
NATURAL SELECTION AND MARKET EFFICIENCY IN A FUTURES MARKET WITH RANDOM SHOCKS

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Even when participants know very little about their environment, the market itself, by serving as a selection process of information, promotes an efficient aggregate outcome. To emphasize the role of the market and the importance of natural selection rather than the strategic actions of participants, an evolutionary model of a commodity futures market is presented, in which there is a continual inflow of unsophisticated traders with predetermined distributions of prediction errors with respect to the fundamental value of the spot price. The market acts as a selection process by constantly shifting wealth from traders with less accurate information to those with more accurate information. Consequently, with probability 1, if the volatility of the underlying spot market is sufficiently small, the proportion of time that the futures price is sufficiently close to the fundamental value converges to one. Furthermore, the width of the interval containing the fundamental value, where the futures price eventually lies, increases as the volatility of the underlying spot market increases. © 2001 John Wiley & Sons, Inc. *Jrl Fut Mark* 21:489–516, 2001

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INTRODUCTION

A financial market is informationally efficient if the market price fully reflects all available information. In economics and finance, traditional methodology usually explains the occurrence of informational efficiency in terms of actions of rational agents. However, Patel, Jayendu, Zeckhauser, and Hendricks (1991) suggested that if the market has a sufficient degree of ability to transfer wealth among traders, this is sufficient to generate an informationally efficient market, regardless of whether traders are rational or not. Here, the biological term *natural selection* takes the form of transferring wealth from less fit traders to more fit ones in the market. This natural selection force in the market promotes an efficient outcome regardless of the rationality of traders.

The idea that markets work even when participants know very little about their environment or about other participants was recognized by Hayek (1945), who marveled at the market price as an economizer of information despite "how little individual participants need to know" (p. 527).¹ Nevertheless, Hayek was never clear about how this evolved. It would seem that there must be an *invisible hand* process that leads the market to an efficient outcome. That is, there is an efficiency pattern that evolves not by conscious design but "instead through the (decentralized) interaction of agents having no such overall pattern in mind" (Nozick, 1994, p. 314). For this efficiency pattern to occur in a market with diverse information, the market must serve as a natural selection device that screens out noisy information and only selects for accurate information.

How the market specifically works as a selection or filtering process in promoting an efficient outcome is illustrated in this article through a dynamic model of a commodity futures market. To illustrate the role of the market as a selection process, it is important that such a model allows for the transfer of wealth among traders and the accumulation of wealth across time. On the one hand, in the oral tradition of economics, it is sometimes argued that agents with inaccurate information may be driven, via bankruptcy, from the market by "natural forces" (Camerer, 1987, p. 982) arising from the presence of more informed agents. On the other hand, there is the frequently provided counterargument that a constant inflow of new poorly informed traders is enough to disrupt any possible market filtering of information.² Therefore, to fully test the idea of

¹This issue is also examined in an experimental context in Smith (1982).

²See Camerer (1987) for a list of arguments (and counterarguments) used to defend economic theories from the criticism that markets are not rational.

natural selection in promoting market efficiency, it is also important that such a model allows for a continual inflow of traders with diverse noisy information signals.

There have been a few studies that examine the relationship between market efficiency and natural selection through wealth distribution. They include Feiger (1978), Figlewski (1978, 1982), and Luo (1998). Feiger's explanation of how an efficient equilibrium can evolve through wealth redistribution relies on traders' rational expectations and uninformed traders learning from market prices to uncover information. Figlewski (1978, 1982) also explored the role of wealth redistribution in determining efficiency in the context of a speculative market with a finite number of traders endowed with differing qualities of information. He assumed that traders maximize their utility and have the ability to make posterior updates of predictions from information gathered in the market. Figlewski (1982) showed that informational efficiency occurs in the long run only in the special situation where all traders have independent information. In a more recent article, Luo (1998) examined the role of natural selection with respect to market efficiency in the context of a futures market with discrete prices. Her article makes a departure from the other articles, in that for the attainment of informational efficiency through natural selection, no rationality is required on the part of traders.

This article is different from the aforementioned rationality approaches. Unlike Feiger (1978) and Figlewski (1978, 1982), to isolate the role of the market as a filtering process, in this article (like Luo, 1998), there is no requirement for rationality on the part of traders. Here traders are not maximizing particular objectives, nor are they receiving feedback from market prices or other traders' behavior. Traders merely act on some predetermined or inherent behavior rules (described in detail in the next section). This article adopts an evolutionary approach of natural selection over these predetermined behavior rules. Unlike Figlewski (1982), informational efficiency can be obtained without the assumption that traders have independent information. One assumption of Figlewski (1978, 1982) that is critical in preventing informational efficiency is the idea that there is a fixed number of traders. In contrast, this article allows for an ongoing entry of new traders, with diverse prediction abilities. This article extends Luo (1998) to allow for continuous prices and traders who both buy and sell. Simulations are conducted to follow wealth redistribution among traders as the futures price converges to the fundamental value. Moreover, unlike Luo (1998), this article, by adding a random shock to the fundamental value in determining the spot price, is able to investigate the relationship between the degree of volatility in

the spot market and the extent of convergence of the futures price to the fundamental value. Although this relationship was explored through simulation by Figlewski (1978), this article is able to characterize this dependence analytically.

To illustrate the idea of natural selection in promoting market efficiency, this article presents a simple and straightforward model. The model is briefly described as follows. Consider a commodity futures market. The commodity is assumed to be nonstorable and must be sold in the spot market at the end of each time period. Correspondingly, the futures contracts are one period in length. Traders enter the market sequentially over time at the beginning of each time period. Traders are engaged in buying or selling contracts to make speculative profits. The spot price consists of the fundamental value plus a random shock to the spot market. The fundamental value is determined in the beginning of each time period before the futures market opens but is unknown to all market participants. The random shock is realized in the spot market at the end of each time period. In each time period, each trader's prediction about the fundamental value together with the trader's wealth provides this trader's demand function for contracts. Each trader's wealth in each time period is defined as the accumulated profits up to that time period. The futures market is a Walrasian market structure. Each time period, the futures price is the futures market clearing price, which equates the aggregate net demand for contracts with the supply of contracts from producers in that time period.

To isolate the role of the market as a selection or filtering process, it is assumed that all traders are unsophisticated in the sense that traders do not adjust their behavior in response to other participants on the basis of their past market experiences. However, it is generally recognized that traders have access to a wide disparity of information. Thus, traders may have information signals with different degrees of reliability. This can be translated into differing abilities of traders in predicting the fundamental value of the spot price.³ Specifically, with the each trader's entry, he is endowed with an initial amount of wealth and a probability distribution of the prediction error with respect to the fundamental value. The endowment of the probability distribution is trader-specific and is fixed in all subsequent time periods. These probability distributions describe

³Furthermore, even if traders have access to the same information, they may disagree on its correct interpretation and may have differing abilities in processing the same information. Of course, this would further add to differing abilities of traders in predicting the fundamental value of the spot price. This would also be consistent with views of bounded rationality (e.g., Simon, 1959, 1986; Vriend, 1996).

differing abilities of traders in predicting the fundamental value of the spot price, and they reflect the diverse noisy information among traders. One trader is more informed (or possesses less noisy information and more accurate information) than another if this trader's probability distribution generates a higher probability of predicting arbitrarily close to the fundamental value than another trader's distribution does.

To easily convey the idea about how the market functions as a filtering process, an intuitive explanation begins with a model with no random shock to the spot market. In this model, traders with better predictions make gains at the expense of their trading counterparts. Therefore, traders with more accurate information tend to accumulate more wealth asymptotically than the traders with less accurate information. As a result, the predictions coming from traders with more accurate information get reflected into the futures price with a greater weight than the predictions from those traders with less accurate information. Over time, traders are constantly entering the market. Some of these traders are not well informed and some are very well informed. If in each time period there is a positive probability that the entering trader possesses more accurate information than any previously entering traders, the filtering process would constantly shift wealth from traders with less accurate information to traders with more accurate information. Thus, the filtering process eventually screens out less accurate information and selects for more accurate information. The futures price will be eventually driven to the fundamental value.

However, with the presence of the random shock to the spot market, this story still works but with less precision. The more volatile the spot market is, the more noisy the information is for which the filtering process selects. As a result, the deviation of the futures market from informational efficiency gets larger. In other words, the width of the interval containing the fundamental value, where the futures price eventually lies, increases as the volatility of the underlying spot market increases.⁴ This latter result is distinctive and reinforces some earlier, similar findings of Figlewski (1978). It is worth noting, however, that Figlewski (1978) in his seminal article had to rely on simulations to illustrate this point; here, this result is achieved in a rigorous analytical way as part of a more general theorem.

One of the key ingredients for the functioning of natural selection in this futures market is each trader's wealth constraint affecting the

⁴A similar descriptive story behind this selection process can be found in Cootner (1967). Another market selection process for producers in an industry is formulated in Luo (1995), where efficient firms are selected for and a perfectly competitive market arises in the long run.

trader's demand and supply of futures contracts. The role of a wealth or budget constraint in producing an efficient outcomes was noted by Becker (1962), who argued that market rationality can result from agents' random choices being subject to budget constraints. Later on, Gode and Sunder (1993, 1997) built on this idea and used experiments to demonstrate that allocative efficiency can be achieved even with zero-intelligence traders and the imposition of a budget constraint where no trader is allowed to sell below the costs or buy above the value. This article similarly finds that random or pervasive irrational behavior on the part of individual traders can coexist with a rational aggregate market. For Gode and Sunder, aggregate market rationality takes the form of allocative efficiency, whereas here aggregate market rationality shows itself in longrun informational efficiency. Nevertheless, there is one key difference between these two sets of models. In Gode and Sunder's allocative efficiency conclusions, the evolutionary idea of natural selection plays no role, whereas here there is a natural selection through time due to a constant reallocation of wealth away from traders with poor information and toward traders with better information.

This article is organized into five sections. The next section describes the commodity futures market. Traders' predetermined behavior rules are defined in the third section. The results of the model are provided in the fourth section along with numerical illustrations. The last section concludes the article.

COMMODITY FUTURES MARKET

Consider a dynamic model of a commodity futures market. Time is discrete and indexed by t , where $t = 1, 2, \dots$. The commodity is nonstorable and must be sold at the end of each time period. Hence, futures contracts are one period in length. The futures market opens at the beginning of each time period. The futures market closes after all transactions in the futures market are completed. Traders participate in the futures market by buying or selling contracts. The aggregate supply of futures contracts from producers at time t , $t = 1, 2, \dots$, denoted as S_t , is randomly determined each time period from an interval $[0, \bar{S}]$ according to a given probability distribution.⁵ After the futures market closes, the spot market opens at the end of each time period. The spot price in each time

⁵To be consistent with the evolutionary framework, producers are also assumed to be irrational in the sense that producers have no understanding of the implication of the past futures prices and have no knowledge of the present futures price.

period is determined in the spot market. The spot price consists of the fundamental value of the spot market and a random shock to the spot market (coming from either the consumers' demand or the producers' supply in the spot market). That is, the spot price at time period t , denoted by P_t , is modeled as

$$P_t = Z_t + \omega_t$$

where Z_t represents the fundamental value of the spot market at time period t . Z_t is determined at the beginning of time period t before traders purchase or sell their contracts, but Z_t is unknown to all the market participants. It is assumed that $\{Z_t\}_{t \geq 1}$ is a random sequence taking values in an interval $[Z, \bar{Z}]$, where $0 < Z < \bar{Z} < \infty$. ω_t is a random shock to the spot market at time period t , and ω_t is realized at the end of time period t . $\{\omega_t\}_{t \geq 1}$ is assumed to be an i.i.d. random sequence taking values in an interval $[-\omega, \omega]$, where $\omega > 0$. ω_t has a symmetric density with $E(\omega_t) = 0$. To prevent the spot price being negative, it is assumed that $\omega \leq Z$.⁶

It is assumed that the random shock at time t , ω_t , is independent of the fundamental value Z_s , for $s = 1, 2, \dots$, and is independent of the aggregate supply of contracts from the producers S_t , for $t = 1, 2, \dots$.

Traders are assumed to enter the market sequentially over time and participate with previously entered traders for the purpose of making speculative profits. At the beginning of time period t , where $t = 1, 2, \dots$, a single trader (called trader t) is allowed to enter the market, and this trader will continue to participate in all future time periods. The fundamental value at time t (Z_t) is determined before trader t enters the market but unknown to all the market participants entering up to time period t . The timing of events for the model is illustrated in Figure 1.

Each trader is assumed to be endowed with an initial wealth V_0 upon entry. Trader t 's prediction or belief about the fundamental value at time s , where $s \geq t$, denoted by b_s^t , indicates that trader t is willing to buy the number of contracts at a price no higher than b_s^t up to whatever the trader's wealth permits and sell the number of contracts at a price no

⁶This decomposition of the spot price is consistent with many economic stories. One such story is as follows. The output level of each of a very large number of producers is determined (although unknown to other producers) at the beginning of the time period, and the total output of all producers is delivered and sold in the spot market at the end of the time period. The intersection of a prespecified consumers' demand curve (determined at the beginning of the time period) with the producers' aggregate supply in the spot market determines the underlying spot price (or fundamental value, Z_t), although unknown to all participants, at the beginning of the time period. However, if a random shock is added to the consumers' demand or the producers' supply at the end of each time period, this spot price (P_t) at the end of the time period becomes $P_t = Z_t + \omega_t$.

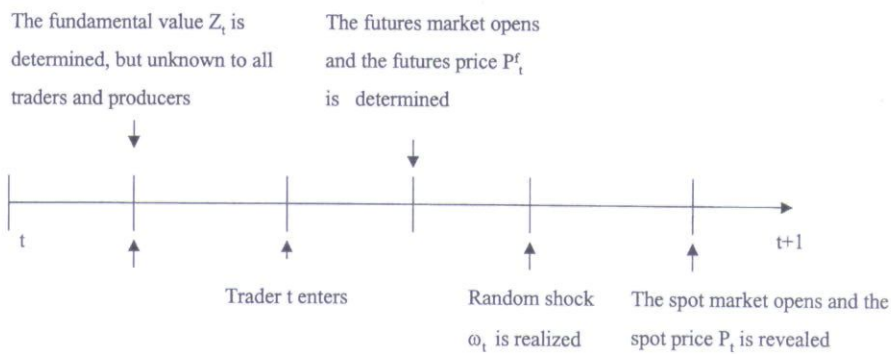


FIGURE 1
Timing of Events.

lower than b_s^t up to whatever the trader's wealth permits him or her to honor.

Specifically, denote trader t 's wealth at the end of time period s , where $s \geq t$, as V_s^t and $V_{t-1}^t = V_0$. Denote the futures price at time s as P_s^f , where $s = 1, 2, \dots$. Given trader t 's prediction at time s , (where $s \geq t$), b_s^t , trader t 's demand for contracts at time period s is V_{s-1}^t/P_s^f if $P_s^f < b_s^t$.⁷ However, if $P_s^f > b_s^t$, trader t short-sells contracts. Because the highest spot price is $\bar{Z} + \omega$, to ensure that each trader honors his or her contracts when he or she is selling contracts, the supply of contracts from each trader is constrained such that this trader has enough money to cover his or her loss. Therefore, if $P_s^f > b_s^t$, the demand for contracts at time period s , where $s \geq t$, is $-V_{s-1}^t/(\bar{Z} + \omega - P_s^f)$. To summarize, trader t 's demand for contracts at time period s , where $s \geq t$, is

$$q_s^t(P_s^f) = \begin{cases} \frac{V_{s-1}^t}{P_s^f} & \text{if } P_s^f < b_s^t \\ \left[-\frac{V_{s-1}^t}{\bar{Z} + \omega - P_s^f}, \frac{V_{s-1}^t}{P_s^f} \right] & \text{if } P_s^f = b_s^t \\ -\frac{V_{s-1}^t}{\bar{Z} + \omega - P_s^f} & \text{if } P_s^f > b_s^t \end{cases} \tag{1}$$

An illustrative demand curve is in Figure 2.

⁷In the following, it is assumed that traders actively use all of their wealth for speculation. However, even if traders withdraw for consumption a constant fraction of their wealth each time period, the results of the article remain the same.

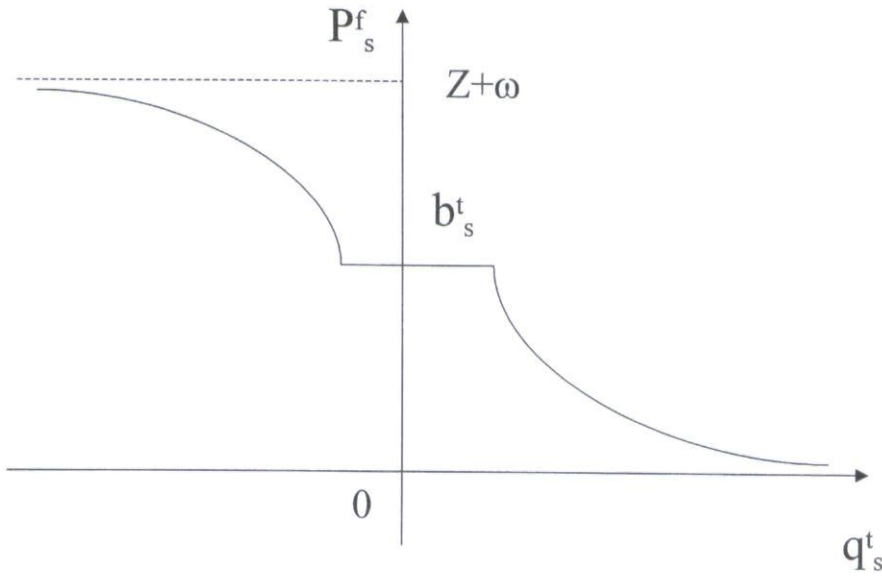


FIGURE 2

Trader t 's demand for contracts at time s .

All payments among all participants in the futures market are settled at the end of each time period. Those who bought futures contracts must pay for the contracts at the futures price and in exchange receive payments for these contracts valued at the spot price. Similarly, those who sold short futures contracts must pay for the contracts at the spot price and in exchange receive payments for these contracts valued at the futures price. Hence, trader t 's profit at the end of time period s is $(P_s - P_s^f)q_s^t$, so trader t 's wealth at the end of time period s is

$$V_s^t = V_{s-1}^t + (P_s - P_s^f)q_s^t$$

The futures price at time s , P_s^f , is the futures market clearing price, determined by the following equality of the aggregate net demand from traders with the aggregate supply of the producers:

$$\sum_{t=1}^s q_s^t(P_s^f) = S_s \quad (2)$$

The solution, P_s^f , to the equation is unique because of the shape of the demand function and the vertical supply from producers. All transactions are executed at the futures market clearing price. If the maximum number of contracts demanded by traders at the futures market clearing price exceeds the number of contracts supplied by producers and if there

are traders whose predictions coincide with the futures market clearing price, the remaining supply could be allocated among those traders proportionately to their wealth. Nevertheless, how the remaining supply is allocated does not affect the article's results.

In fact, this futures market mechanism is a Walrasian market. It is evident that the more wealth a trader has, the more contracts that this trader demands and, consequently, the more influence this trader's prediction has over the futures price.

TRADERS' PREDETERMINED BEHAVIOR RULES

Given that the intention is to illustrate how the market functions as a filtering process that screens out traders with less accurate information and selects for traders with more accurate information, the role of the market needs to be isolated and highlighted. Therefore, with respect to modeling traders' predictions, the guiding principle is to model traders' behavior as unsophisticated. That is, traders will be modeled as unresponsive to other participants' behavior and the futures market environment.⁸ To incorporate information about the fundamental value into the traders' predictions, one way of modeling traders' predictions is to assume that each trader's prediction errors with respect to the fundamental value obey his or her own inherent or preprogrammed stationary distribution. This probability distribution function characterizes each trader's ability to predict the fundamental value, and it reflects the accuracy level of the information this trader possesses. Different traders may have different abilities in predicting the fundamental value or different accuracy levels of information. As a result, different traders' prediction errors may have different probability distribution functions. This way of modeling a trader's ability is analogous to how the performance of a machine is measured by the probability distribution of its precision in meeting particular standards. Keeping each trader's distribution of prediction errors fixed through time is consistent with traders having systematical prediction biases.

Specifically, let b_s^t denote trader t 's prediction about the fundamental value at time period s , where $s \geq t$, and let $b_s^t = Z_s + u_s^t$, where u_s^t is trader t 's prediction error with respect to the fundamental value at time s .⁹

⁸Of course, traders with more sophisticated behavior could be added to the model, but this would make it more difficult in isolating the filtering role of the market.

⁹This modeling of predictions is the same as those of Grossman (1976, 1978), Figlewski (1978, 1982), and Hellwig (1980).

u_s^t may be correlated across traders. (This contrasts with Figlewski (1978, 1982), where to achieve informational efficiency in the long run, u_s^t must be independent across traders.) It is assumed that trader t 's prediction error at time s ($s \geq t$), u_s^t , is independent of Z_s and for all $s \geq t$, $u_s^t \in [-u, u]$. To ensure that the highest prediction does not exceed the highest spot price and the lowest prediction is positive, it is assumed that $u \leq \omega$. It is also assumed that for each t , u_s^t obeys the same probability distribution for all $s \geq t$. Hence, define a vector for trader t , where $t = 1, 2, \dots$, as follows: For any given $\varepsilon' > 0$,

$$\theta_1^t = \Pr(u_s^t \geq \varepsilon'), \theta_2^t = \Pr(-\varepsilon' \leq u_s^t \leq \varepsilon') \quad \text{and} \quad \theta_3^t = \Pr(u_s^t \leq -\varepsilon').$$

The variables θ_1^t , θ_2^t , and θ_3^t define the probability of overpredicting, predicting correctly, and underpredicting the fundamental value for trader t , respectively. This vector $(\theta_1^t, \theta_2^t, \theta_3^t)$ characterizes trader t 's probability distribution of his or her prediction error with respect to the fundamental value, which reflects trader t 's ability to predict the fundamental value.¹⁰ Upon the beginning of trader t 's entry period, trader t 's vector (θ_1^t, θ_2^t) is randomly taken from a set $\{(\theta_1, \theta_2) \in (0, 1) \times (0, 1); \theta_1 + \theta_2 < 1\}$, and $\theta_3^t = 1 - \theta_1^t - \theta_2^t$. Trader t 's vector $(\theta_1^t, \theta_2^t, \theta_3^t)$ is determined in the beginning of time period t and is fixed in any subsequent time period. This means that there is no adaptive learning or any strategic usage of any information from the markets or other markets' participants among all traders. Trader t acts on his or her vector $(\theta_1^t, \theta_2^t, \theta_3^t)$ each time period. In other words, in each time period trader t 's prediction error is generated from his or her vector; hence, his or her prediction is determined. Given a trader's prediction at time s , this trader's demand for the number of contracts at time s is characterized by Equation (1).

The sequence of traders' vectors $\{(\theta_1^t, \theta_2^t)\}_{t \geq 1}$ is independently and identically distributed according to the distribution function $F(\cdot)$ across traders $t = 1, 2, \dots$.

Furthermore, it is assumed that in each time period there is a positive probability that an entering trader has an arbitrarily high probability of predicting arbitrarily close to the fundamental value. That is, for any given small positive ε ,

$$\int_{1-\varepsilon < \theta_2 < 1} dF(\theta_1, \theta_2) > 0$$

¹⁰No assumptions are made with respect to the type of distribution of the prediction error. This contrasts with most publications, including Figlewski (1978, 1982), where normality is assumed.

In addition, it is also assumed that the random shock at time s ($s \geq 1$), ω_s , is independent of all traders' prediction errors at all time periods, u_s^t , where $s' \geq t \geq 1$.

The futures price at time s , P_s^f , is the futures market clearing price at time s , which is determined by Equation (2). As can be seen the futures price, P_s^f is a function of $S_1, S_2 \dots S_s; Z_1, Z_2 \dots Z_s; \omega_1, \omega_2 \dots \omega_{s-1}$; and $u_1^t, u_2^t \dots, u_s^t$, for all $t \leq s$; and V_0 .

NUMERICAL EXAMPLES AND RESULTS

To see how the futures price moves over time in this market, a set of simulations is now conducted. For $t = 1, 2 \dots$, define $\theta_1^t = \Pr(u_s^t > 0.10)$, $\theta_2^t = \Pr(-0.10 \leq u_s^t \leq 0.10)$, and $\theta_3^t = \Pr(u_s^t < -0.10)$. The vector $(\theta_1^t, \theta_2^t, \theta_3^t)$ characterizes trader t 's probability distribution of his or her prediction error with respect to the fundamental value.

Consider a futures market with a random shock, ω_s , where $s \geq 1$ is a random draw from an interval, $(-\omega, \omega) = (-3, 3)$, according to a symmetric doubly truncated normal distribution where the density function is $\sigma^{-1}Z(w_t/\sigma)[2\Phi(B/\sigma) - 1]^{-1}$, with parameter σ and $B = 2.999$. $Z(\cdot)$ is the unit normal probability density function, and $\Phi(\cdot)$ is the corresponding cumulative distribution function.¹¹ As σ goes up (down), the variation in random shock also goes up (down). Other detailed characteristics of the market are described by the four points in Appendix A.

First, with $\sigma = 5.0$, 100 simulations are conducted, and the market is followed from time periods 1 to 3,000. The histogram in Figure 3(a) shows that at time period 500, on average across 100 simulations, the percentages of time that $|P_s^f - Z_s| \geq 0.10$ and $|P_s^f - Z_s| < 0.025$ are 40% and 16%, respectively. By time period 3,000, on average across 100 simulations, the percentage of time that $|P_s^f - Z_s| \geq 0.10$ has decreased to 28%, whereas the percentages of time that $|P_s^f - Z_s| < 0.025$ has increased to 19%. That is, as time goes by there is a lower proportion of time that $|P_s^f - Z_s|$ is greater than 0.10, and there is a higher proportion of time that $|P_s^f - Z_s|$ is less than 0.025.

However, if the variation in the random shock gets smaller, does the futures price move closer to the fundamental value?

Now a smaller variance in the random shock is chosen to conduct the second set of simulations. With all other aspects of the model remaining identical, $\sigma = 5.0$ is reduced to $\sigma = 1.5$, 100 simulations are conducted, and the market is followed from times 1 to 3,000. The histogram of the

¹¹The truncation keeps the upper bound of the spot price, P_s , from exceeding the highest bid, 10.

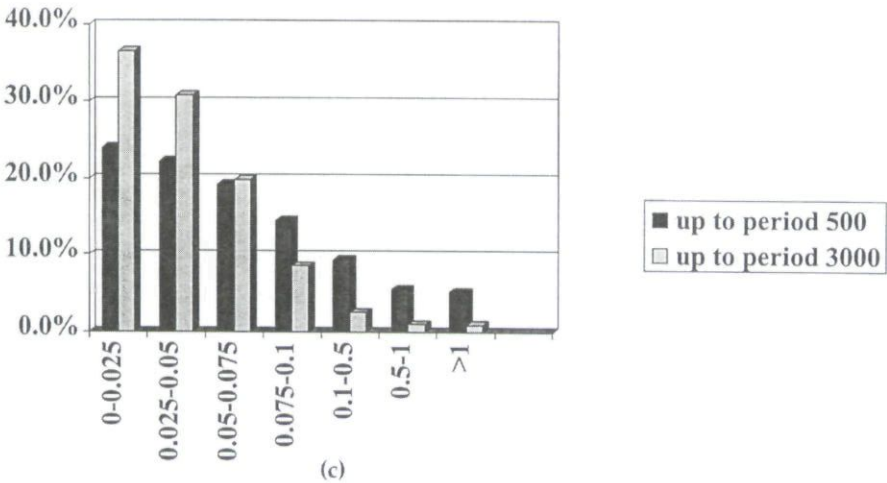
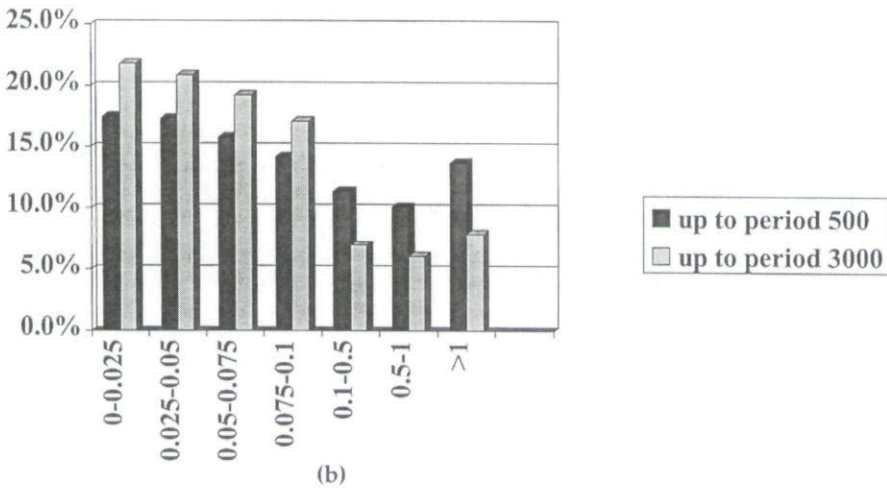
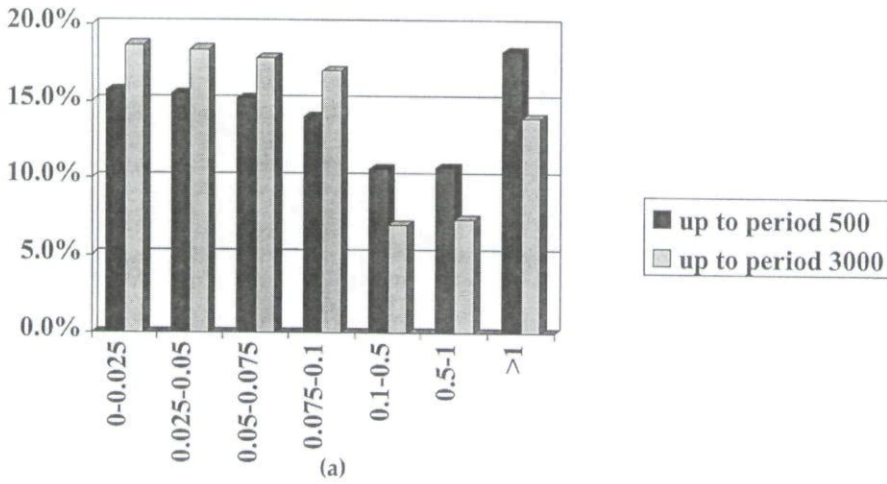


FIGURE 3

Histogram of $|P_t^f - Z_t|$ as a percentage of times: (a,b) ω_t obeys a truncated normal distribution (a, $\sigma = 5$, b, $\sigma = 1.5$) and (c) there is no random shock to the economy.

absolute deviation of the futures price from the fundamental value as a percentage of times is shown in Figure 3(b). Figure 3(b) shows the clustering of the futures price about the fundamental values as time goes by. Furthermore, by time period 3,000, on average across 100 simulations, the percentages of time that $|P_s^f - Z_s| \geq 0.10$ and $|P_s^f - Z_s| < 0.025$ are 21% and 22%, respectively. This shows that, in comparison to the simulations with $\sigma = 5.0$, with $\sigma = 1.5$ there is a larger proportion of time that $|P_s^f - Z_s| < 0.025$.

Now suppose that no random shock is assumed in the spot market (i.e., $\omega_t = 0$ for $t = 1, 2, \dots$). A third set of simulations is conducted with the model described by the four assumptions in Appendix A along with no random shock assumption. Again, 100 simulations are conducted, and the market is followed from times 1 to 3,000.

As expected, with no random shocks convergence is much faster. The histogram of Figure 3(c) shows that by time period 3,000, on average across 100 simulations, the percentages of time that $|P_s^f - Z_s| \geq 0.10$ and $|P_s^f - Z_s| < 0.025$ are 4% and 37%, respectively. This shows that with no random shock, in comparison to the previous simulations with random shocks, there is an even larger proportion of time that $|P_s^f - Z_s| < 0.025$.

This provides good evidence that the smaller the volatility of the random shock to the spot market is, the higher the proportion of time that the futures price lies in a small interval containing the fundamental value as time increases. This is precisely stated in the following theorem:

Theorem 1

With probability 1, the following occurs: For any given positive ε , there exists a positive number $\bar{k}(\varepsilon)$ such that, for the i.i.d. random shock sequence $\{\omega_t\}_{t \geq 1}$ with $\ln \underline{Z} - E(\ln(\underline{Z} + \omega_t)) \leq \bar{k}(\varepsilon)$ and for any given positive $\varepsilon' > 0$, there exists a time period $T'(\varepsilon', \varepsilon)$ such that for $T \geq T'(\varepsilon', \varepsilon)$

$$\frac{\#\{t \leq T : P_t^f \in [Z_t - \varepsilon, Z_t + \varepsilon]\}}{T} \geq 1 - \varepsilon'$$

For the proof, see Appendix B.

As shown in the appendix, the term $\ln \underline{Z} - E[\ln(\underline{Z} + \omega_t)]$ is a function of all the moments of the random shock ω_t . This term reflects the volatility of the random shock ω_t . For example, if the random shock is

normally distributed, the term $\ln \underline{Z} - E[\ln(\underline{Z} + \omega_t)]$ is an increasing function of the variance of the random shock ω_t . Hence, Theorem 1 essentially says that, if the volatility of the random shock is sufficiently small, the proportion of time that the futures price is sufficiently close to the fundamental value of the spot price converges to one with probability 1. Theorem 1 also indicates that with probability 1, the smaller the volatility of the random shock is, the smaller the width of the interval is (containing the fundamental value), in which the futures price eventually lies. This further suggests that if there is no random shock in any time period, then with probability 1 the width of the interval is arbitrarily small. This is stated formally in the following corollary, which can be established directly from the theorem:¹²

Corollary 1

If $\omega_t = 0$ for all t , then with probability 1 for any given $\varepsilon, \varepsilon' > 0$, there exists a time period $T^*(\varepsilon', \varepsilon)$ such that for $T \geq T^*(\varepsilon', \varepsilon)$

$$\frac{\#\{t \leq T : P_t^f \in [Z_t - \varepsilon, Z_t + \varepsilon]\}}{T} \geq 1 - \varepsilon'$$

Although the convergence of the futures price is rigorously proven in Appendix B, the intuitive reason for convergence deserves further examination. In other words, what is the cause of the convergence of the futures price? For the purpose of dealing with this question, a trader is said to be more informed or possess more accurate information or less noisy information than others if this trader has a higher probability of predicting arbitrarily close to the fundamental value than others do. Because there is no learning or strategic behavior by participants in this model, such that traders possessing less accurate information become the ones possessing more accurate information, the occurrence of convergence must be due to the constant redistribution of wealth among all traders. This constant redistribution process must be in favor of traders with a higher probability of predicting arbitrarily close to the fundamental value. That is, the proportion of wealth owned by traders with more accurate information increases relative to the proportion of the wealth owned by traders with less accurate information. Consequently, the

¹²This corollary is the same as Theorem 1 in Luo (1998), except the prices in this article are continuous.

more informed traders have more influence over the futures price than the less informed traders. It is the economic natural selection or the filtering process that screens out traders with less accurate information and selects for more accurate information.

To look at this issue, the distribution of wealth by trader types, described by θ_2^t , is examined for the simulation model with $\sigma = 1.5$ and the four assumptions in Appendix A. The distribution of the proportion of the wealth across all trader types at time periods 1,000, 2,000, and 3,000 is shown in Table I. The corresponding distribution of all trader types (here characterized by θ_2^t) at time periods 1,000, 2,000, and 3,000 is in Table II.

As seen in Table I, the proportion of the wealth owned by traders with θ_2^t lying in an interval (0.95, 1.00) increases to 18.2% at time period 3,000 from 3.5% at time 1,000. In the meantime, the proportion of wealth owned by traders with θ_2^t lying in an interval (0, 0.50) decreases to 20.3% at time 3,000 from 37.8% at time 1,000. From Table II, it can be seen that the proportion of traders with θ_2^t lying in an interval (0.95, 1.00) is relatively stable at about 0.05% over time. Both tables suggest that over time, more and more wealth is shifted to the traders with more accurate information from the traders with less accurate information as

TABLE I
Distribution of the Proportion of Wealth by Trader Types

Trader Types (θ_2^t)	At Time 1,000	At Time 2,000	At Time 3,000
0-0.50	0.378	0.331	0.203
0.50-0.75	0.314	0.291	0.229
0.75-0.90	0.228	0.228	0.229
0.90-0.95	0.045	0.083	0.156
0.95-1.00	0.035	0.067	0.182

TABLE II
Distribution of Trader Types

Trader Types (θ_2^t)	At Time 1,000	At Time 2,000	At Time 3,000
0-0.50	0.8270	0.8363	0.8327
0.50-0.75	0.1552	0.1456	0.1487
0.75-0.90	0.0159	0.0161	0.0164
0.90-0.95	0.0015	0.0015	0.0016
0.95-1.00	0.0005	0.0005	0.0005

more traders with more accurate information enter the economy. This redistribution of wealth process goes on and on; eventually, the group of traders with θ_2^t arbitrarily close to 1 comes to dominate the markets and drives the futures price towards the fundamental value.

CONCLUSIONS

The idea that the market functions as a filtering process of information, promoting an efficient outcome, has been illustrated with a dynamic model of a commodity futures market. To emphasize the concept of natural selection, the model allows for the continual inflow of traders with a wide spectrum of noisy information signals, the transfer of wealth among traders, and the accumulation of wealth across time.

To highlight the role of the market as a selection process of information rather than the strategic moves of participants, traders are modeled as not being responsive to past market experiences. The model recognizes that traders' predictions reflect a wide disparity of information. Each trader is endowed with a probability distribution of the prediction error with respect to the fundamental value. Because in each time period there is a positive probability that a trader enters with a higher probability of predicting arbitrarily close to the fundamental value than any previously entered traders, over time as more traders enter the markets, if the volatility of the random shock is sufficiently small, then the proportion of time that the futures price is sufficiently close to the fundamental value of the spot price converges to one. Nevertheless, the interval containing the fundamental value, where the futures price eventually lies, is influenced by the underlying volatility generated from the spot market. In other words, the accuracy of the information for which the market can eventually select depends on the volatility generated from the random shock in the spot market. The more volatile the spot market is, the noisier the information is that gets selected. As a result, the futures market moves further away from informational efficiency.

The explanation for the cause of the convergence of the futures price to the fundamental value, which is also supported by the simulations, is rather intuitive. That is, the market as a filtering process constantly shifts wealth from traders with less accurate information to traders with more accurate information. Over time, as more traders enter the market with more accurate information eventually, the traders with an arbitrarily high probability of predicting the fundamental value come

to dominate the markets and drive the futures price toward the fundamental value of the spot market.

APPENDIX A

1. The fundamental value of the spot price at time $s(z_s)$, where $s = 1, 2, \dots$, is assumed to have a uniform distribution with its support $[Z, \bar{Z}] = [3, 7]$.
2. The supply of contracts brought to the futures market by the producers at time i , S_i , is randomly drawn from the interval $(3, 7)$ according to a uniform distribution. This random draw of S_i is independent of the fundamental value Z_s for all $s \geq 1$.
3. Each trader on entry is endowed with initial wealth $V_0 = 0.001$.

Finally, all traders' predictions are modeled as being taken from a given distribution each time period. That is,

4. For $t = 1, 2, \dots$, $(\theta_1^t, \theta_2^t, \theta_3^t)$ is drawn randomly and independently according to a uniform distribution from a cube defined by $\{(\theta_1, \theta_2, \theta_3) \in (0, 1) \times (0, 1) \times (0, 1): \theta_1 + \theta_2 + \theta_3 = 1\}$. Trader t 's prediction error in each time period is randomly and independently generated from the vector $(\theta_1^t, \theta_2^t, \theta_3^t)$ and trader t 's prediction error at time s ($s \geq t$), $u_s^t \in [-3, 3]$.

APPENDIX B

The purpose of this appendix is to provide a sketch of the proof of the result in Theorem 1. The result in Theorem 1 is proven by way of contradiction. Roughly speaking, if the result is not true or, in other words, if the futures price stays infinitely often away from the arbitrarily small interval containing the fundamental value, then there would exist a trader, trader I , say with a sufficiently large θ_2^I or sufficiently small θ_1^I and θ_3^I such that this trader's wealth grows exponentially over time, and eventually this trader's average wealth over time explodes to infinity. This contradicts the fact that the average total wealth in the market over time is bounded from above by a constant. Because of the complexity and length of the proof, the proof is broken into two lemmas and one theorem. Lemma 1 shows the existence of such a trader with sufficiently large θ_2^I or sufficiently small θ_1^I and θ_3^I . Lemma 2 derives the lower bound for any trader's average logarithm wealth over time. This lower bound is made

used by the proof of Theorem 1 to show that under its contrapositive, the wealth for trader I presented in Lemma 1 would grow exponentially. What follows begins with a sketch of the proof of Lemma 1, followed by Proposition 1, which is required by Lemma 2. This is followed by a sketch of the proof of Lemma 2. The proof of Theorem 1 makes use of these two lemmas.

Lemma 1

For any given $\delta > 0$ and for any given $\varepsilon > 0$,

$$\Pr \left(\exists \text{ trader } I \text{ such that } \left(\theta_2^I - 1 + \frac{1}{2}\delta \right) \ln \left(\frac{\bar{Z}}{\bar{Z} - \varepsilon} \right) + \theta_1^I \ln \left(\frac{\underline{Z}}{\underline{Z} + u} \right) + \theta_3^I \ln \left(\frac{\omega}{\bar{Z} + \omega} \right) + \theta_2^I \ln \left(\frac{\omega}{\omega + \varepsilon'} \right) - \bar{k} > 0 \text{ for some positive } \varepsilon' < \varepsilon \text{ and some positive } \bar{k} \right) = 1.$$

This proof is done under the assumption that $(\theta_1^t, \theta_2^t, \theta_3^t)$ is independently and identically distributed across time $t = 1, 2, \dots$, along with the second Borel Cantelli Lemma (see Billingsley, 1995, p. 83). The details of the proof are available from the author on request.

Proposition 1

Let $\{X_t\}_{t \geq 1}$ be a random sequence with a finite expectation $E(X_t)$ and $|X_t| \leq M_1$, where M_1 is a finite positive number. Let Y_t , where $t = 1, 2, \dots$, be a random variable with $|Y_t| \leq M_2$, where M_2 is a finite positive number. If X_t is independent of $Y_1, Y_2, \dots, Y_t$, then for any given $\varepsilon > 0$,

$$\lim_{T \rightarrow \infty} \Pr \left(\left| \frac{\sum_{t=1}^T (X_t Y_t)}{T} - \frac{\sum_{t=1}^T [Y_t E(X_t)]}{T} \right| < \varepsilon \right) = 1$$

The proof is available from the author on request. To establish Lemma 2, the following definitions are needed:

Definition 1. For any given positive ε and $\varepsilon' (\varepsilon' < \varepsilon)$, to describe the position of the futures price relative to the fundamental value plus ε , or

minus ε , or plus ε' , or minus ε' , define l_k , a_k , m_k , $\overline{m}_k$, and $\underline{m}_k$, where $k = 1, 2, \dots$, as

$$l_k = \begin{cases} 1 & \text{if } P_k^f < Z_k - \varepsilon \\ 0 & \text{otherwise} \end{cases} \quad a_k = \begin{cases} 1 & \text{if } P_k^f > Z_k + \varepsilon \\ 0 & \text{otherwise} \end{cases}$$

$$m_k = \begin{cases} 1 & \text{if } Z_k - \varepsilon' \leq P_k^f \leq Z_k + \varepsilon' \\ 0 & \text{otherwise} \end{cases}$$

$$\overline{m}_k = \begin{cases} 1 & \text{if } Z_k + \varepsilon' < P_k^f \leq Z_k + \varepsilon \\ 0 & \text{otherwise} \end{cases}$$

and

$$m_k = \begin{cases} 1 & \text{if } Z_k - \varepsilon \leq P_k^f < Z_k - \varepsilon' \\ 0 & \text{otherwise} \end{cases}$$

Definition 2. For any given $\varepsilon' > 0$, to describe the position of trader t 's (where $t = 1, 2, \dots$) prediction error relative to $-\varepsilon'$ and ε' , define x_k^t , g_k^t , and y_k^t , where $k \geq t$, as follows:

$$x_k^t = \begin{cases} 1 & \text{if } u_k^t > \varepsilon' \\ 0 & \text{otherwise} \end{cases} \quad g_k^t = \begin{cases} 1 & \text{if } -\varepsilon' \leq u_k^t \leq \varepsilon' \\ 0 & \text{otherwise} \end{cases}$$

and

$$y_k^t = \begin{cases} 1 & \text{if } u_k^t < -\varepsilon' \\ 0 & \text{otherwise} \end{cases}$$

Definition 3. Trader t (where $t = 1, 2, \dots$) is referred to as a buyer at time period k if trader t is purchasing contracts at time period k (which occurs whenever $b_k^t > P_k^f$); trader t is referred to as a seller at time period k if trader t is selling contracts at time period k , (which occurs whenever $b_k^t < P_k^f$); trader t is referred to as a marginal buyer if trader t is actually purchasing contracts when $b_k^t = P_k^f$; and trader t is referred to as a marginal seller if trader t is actually selling contracts when $b_k^t = P_k^f$. To describe whether trader t at one time period participates as a buyer, a seller, a marginal buyer, or a marginal seller, define four random variables B_k^t , M_k^t , N_k^t , and S_k^t , where $k \geq t$, as

$$B_k^t = \begin{cases} 1 & \text{if } b_k^t > P_k^f \\ 0 & \text{otherwise} \end{cases} \quad M_k^t = \begin{cases} 1 & \text{if } b_k^t = P_k^f, q_k^t > 0 \\ 0 & \text{otherwise} \end{cases}$$

$$N_k^t = \begin{cases} 1 & \text{if } b_k^t = P_k^f, q_k^t < 0 \\ 0 & \text{otherwise} \end{cases}$$

and define $S_k^t = 1 - B_k^t - M_k^t - N_k^t$.

Lemma 2

Consider trader t 's wealth at time T , where $t = 1, 2, \dots$, for any given $\varepsilon > 0$ and for any $\varepsilon' \in (0, \varepsilon)$

$$\lim_{T \rightarrow \infty} Pr \left(\frac{\ln V_T^t}{T - t + 1} > \frac{\sum_{k=1}^T (l_k + a_k) g_k^t}{T - t + 1} \ln \left(\frac{\bar{Z}}{\bar{Z} - \varepsilon} \right) \right. \\ \left. + \frac{\sum_{k=t}^T x_k^t}{T - t + 1} \ln \left(\frac{\underline{Z}}{\underline{Z} + u} \right) + \frac{\sum_{k=t}^T y_k^t}{T - t + 1} \ln \left(\frac{\omega}{\bar{Z} + \omega} \right) \right. \\ \left. + \frac{\sum_{k=t}^T g_k^t}{T - t + 1} \ln \left(\frac{\omega}{\omega + \varepsilon'} \right) + (E(\ln(\underline{Z} + \omega_k)) - \ln \underline{Z}) \right) = 1$$

There are five steps to the proof. Step 1 defines precisely the formula for trader t 's wealth at time $T(V_T^t)$:

$$V_T^t = \prod_{k=t}^T \left\{ \left(\frac{P_k}{P_k^f} \right)^{C_k} \left(\frac{\bar{Z} + \omega - P_k}{\bar{Z} + \omega - P_k^f} \right)^{D_k} \left(1 - r_k + r_k \frac{P_k}{P_k^f} \right)^{H_k} \right. \\ \left. \times \left(1 - r'_k + r'_k \left(\frac{\bar{Z} + \omega - P_k}{\bar{Z} + \omega - P_k^f} \right) \right)^{F_k} \right\} V_{t-1}^t \quad (A-1)$$

Where $C_k = g_k^t(l_k + \underline{m}_k + m_k B_k^t) + x_k^t(l_k + \underline{m}_k + m_k + (a_k + \bar{m}_k) B_k^t) + y_k^t(l_k + \underline{m}_k) B_k^t$, $D_k = g_k^t(a_k + \bar{m}_k + m_k S_k^t) + y_k^t(a_k + \bar{m}_k + m_k + (l_k + \underline{m}_k) S_k^t) + x_k^t(a_k + \bar{m}_k) S_k^t$, $H_k = g_k^t m_k M_k^t + y_k^t(l_k + \underline{m}_k) M_k^t + x_k^t(a_k + \bar{m}_k) M_k^t$, and $F_k = g_k^t m_k N_k^t + y_k^t(l_k + \underline{m}_k) N_k^t + x_k^t(a_k + \bar{m}_k) N_k^t$.

By taking logarithms of both sides of Equation (A-1), setting $V_{t-1}^t = V_0$, and making use of the concavity of the function $\ln(\cdot)$, Step 2 establishes a lower bound for $\ln V_T^t / (T - t + 1)$:

$$\frac{\ln V_T^t}{T - t + 1} \geq \frac{1}{T - t + 1} \left\{ \sum_{k=t}^T \left((C_k + H_k r_k) \ln \left(\frac{P_k}{P_k^f} \right) \right) \right. \\ \left. + \sum_{k=t}^T \left((D_k + F_k r'_k) \ln \left(\frac{\bar{Z} + \omega - P_k}{\bar{Z} + \omega - P_k^f} \right) \right) + \ln V_0 \right\}$$

As can be seen, there are three components to this lower bound, two of which are random. Steps 3 and 4 determine probabilistic lower bounds for the first two random components of this lower bound. This is done with a

Taylor expansion of $\ln(P_k) = \ln(Z_k + \omega_k)$ and $\ln(\bar{Z} + \omega - P_k) = \ln(\bar{Z} + \omega - (Z_k + \omega_k))$ and Proposition 1 to produce the following two equations:

$$\begin{aligned} \lim_{T \rightarrow \infty} Pr & \left(\frac{\sum_{k=t}^T \left((C_k + H_k r_k) \ln \left(\frac{P_k}{P_k^f} \right) \right)}{T - t + 1} > \frac{\sum_{k=t}^T \left(g_k^t l_k \ln \left(\frac{\bar{Z}}{\bar{Z} - \varepsilon} \right) \right)}{T - t + 1} \right. \\ & + \frac{\sum_{k=t}^T \left(x_k^t m_k \ln \left(\frac{\underline{Z}}{\underline{Z} + \varepsilon'} \right) \right)}{T - t + 1} + \frac{\sum_{k=t}^T \left(g_k^t m_k (B_k^t + r_k M_k^t) \ln \left(\frac{\underline{Z}}{\underline{Z} + \varepsilon'} \right) \right)}{T - t + 1} \\ & + \frac{\sum_{k=t}^T \left(x_k^t (a_k + \bar{m}_k) (B_k^t + r_k M_k^t) \ln \left(\frac{\underline{Z}}{\underline{Z} + u} \right) \right)}{T - t + 1} \\ & \left. + \frac{\sum_{k=t}^T \left((C_k + H_k r_k) (E[\ln(Z + \omega_k)] - \ln Z) \right)}{T - t + 1} \right) = 1 \end{aligned}$$

and

$$\begin{aligned} \lim_{T \rightarrow \infty} Pr & \left(\frac{\sum_{k=t}^T \left((D_k + F_k r'_k) \ln \left(\frac{\bar{Z} + \omega - P_k}{\bar{Z} + \omega - P_k^f} \right) \right)}{T - t + 1} \right. \\ & > \frac{\sum_{k=t}^T \left(g_k^t a_k \ln \left(\frac{\bar{Z} + \omega - \underline{Z}}{\bar{Z} + \omega - (\underline{Z} + \varepsilon)} \right) \right)}{T - t + 1} + \frac{\sum_{k=t}^T \left(y_k^t m_k \ln \left(\frac{\omega}{\omega + \varepsilon'} \right) \right)}{T - t + 1} \\ & + \frac{\sum_{k=t}^T \left(g_k^t m_k (S_k^t + r'_k N_k^t) \ln \left(\frac{\omega}{\omega + \varepsilon'} \right) \right)}{T - t + 1} \\ & + \frac{\sum_{k=t}^T \left(y_k^t (l_k + \underline{m}_k) (S_k^t + r'_k N_k^t) \ln \left(\frac{\omega}{\bar{Z} + \omega} \right) \right)}{T - t + 1} \\ & \left. + \frac{\sum_{k=t}^T \left((D_k + F_k r'_k) (E(\ln(\omega - \omega_k)) - \ln \omega) \right)}{T - t + 1} \right) = 1 \end{aligned}$$

Step 5 substitutes these two probabilistic lower bounds into the lower bound of $\ln V_T^I / (T - t + 1)$ and simplifies it to give the results of Lemma 2. The details of the proof are available from the author on request.

Proof of Theorem 1

This is shown by way of contradiction. Suppose that Theorem 1 is not true, so with a strictly positive probability the following occurs: There exists an ε_0 such that, for all $\bar{k}'$, it is possible to find an i.i.d. random sequence $\{\omega_k\}_{k \geq 1}$, with $\ln \underline{Z} - E[\ln(\underline{Z} - \omega_k)] \leq \bar{k}'$, for which there exists a $\delta_0 > 0$ and a subsequence $n_1, n_2 \dots n_T \dots$, such that for all $T \geq 1$, $\#\{k \leq n_T : P_k^f \notin [Z_k - \varepsilon_0, Z_k + \varepsilon_0]\} / n_T > \delta_0$. Since Lemma 1 implies that for this $\delta_0 > 0$ and for this $\varepsilon_0 > 0$,

$$\begin{aligned} &Pr(\exists \text{ trader } I \text{ such that, } \left(\theta_2^I - 1 + \frac{1}{2}\delta_0\right) \ln\left(\frac{\bar{Z}}{\bar{Z} - \varepsilon_0}\right) \\ &+ \theta_1^I \ln\left(\frac{\underline{Z}}{\underline{Z} + u}\right) + \theta_3^I \ln\left(\frac{\omega}{\bar{Z} + \omega}\right) + \theta_2^I \ln\left(\frac{\omega}{\omega + \varepsilon_0'}\right) \\ &- \bar{k} > 0 \text{ for some positive } \varepsilon_0' < \varepsilon_0 \text{ and some positive } \bar{k}) = 1 \quad (\text{A-2}) \end{aligned}$$

and using the notation in Definitions 1, 2, and 3 after replacing ε and ε' with ε_0 and ε_0' , respectively, in the Definitions 1, 2, and 3, the statement can be restated as the following. With a strictly positive probability, the following occurs: There exists an ε_0 such that, for all $\bar{k}'$, it is possible to find an i.i.d. random sequence $\{\omega_k\}_{k \geq 1}$, with $\ln \underline{Z} - E[\ln(\underline{Z} + \omega_k)] \leq \bar{k}'$, for which there exists a $\delta_0 > 0$ and a subsequence $n_1, n_2 \dots n_T \dots$, such that for all $T \geq 1$,

$$\frac{\sum_{k=1}^{n_T} (l_k + a_k)}{n_T} > \delta_0 > 0 \quad (\text{A-3})$$

Consider trader I . The following shows that trader I 's wealth grows exponentially, whereas the aggregate wealth can only grow arithmetically. This is a contradiction.

Because trader I 's prediction error u_k^I is independently and identically distributed across time k , the Strong Law of Large Number implies that with probability 1, as $T \rightarrow \infty$,

$$\frac{\sum_{k=1}^{n_T} g_k^I}{n_T - I + 1} \rightarrow \theta_2^I, \quad \frac{\sum_{k=1}^{n_T} x_k^I}{n_T - I + 1} \rightarrow \theta_1^I$$

and

$$\frac{\sum_{k=1}^{n_I} y_k^I}{n_T - I + 1} \rightarrow \theta_3^I \quad (\text{A-4})$$

Furthermore,

$$\begin{aligned} \sum_{k=1}^{n_I} [(l_k + a_k) g_k^I] &= \sum_{k=1}^{n_I} [(l_k + a_k) (1 - y_k^I - x_k^I)] \\ &\geq \sum_{k=1}^{n_I} [(l_k + a_k) - (1 - g_k^I)] \\ &= (n_T - I + 1) \left[\left(\frac{\sum_{k=1}^{n_I} (l_k + a_k)}{n_T - I + 1} + \frac{\sum_{k=1}^{n_I} g_k^I}{n_T - I + 1} \right) - 1 \right] \end{aligned} \quad (\text{A-5})$$

Therefore, Equations (A-2), (A-4), and (A-5) imply that

$$\begin{aligned} &Pr \left(\lim_{T \rightarrow \infty} \left(\frac{\sum_{k=1}^{n_I} [(l_k + a_k) g_k^I]}{n_T - I + 1} \right) \ln \left(\frac{\bar{Z}}{\bar{Z} - \varepsilon_0} \right) + \frac{\sum_{k=1}^T x_k^I}{n_T - I + 1} \ln \left(\frac{\underline{Z}}{\underline{Z} + u} \right) \right. \\ &\quad + \frac{\sum_{k=1}^T y_k^I}{n_T - I + 1} \ln \left(\frac{\omega}{\bar{Z} + \omega} \right) + \frac{\sum_{k=1}^T g_k^I}{n_T - I + 1} \ln \left(\frac{\omega}{\omega + \varepsilon'_0} \right) \\ &\quad > \left(\theta_2^I - 1 + \frac{\sum_{k=1}^{n_I} (l_k + a_k)}{n_T - I + 1} \right) \ln \left(\frac{\bar{Z}}{\bar{Z} - \varepsilon_0} \right) + \theta_1^I \ln \left(\frac{\underline{Z}}{\underline{Z} + u} \right) \\ &\quad \left. + \theta_3^I \ln \left(\frac{\omega}{\bar{Z} + \omega} \right) + \theta_2^I \ln \left(\frac{\omega}{\omega + \varepsilon'_0} \right) \middle| A \right) = 1 \end{aligned} \quad (\text{A-6})$$

where A denotes the event that there exists trader I , such that $(\theta_2^I - 1 + 1/2\delta_0) \ln[\bar{Z}/(\bar{Z} - \varepsilon_0)] + \theta_1^I \ln[\underline{Z}/(\underline{Z} + u)] + \theta_3^I \ln(\omega/(\bar{Z} + \omega)) + \theta_2^I \ln(\omega/(\omega + \varepsilon'_0)) - k > 0$, for some positive $\varepsilon'_0 < \varepsilon_0$ and some positive $\bar{k}$.

Since if $\sum_{i=1}^{n_I} (l_k + a_k) / n_T > \delta_0$, then as $T \rightarrow \infty$,

$$\frac{\sum_{k=I}^{n_I} (l_k + a_k)}{n_T - I + 1} > \frac{1}{2} \delta_0 \quad (\text{A-7})$$

Applying Bayes rule to Equations (A-3) and (A-6) and using Equation (A-7) it follows that

$$\begin{aligned} & Pr \left(\lim_{T \rightarrow \infty} \left(\frac{\sum_{k=I}^{n_I} [(l_k + a_k) g_k^I]}{n_T - I + 1} \right) \ln \left(\frac{\bar{Z}}{\bar{Z} - \varepsilon_0} \right) + \frac{\sum_{k=I}^T x_k^I}{n_T - I + 1} \right. \\ & \ln \left(\frac{\underline{Z}}{\underline{Z} + u} \right) + \frac{\sum_{k=I}^T y_k^I}{n_T - I + 1} \ln \left(\frac{\omega}{\bar{Z} + \omega} \right) + \frac{\sum_{k=I}^T g_k^I}{n_T - I + 1} \ln \left(\frac{\omega}{\omega + \varepsilon'_0} \right) \\ & > \left(\theta_2^I - 1 + \frac{1}{2} \delta_0 \right) \ln \left(\frac{\bar{Z}}{\bar{Z} + \varepsilon_0} \right) + \theta_1^I \ln \left(\frac{\underline{Z}}{\underline{Z} + u} \right) \\ & \left. + \theta_3^I \ln \left(\frac{\omega}{\bar{Z} + \omega} \right) + \theta_2^I \ln \left(\frac{\omega}{\omega + \varepsilon'_0} \right) \middle| (A, E) \right) = 1 \quad (\text{A-8}) \end{aligned}$$

where E represents the following event. With a strictly positive probability, the following occurs: There exists an ε_0 such that for all $\bar{k}'$, it is possible to find an i.i.d. random sequence $\{\omega_k\}_{k=1}$, with $\ln \underline{Z} - E[\ln(\underline{Z} + \omega_k)] \leq \bar{k}'$, for which there exists a $\delta_0 > 0$ and a subsequent $n_1, n_2 \dots n_T \dots$, such that for all T , $\sum_{i=1}^{n_I} (l_k + a_k) / n_T > \delta_0$.

Furthermore, noticing that for trader I there exists some positive $\lambda > 0$, such that

$$\begin{aligned} & \left(\theta_2^I - 1 + \frac{1}{2} \delta_0 \right) \ln \left(\frac{\bar{Z}}{\bar{Z} + \varepsilon_0} \right) + \theta_1^I \ln \left(\frac{\underline{Z}}{\underline{Z} + u} \right) \\ & + \theta_3^I \ln \left(\frac{\omega}{\bar{Z} + \omega} \right) + \theta_2^I \ln \left(\frac{\omega}{\omega + \varepsilon'_0} \right) - \bar{k} > \lambda \quad (\text{A-9}) \end{aligned}$$

Equations (A-8) and (A-9) imply that

$$\begin{aligned}
 & Pr \left(\lim_{T \rightarrow \infty} \left(\frac{\sum_{k=I}^{n_I} [(l_k + a_k) g_k^I]}{n_T - I + 1} \right) \ln \left(\frac{\bar{Z}}{\bar{Z} - \varepsilon_0} \right) \right. \\
 & \quad + \frac{\sum_{k=I}^T x_k^I}{n_T - I + 1} \ln \left(\frac{\underline{Z}}{\underline{Z} + u} \right) + \frac{\sum_{k=I}^T y_k^I}{n_T - I + 1} \ln \left(\frac{\omega}{\bar{Z} + \omega} \right) \\
 & \quad \left. + \frac{\sum_{k=I}^T g_k^I}{n_T - I + 1} \ln \left(\frac{\omega}{\omega + \varepsilon'_0} \right) - \bar{k} > \lambda | (A, E) \right) = 1 \quad (\text{A-10})
 \end{aligned}$$

Equation (A-10) further implies that

$$\begin{aligned}
 & Pr \left(\lim_{T \rightarrow \infty} \left(\frac{\sum_{k=I}^{n_I} [(l_k + a_k) g_k^I]}{n_T - I + 1} \right) \ln \left(\frac{\bar{Z}}{\bar{Z} - \varepsilon_0} \right) \right. \\
 & \quad + \frac{\sum_{k=I}^T x_k^I}{n_T - I + 1} \ln \left(\frac{\underline{Z}}{\underline{Z} + u} \right) + \frac{\sum_{k=I}^T y_k^I}{n_T - I + 1} \ln \left(\frac{\omega}{\bar{Z} + \omega} \right) \\
 & \quad \left. + \frac{\sum_{k=I}^T g_k^I}{n_T - I + 1} \ln \left(\frac{\omega}{\omega + \varepsilon'_0} \right) - \bar{k} > \lambda \right) > 0 \quad (\text{A-11})
 \end{aligned}$$

Lemma 2 implies that for trader I at time n_T , for this ε_0 and for $\varepsilon'_0 \in (0, \varepsilon_0)$,

$$\begin{aligned}
 & \lim_{T \rightarrow \infty} Pr \left(\frac{\ln V_{n_I}^I}{n_T - I + 1} > \frac{\sum_{k=I}^{n_I} [(l_k + a_k) g_k^I]}{n_T - I + 1} \ln \left(\frac{\bar{Z}}{\bar{Z} - \varepsilon_0} \right) \right. \\
 & \quad + \frac{\sum_{k=I}^{n_I} x_k^I}{n_T - I + 1} \ln \left(\frac{\underline{Z}}{\underline{Z} + u} \right) + \frac{\sum_{k=I}^{n_I} y_k^I}{n_T - I + 1} \ln \left(\frac{\omega}{\bar{Z} + \omega} \right) \\
 & \quad \left. + \frac{\sum_{k=I}^{n_I} g_k^I}{n_T - I + 1} \ln \left(\frac{\omega}{\omega + \varepsilon'_0} \right) + (E(\ln(\underline{Z} + \omega_k)) - \ln \underline{Z}) \right) = 1 \quad (\text{A-12})
 \end{aligned}$$

Equations (A-11) and (A-12) imply that for any i.i.d. random shock sequence $\{\omega_k\}_{k \geq 1}$ with $(\ln \underline{Z} - E(\ln(\underline{Z} + \omega_k))) \leq \bar{k}$, $\lim_{T \rightarrow \infty} \Pr(\ln V_{n_T}^I / (n_T - I + 1) > \lambda) > 0$. This further implies that for any given $F > 0$, for any i.i.d. random shock sequence $\{\omega_k\}_{k \geq 1}$ with $(\ln \underline{Z} - E(\ln(\underline{Z} + \omega_k))) \leq \bar{k}$,

$$\lim_{T \rightarrow \infty} \Pr\left(\frac{V_{n_T}^I}{n_T - I + 1} > F\right) > 0 \quad (\text{A-13})$$

However, because the maximum amount of wealth, which is injected into the market each time period, is bounded from above by a constant $[\bar{S}(\bar{Z} + \omega) + V_0]$, the average wealth (across time) of all the traders is bounded from above by this constant. This contradicts Equation (A-13).

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