximation solution.

Consider also the type of HARA-utility functions with HARA $-U''(f_H)/U'(f_H) = 1/(a + b \cdot f_H)$ with a and $b \in \mathrm{IR}^{\geq 0}$ and $a + b > 0$ in the special case of constant relative risk aversion (HARA-utility functions with $a = 0$). Using the special properties of utility functions with constant relative risk aversion, we get from Equation 7

$$CE_A \cdot \left(1 - \frac{C_A}{CE_A}\right) = CE_h \cdot \left(1 - \frac{\Pi_A}{CE_h}\right). \tag{14}$$

A strategy X is not better than the approximation solution A, if $CE_A \cdot (1 - C_A/CE_A) \geq CE_h \cdot (1 - C_X/CE_h)$ because $CE_X \leq CE_h$. With $CA = 0$, once again we get Inequation 13.

As was mentioned before, Π_A can be calculated numerically for all other utility functions. Moreover, it remains true that in any case an alternative strategy X with $1 - CE_X/CE_h > 1 - CE_A/CE_h$ or, equivalently, $CE_X/CE_h < CE_A/CE_h$ is inferior in comparison to our approximation solution A.

Altogether, an investor has at his disposal an adequate number of instruments for checking the adequacy of the approximation procedure (Equation 10). In the following section, the actual use of the procedure is tested with two examples.

HEDGING UNCERTAIN CASH FLOWS FROM CORN PRODUCTION IN IOWA

Consider the situation outlined in the introduction in which an entrepreneur manufacturing a product encounters uncertainty with regard to sales volume and sales price. The specific constellation examined here refers to an example given by Sakong, Hayes, and Hallam (1993), who analyzed the hedging behavior of a corn producer in Iowa. In this

[22]An index similar to CE_A/CE_h was also used by Kroll et al. (1984) to check the quality of mean-variance optimal solutions. An interpretation in terms of relative calculation costs, however, was not given. The authors merely expressed in percentages by how much the actual expected utility optimum was missed. Thus, the question was never raised concerning the quality of approximation, that is, whether or not it could be deemed adequate.

context, the uncertain sales volume is denoted $\tilde{y}$, and the uncertain price of a bushel of corn is denoted $\tilde{p}$. Furthermore, if the production of an amount $\tilde{y}$ at $t = 1$ leads to costs in the amount of $C(\tilde{y})$, the following will be generated for the unhedged cash flow at $t = 1$:

$$f(\tilde{y}, \tilde{p}) = \tilde{p} \cdot \tilde{y} - C(\tilde{y}) \tag{15}$$

In addition, two types of derivatives will be available in the market with underlying price $\tilde{p}$. These are short forward contracts entitling the sale of a bushel of corn for the forward price $p^{(F)}$ at $t = 1$ and the corresponding put options with a strike price to the same price $p^{(F)}$. As a result, the derivatives yield the following payment structures at $t = 1$ under consideration of the option premium $\bar{v}$ at maturity and an interest rate of zero:

$$f^{(1)}(\tilde{p}) = p^{(F)} - \tilde{p} \tag{16}$$

$$f^{(2)}(\tilde{p}) = \max\{p^{(F)} - \tilde{p}, 0\} - \bar{v} \tag{17}$$

The assumption of an unbiased evaluation will result in $p^{(F)} = E[\tilde{p}] =: \bar{p}$ and $\bar{v} = E[\max\{p^{(F)} - \tilde{p}, 0\}]$, which will ultimately lead to a cash flow hedged by quantities $x^{(1)}$ and $x^{(2)}$:

$$\begin{aligned} &f(\tilde{y}, \tilde{p}) + x^{(1)} \cdot f^{(1)}(\tilde{p}) + x^{(2)} \cdot f^{(2)}(\tilde{p}) \\ &\quad = \tilde{p} \cdot \tilde{y} - C(\tilde{y}) + x^{(1)} \cdot (\bar{p} - \tilde{p}) + x^{(2)} \cdot (\max\{\bar{p} - \tilde{p}, 0\} - \bar{v}) \end{aligned} \tag{18}$$

The objective is now to calculate the optimal values for $x^{(1)}$ and $x^{(2)}$ under the assumption of constant absolute risk aversion. On the simplified premise $C(\tilde{y}) = 0$, Sakong et al. assumed the following relationship between sales volume and future spot price:

$$\tilde{y} = \rho \cdot \tilde{p}^{\eta} \cdot (1 + \tilde{\kappa}) \tag{19}$$

In this context, $\eta(-1 < \eta \leq 0)$ denotes the price elasticity of the sales volume and $\tilde{\kappa}$ the specific production uncertainty assumed to be independent of $\tilde{p}$ and $\tilde{y}$. Both $\tilde{p}$ and $\tilde{\kappa}$ are supposed to be normal variables, with $\tilde{\kappa}$ having an expectation value of zero. Thus, the conditional random variable $\tilde{y}|p$ is also normally distributed. Following empirical results obtained by Sakong et al., we assume an expected price of $\mu_p = \$2.92$ with a variance of $\sigma_p^2 = 0.256$. The expected sales volume amounts to $\mu_y = 20{,}000$ bushels with a variance of $\sigma_p^2 = 10^7$. The parameter ρ and the variance of $\tilde{\kappa}$ have to be aligned in accordance with the following scenarios for η.

Because

$$\frac{df}{dp} = \frac{\partial f}{\partial y} \cdot \frac{\partial y}{\partial p} + \frac{\partial f}{\partial p} = \Bigg(p - \underbrace{C'(y)}_{=0}\Bigg) \cdot \rho \cdot \eta \cdot p^{\eta - 1} \cdot (1 + \kappa) + y$$
$$= \eta \cdot \rho \cdot p^{\eta} \cdot (1 + \kappa) + y = (\eta + 1) \cdot y > 0 \tag{20}$$

for positive quantity y and

$$\frac{d^2 f}{dp^2} = (\eta + 1) \cdot \rho \cdot \eta \cdot p^{\eta - 1} \cdot (1 + \kappa) = (\eta + 1) \cdot \eta \cdot \frac{y}{p} < 0 \tag{21}$$

for positive quantity y, positive price p, and $\eta < 0$, a decreasing strictly convex progression of h occurs in accordance with Theorem 2. In this context, it can be supposed (without the use of numerical procedures) that positive quantities of both short forwards and put options will be optimal. This is one of a number of additional advantages of the procedure presented here, as it is possible to see from the structure of the underlying cash flow alone how the hedge cash flow ought to develop in qualitative terms.

The first scenario deals with the case $\eta = 0$, that is, the stochastic independence between price and sales. For $\tilde{y}|p = \tilde{y}$ to have the required expectation value, it must hold that $\rho = 20{,}000$. Moreover, $\tilde{\kappa}$ has a variance of 10^7. In this case, the examination is based on different risk-aversion parameters $a = 0.00015$, $a = 0.0003$, and $a = 0.00045$.

In the second scenario, the case $\tilde{\kappa}$ is presented, that is, the case of perfect interdependence between price and sales. Consequently, $\tilde{y}|p$ is fixed for a given p at a value of $\rho \cdot p^n$. Because the hedging behavior will be analyzed with regard to different price elasticities $\eta = -0.1$, $\eta = -0.5$, and $\eta = -0.9$, it is possible, because of the required expectation value of $\tilde{y}$, to calculate the parameter ρ by means of $\rho = 20{,}000/E[\tilde{p}^n]$ for each case. However, the variance of the sales volume will now in general deviate from the empirically observed value of 10^7. Calculations here refer to the risk-aversion parameter $a = 0.00015$.

Finally, the general case of stochastic interdependence between sales volume and price is discussed in the third scenario. Similar to the second scenario, ρ is established at a fixed value $\rho = 20{,}000/E[\tilde{p}^{-0.5}] = 33{,}767$. The variance of production uncertainty is determined by the assumption of an elasticity $\eta = -0.5$ according to[23]

$$\mathrm{var}[\tilde{\kappa}] = \frac{\mathrm{var}[\tilde{y}]}{\rho^2 \cdot E[\tilde{p}^{2 \cdot \eta}]} + \frac{E^2[\tilde{p}^{\eta}]}{E[\tilde{p}^{2 \cdot \eta}]} - 1 \approx 0.0164 \tag{22}$$

[23]See Sakong et al. (1993, Footnote 16). Note the small difference between our calculation, $\rho = 33{,}767$, and the calculation of Sakong et al., $\rho = 33{,}775.513$. The reason is that Sakong et al. only used values from \$1.20 to \$4.54 for the normal variable $\tilde{p}$.

In this scenario, the examination deals with sectionally constant values of absolute risk aversion. For sales volumes not exceeding 40,000 bushels, $a = 0.0009$ is chosen; for sales volumes ranging between 40,000 and 45,000 bushels (inclusive), $a = 0.00045$ is chosen; and for those in excess of 45,000 bushels, $a = 0.00015$ is chosen.

Sakong et al. (1993) constructed the utility function by presupposing for each subinterval an exponential utility function $U_i(f_H) = c_i - d_i \cdot \exp[-a_i \cdot (f_H - 45{,}000)]$. $i = 3$ indicates the region for values not exceeding 40,000, $i = 2$ stands for the region (40,000; 45,000], and $i = 1$ stands for those values in excess of 45,000. The parameters c_i and d_i were chosen in such a way as to ensure that the utility function is continuously differentiable with regard to the total domain of f_H. Starting with $c_1 = 2$ and $d_1 = 3$, one can easily calculate $c_2 = 0$, $d_2 = 1$, $c_3 = -4.7439$, and $d_3 = 0.0527$. To our surprise, Sakong et al. received the values $c_2 = 0$, $d_2 = 3$, $c_3 = 1{,}458{,}742.7$, and $d_3 = 180.034626$, whereby their total utility function is neither differentiable nor continuous. Because of these flawed calculations for the third scenario, the parameter values computed in this article are used instead of the values proposed by Sakong et al.

It is now possible to carry out a numerical examination of the scenarios. Sakong et al. (1993) did so by applying a specific grid search.[24] Table I shows the approximation solutions on the basis of our article and the results of our application of the grid search algorithm for the three scenarios. The results determined by Sakong et al. always differ from our solutions using the described grid search algorithm. Their solutions seem to be inferior because they lead to smaller values of expected utility. Moreover, the table presents the corresponding index values CE_X/CE_h for both approximation procedures and the no-hedge alternative as a means of comparison. The last two columns provide computing times for the two approximation procedures for given optimal payment structures $h(p)$.

The positive values for all decision variables $x^{(1)}$ and $x^{(2)}$ confirm the result of the general analysis in Inequations 20 and 21. The very good

[24]The numerical procedure was explained by Sakong et al. (1993, p. 415). Resumed, the algorithm works as follows. The procedure requires a rough grid (ranging in this case from −10,000 to 10,000 with a step size of 1,000) to start. The grid point leading to the highest expected utility is defined as the center of a new grid. If this new center is a corner solution of the original grid, the new grid is built up around the new center with the original step size (of 1,000). This procedure will be repeated until an interior solution has been reached. This interior solution defines the center of a grid with reduced step size (in this case, 100). Again, the described procedure to determine an interior solution with the highest expected utility will be executed with respect to the second grid. Finally, the newly calculated interior grid point defines the center of a third grid with further reduced step size (in this case, 10). Using the procedure again with the underlying third grid leads to the grid solution under consideration.

TABLE I

Three Scenarios Under Consideration

First Scenario: Stochastic Independence Between Price and Sales									
a	$x^{(1)G}$	$x^{(2)G}$	$x^{(1)A}$	$x^{(2)A}$	$\frac{CE_G}{CE_h}$	$\frac{CE_A}{CE_h}$	$\frac{CE_o}{CE_h}$	T^G	T^A
0.00015	14,780	1,680	14,786.8	1,666.4	99.999%	99.999%	90.395%	26,568	49
0.00030	9,560	3,380	9,589.5	3,314.5	99.987%	99.987%	85.948%	25,547	45
0.00045	4,850	4,580	4,904.1	4,408.5	99.959%	99.959%	84.854%	16,452	48
Second Scenario: Perfect Interdependence Between Price and Sales									
η	$x^{(1)G}$	$x^{(2)G}$	$x^{(1)A}$	$x^{(2)A}$	$\frac{CE_G}{CE_h}$	$\frac{CE_A}{CE_h}$	$\frac{CE_o}{CE_h}$	T^G	T^A
−0.1	17,650	700	17,647.3	705.4	100%	100%	89.012%	463	0.44
−0.5	9,010	1,990	9,010.6	1,978.8	99.998%	99.998%	96.362%	244	0.57
−0.9	1,640	720	1,640.4	719.2	100%	100%	99.859%	219	0.57
Third Scenario: General Case of Interdependence Between Price and Sales									
η	$x^{(1)G}$	$x^{(2)G}$	$x^{(1)A}$	$x^{(2)A}$	$\frac{CE_G}{CE_h}$	$\frac{CE_A}{CE_h}$	$\frac{CE_o}{CE_h}$	T^G	T^A
−0.5	1,090	2,110	1,166.7	1,938.8	99.968%	99.967%	97.212%	1,274	75

Note: The first scenario characterizes the situation where price $\tilde{p}$ and sales volume $\tilde{y}$ are assumed to be stochastic-independent; that is, price elasticity η is zero. In this scenario, the first column contains the three underlying risk-aversion parameters. The second scenario represents the case $\tilde{\kappa} = 0$, so that sales volume depends on price $\tilde{p}$ according to $\tilde{y} = \rho \cdot \tilde{p}^{\eta}$. Give a risk aversion parameter $a = 0.00015$, three different values for the price elasticity η are distinguished. The third scenario covers the situation where sales volume $\tilde{y}$ depends stochastically on price $\tilde{p}$ with price elasticity $\eta = -0.5$, $\rho = 33{,}767$, and, thus, a variance of the specific production uncertainty, var$[\tilde{\kappa}] = 0.0164$. G indicates the solution according to the grid search algorithm. With A, the approximation solution presented in this article is named. Moreover, the fractions CE/CE_h are index values to assess the quality of each solution, whereby CE_o denotes the certainty equivalent of the nohedge alternative. Finally, T^G and T^A indicate times in seconds needed by the corresponding approximation method. According to this table, both approximation procedures work very accurately, but procedure A is on average more than 320 times faster than G.

performance of the no-hedge alternative in the sixth case ($\eta = -0.9$) is a direct result of a price elasticity near -1. Such price elasticities of demand suggest only minor fluctuations in overall cash flow due to the change in sales price. Besides this, all index values indicate that there is essentially no need for the use of additional financial derivatives depending on $\tilde{p}$. Furthermore, in all seven situations both approximation procedures nearly lead to the same index values. This demonstrates that the approximation approach proposed in this article may work as precisely as other more direct numerical approximation procedures. The most remarkable result of Table I may be that the overall computing time for the grid search amounts to about 20 hr on a personal computer with a Pen tium II processor (233 MHz), whereas the calculation by our approximation procedure only took about 219 s. On average, our approximation procedure proves to be about 324 times faster than the grid search algorithm applied by Sakong et al. (1993). In particular, for constant absolute risk aversion the approximation procedure suggested in this article is highly efficient. Nevertheless, as typical for numerical analyses, there may be certain cases, such as the last one in Table I, where our procedure does not work quite as well as in other situations. Despite this, the production example clearly indicates the power of our approximation procedure. Instead of extending the production example, we now take a closer look at the tender example to further elaborate on the efficiency of our approximation procedure.

SCOUTS AND INTERNATIONAL INVITATIONS FOR TENDERS[25]

As sketched in the beginning of this article, we consider an American entrepreneur participating in an international invitation for tenders with an offer in DM at $t = 0$ as one of m different bidders. $\tilde{f}_H$ denotes the entrepreneur's cumulated total payoff in dollars from his participation in the tender as well as from any transactions on the foreign exchange markets that may be carried out until $t = 2$. Moreover, $\tilde{\gamma}$ is defined as outlined previously. Because the entrepreneur cannot be sure of winning the contract, it may be advantageous for him to have access to a hedging device with a payment structure depending on $\tilde{\gamma}$. In particular, he might be interested in using so-called SCOUTs. In our case, a SCOUT consists of one standardized put option for DM 1, acquired by all m bidders in the tender for a price in dollars of $v_0^{(P)}$ with a strike price of $e_{0,2}^{(F)}$ and maturity at time $t = 2$. We assume that S SCOUTs are purchased by the m bidders. All m participating bidders must then pay the same amount $(S \cdot v_0^{(P)})/m$ at time $t = 0$, and only the winner gets all S currency options at time $t = 1$ with the announcement of the outcome of the tender.

Obviously, SCOUTs may help to save money at time $t = 0$ by simultaneously offering protection against exchange rate deterioration in the case of winning the tender. In fact, they were introduced by Midland Bank in 1987,[26] but despite their interesting features, at least until 1999

[25]The decision problem of hedging currency risk exposures from participating in an international invitation for tenders was first analyzed by Eaker and Grant (1985). See also Steil (1993) for a numerical treatment of the problem. Neither article allows for the simultaneous use of currency forwards and options for hedging purposes or considers SCOUTs.

[26]See, for example, Jacque (1996, p. 254).

not a single SCOUT was sold.[27] In what follows, we want to analyze numerically an entrepreneur's decision problem to explain this lack of practical attractiveness.

To this end, we allow for the use of traditional currency forwards and put options in addition to the purchase of SCOUTs.

We thus present an example for a sensitivity analysis that is characterized by the calculation of optimal hedging behavior for a large number of different situations. Hence, it pays to implement fast approximation procedures like the one suggested in this article. After having shown that SCOUTs are not viable as a hedging device, we focus on the optimal use of currency forwards and put options in the absence of engagements in SCOUTs. This enables us to fulfil some more goals.

First, the construction of the situations considered in the tender example highlights how utility functions can be calculated on the basis of relative RDs specified by the decision maker. This can be done not only for constant absolute risk aversion but also for decreasing absolute risk aversion. Second, we show that at least for the tender example, simple variance minimization may lead to rather poor results, emphasizing the necessity of good approximation procedures. Third, some rough comparative statics based on the numerical analysis are derived. Moreover (and similar to the production example), numerical optimizations will indicate that there is in general no need to engage in more than just two derivatives to hedge foreign exchange risks in the tender example. For example, the purchase of additional put options with varying strike prices does not seem necessary.

To these ends, $x^{(1)}$ represents the amount of foreign exchange sold forward at $t = 0$ with maturity at $t = 2$ at a current forward rate $e_{0,2}^{(F)}$. Moreover, $x^{(2)}$ describes the number of usual currency put options for DM 1 each, purchased at $t = 0$ for an option premium in dollars of $v_0^{(P)}$ with a strike price of $e_{0,2}^{(F)}$ and maturity at $t = 2$. As in the preceding section, we assume that both derivatives cannot be used to influence entrepreneurial expected cash flow in domestic currency. Therefore, the entrepreneur can and will carry out a perfect hedge at $t = 1$. This means that at $t = 1$, the currency put options (possibly including the SCOUTs) will be disposed of again at the then prevailing option price $\tilde{v}^{(P)}$ and that on award of the tender, additional currency amounting to $B - x^{(1)}$ will be sold with maturity at $t = 2$ at the then current forward rate $\tilde{e}_{1,2}^{(F)}$. Conversely, when the award is not obtained, DM $x^{(1)}$ is bought at the aforementioned current forward rate with expiration at $t = 2$. With

[27]See Edwardes (1999).

domestic interest rates $r_{0,1}^{(A)}$ from $t = 0$ to $t = 1$ and $r_{1,2}^{(A)}$ from $t = 1$ to $t = 2$, it is possible to describe $\tilde{f}_H$ from the point of time $t = 0$ as

$$\begin{aligned}\tilde{f}_H = {} & \tilde{y} \cdot (B \cdot \tilde{e}_{1,2}^{(F)} - C + S \cdot \tilde{v}_1^{(P)} \cdot (1 + r_{1,2}^{(A)})) \\ & - [(S \cdot v_0^{(P)})/m] \cdot (1 + r_{0,1}^{(A)}) \cdot (1 + r_{1,2}^{(A)}) \\ & - x^{(2)} \cdot [v_0^{(P)} \cdot (1 + r_{0,1}^{(A)}) \cdot (1 + r_{1,2}^{(A)}) - \tilde{v}_1^{(P)} \cdot (1 + r_{1,2}^{(A)})] \\ & + x^{(1)} \cdot (e_{0,2}^{(F)} - \tilde{e}_{1,2}^{(F)}) \end{aligned} \tag{23}$$

For any given parameter values, the entrepreneur's aim is to maximize the expectation value of an exponential utility function defined in $\tilde{f}_H$ by a suitable choice of $x^{(1)}$ and $x^{(2)}$. In this context, we assume is $\tilde{\gamma}$ is stochastically independent of all future prices on the foreign exchange markets. Because all bids are denominated in DM, there is no reason why the tender outcomes should be dependent on the situation in the foreign currency markets. Moreover, the period of time between offer submission and award ($t = 0$ to $t = 1$) will be 3 months, but the period of time between award and delivery as well as payment ($t = 1$ to $t = 2$) will be 21 months, that is, seven quarters.

In what follows, the optimal behavior at $t = 0$ will be determined numerically. This requires a more detailed description of the relevant parameters.

Data Concerning Capital and Foreign Exchange Markets

For the sake of simplification, a flat interest yield curve is assumed to exist in (perfect) domestic and international capital markets. The corresponding 3-month interest rates in Germany and the United States for risk-free lending and borrowing are fixed at 0.863745% as the DM interest rate $r_{0,1}^{(G)}$ and at 1.36252% as the dollar interest rate $r_{0,1}^{(A)}$ on the basis of the Euro money market rates (issued in March 1998). The exchange rate e_0 between dollars and DM at $t = 0$ is based on data of March 1998 and is quantified with \$0.5459/DM. A logarithmic normal distribution is adopted for future spot exchange rates. This probability distribution is not only useful in a number of ways, but it also forms the basis for the evaluation of currency put options by way of the Garman and Kohlhagen (1983) formula. The application of this formula is permissible because of the assumed exclusion of speculation alone, even if a continuous trade in securities is not possible. This is because the impossibility of realizing expected returns from transactions in foreign exchange markets directly implies a risk-neutral market evaluation. The random variable can $\tilde{v}_1^{(P)}$ can thus be traced back to the spot rate $\tilde{e}_1$ prevailing at $t = 1$. On the basis of the assumed perfect capital and foreign exchange markets, the theory of covered interest rate parity has to be applicable at each point in time. For this reason and also because of the flat interest rate structure, the forward rate $\tilde{e}_{1,2}^{(F)}$ can thus also be expressed as a function of $\tilde{e}_1$ as well as of the interest rates $\tilde{r}_{1,2}^{(G)}$ and $r_{1,2}^{(A)}$ from $t = 1$ to $t = 2$ in Germany and the United States, respectively:

$$\tilde{e}_{1,2}^{(F)} = \tilde{e}_1 \cdot \frac{1 + r_{1,2}^{(A)}}{1 + r_{1,2}^{(G)}} = \tilde{e}_1 \cdot \left(\frac{1 + r_{0,1}^{(A)}}{1 + r_{0,1}^{(G)}}\right)^7 \tag{24}$$

As a result of the assumed impossibility of speculation by way of currency forwards, not only the covered but also the uncovered interest rate parity is valid at each point in time. The spot rate expected for $t = 1$ can, therefore, be determined as follows:

$$E[\tilde{e}_1] = e_0 \cdot \frac{1 + r_{0,1}^{(A)}}{1 + r_{0,1}^{(G)}} 0.5459 \cdot \frac{1.0136252}{1.00863735} \approx \$\ 0.5432/\text{DM} \tag{25}$$

To describe the probability distribution of $\tilde{e}_1$, the volatility of future exchange rates finally needs to be specified. This volatility denotes the standard deviation s of the one-periodic[28] logarithmic exchange rate[29] with normal distribution.

A simple reference for the estimation of exchange rate volatility is provided by implied volatilities that can be deduced from the observable prices for currency put options on the basis of the evaluation formula by Garman and Kohlhagen (1983). For a 3-month period, a value of approximately 4.53% is obtained for the beginning of March 1998. This value was estimated as the arithmetic mean of the implied volatilities of currency put options, each with a life of 1, 3, 6, and 12 months, respectively. Prices were based on those quoted in the German newspaper *Handelsblatt* on March 26, 1998.[30]

[28]Here, the period from $t = 0$ to $t = 1$; that is, one period corresponds to one quarter.

[29]In general, the standard deviation of the logarithm of the exchange rate is proportional to the square root of the respective residual maturity with proportionality factor s.

[30]Indeed, this value corresponds to the greatest possible extent to that obtained with other sources. For implied volatility with regard to the $/DM exchange rate, for example, the Federal Reserve Bank of New York issued values ranging from 4.75 to 4.95% for the period March to August 1998 and adjusted to the quarter. This 3-month volatility estimated with historical data amounts to 4.46%, a value based on the annual $/DM exchange rates for the period 1953 to 1997. An analysis of the data in March 1999 underpins the (almost always existing) permanence of the volatilities that were considered, although there was the introduction of the European Monetary Union at the beginning of 1999. Moreover, rather than with the DM, the American entrepreneur's decision problem could be expressed with the Euro as the foreign currency without any change because there is a fixed relationship between the DM and Euro.

Data Referring to Invitations for Tenders

First, the value of \$125,000,000 is arbitrarily chosen. This amount represents the production costs incurred by the entrepreneur until $t = 2$ in the manufacturing of his good. To determine suitable and fairly plausible values for bid B, the insecure sales return of the American entrepreneur in the case of his being awarded the contract is chosen as a starting point:

$$\frac{B \cdot \tilde{e}_2 - C}{B \cdot \tilde{e}_2} = 1 - \frac{C}{B \cdot \tilde{e}_2} \tag{26}$$

Its expectation value is

$$\overline{SR} := 1 - \frac{C}{B} \cdot E\left[\frac{1}{\tilde{e}_2}\right] \tag{27}$$

Three different values will be introduced for $\overline{SR}$: 5, 15, and 25%. The corresponding individual offers B are DM 232,176,000, DM 259,491,000, and DM 294,090,000. If the sales returns typical for American industrial enterprises are considered,[31] it can be assumed that effective sales returns for individual orders will hardly move outside this scale. On the contrary, 25% would more likely represent an extremely high value for the expected sales return under discussion.

Second, it is necessary to define the probability ϕ to be considered for winning the contract as seen from $t = 0$. Our main focus refers to the sensibility of engaging in SCOUTs. In particular, we want to confront their use with the certainty equivalents attainable by the entrepreneur in the case of refraining from any engagements in derivatives at all. For this comparison, it is easy to show the following relationship: If the no-hedge alternative is superior for some success probability ϕ, it is for any probability $\phi^+ < \phi$.[32] For SCOUTs to be viable in the case of participation of m bidders, they thus must be better than the no-hedge alternative for the bidder with the lowest success probability. For m bidders with homogeneous expectations, the highest possible minimum success probability will be $1/m$. We, therefore, restrict our attention to the case $\phi = 1/m$ and

[31]According to the information given by the *Financial Times* on July 12, 1998, aftertax sales returns for companies in the Standard & Poor's 500 were close to 7%. Moreover, runaways occur with some organizations, such as Compaq, which is by all means able to show, according to Reuters information on July 10, 1997, sales returns ranging around 25%, just defining a maximum for our numerical analysis. As a further example, in 1995 one of the world's leading engineering companies, ABB, had sales returns that came to 9.7%. However, average sales returns on sizable individual orders are certainly larger than those calculated on company or divisional levels. With the latter, costs are included that have to be qualified as overhead with regard to each individual order. For this reason alone, in comparison to empirically observable peculiarities of entrepreneurial sales returns, the values to be determined for $\overline{SR}$ have to be slightly increased. This should certainly give some plausibility to the value range for $\overline{SR}$ used as a basis for our example.

[32]The proof is available from the authors on request.

consider for m all integers from 2 to 10. Third, because the engagement in a SCOUT cannot be optimized independently by any entrepreneur, we must define fixed values for S. To cover a broad range of possible scenarios, we allow for the cases $S \in \{-0.5 \cdot B, 0, 0.5 \cdot B, B, 1.5 \cdot B, 2 \cdot B\}$.[33]

Entrepreneurial Risk Aversion

Finally, the entrepreneur's risk aversion a needs to be specified. The extent of entrepreneurial risk aversion can be described as the amount by which the certainty equivalent of an uncertain cash flow in domestic currency lies below the respective expectation value. In concrete terms, we assume that in a situation without any engagement in forwards or options, RDs of 10, 30, 50, 70, and 90% are levied on the expectation value of the dollar payoff from the tender participation to derive the matching certainty equivalent. On account of these discount rates, the relevant risk aversion can be calculated for each of the scenarios observed. This eventually leads to $1 \cdot 3 \cdot 9 \cdot 5 = 135$ different values for a altogether, all of which were computed for our analysis. Resulting values range from $a = 5.4524 \cdot 10^{-7}$ (for 25% expected sales return, 10% success probability, and 10% RD) to $6.1873 \cdot 10^{-5}$ (for 15% expected sales return, 50% success probability, and 90% RD).[34,35] One may wonder if

[33]$S = -0.5 \cdot B$ means that the entrepreneur sells short SCOUTs. Certainly, this is not possible. However, to better assess the welfare implications of positive engagements in SCOUTs, we consider this case in addition to situations with nonnegative values for S.

[34]All 135 values for a and all other numerical results of this second example are available in detail from the authors on request. The calculation of all 135 values for a took approximately 206 min on a personal computer with an Intel Pentium II processor (233 MHz).

[35]In principle, it is not difficult to allow for other risk preferences than constant absolute risk aversion. For example, we could assume constant relative risk aversion amounting to $1/b$, that is, if we abstract from the logarithmic utility a utility function of the kind $U^+(f_H) = 1/(b-1) \cdot (b \cdot f_H)^{1-1/b}$ $(0 < b \neq 1)$. There are just two issues that should briefly be addressed. First, because (for almost all possible values of b) the domain of U^+ is restricted to positive outcomes, it is necessary to define a lower bound $\underline{f}_H$ below which the utility function must be replaced by another, U^-, for example, one with constant absolute risk aversion. The transition from the first part of the compound utility function to the second one should be designed in such a way as to assure continuous differentiability for all values $f_H \in \mathrm{IR}$ This leads to a utility function U with

$$U(f_H) = \begin{cases} U^+(f_H) & (f_H \geq \underline{f}_H); \\ U^-(f_H) = (b \cdot \underline{f}_H)^{1-1/b} \cdot [b/(b-1) - \exp((\underline{f}_H - f_H)/(b \cdot f_H))] & (f_H < \underline{f}_H). \end{cases}$$

Moreover, in contrast to a situation with constant absolute risk aversion, the parameter b of the utility function U^+ can only be calculated if the entrepreneur's (secure) terminal wealth ΔW_2 at time $t = 2$ from sources other than the international tender participation under consideration is explicitly specified. For example, in the case of $\overline{SR} = 5\%$, RD = 30%, $\phi = 50\%$, and additional safe entrepreneurial earnings of twice the expected payoffs from tender participation, we get $1/b = 0.862$, and for RD = 70%, *ceteris paribus* we arrive at $1/b = 2.186$, when the transition from U^- to U^+ takes place at a value for f_H with no cash flows from the tender participation; that is, $\underline{f}_H = \Delta W_2 = 2 \cdot \phi \cdot (B \cdot E[\tilde{e}_2] - C)$. Values of $1/b$ of this kind fit comparatively well into those orders of magnitude that are the result of empirical studies or serve as a basis for other theoretical analyses. See, for example, Friend and Blume (1975) and the studies cited by Pannel, Malcolm, and Kingwell (2000).

expected utility maximizing behavior is the adequate approach in the case of large companies participating in an international tender because of the potential separation of ownership and control. Even under such circumstances, the expected utility approach can be justified either if we interpret the underlying utility function as a representative one for all investors and suppose the company's manager to act in the interest of the owners or if we assume that the (opportunistic) manager gets a fixed share α (possibly plus some fixed payments) of the company's overall net cash flow. In the last case, costs $C = \$125{,}000{,}000$ must be viewed as only a fraction α of overall costs without any further changes in our analysis. Third, strictly concave objective functions could be justified somewhat more indirectly by the work done by Froot, Scharfstein, and Stein (1993). Our RDs would then become some kind of transaction costs discounts.

Results

For all 135 scenarios, we first calculated optimal entrepreneurial engagements $x^{(1)}$ and $x^{(2)}$ in forwards and options for $S \in \{-0.5 \cdot B, 0, 0.5 \cdot B, B, 1.5 \cdot B, 2 \cdot B\}$ by means of our approximation procedure. This led to 810 optimizations for which we additionally determined the corresponding certainty equivalents CE_A and index values CE_A/CE_h. In this regard, CE_h is defined as the resulting certainty equivalent when the optimal fictitious cash flow is realized for a given value of S. Moreover, we computed the certainty equivalent of the no-hedge alternative $x^{(1)} = x^{(2)} = S = 0$ for each scenario. In all 135 scenarios, the no-hedge alternative leads to higher index values than any positive entrepreneurial engagement in SCOUTs under consideration, whereby the high quality of our approximation solutions is indicated by their average index value of 99.89% for all scenarios with $S \geq 0$ and $CE_h > 0$.[36] This result may explain to some extent why SCOUTs are of only minor practical importance despite their ability to complete hedging possibilities.[37] Indeed, in the case of $S = -0.5 \cdot B$, for most scenarios we got the highest certainty

[36]Situations with $S = -0.5 \cdot B$ are of no practical relevance and were only used for illustrative purposes. Remarkably, in many cases with $S > 0$, we got $CE_h < 0$. By construction, we then may immediately conclude the superiority of the no-hedge alternative so that the calculation of index values is not necessary any more.

[37]We may generalize our analysis to the case where only $m^{(S)} < m$ bidders participate in purchasing SCOUTs. If none of them wins the contract, the SCOUTs are equally divided among them. It is easy to show that our results regarding the lack of attractiveness of engagement in SCOUTs hold for this generalized setting as well. To be precise, in all scenarios under consideration with $S > 0$ and $m^{(S)} \in \{2, 3 \ldots 10\}$, the no-hedge alternative is superior to an engagement in SCOUTs for any $m \geq m^{(S)}$. The proof is available from the authors on request.

equivalents of the hedging strategies under consideration, which underpins our assessment of the insufficient attractiveness of engagements in SCOUTs.

It took 1,224 min to carry out all 810 optimizations and 945 calculations of certainty equivalents ($\approx$1 min 30 s for each optimization case) on a personal computer with an Intel Pentium II (233 MHz). If instead we had used the grid programming approach demonstrated in the section Hedging Uncertain Cash Flows From Corn Production in Iowa to directly approximate optimal hedging strategies, computing time would have been about 40 times higher.[38] Moreover, the numerical solution of such an optimization problem with the FindMinimum command of the program Mathematica completely failed.

Our results imply that we must only take care of situations with $S = 0$. For this case, our approximation procedure leads to an average index value of 99.458%, which corresponds with an average entrepreneurial certainty equivalent of \$2,576,607.4. The average certainty equivalent in the case of perfectly reproducing the optimum payment structure h is \$2,578,276.0, so that the entrepreneur's average maximum gain from the use of better approximation procedures or additional hedging instruments (not depending on $\tilde{y}$) is restricted to approximately \$1,668.6. Moreover, the average gain of the use of our approximation procedure in comparison to simply refraining from hedging at all is \$32,603.0. All index values remain unchanged, and all certainty equivalents must be multiplied by α if we use dollar costs $\alpha \cdot C$ instead of C as our starting point.

From our numerical analysis, we are able to derive some additional conclusions with respect to the optimum use of forwards and options in the case of participation in international invitations for tenders. Both variables $x^{(1)}$ and $x^{(2)}$ react very sensitively in the case of parameter variation. In particular, $x^{(1)}$ seems to be increasing in $\overline{SR}$ and decreasing in RD,[39] whereas the opposite is true for $x^{(2)}$. In general, the absolute values of both variables are rising in ϕ. In fact, our low values for $\phi (\phi \leq 0.5)$ may explain the good performance of the no-hedge alternative even in comparison to a situation without any engagement in SCOUTs. Therefore, we analyzed the situation $S = 0$ also for $\phi \in \{0.6, 0.7, 0.8, 0.9\}$ which led to $3 \cdot 5 \cdot 4 = 60$ additional optimizations. For these

[38]This result was derived for the special case $S = B$, $\overline{SR} = 5\%$, RD = 10%, and $\phi = 50\%$. The grid search routine applied by Sakong et al. (1993; with step sizes of 1,000,000, 100,000, and 10,000) leads to 1,321 grid points that need to be calculated. This takes about 3,680.45 s, which is 40.6 times higher than with the procedure suggested in this article.

[39]Indeed, for RD $\geq$ 50%, we almost always get $x^{(1)} < 0$.

60 scenarios, the average gain from implementing our approximation solutions instead of the no-hedge alternative in the case of $S = 0$ amounts to \$422,515.07.

Finally, we want to return to the question of the efficiency of the pure variance-minimization approach. As it is easy to verify, variance minimization for $S = 0$ can be achieved by $x^{(1)} = \phi \cdot B$ and $x^{(2)} = 0$ because of the independence of $\tilde{\gamma}$ and $\tilde{e}^{(f)}_{1,2}$.[40] This solution can be called the (variance-minimizing) pure forward hedge, and index values for this pure forward hedge were determined for $\phi \in \{10\%, 11.1\%, 12.5\%, 14.3\%, 16.7\%, 20\%, 25\%, 33.3\%, 50\%, 60\%, 70\%, 80\%, 90\%\}$, RD $\in \{10\%, 30\%, 50\%, 70\%, 90\%\}$, and $\overline{SR} \in \{5\%, 15\%, 25\%\}$. In Table II, we present the average index values for all combinations of ϕ and RD.[41]

As can be seen, the performance of the pure forward hedge becomes poorer with increasing RDs[42] and, somewhat surprisingly, increasing success probabilities. In fact, especially for RDs greater than 30%, the pure forward hedge does not seem to be able to sufficiently approximate the true expected utility maximum, whereas our approximation procedure leads to an average index value of 99.57% for the scenarios underlying Table II. Moreover, even for RD = 30%, there is at least one case with $\overline{SR} = 15\%$ (not separately presented in Table II) where the pure forward hedge only leads to an index value smaller than 94%. For the model context presented in this section, a pure forward hedge, therefore, can only be recommended without hesitation in situations with RD $\leq$ 10%.

In summary, once again our approximation procedure seems to be very efficient at generating good hedging recommendations.

[40] See, for example, Wolf (1987, Theorem 3, pp. 152–153).

[41] The reader may be surprised that there are some negative (average) index values. They indicate situations where the entrepreneur would actually prefer to refrain from participating in the tender if he would simultaneously be forced to conduct the pure forward hedge. By construction, it is guaranteed that the entrepreneur will be willing to participate in the tender if he is allowed to freely determine $x^{(1)}$ and $x^{(2)}$. So in these situations, variance minimization leads to rather poorly performing recommendations.

[42] The same result can be derived with regard to the production example. The first three cases of Table I correspond with RDs in the terminology of this section of 19.8, 33.44, and 43.72%, respectively. The variance-minimizing engagement in derivatives once again is the pure forward hedge and is characterized by the forward sale of 20,000 bushels and no activities in options, thus leading to index values of 99.2, 91.48, and only 62.48%, respectively. This result thus confirms the possible shortcomings of pure variance-minimization approaches as approximation procedures for expected utility maximization for other than quadratic utilities. Moreover, further numerical analyses performed by us reveal that this assessment of the approximation quality of variance-minimization approaches even holds in the case of the two examples of (eventually) constant relative risk aversion presented in Footnote 35. The (variance-minimizing) pure forward hedge amounts to index values of 98.36 and 88.68%, whereas the approximation solutions presented in this article lead to 99.98 and 99.76%.

TABLE II

Average Index Values of the Pure Forward Hedge for All Combinations of ϕ and RD

	RD = *10%*	*RD* = *30%*	*RD* = *50%*	*RD* = *70%*	*RD* = *90%*
ϕ = 10.0%	100.00%	99.83%	98.85%	94.52%	64.60%
ϕ = 11.1%	99.99%	99.81%	98.72%	93.93%	61.39%
ϕ = 12.5%	99.99%	99.78%	98.55%	93.21%	57.32%
ϕ = 14.3%	99.99%	99.75%	98.33%	92.27%	51.99%
ϕ = 16.7%	99.99%	99.71%	98.04%	91.01%	44.67%
ϕ = 20.0%	99.99%	99.65%	97.62%	89.22%	34.00%
ϕ = 25.0%	99.99%	99.55%	96.97%	86.47%	17.03%
ϕ = 33.3%	99.98%	99.38%	95.82%	81.68%	−14.10%
ϕ = 50.0%	99.97%	99.00%	93.22%	71.11%	−90.37%
ϕ = 60.0%	99.97%	98.74%	91.41%	63.98%	−150.31%
ϕ = 70.0%	99.96%	98.40%	89.29%	55.93%	−230.42%
ϕ = 80.0%	99.95%	97.92%	86.52%	46.07%	−352.06%
ϕ = 90.0%	99.93%	96.77%	81.73%	30.97%	−5,974.40%

Note: Average index values of the pure forward hedge for given success probability ϕ and *RD* in the tender example. The respective average is calculated over all possible realizations of the expected sales return *SR*. According to the results of this table, the performance of the pure forward hedge *ceteris paribus* becomes poorer with increasing ϕ and RD.

CONCLUSION

We presented a new approximation procedure for calculating optimal pure hedging strategies and illustrated its application with two examples. For our examples, this new approximation method on average proved to be 40 to 300 times faster than simple grid search procedures. It thus seems to be particularly well suited for conducting complex numerical sensitivity analyses with hundreds or even thousands of different parameter constellations.

Our procedure can be applied to certain situations with multiple exposures as well. As pointed out in Footnote 2, it does not lead to any additional difficulty if the decision maker faces more than just one nonhedgable random variable. For the tender example, this means that it would cause no particular problems to examine situations where an American entrepreneur is simultaneously participating in several invitations for tenders with all contracts denominated in the same foreign currency. Other aspects of the flexibility of our approach, such as the possibility of basing the analysis on different risk preferences, have already been mentioned. Moreover, numerical analyses may help banks to design more successful financial innovations. For example, in principle it can easily be tested if a modified SCOUT of the following type might be attractive for firms: The winner of the contract has to reimburse at time

$t = 1$ all other SCOUT participants for their shares of the SCOUT premium as paid at time $t = 0$. With this new design, the problem is circumvented that entrepreneurs with low success probability are not willing to participate in the acquisition of SCOUTs. It would be interesting to analyze the viability of this modified SCOUT with the method developed in this article.

After all, the two main problems with regard to the proposed approximation procedure might be that there are no straightforward extensions of our approach to allow for speculation possibilities and for the use of derivatives depending on more than one random variable. However, as was shown in our examples, our approximation method can be applied to very different decision problems discussed in the literature, and it usually works quite well. Therefore, we think that there remain enough possibilities for the use of our approach in solving complex entrepreneurial hedging problems.

APPENDIX

Derivation of 5

The necessary Condition 3, yields for a quadratic utility function, $U(f_H) = a_1 \cdot f_H - a_2 \cdot f_H^2$:

$$a_1 - \lambda = 2 \cdot a_2 \cdot E_\gamma[f(\tilde{\gamma}, r) + h(r))|r]$$

$$\Leftrightarrow h(r) = \frac{a_1 - \lambda}{2 \cdot a_2} - E_\gamma[f(\tilde{\gamma}, r)|r] \tag{A1}$$

If Condition 4 is also applied, the result is $\lambda = a_1 - 2 \cdot a_2 \cdot E[f(\tilde{\gamma}, \tilde{r})]$, and thus 5 is directly obtained from A1.

Derivation of 6

Using Condition 3 yields for an exponential utility function, $U(f_H) = -\exp(-a \cdot f_H)$:

$$a \cdot E_\gamma[\exp(-a \cdot (f(\tilde{\gamma}, r) + h(r))|r] = \lambda$$

$$\Leftrightarrow h(r) = -\frac{1}{a} \cdot \ln\left(\frac{\lambda}{a}\right) + \frac{1}{a} \cdot \ln(E_\gamma[\exp(-a \cdot f(\tilde{\gamma}, r))|r]) \tag{A2}$$

Condition 4 particularly implies $\ln(\lambda/a) = E_\gamma[\ln(E_\gamma[\exp(-a \cdot f(\tilde{\gamma}, \tilde{r}))| \tilde{r}])]$, so that the optimal cash flow is finally determined according to 6.

Derivation of 8

With $f_A(\tilde{\gamma}, \tilde{r}) := f(\tilde{\gamma}, \tilde{r}) - C_A$ and $\Pi_A^+ := \Pi_A - C_A$, Equation 7 is equivalent to $E[U(f_A(\tilde{\gamma}, \tilde{r}) + h(\tilde{r}) - \Pi_A^+)] = E[U(f_A(\tilde{\gamma}, \tilde{r}) + h(\tilde{r}) + \varepsilon_A(\tilde{r}))]$. Because 8 is obvious in the case $\Pi_A^+ < 0$,, it can be assumed $\Pi_A^+ \geq 0$. The application of Taylor's theorem to both sides of the latter equation leads to the two inequalities given next. The development of the right side, when $0 < \alpha(\tilde{\gamma}, \tilde{r}) < 1$ is considered, results in

$$
\begin{aligned}
&E[U(f_A(\tilde{\gamma}, \tilde{r}) + h(\tilde{r}) + \varepsilon_A(\tilde{r}))] = E[U(f_A(\tilde{\gamma}, \tilde{r}) + h(\tilde{r}))] \\
&\quad + E\,\underbrace{[U'(f_A(\tilde{\gamma}, \tilde{r}) + h(\tilde{r}) + \alpha(\tilde{\gamma}, \tilde{r}) \cdot \varepsilon_A(\tilde{r})) \cdot \varepsilon_A(\tilde{r})]}_{\geq U'(f_A(\tilde{\gamma}, \tilde{r}) + h(\tilde{r}) + \varepsilon_A(\tilde{r})) \cdot \varepsilon_A(\tilde{r})} \\
&= E[U(f_A(\tilde{\gamma}, \tilde{r}) + h(\tilde{r}))] + \operatorname{cov}[U'(f_A(\tilde{\gamma}, \tilde{r}) + h_A(\tilde{r})), \varepsilon_A(\tilde{r})] \\
&\geq E[U(f_A(\tilde{\gamma}, \tilde{r}) + h(\tilde{r}))] - \operatorname{stddev}[U'(f_A(\tilde{\gamma}, \tilde{r}) \\
&\quad + h_A(\tilde{r}))] \cdot \operatorname{stddev}[\varepsilon_A(\tilde{r})]
\end{aligned} \tag{A3}
$$

The left side develops correspondingly with $0 < \beta(\tilde{\gamma}, \tilde{r}) < 1$:

$$
\begin{aligned}
&E[U(f_A(\tilde{\gamma}, \tilde{r}) + h(\tilde{r}) - \Pi_A^+)] \\
&\quad = E[U(f_A(\tilde{\gamma}, \tilde{r}) + h(\tilde{r}))] - \Pi_A^+ \cdot \underbrace{E[U'(f_A(\tilde{\gamma}, \tilde{r}) + h(\tilde{r}))]}_{\geq E[U'(f(\tilde{\gamma}, \tilde{r}) + h(\tilde{r}))] = \lambda} \\
&\quad + \frac{(\Pi_A^+)^2}{2} \cdot E[U''(f_A(\tilde{\gamma}, \tilde{r}) + h(\tilde{r}) - \beta(\tilde{\gamma}, \tilde{r}) \cdot \Pi_A^+)] \\
&\quad \leq E[U(f_A(\tilde{\gamma}, \tilde{r}) + h(\tilde{r}))] - \lambda \cdot \Pi_A^+
\end{aligned} \tag{A4}
$$

As both sides are equal as a result of 7, Inequality 8 is directly obtained.

Proof of Theorem 1

Let $\hat{h}(r)$ be defined in such a way that $h(r) = \hat{h}(r) - E[\hat{h}(\tilde{r})]$. Each number $z \in \mathbb{Z}$ and $i \in \mathbb{N}$ defines a value $r_{i,z} := z/2^i$ and set $W_{i,z} := [r_{i,z}, r_{i,z+1})$ with the property $\cup_{z \in \mathbb{Z}} W_{i,z} = \mathbb{R}$ for each $i \in \mathbb{N}$. In addition, Z_i denotes the following finite subset of $\mathbb{Z}$: $Z_i := \{z \in \mathbb{Z} \mid \overline{W}_{i,z} := [r_{i,z}, r_{i,z+1}] \subset [-i-1, i+1]\}$. With these notations and additional $h_{i,j,z} := h(r_{i,z}) \cdot 2^j$, it is possible to construct the following portfolio of puts $\hat{g}_{i,j,z}$ for each $i, j \in \mathbb{N}$ $(j > i)$ and $z \in \mathbb{Z}$. $\hat{g}_{i,j,z}$ consists of

- $h_{i,j,z}$ puts with strike price $r_{i,z} - 1/2^j$.
- $h_{i,j,z}$ short puts with strike price $r_{i,z}$.

- $h_{i,j,z}$ short puts with strike price $r_{i,z+1} - 1/2^j$.
- $h_{i,j,z}$ puts with strike price $r_{i,z+1}$.

The resulting cash flow of this portfolio for a given $\in \mathbb{R}$ can easily be transformed to

$$\hat{g}_{i,j,z}(r) = \begin{cases} 0, & \text{if } r \le r_{i,z} - \dfrac{1}{2^j} \\ h_{i,j,z} \cdot \left(r - r_{i,z} + \dfrac{1}{2^j}\right), & \text{if } r_{i,z} - \dfrac{1}{2^j} < r \le r_{i,z} \\ h_{i,j,z} \cdot \dfrac{1}{2^j}, & \text{if } r_{i,z} < r \le r_{i,z+1} - \dfrac{1}{2^j} \\ h_{i,j,z} \cdot (r_{i,z+1} - r), & \text{if } r_{i,z+1} - \dfrac{1}{2^j} < r \le r_{i,z+1} \\ 0, & \text{if } r > r_{i,z+1} \end{cases} \tag{A5}$$

For an arbitrary $r \in \mathbb{R}$ and sufficiently large $i \in \mathbb{N}$, there exists an index $z_i \in \mathbb{Z}$ with $r \in W_{i,z_i}$. Moreover, a value $j_0 \in \mathbb{N}$ can be determined so that each $j \in \mathbb{N}^{\ge j_0}$ implies $r < r_{i,z+1} - 1/2^j$. Therefore, $\hat{g}_{i,j,z_i}(r) = h_{i,j,z} \cdot 1/2^j = \hat{h}(r_{i,z})$. Furthermore, $\hat{g}_{i,j,z}(r) = 0$, if $z \neq z_i$ and j is sufficiently high. This in turn implies that the portfolio $\hat{f}^{(i,j)} := \Sigma_{z \in Z_i} g_{i,j,z}$ has the following property:

$$\lim_{j \to \infty} \hat{f}^{(i,j)}(r) = \hat{h}(r_{i,z_i}) \tag{A6}$$

The characteristic $\lim_{i \to \infty} r_{i,z_i} = r$ and the continuity of h lead to

$$\lim_{i \to \infty} \lim_{j \to \infty} \hat{f}^{(i,j)}(r) = \hat{h}(r) \tag{A7}$$

Under the consideration of an unbiased evaluation, this result of pointwise convergency remains valid for the total cash flows h and $f^{(i,j)} = \hat{f}^{(i,j)}(r) - E[\hat{f}^{(i,j)}(\tilde{r})]$. Thus, the convergence also persists under the expectation integrals:

$$\lim_{i \to \infty} \lim_{j \to \infty} E[h(\tilde{r}) - f^{(i,j)}(\tilde{r}))^2] = 0 \tag{A8}$$

Proof of Theorem 2

The derivation of both sides in Condition 3 with respect to r leads to

$$h'(r) = -\frac{E_\gamma\left[U''(f(\tilde{\gamma}, r) + h(r)) \cdot \dfrac{df}{dr}(\tilde{\gamma}, r) \middle| r\right]}{E_\gamma[U''(f(\tilde{\gamma}, r) + h(r))|r]} \tag{A9}$$

Thus $h'(r)$ always has the inverse sign of $df/dr(\gamma, r)$.

Deriving both sides twice in Condition 3 with respect to r results in

$$h''(r) = -\frac{E_\gamma\left[U'''(f(\tilde{\gamma}, r) + h(r)) \cdot \left(\frac{df}{dr}(\tilde{\gamma}, r) + h'(r)\right)^2 \middle| r\right]}{E_\gamma[U''(f(\tilde{\gamma}, r) + h(r))|r]}$$
$$- \frac{E_\gamma\left[U''(f(\tilde{\gamma}, r) + h(r)) \cdot \frac{d^2f}{dr^2}(\tilde{\gamma}, r) \middle| r\right]}{E_\gamma[U''(f(\tilde{\gamma}, r) + h(r))|r]} \qquad \text{(A10)}$$

Both summands have a nonnegative sign in the case of $P_\gamma(d^2f/dr^2(\tilde{\gamma}, r_0) < 0|r_0) > 0$, with the second summand even possessing a positive sign.

If, however, the condition $P_\gamma(d^2f/(d\gamma dr)(\tilde{\gamma}, r_0) \neq 0|r_0) > 0$ is used, the following holds:

$$\frac{df}{dr}(\tilde{\gamma}, r) + h'(r) = \frac{df}{dr}(\tilde{\gamma}, r) - \frac{E_\gamma\left[U''(f(\tilde{\gamma}, r) + h(r)) \cdot \frac{df}{dr}(\tilde{\gamma}, r) \middle| r\right]}{E_\gamma[U''(f(\tilde{\gamma}, r) + h(r))|r]}$$
$$= \frac{df}{dr}(\tilde{\gamma}, r) - \frac{df}{dr}(\hat{\gamma}, r) \cdot \frac{E_\gamma[U''(f(\tilde{\gamma}, r) + h(r))|r]}{E_\gamma[U''(f(\tilde{\gamma}, r) + h(r))|r]}$$
$$= \frac{df}{dr}(\tilde{\gamma}, r) - \frac{df}{dr}(\hat{\gamma}, r) \neq 0, \qquad \text{(A11)}$$

because according to the assumption, $df/dr(\gamma, r)$ is not constant with regard to γ.

BIBLIOGRAPHY

Adler, M., & Detemple, J. (1988). Hedging with futures in an intertemporal portfolio context. Journal of Futures Markets, 8, 249–269.

Arrow, K. J. (1971). Essays in the theory of risk-bearing. Chicago: Markham.

Benninga, S., Eldor, R., & Zilcha, I. (1983). Optimal hedging in the futures market under price uncertainty. Economics Letters, 13, 141–145.

Benninga, S., Eldor, R., & Zilcha, I. (1985). Optimal international hedging in commodity and currency forward markets. Journal of International Money and Finance, 4, 537–552.

Briys, E., & Schlesinger, H. (1993). Optimal hedging when preferences are state dependent. Journal of Futures Markets, 13, 441–451.

Broll, U., & Wahl, J. E. (1992). Hedging with synthetics, foreign-exchange forwards, and the export decision. Journal of Futures Markets, 12, 511–517.

Broll, U., & Wong, K. P. (1999). Hedging with mismatched currencies. Journal of Futures Markets, 19, 859–875.

Cass, D., & Stiglitz, J. E. (1970). The structure of investor preferences and asset returns, and separability in portfolio allocation—A contribution to the pure theory of mutual funds. Journal of Economic Theory, 2, 122–160.

Eaker, M. R., & Grant, D. (1985). Optimal hedging of uncertain and long term foreign exchange exposure. Journal of Banking and Finance, 9, 221–231.

Edwardes, W. (1999). So what's the big IDEA? Unpublished manuscript.

Friend, I., & Blume, M. E. (1975). The demand for risky assets. American Economic Review, 65, 900–921.

Froot, K. A., Scharfstein, D. S., & Stein, J. (1993). Risk management: Coordinating corporate investment and financing policies. Journal of Finance, 48, 1629–1658.

Garman, M. B., & Kohlhagen, S. W. (1983). Foreign currency option values. Journal of International Money and Finance, 2, 231–237.

Jacque, L. L. (1996). Management and control of foreign exchange risk. Boston: Kluwer.

Kraus, A., & Litzenberger, R. H. (1976). Skewness preference and the valuation of risk assets. Journal of Finance, 31, 1085–1100.

Kroll, Y., Levy, H., & Markowitz, H. M. (1984). Mean-variance versus direct utility maximization. Journal of Finance, 39, 47–61.

Lagerstam, C. (1990). Hedging of contracts, anticipated positions, and tender offers: A study of corporate foreign exchange rate risk and/or price risk. Stockholm, Sweden: Stockholm School of Economics.

Lei, L.-F., Liu, D., & Hallam, A. (1995). Solving for optimal futures and options positions using a simulation-optimization technique. Journal of Futures Markets, 15, 559–571.

Lien, D. (2000). Production and hedging under Knightian uncertainty. Journal of Futures Markets, 20, 397–404.

Merton, R. C. (1972). An analytic derivation of the efficient portfolio frontier. Journal of Financial and Quantitative Analysis, 7, 1851–1872.

Pannel, D. J., Malcolm, B., & Kingwell, R. S. (2000). Are we risking too much? Perspectives on risk in farm modelling. Agricultural Economics, 23, 69–78.

Park, T., & Antonovitz, F. (1990). Basis risk and optimal decision making for California feedlots. Journal of Futures Markets, 10, 259–271.

Poitras, G. (1993). Hedging and crop insurance. Journal of Futures Markets, 13, 373–388.

Rothschild, M., & Stiglitz, J. E. (1970). Increasing risk: I. A definition. Journal of Economic Theory, 2, 225–243.

Rubinstein, M. E. (1973). The fundamental theorem of parameter-preference security valuation. Journal of Financial and Quantitative Analysis, 8, 61–69.

Sakong, Y., Hayes, D. J., & Hallam, A. (1993). Hedging production risk with options. American Journal of Agricultural Economics, 75, 408–415.

Steil, B. (1993). Currency options and the optimal hedging of contingent foreign exchange exposure. Economica, 60, 413–431.

Wolf, A. (1987). Optimal hedging with futures options. Journal of Economics and Business, 39, 141–158.

Working, H. (1962). New concepts concerning futures markets and prices. American Economic Review, 52, 431–459.